Flight Dynamics II (Stability)

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Module No. # 11

Lateral-Directional Dynamic Modes

Lecture No. # 36

Lateral-Directional Flying Qualities, Routh's Stability Criterion

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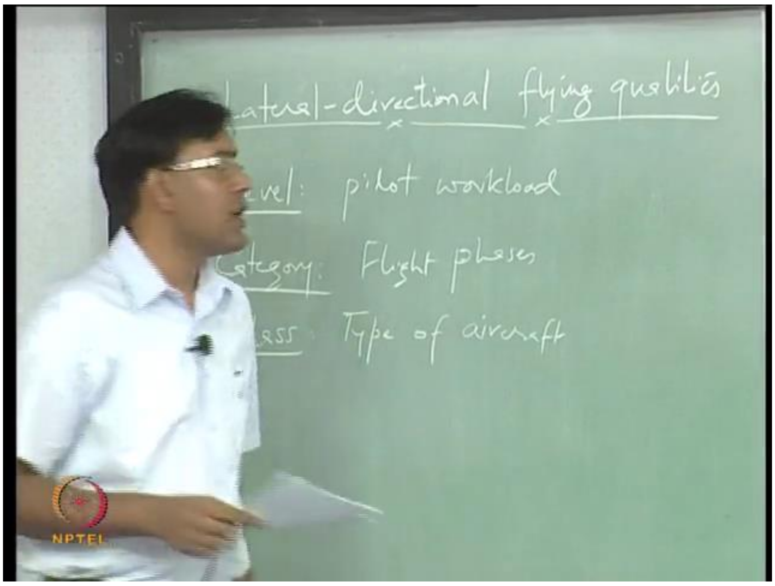

In the last class we talked about approximate formula for lateral directional modes of aircraft from the level cruise condition, and we wrote formula for the frequency and damping and the time constant of aircraft in different modes. Now, let us look at, according to the flying and handling qualities where all they should be, your aircraft has to be classified as a good one.

These qualities are actually not specific to an operating condition, in general any flying condition, if your characteristic parameters are lying within limits then you can say that the aircraft is good for flying and handling both. Handling is in terms of the load on the pilot work load.

So, we had three categories in which we classified the dynamic behavior of aircraft and one was level, level was related to the pilot work load. Category was depending upon the flight phases; terminal or non-terminal and class. Class was the type of aircraft. Is it not?? So, let us look at some figures for each of the lateral directional modes of aircraft.

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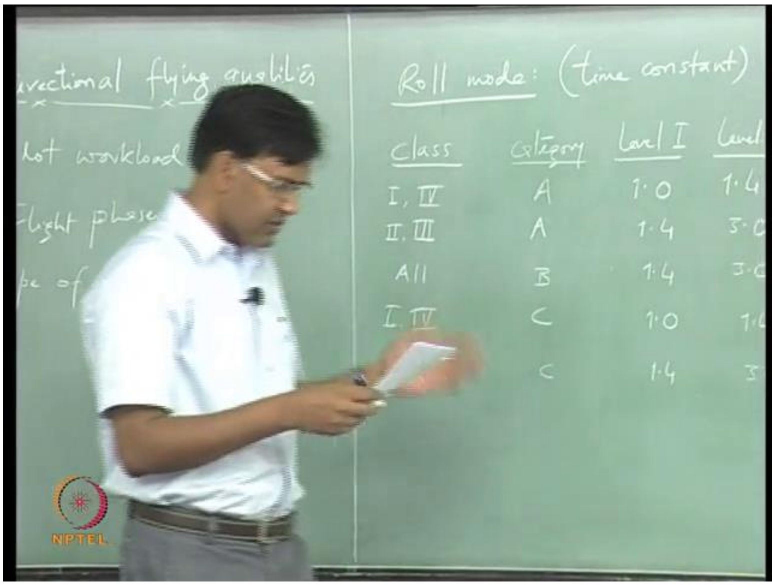

So, the first mode is the roll mode and let us see what are, the number is going to be only specified in terms of the time constant of the motion. We have only one characteristic parameter defining the motion and that is the time constant (for first order roll dynamics). So, class, category, level 1, level 2, level 3. Now, these numbers are given as standards. You have to follow this set of numbers, and you can find them in any book, they will be same they are not different. Class 1, 4: 1 is small aircraft, small general aviation aircraft, 4 is fighter, with high maneuverability, and this category A is for small maneuvers, not very high requirement on the maneuverability, and this time constant is what I am writing here.

