Flight Dynamics – II (Stability)

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Module No. # 11

Lateral – Directional Dynamic Modes

Lecture No. # 34

Pure Rolling Motion, Pure Yawing Motion, Spiral Approximation

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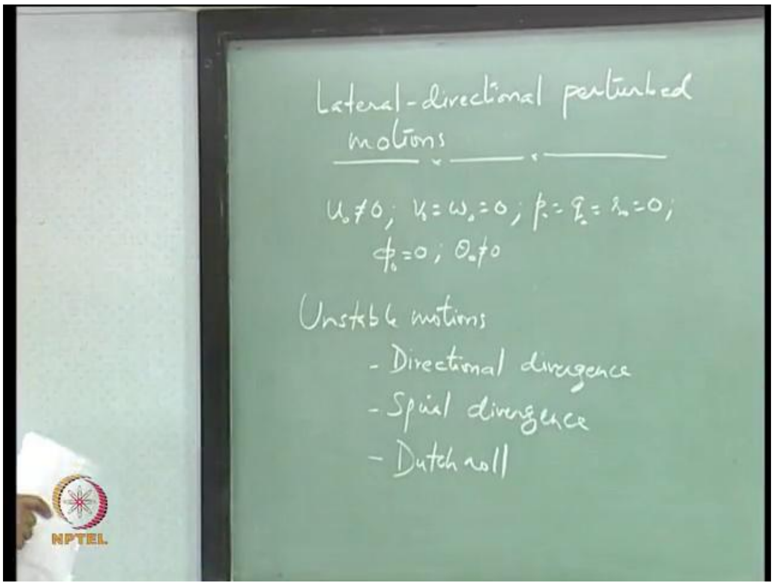

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So, we started talking about lateral directional perturbedmotion from the equilibrium state which was this ....Refer the slide above.

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... u0 is non 0, v0 w naught are 0, p 0 q 0 r 0 rates are 0 and bank angle is 0, theta naught is non 0. So, we were looking at this equilibrium state and the perturbed motion of aircraft from this equilibrium state.

In the last class, we started looking at lateral directional perturbed motions and we talked about the unstable motions that are possible and they were: directional divergent motion, spiral divergence and dutch roll ....

Remember, we are talking about here unstable motions. These modes could be stable .. but in general, you will worry about the unstable modes when you are designing your aircraft.

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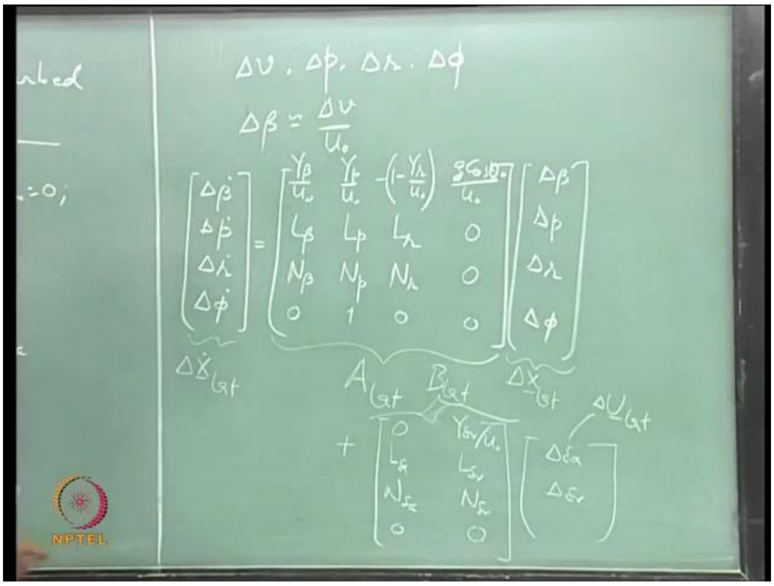

So, these motions are actually coupled motions in lateral directional variables and those variables are: .... side velocity, roll rate, yaw rate and bank angle, .. perturbation in these state variables.

