Flight Dynamics – II (Stability)

Prof. Nandan Kumar Sinha

Department of Aerospace Engineering

Indian Institute of Technology, Madras

Module No. # 09

Perturbed (Linear) Aircraft Model

Lecture No. # 28

Aerodynamic Force and Moment Derivatives

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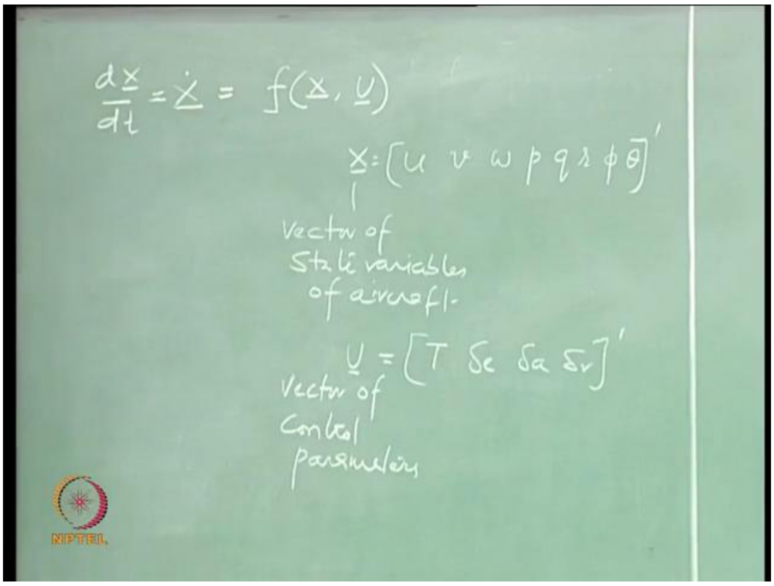

So, let me go back to what we did in the last class so that it is more clear. We have seen that our equations are in this form, where $\underline{x}$ is the vector of state variables ..... eight state variables and $\underline{U}$ is the vector of control parameters. We will keep only four basic controls in this vector. So, all the equations put into this compact form.

Now, what we are trying to do is, we are trying to linearize these equations around an equilibrium flying condition which is a longitudinal trim.

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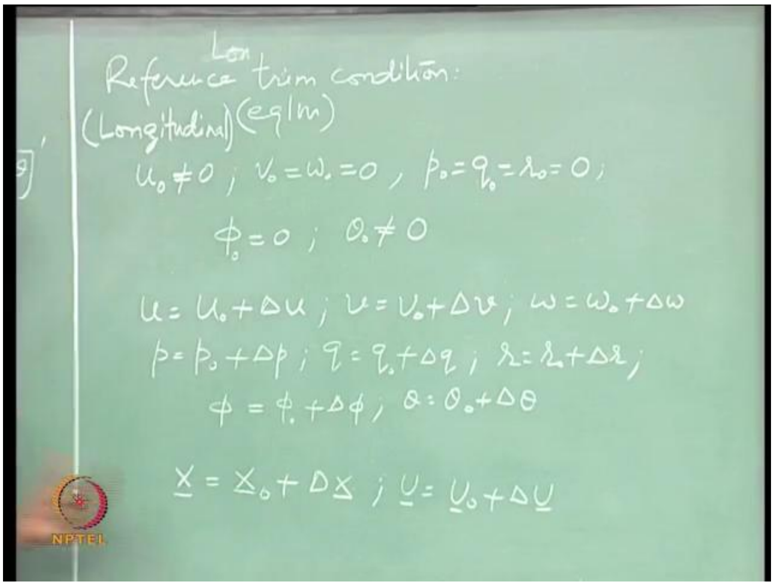

Trim or equilibrium, sometimes people get confused with this, because trim is related to locking your control surfaces. Is it not? So, in a general sense people do not like to use trim for equilibrium condition, but for an aircraft it is a valid word.

We are denoting our equilibrium condition with this subscript '0'. So, u naught is non zero, other two velocities are 0, rates are all 0 and it is a longitudinal flying condition, so the wing bank angle is also 0.