Now, level 3 is the one where the pilot will feel the excessive work load. So, for that class or for that level you have to have time constant which is roughly about 10. ... Class 2 and class 3 aircraft in category A, these numbers are a little different 1.43 and 10. All class category B, ... class 1 and 4 category C aircraft, 2 and 3 category C. So, these are the values which you must satisfy, through the design parameters and then you can classify your aircraft to be having a good dynamic behavior in roll.

Let us try to write the limits in the other two cases and not the whole table which you will find in any other book. So, any book you pick up these values are going to be same.

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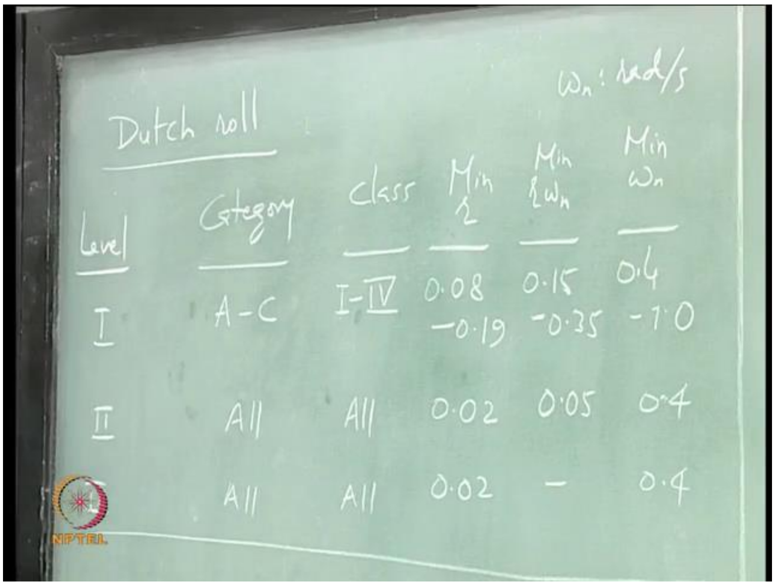

These are some specifications for the aircraft designer. Dutch roll, category class, minimum damping ratio, minimum real part of the Eigenvalue, and minimum omega n. omega n should be in radian per second. .... Here I am writing level. So, level 1, category A to C. Now, this is slightly summarized, I am not giving you the whole table here. Remember, this is not the range these are the minimum values. For detailed table, you should look into the book ..... So, you can look at this value of the damping ratio. Category A aircraft, level 1 plus class 1, minimum of damping should be 0.08. So, aircraft should have damping which is more than 0.08 to have good flying characteristic in Dutch roll. ....

So, you have to look at what kind of aircraft you are designing, and depending upon that, you can identify your class category and the level and then look at the appropriate numbers; that number should be satisfied from the design.

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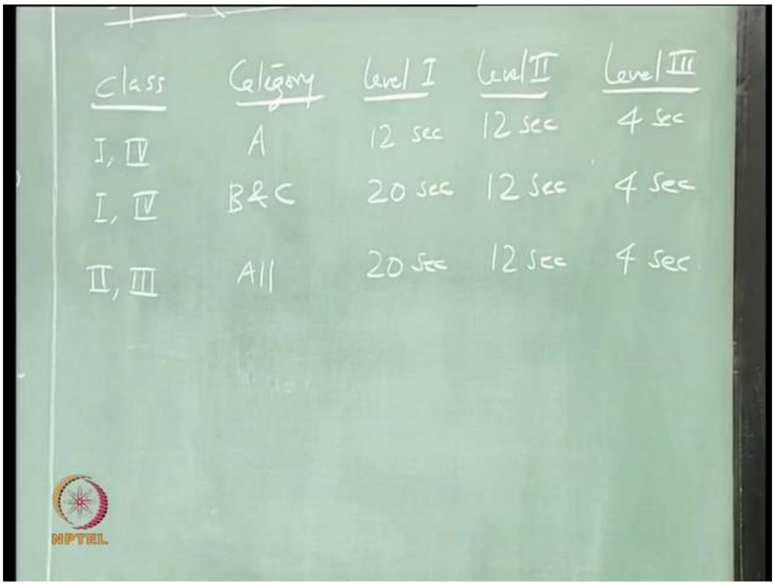