This delta v can also be written in terms of delta beta...We are talking about wind disturbance... Disturbance is coming from the wind. Will talk about this angle delta beta which is this ....

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If you look at the equations, equations will consist of derivative of these variables (Audio not available: 04:15-04:28) ..... with respect to time that is equal to Y beta over u naught, Yp over u naught .... minus of 1 minus Yr over u naught, g cost theta naught over u naught, L beta Lp Lr 0, N beta Np Nr 0 and delta, this phi dot is actually equal to delta p ... We have done this derivation earlier ...

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This is my A matrix for lateral directional perturbed motion. ......

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Plus .... delta delta a, delta delta r control inputs... In lateral directional motion, 0 Y delta r over u naught, L delta a, L delta r, N delta a, N delta r. This is our B matrix and this is u delta or delta u. So, you have the equations written in this .....

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compact form.

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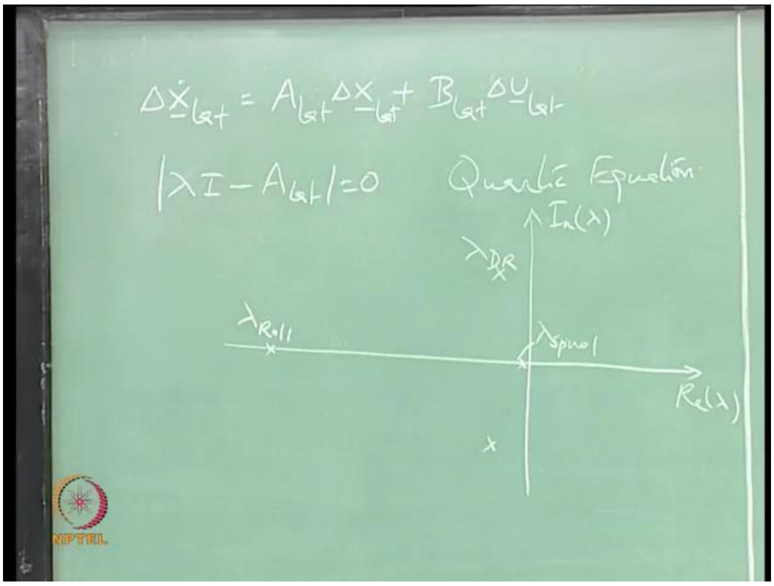

Now, we have to repeat the procedure. What we did, we solve this matrix ($A_{lat-dir}$) for the Eigenvalues (Refer Eq(1))

$$\Delta \dot{\underline{x}}_{Lat-Dir} = A_{Lat-Dir}\Delta \underline{x}_{Lat-Dir} + B_{Lat-Dir}\Delta \underline{U}_{Lat-Dir}$$
$$\left|\lambda I - A_{Lat-Dir}\right| = 0 \qquad (1)$$

This gives me the characteristic equation which is a Quartic equation (Refer Eq(1)). When we solve this equation, we get four Eigenvalues and you know where the Eigenvalues are located.

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Now, this is the case when this spiral mode is stable ... this lambda spiral, dutch roll and ....

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What we will do in this class is we will look at simpler motions as we did in the case of longitudinal perturbed dynamics. Here also, we will look at some motions which you can simulate in the wind tunnel. It is very useful to give a motion, a three-dimensional motion if you give to the aircraft it may not be able to get any parameter from your results ...

The advantage of studying the constrained motions is that you can extract some parameters of the aircraft ..., geometric or aerodynamic parameters.