Let us say the pitch angle is not zero, so that I am talking about all kind of flying conditions in the longitudinal plane. Now, we want to linearise our equations of motions around this longitudinal equilibrium condition. .... I said that we can introduce perturbation, you know, from this equilibrium condition. So, perturbation can be introduced like this (Refer slide above)..... So, my new variables are these.

So, aircraft flying in perturbed condition, which is different from equilibrium flying condition, and the perturbation are these, perturbations in each variable. ..... (Refer slide above).

5:56

So in general, I can write this x as sum of this equilibrium condition, plus the variation in x from the equilibrium condition. Similarly, I can also write for u, the new condition, perturbed condition. Now, let us try to substitute these variables in this equation 1.

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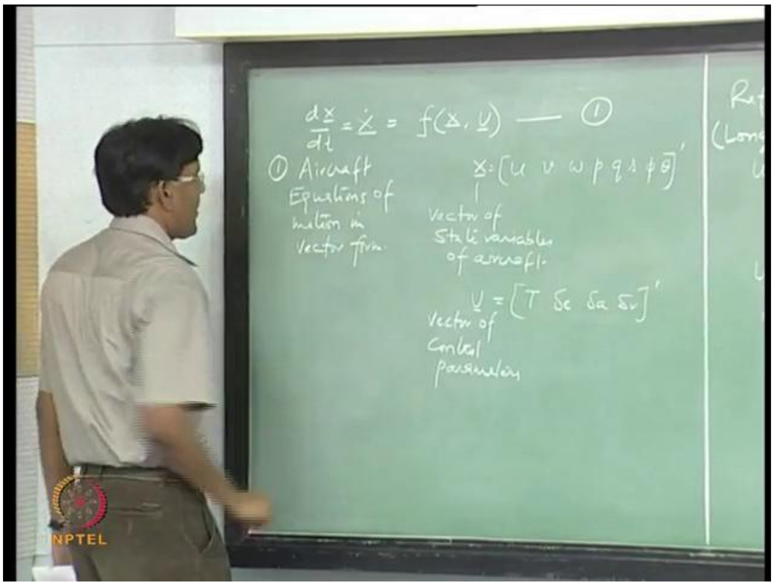

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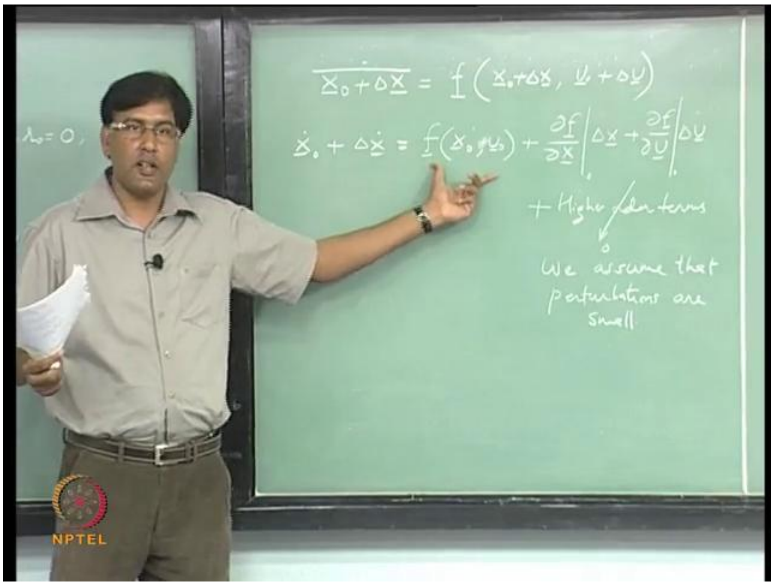

Now, let us expand this function, using Taylor series expansion.

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Higher order terms which will have square or cubic of this perturbed, perturbation variable.. . I am assuming that the perturbations are small ... In that case, we can drop these (higher order) terms .... So, what you have is this equal to this after we drop this..., and we assume that and we drop this, we assume that, so looking at only small motion about the equilibrium condition. (())

Now, this is equal to this identically, or even otherwise, because we are talking about this as the equilibrium condition. This function is going to be 0. Is it not?