Now, spiral mode is usually written in terms of the time to double the amplitude, because it is considered to be unstable divergent motion and we have to look at how much time it takes for the disturbances to double, right. So, usually this is specified in terms of time to double the amplitude. What it means that as long as its stable there is no problem. If it is unstable then we have to look at these numbers .... class category level 1 2 and 3, ... 12 seconds. .... So, if you want to increase the pilot work load, let us say pilot can afford to increase the work load then this number is only 4 seconds. You can actually take care of that aircraft with increased pilot work load. In only 4 seconds, the perturbation grows by double, that is what it means. Class 1 and 4 category B and C, remember these numbers give you the time that you can take to control the aircraft in this particular mode. .... This is summary. So if you want to look at it very carefully you should look into any book on flight dynamics or aircraft design.

Let us move on to something that we have not talked about much. Remember we said that, we are getting quartic equations in lambda, and the coefficients are going to be functions of the parameters of the aircraft. And, to solve that quartic equation is a really difficult thing, even if I have to solve for the numerical values. Let us say the coefficients are given some numbers, even then finding out any information from there may be difficult. Is it not?

So, how would one in earlier days have come down to any approximate formula for stability, even computers are not there then how would you find out the stability of aircraft by just looking at the coefficients of the quartic equations, that is what we will discuss today.

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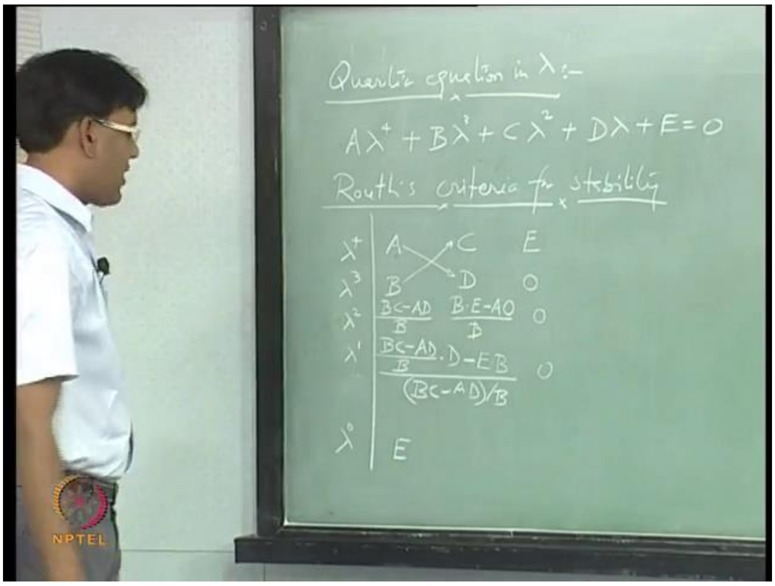

So, let us say you have a quartic equation which is A lambda 4 plus B lambda 3 plus C lambda square plus D into lambda plus E equal to zero. These are our characteristic equations. If I can solve for the Eigenvalues (lambda) numerically, then we know the stability is depending upon where the Eigenvalues are lying in the complex plane.

Without even doing that we have to solve the equation first for the Eigenvalues then only we can comment on the stability. And let us say the coefficients are not numbers to start with. When we are starting, we are designing the aircraft then the coefficients are functions of parameters and not the real numbers, then the Eigenvalues, the formula for the Eigenvalues will be really large, huge formula incorporating all the parameters of aircraft. So, then what do you do? Whatever we have done so far is based on some assumptions. We have come down to a matrix which we could solve and find out the approximate formula for the dynamic characteristics in different modes, we made some assumptions.