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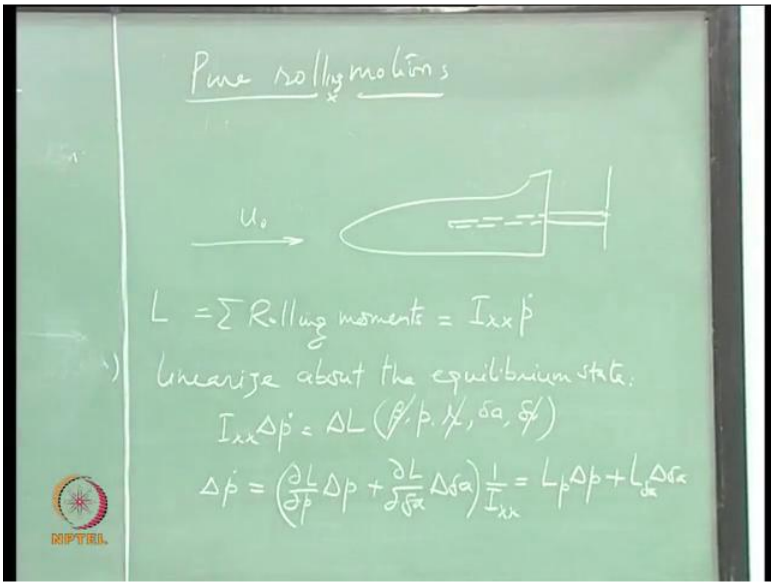

So, let us look at some constrained motions. Pure rolling motion. So, what you are doing is you are mounting your aircraft, model of aircraft in the wind tunnel, so that it has only one degree of freedom... So, this is only one degree of freedom of motion of this aircraft in the wind tunnel. So, wind is coming like this. Let us say we can provide some disturbance, so that it can start rolling. Now, I want to look at that motion ...

$$I_{xx}\dot{p} = L;\ p = \underbrace{p_0}_{0} + \Delta p, L = \underbrace{L_0}_{0} + \Delta L = \frac{\partial L}{\partial p}\Delta p + \frac{\partial L}{\partial \delta a}\Delta \delta a$$
$$\Delta \dot{p} = L_p \Delta p + L_{\delta a}\Delta \delta a \qquad (2)$$

So, since it is constrained to only roll, the equation would be .....Refer Eq(2).

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Sum of all external rolling moments is equal to Ixx into p dot. We want to linearise this equation and we want to study the linear or the perturbed motion about an equilibrium state, which is this; ... there is only u naught along the X axis of the aircraft in the wind tunnel and this aircraft is perfectly aligned with the fixed frame of reference to start with. That is the assumption that we are going to make... This is nothing but the rolling moment L prime. When you linearise this equation of motion, what you get is .... this Refer Eq(2)?

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This L is going to be a function of various parameters and those parameters are angle of side slip, the roll rate, what else? The yaw rate and the control inputs. So, since I am not providing any degree of freedom to this aircraft in the yaw direction, I can drop beta and r from here and also delta r. So, this equation is ....Refer Eq(2).

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Can be expanded like this ....

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... Change in rolling moment due to the change in roll rate, change in rolling moment due to aileron deflection and this is equal to ....Refer Eq(2).

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So, this is my equation of perturbed motion in roll about the equilibrium state that we are talking about.

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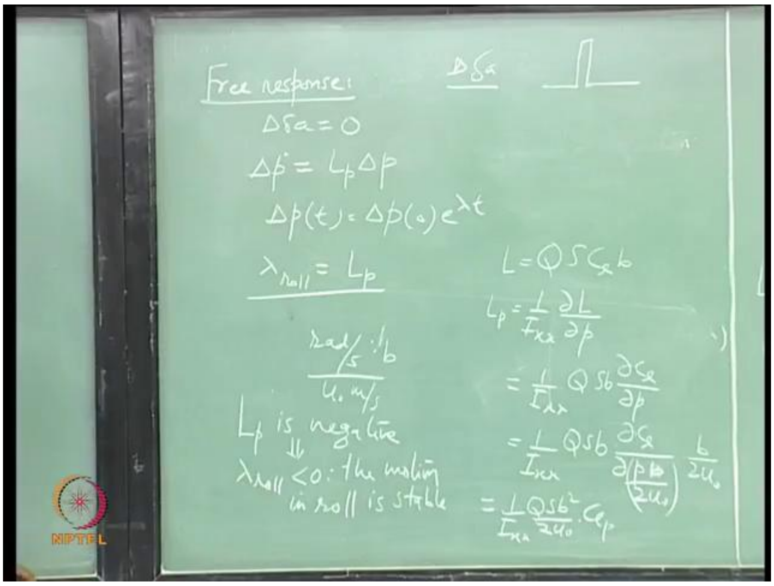