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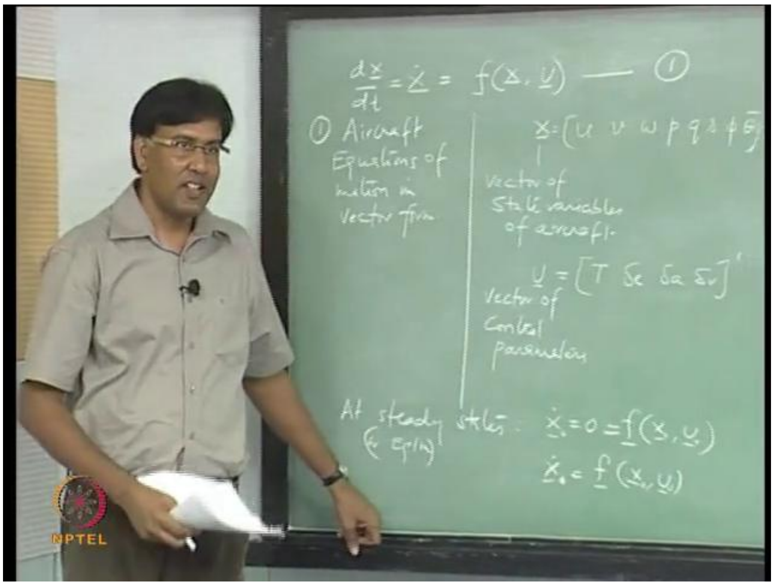

So, when we are talking about equilibrium condition or the steady state of the aircraft, what it means is, ... change in, time rate of change of variable is going to be 0. That is what defines the steady state. So, this is going to be 0. So, automatically you are solving this equal to 0 for the equilibrium conditions, and also, now think about any condition, not just the equilibrium condition, ... this is true. This is in general true. So, this (Refer slide above) is also true.

(())

In the last class, I directly did it. I am trying to explain how we get to this.

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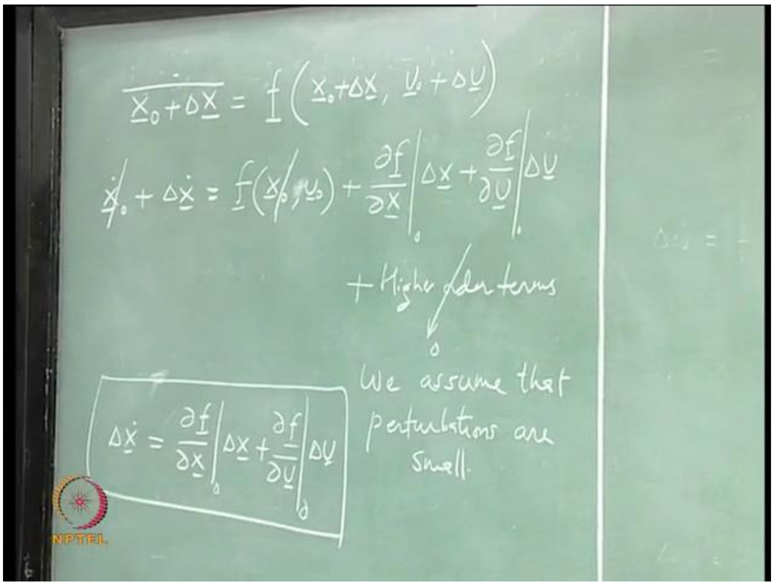

So, we can set, you know, this is equal to this (Refer slides above) or even otherwise, if you talk about steady state, this is 0 and this is 0. The difference here is that you may be flying any condition. Let us say, you are not flying in equilibrium condition but may be a maneuver where you still want to look at the perturbed motion of the aircraft. There this formulation is useful, because you are not dropping these two, because you are at equilibrium states, where this has to be equal to zero, and this also has to be equal to 0, but also for the reason that this is equal to this. So, this is what you can use for linearising your equations of motion, and we already finished linearising the force equation and one moment equation. I am going to write down the final form of the linearised equation.