Those assumptions also are good, but let us say we want to look at this quartic equation then what should we do? And try to find out the stability in terms of the parameters of the aircraft, when the aircraft will become unstable and what kind of arrangements of parameters should we have, so that aircraft is stable. So, we can define a boundary based on such relations that we get. So, let us look at this equation. .... So, A into lambda raised to 4 plus B into lambda raised to 3, C lambda squared, D lambda plus E , right. For our aircraft, these coefficients A B C D and E are functions of aircraft design parameters.

See you can arrive at a number only when the numbers are known of the parameters, but let us say I am at a stage where I cannot give any numbers to those parameters. For example Cm alpha, if I do not want to give any number to that, but I want to look at how Cm alpha is going to affect the stability of the aircraft, then we have to work with this equation in its full form without giving them (coefficients) any number.

So, there is this criteria called Routh's criterion for stability. This is very powerful and people have used it a lot. Even today it will be important. We cannot always convert a problem to an Eigenvalue problem and solve it on Matlab to get some results, that is not a good idea when you are still at the design stage. .....

This is extremely powerful tool even today, you try solving this equation in Mathematica, for example, it is, Mathematica is a symbolic tool which can probably solve this, but you think about how many parameters each of these Eigenvalues will have. We have tried, it is not possible to actually solve on Mathematica also. You have four by four matrix, you are solving an Eigenvalue problem, it is a terrible problem. Routh's criteria for stability, it first asks you to make an array in this fashion, so for specific to this equation. Write the powers of lambda in the first column and write the coefficients like this.

So, first you start with this lambda raised to 4, A is the first element here, then C and, this is how you have to construct the array. I cannot answer you how he arrived at this, but this is how you should go about doing this. To get the first term of this row, what you do is, you multiply this element with this element and subtract the product of this element and this element, and divide by this. So, this term is BC minus AD over B. Now, to get the second term of this row, multiply this by this, subtract the product of this into this, divide by B. So, B into E minus A into 0 over B. So, that is E.

Now, repeat this exercise. Now, the first element of this is going to be this into this minus this into this over this. So, let us try to write that. B C minus A D over B into D. So, this is E. So, A into 0 is 0. .... This is going to be E because if you multiply this, these two terms, we are going to get this into E minus 0 divided by this. So, what you get is E. Now, criteria for stability is, first condition is that, all of these coefficients must be positive or having the same sign. If they are having the same sign, if it is negative then you can take it to the right side and all of them will become positive. So, coefficients of these powers of lambdas must be positive.

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So, first condition is, coefficients of, so remember, this is also carrying this into E into lambda to the power 0. Must have all same sign. Any change in sign will indicate instability. If it is negative, I can take it to the right hand side and all the coefficients in the quartic will be positive, that is condition one.

Second condition is, but this is not enough, can be taken as a necessary condition, to start with, to start with if you are saying that there is change in sign in between, so the aircraft is unstable in this particular flying condition for which we have written this quartic equation.

So, this is Routh's table, all elements in column 1 of Routh's table must have same sign, actually positive. We said that if all of them are negative, all the coefficients A B C D and E, then we can take it to the right hand side and then this is how you start writing the elements of the Routh's table. All elements in column one of Routh's table must have positive sign. Change in sign of the elements ..... (indicates) instability. You will have as many roots with positive real part as a number of sign changes.

We will look at some simple examples after which I will talk about one example, where we will directly work with the parameters of the aircraft, and somebody did actually this job for this quartic equation and he found the stability boundaries. Very interesting results; the results are in the name of the person who derived those results, it is called Phillips' stability criterion. So, very remarkable achievement the way you remember Lanchester, similar to that, for a more complicated problem actually.