If you want to look at the free response, control free response, now there are two ways you can actually simulate control free response. What you mean by control free response is that you are probably either having the disturbance in the wind itself. Wind is going to come and hit the aircraft and go. It is an impulse.

... Whenever we are trying to simulate the motion around an equilibrium state, we have to either do something with the initial conditions, you can assume a change from the initial condition ... You can assume a perturbation in the initial condition itself and look at what is the transient in perturbed motion.

So, there are two ways of doing this. One is to give a perturbation in the initial condition and simulate the aircraft linearised equations of motion. Second thing is you can give an impulse or pulse input to, ... pulse of very small duration to the control input. So, it acts like a disturbance. If you are giving a pulse to aileron and that will initiate the roll motion. So, it acts as a perturbation.

So, we are here assuming that this ($\Delta\delta a$) is 0 and we want to look at the stability properties of this motion ......

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... The response is of this form ...... $\Delta p(t) = e^{L_p t} \Delta p(0)$.

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Wherein this lambda is the Eigenvalue corresponding to this constrained roll motion and is equal to .... $\lambda = L_p$.

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We said that this is a constrained system, but yawing is not there, there is no degree of freedom in yaw for this particular model. (())

In flight, yes it will be always coupled. Whenever you have delta beta, you will have both roll and yaw, but here this model is constrained to have only one degree of freedom in motion and that is roll. So, this experiment I am conducting in wind tunnel, so that I can extract some parameter from the transients. (())

But look at the usefulness of this. Because now you are able to extract one Eigenvalue, this is, this may be a rough estimate and sometimes very good estimate actually. In case of roll, actually this is a good estimate. You are able to get this one Eigenvalue and once you have this Eigenvalue, now you are only left with a cubic equation which you can solve and find out other Eigenvalues.

So, that is one advantage. You know from the response, you are trying to extract some parameters. Sometimes, you can also find out what this Ixx is from the response. So, this Eigenvalue curve corresponding to this particular motion, constraint motion is equal to Lp.

What is Lp? L is QS Cl into b ...Q is the dynamic pressure, half rho u naught squared. This is the wing plan form area, Cl is the roll moment coefficient, b is the span of the wing. Lp is .....

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Now, this quantity is, of course, not a number. It has some unit because of this p appearing here. So, what you can do to get rid of that .....

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p is having a unit radian per second. So, I have to multiply this by or divide it by

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u0, so that the second goes and then you have a meter left. So, you can multiply this by the wing span (non-dimensionalization factor $2b/u_0$ (sec)* p(rad/sec)). That is how you will get non-dimensional coefficients.

So, here it is non-dimensionalised like this ....

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You should be very careful. Sometimes, ...sometimes this will be given in units, sometime not. If it is not, there is no unit attached to it, then you have to multiply, when you are trying to find out Lp, you have to multiply this Clp by this factor and it makes a huge difference...

So, this is my roll mode Eigenvalue. Lp is usually negative, so the roll mode is stable. lambda has to lie in the left half complex plane for the motion to be stable and Lp is negative .....

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Let us look at another case when you are providing only degree of freedom in yaw.