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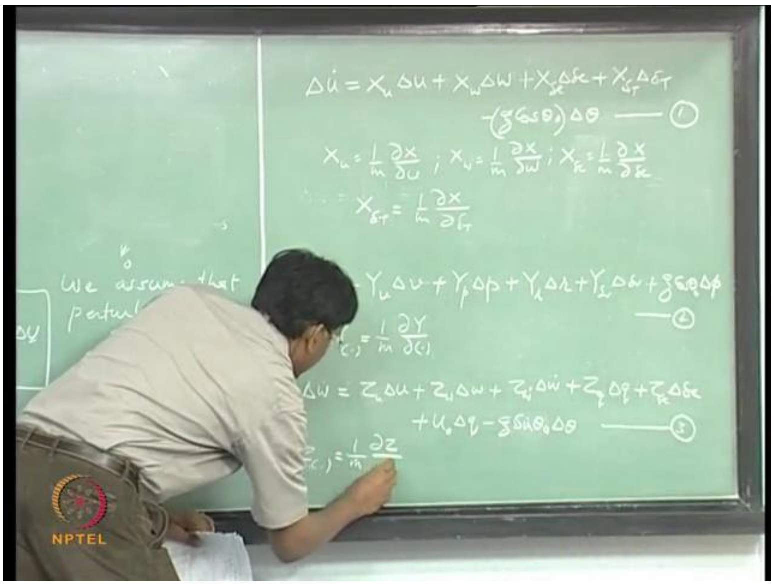

Second equation ...... is the equation, the force equation in the Y- direction.

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In general, these forces are going to be function of all the variables, but we are trying to retain only those derivatives which are important. So, this derivative you see is the change in the force in the normal direction with respect to the pitch rate.

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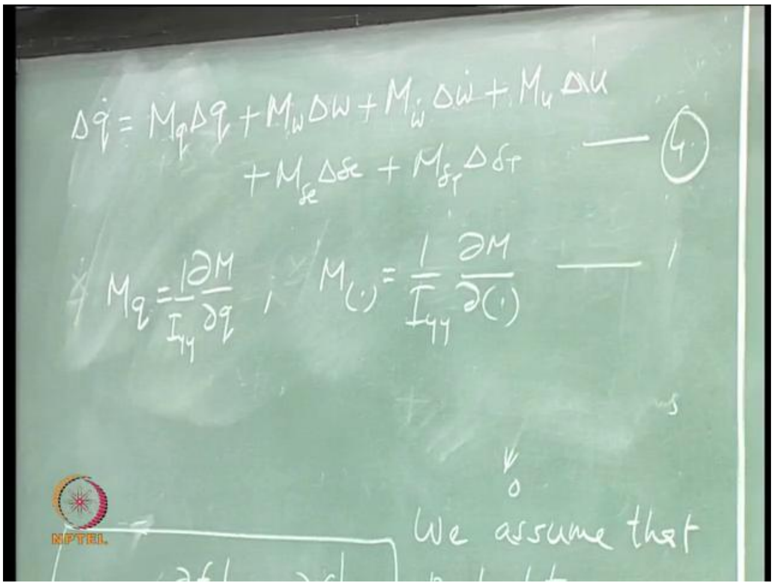

M is the pitching moment. This derivative is .... rate of change of pitching moment with respect to the pitch rate ... defined like this. So, we can write in general .... these derivatives in this form.