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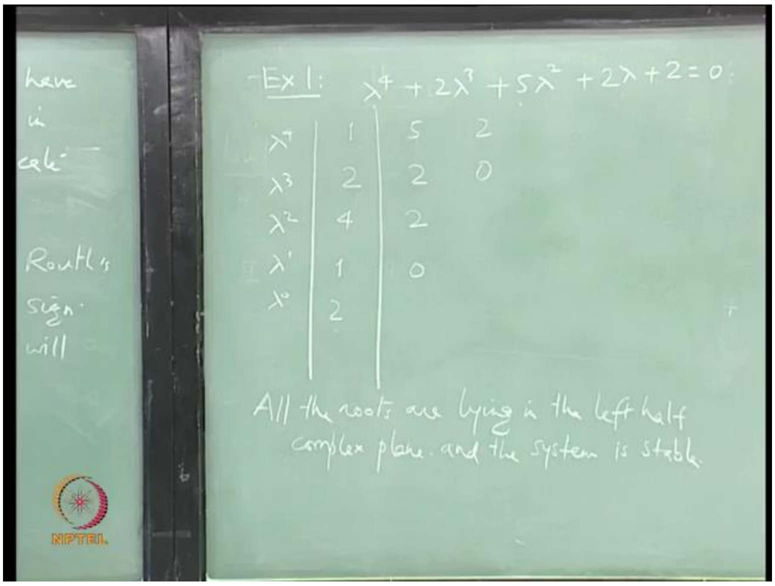

So, let us look at simpler examples. Example one, ... we have this equation given. Now, construct the arrays. The most interesting part of this is that you can write higher order equations also in lambdas and you can carry out this exercise. 1, 5 and 2 will appear in the first row, second row is 2 and 2, third row first element 2 into 5, 10, minus 1 into 2, 10 minus 2, 8, over 2, right, so 4. Second element 2 into 2, minus 1 into 0 divided by 2. So, you have 2 as the second element of the third row and similarly you can find out the other elements, right. Look at this. The first criteria is that, condition is that, all the coefficients of this equation must have the same sign which is all right in this case.

But we cannot comment on stability at that point. But you can comment on instability if there is a sign change here in the equation itself then you can say that the aircraft is unstable. So, look at this. So, that condition, condition one is satisfied and condition two is also satisfied.

Student: (( ))

If there is a change in sign how do you say its unstable? Now you have to go back and read.

Student: (( ))

Yes

Student: (( ))

Yes right

Student: (( ))

Yeah ah

Student: (( ))

No, this condition is strong and it is not about possibility. If there is change in sign, there will be positive real roots depending upon the number of changes in sign. So, to find the proof of that you have to go back and look at the derivations.

Student: (( ))

As many positive real roots as the number of sign changes.

Student: (( )).

Not.

Student: (( ))

Does it say like that?

Student: (( ))

So, if the elements in column one are all having positive sign, that is a second condition that you have to satisfy after this, then what it means is the Eigenvalues are going to lie in the left half complex plane, all four Eigenvalues. .... System is your aircraft. ....

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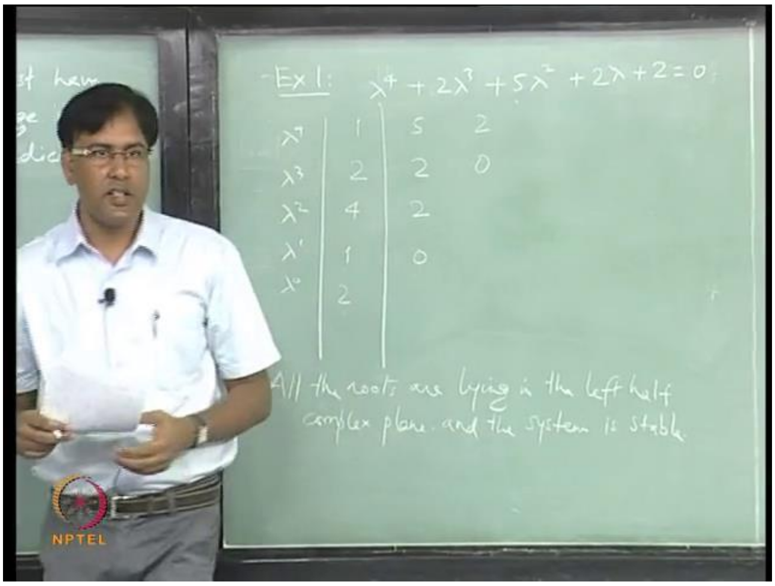

Let us look at example two, where we have change .... in sign in the first column. .... 1, 2, 1, 3 and 4 .... Because there a change in sign, and change in sign twice, one is from positive to negative and from negative to positive. Then as you are saying, there could be two 2 roots with positive real part, ... and the system is unstable. ....