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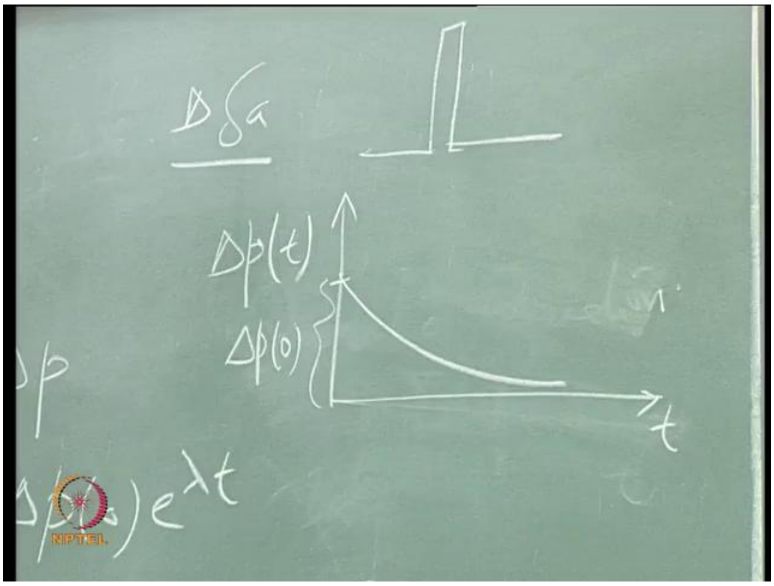

Pure yawing motion. Before doing that let us also plot the response here. So, I am plotting the time response of this delta p and let us say this is the initial delta p. How will the response look like? It is going to decay exponentially. ....

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pure yawing motion.

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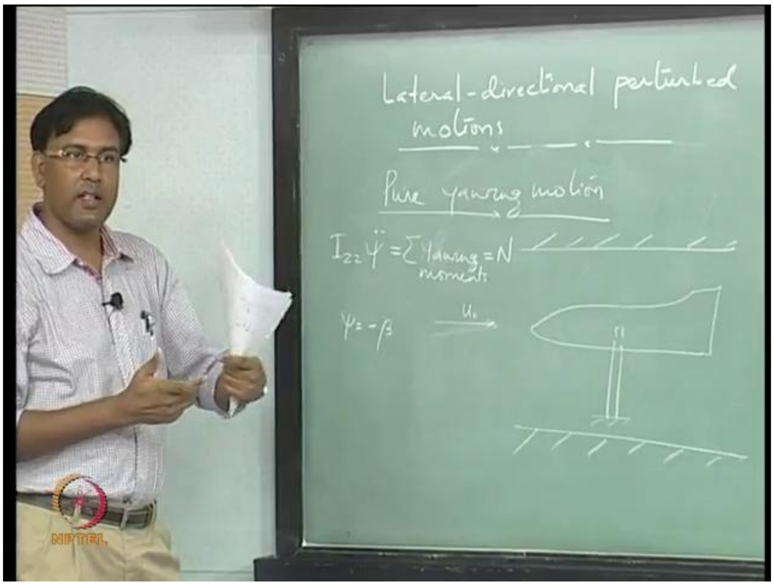

So, again I have some arrangement made in the wind tunnel which only provides degree of freedom in yaw. ... Can freely rotate about the Z axis but no other motion is possible. So, the equation of motion now is ..... this.

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Sum of yawing moments ......

$$I_{zz}\dot{r} = N(r, \beta, \underbrace{p}_{0}, \underbrace{\delta a}_{0}, \delta r); \Delta r = \Delta\dot{\psi} = -\Delta\dot{\beta}$$
$$I_{zz}\Delta\dot{r} = \Delta N(r, \beta, \underbrace{p}_{0}, \underbrace{\delta a}_{0}, \delta r) \qquad (3)$$

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Let us again repeat the procedure. We have to linearise this. So, we are assuming that aircraft, again the aircraft axes are aligned with the fixed frame of reference so that this psi is also minus beta.

Do you see that? You are rotating your aircraft about the Z axis? So, X axis is moving towards Y ... and then it will start making an angle. The X axis of the aircraft will start making an angle with the flow direction right. That will be beta and that beta is negative ... So, psi is actually equal to minus of side slip angle. Yaw is, yaw angle is equal to the minus of the slide slip angle.