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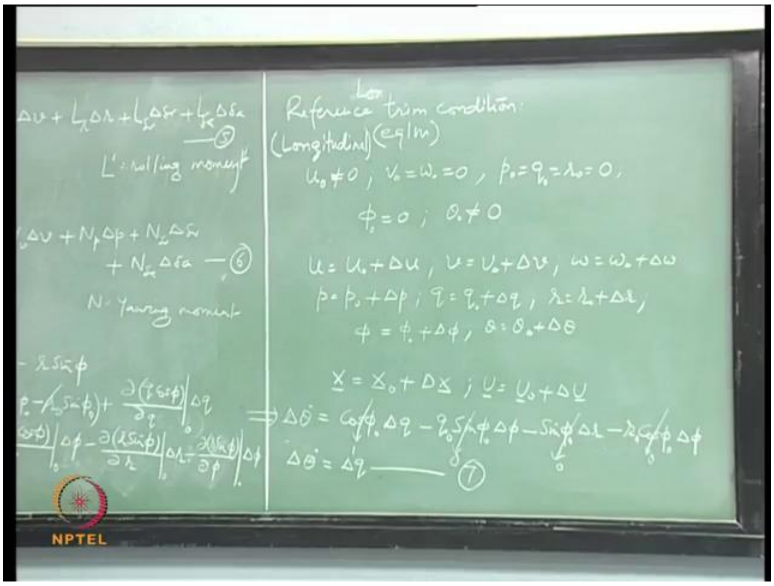

Rate of change of rolling moment with respect to the variable ... this is how you will define it. So, I have been using L prime for rolling moment. We will keep this to distinguish it from lift.

(No audio from 21:37 to 23:12)

Remember to stimulate the accurate motion of aircraft. If my wind tunnel measurements gives me this (derivatives) as, lets say I am measuring this (derivatives), and this (derivatives) is not a small value. Then I have to include that in the equations of motions. So, whatever data comes to you from the wind tunnel measurements, you have to keep it as it is, and then you have to write down the equations of motions accordingly, whether it is perturbed equations or the full form of equations.

So, you have to write your equations of motions in such a fashion that you can include all the data that is provided to you, and this data is coming from experiments. So, somebody is, somebody doing experimental measurements is going to give you the aerodynamic database, which will have all these derivatives or all the forces as functions of all variables? Whatever we can measure, and then you need to model them ... because you want to study the motion of aircraft using the data given.

That is not our job. That part is, someone else is doing that job. Someone else is collecting the data, he is giving you the data and he is asking you to look at the aircraft, or stimulate aircraft equations, aircraft behavior ... using the (aerodynamic) database given to you. So, if I want to simulate the perturbed behavior, then I have to use these perturbed equations.

In general, if you want to look at the aircraft behavior, you know, trying to figure out the trajectory or something like that, then you have to use the original form of the equations ...

Linearised forms of equations are normally used for looking at the dynamic stability properties of the aircraft. So, we already have six equations linearised, we have two more equations or let us…

(No audio from 25:52 to 27:46)

So, this is equal to this from this equation. So, I can cancel these out and let us look at what is left ....

$$\Delta\dot{\theta} = \Delta q; \quad \Delta\dot{\phi} = \Delta p \qquad (1)$$

(No audio from 28:06 to 29:23)

And now use this reference flight condition, and see what is left. So, phi naught is 0, cos phi naught is 1, this is 0. So, what is left is .... (()) but this r naught is 0.

Now, you can repeat this exercise, and you will find that delta phi dot is equal to delta p (Refer Eq(1)). So, we have linearised all our equations around this (our) particular flying condition. So, if you have any other flying condition, you have to repeat this exercise, because here what we have assumed is that alpha naught is 0, but alpha naught may not be 0 in longitudinal flight.

Now, let us start looking at some of these derivatives. See, what we are interested in finding out is. We want to find out how the dynamics of aircraft is getting affected because of the change in aerodynamic forces?

So, we get some idea about how we are going to let us say design our aircraft, so that we can control these derivatives. Is it not?

So, first thing is, we should know what these derivatives are? So, we will start looking at them separately, not really looking at all of them, but some of them. Then you will know how to find others.