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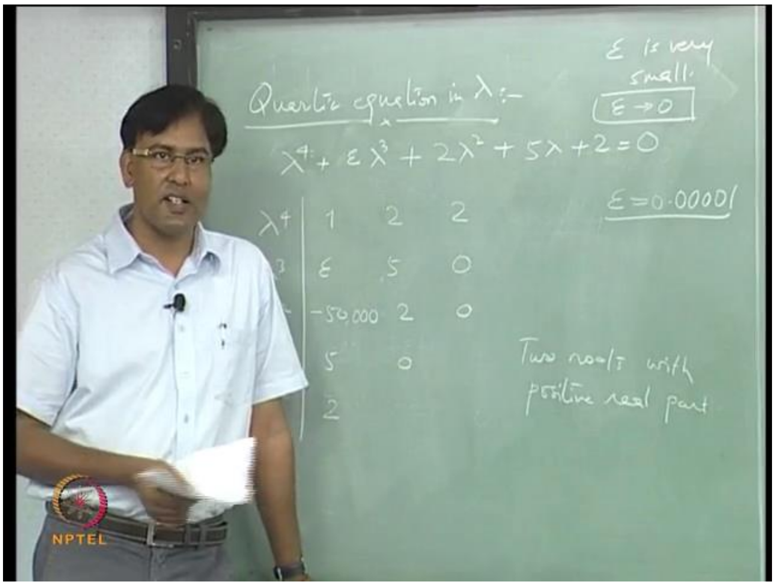

Now, let us look at one case when one of the coefficients is 0, then how do we deal with such situations. Actually, this theory is much more than what we are doing here. It can even tell you about the complex conjugate Eigenvalues. So, those cases we are not trying to consider here.

.... So, there is one term which is missing here. I am taking that as 0 into that term. The coefficient of that term is 0. We call this term as, this coefficient is epsilon, epsilon is very small. Actually, you have to take a limit like this (epsilon approaches zero) eventually. ....

So, we will assume that this epsilon is a positive number which is extremely small, but you can keep this epsilon in here and still work with this, so that finally, you take the limit, you can take this limit (epsilon approaches zero). With this value of epsilon what we get is this .... and this limit is from the positive side. So, epsilon is a positive number, but in the limit, you have to do this… Because this is positive, the change in sign again here we see is twice, positive to negative and negative to positive.

So, two roots with ..... What we are doing here is only basic examples. But the biggest advantage with this is that you can find the stability in terms of the parameters of the aircraft, and this one example which is there in Doctor Bandu Pamadi's book is what I am going to talk about in the next class. We will look at that example, and that example is about the rolling maneuver of aircraft. So, it is actually complicated, you think about the 8 equations that you have to talk about when you talk about coupled motions, rolling is a coupled motion, roll maneuver.

So, what you have to do to study the stability of the aircraft in that maneuver, we have to look at all 8 equations. And 8 equations linearized means you get an equation which will have the lambda to the power 8. So, it is very complicated, and then how do you arrive at any relation which will relate to the stability of the aircraft in that particular motion. You cannot do it actually, you have to make some simplifications, go on making simplifications till you arrive at a point where you can solve it analytically, is it not?.

But it is very powerful because you want all the information at the design stage where you are not giving your parameters any numbers. If you have designated the numbers then we know how to solve it, now we have mathematical tools also available to do that, but you think about the earlier days when people have to work with this.

Student: (( ))

That is the first condition, necessary condition is that all the coefficients must be having the same sign. If the first stage itself, you see that there is a change in sign, then its unstable without going to the next step.

Student: (( )).

Yes. So, we will look at this example tomorrow.