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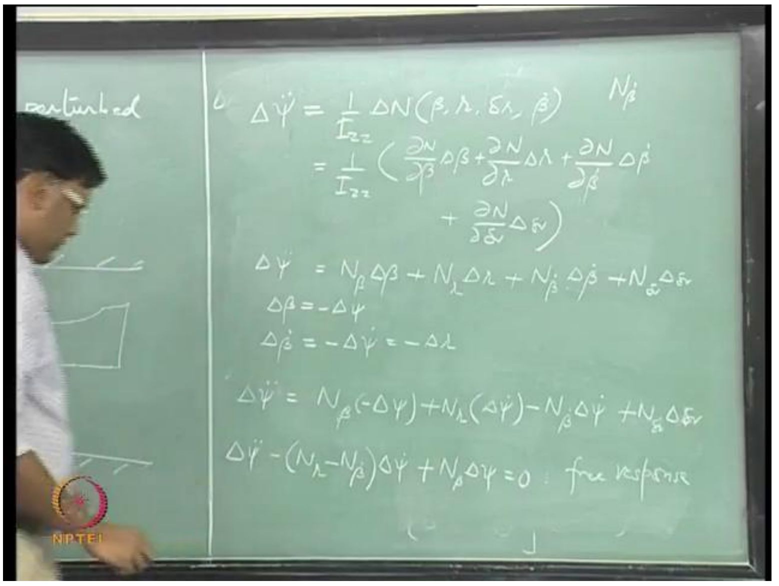

Let us try to expand .....

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This is going to be function of beta, r, in general roll rate also, but here the motion is constrained, so will not consider the roll rate, beta, r and delta r. If there is some contribution coming from beta dot, then we have to include that also. But that contribution is usually small. ... I am talking about the contribution which is N beta dot.

So, let us look at the expanded form .....

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This delta beta is minus of delta psi, ... and this is possible only when we are talking about small angles ...We are not talking about large angles here .....

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This is also true. We are talking about rate of change of angle about the Z axis that is equal to the yaw rate.

So, let us write everything in terms of this angle psi ....

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Let us collect the common terms. What you get is .....

$$\Delta\ddot{\psi} - \underbrace{(N_r - N_{\dot{\beta}})}_{2\zeta_{DR}\omega_{nDR}}\Delta\dot{\psi} + \underbrace{N_{\beta}}_{\omega_{nDR}^2}\Delta\psi = 0 \qquad (4)$$

for the free response. So, I am setting this delta delta r to be 0 to look at the free response.

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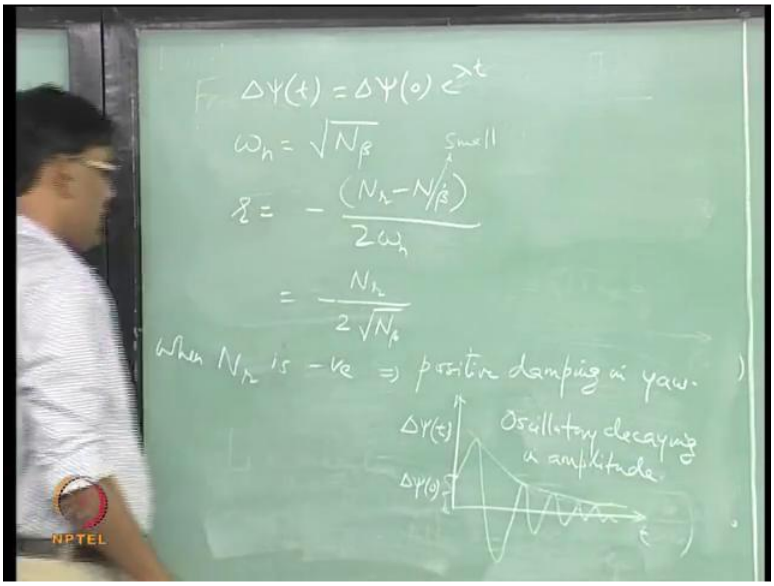

Now, again, I can assume a form of solution which is a general form ... and that will result into a quadratic equation in lambda, ... and we can find out what is the frequency and damping of this motion. This comes from here and ....Refer Eq(4).