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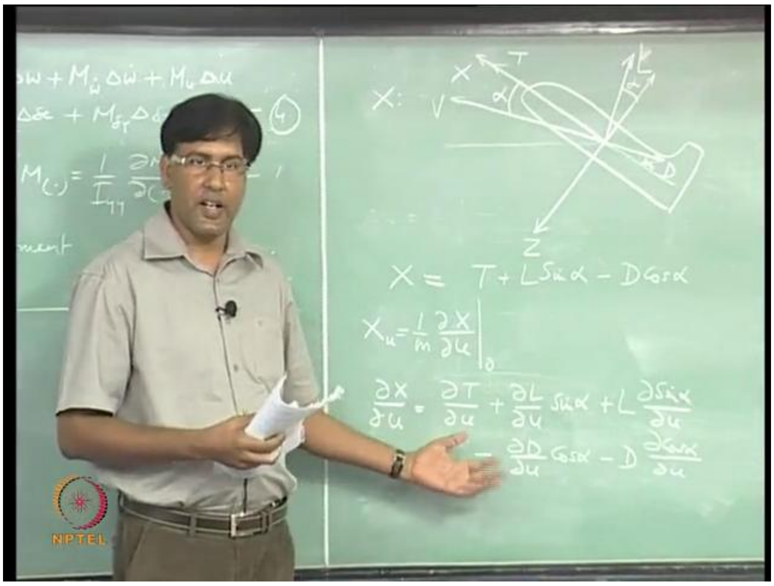

$$X = T + L\sin\alpha - D\cos\alpha$$

$$X_u = \frac{1}{m}\frac{\partial X}{\partial u} = \frac{\partial T}{\partial u} + \frac{\partial L}{\partial u}\sin\alpha + L\frac{\partial \sin\alpha}{\partial u} - \frac{\partial D}{\partial u}\cos\alpha - D\frac{\partial \cos\alpha}{\partial u} \quad (2)$$

We will first try to write down what this force X is, and afterwards, we are going to find the derivatives by taking the derivative. So, I will only write down the forces. So, X is the sum of all the forces, you know, including the thrust, if you think that the thrust is also going to change with respect to the variables of the aircraft and you have to include that also. So, here we will try to keep that if the thrust is not changing with the perturbation in forward velocity, then we can drop that...

33:40

So, aircraft is actually climbing.

(No audio from 28:06 to 29:23)

34:18

So, if I say that the thrust is acting along the body fixed X axis, then this force X is ... T plus L, component of lift is going to act towards the body fixed X axis ...(Refer Eq(2)).

I want to find out this derivative Xu which is .... (Refer Eq(2)). Remember, I already said that these derivatives have to be evaluated at the equilibrium flight condition. So, I am going to evaluate this at equilibrium flying condition which is given as this. Here, alpha naught is 0 because of this, w naught is zero, so alpha naught is 0. (())

$$X_u = \left.\frac{\partial T}{\partial u}\right|_0 + \left.\frac{\partial L}{\partial u}\right|_0 \sin\alpha_0 + L_0\cos\alpha_0 \left.\frac{\partial \alpha}{\partial u}\right|_0 - \left.\frac{\partial D}{\partial u}\right|_0 \cos\alpha_0 + D_0 \sin\alpha_0 \left.\frac{\partial \alpha}{\partial u}\right|_0 \qquad (3)$$

So, you can general, generally also you can find this derivative, and you can write the general form, and then keep on evaluating this derivative at different equilibrium conditions. (()) Is that what you were saying? (())

What you are saying is, your alpha naught ... so, you get only expanded form. (No audio from 36:39 to 37:46) Now, you have to remember one thing, you know, we are talking about pre-stall regime of flight.

In general, if you are talking about all kind of flight regimes where you are also trying to go into post stall regimes, then these derivatives can become non-linear functions of the variables itself, but here, they are going to be constant.... Also, in the pre-stall regime when alpha is roughly about fifteen to twenty degrees from 0 to 15, let us say, or may be little more than that, we can make assumptions like sine alpha is equal to alpha. Is it not? So, when we are talking about pre-stall flight regimes, it gets much simplified, you know, it is not complicated, and these derivatives are going to be constants.

So, X u ....(Refer Eq(3)) let us also try to expand these terms. .... Lift is equal to half rho V squared S into CL. Here, both CL and V are functions of u V squared ....(Refer Eq(4)) .