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So, this is an oscillatory mode. We are talking about response of a second order system.

So, this (N beta dot) is usually very small. We can neglect this and find out expression for the damping ratio which is this: Minus of Nr over two, square root of N beta. This we have seen already earlier. What is this N beta? N beta is the yaw stiffness term .. or the directional stability term ... and that gave me only the frequency .... We had no idea of damping earlier and that is where we were talking only about the static stability.

Now, you have damping in the model. So, now you can look at how the amplitude is growing or decaying in time. So, Nr is ..., when Nr is negative, this quantity is positive, ... and so the motion would look something like this ...Refer to the slide above. roughly.

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... Oscillatory decaying ..... two of the Eigenvalues we have found ... or three Eigenvalues corresponding to two modes, approximately.

So, one is a pure motion in yaw, the other one is pure motion in roll. Now, what we can do is, look at the actual equations of motion, try to solve the quartic equation in lambda, find out the Eigenvalues from there and see how close these values are to the actual values. That you can do numerically. ... But still if we want to make some assumption and find out approximate formula, so that we can relate the dynamic mode with some of the design parameters, then we still have to do things analytically. I hope you get that point.

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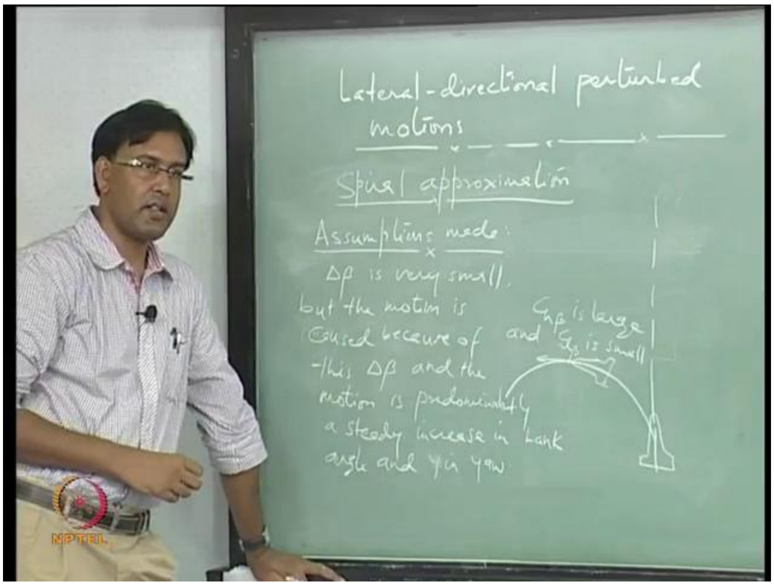

So, let us look at spiral approximation first. If you remember, in the last class, we talked about this motion. So, your aircraft is actually on a cruise flight condition and, a perturbation hits the aircraft, a gust can hit the aircraft from the side. What aircraft does is, this is when Cn beta is large and Cl beta is small, but both of them are providing you stiffness. What I mean to say is this Cl beta is small but its negative. That is what was required for static stability and this Cn beta is positive.

So, what you are getting is a motion which is in delta beta. Delta beta is, it is because of the sideslip, perturbation in sideslip, and that sideslip is constantly being killed because of this large stiffness in yaw...But it is giving you a roll because this Cl beta is low... So, the aircraft is following a curved path which is something like this. The nose is always in the direction of the velocity vector.

So, beta is constantly being killed because of this Cn beta, large Cn beta. Roll is, the aircraft is banking because of low Cl beta. So, it is like you are flying like this and there is a perturbation from side, side velocity. There is a beta introduced and then aircraft starts banking, and this banking is so slow that there is not much change in the roll rate as such.

So, it is a steady change in bank angle and the beta is getting killed. So, it is following the velocity vectors, starts doing this and, as it progresses on this curved path because of increase bank angle, this term becomes tighter and tighter. So, it is doing this and then starts, ... and also it will start falling down. We have understood that.

Whenever there is a change in bank angle, the lift produced will not be enough to balance the weight. So, it will start doing this banking and it is also going down. That is the spiral divergent motion. In a critical case, actually it can also take a dive that is called the spiral dive. So, let us try to write down the approximations. Assumptions made here while deriving approximate formula for the Spiral mode Eigenvalue.

So, assumption made here is that delta beta is very small, even though this is the main reason why we are getting this motion. This is a small quantity getting killed due to this large stiffness in yaw.....

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Aircraft is yawing, it is taking a turn. So, yaw is involved, even though side slip is very small or it is getting reduced because of this large Cn beta, it is yawing. We are deviating from its original direction in the XY plane. So, there is yaw motion involved with a slow change in bank angle and that is coming from this delta beta.

So, now you can use your aircraft perturbed equations of motion in lateral directional motion and try to use these assumptions and derive the approximate formula for the spiral mode Eigenvalue. So, there is no side force as such generated. So, I can drop the delta beta term, the first term which is about the force equation in the sideways direction.

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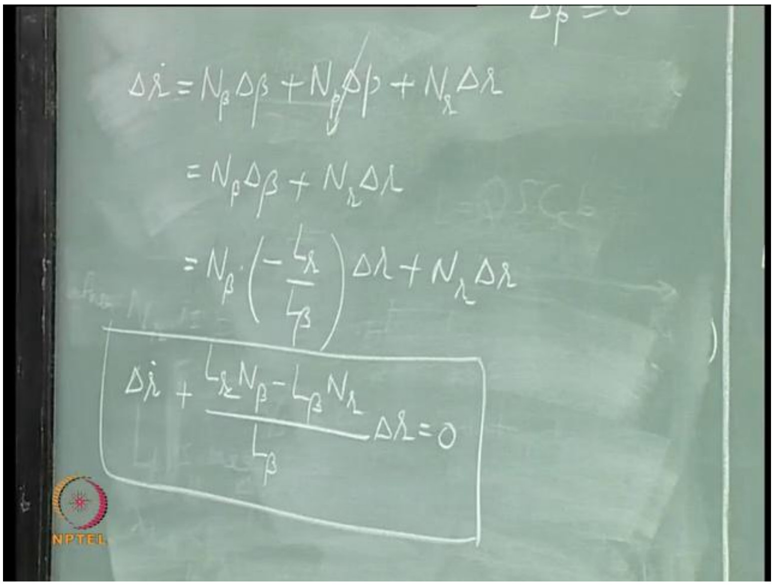

So, once you do that, what you are left with is .... this delta p dot .... and

$$\Delta\dot{p} = 0 = L_r \Delta r + L_\beta \Delta\beta \Rightarrow \Delta\beta = -\frac{L_r}{L_\beta}\Delta r$$
$$\Delta\dot{r} = N_\beta \Delta\beta + N_r \Delta r = N_\beta\left(-\frac{L_r}{L_\beta}\right)\Delta r + N_r \Delta r \Rightarrow \Delta\dot{r} + \frac{L_r N_\beta - L_\beta N_r}{L_\beta}\Delta r = 0 \qquad (5)$$

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There is steady increase in the bank angle. So, I can set this delta p or the derivative of this delta p dot to 0. So, steady change in bank angle, I .. can set these two parameters to 0 and gives you this ...Refer to the slide above.

Let us look at the equation in the yaw ..... Refer to Eq(5).

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So, we are not including the control terms here because we are looking at the free motion. So, this you can drop because delta p is ... Refer to Eq(5)..

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And find out this delta beta from this equation ...Refer to Eq(5).

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So, this is a first order equation. ...You get a single real Eigenvalue depending upon which side of the imaginary axis it lies, can be unstable or a stable motion. So, we will stop at this and we will start from here tomorrow.