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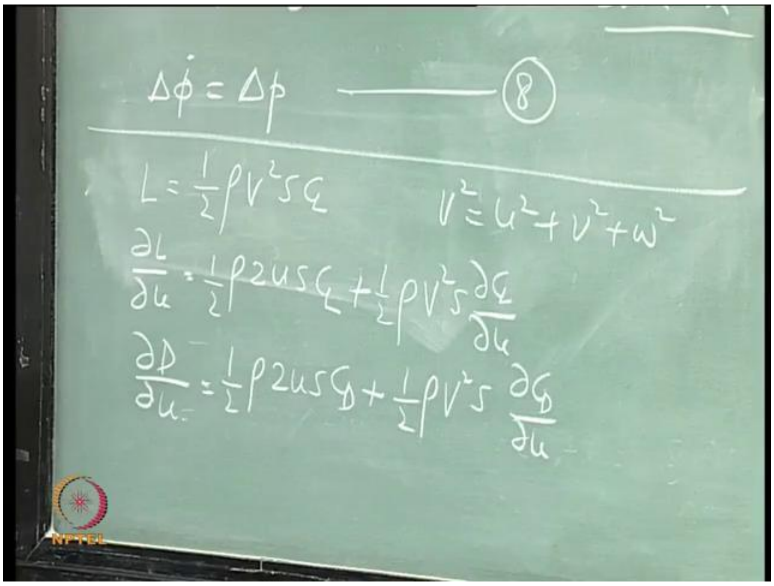

$$L = \tfrac{1}{2}\rho V^2 SC_L; \quad V^2 = u^2 + v^2 + w^2$$

$$\frac{\partial L}{\partial u} = \tfrac{1}{2}\rho 2uSC_L + \tfrac{1}{2}\rho V^2 S\frac{\partial C_L}{\partial u} \qquad (4)$$

$$\frac{\partial D}{\partial u} = \tfrac{1}{2}\rho 2uSC_D + \tfrac{1}{2}\rho V^2 S\frac{\partial C_D}{\partial u}$$

Similarly, I can write the derivative of the drag with respect to u ..... is this right? And what is this derivative? What is Alpha?

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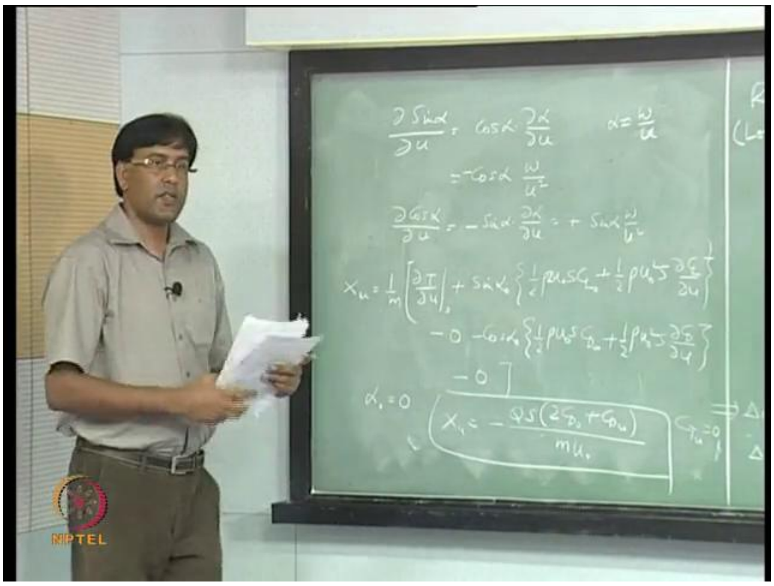

$$\alpha \approx \frac{w}{u}$$
$$\frac{\partial \sin\alpha}{\partial u} = -\cos\alpha.\frac{w}{u^2}; \quad \frac{\partial \cos\alpha}{\partial u} = \sin\alpha.\frac{w}{u^2}$$
$$\frac{\partial D}{\partial u} = \frac{\partial \frac{1}{2}\rho(u^2+v^2+w^2)SC_D}{\partial u} = \frac{1}{2}\rho 2uSC_D + \frac{1}{2}\rho(u^2+v^2+w^2)S\frac{\partial C_D}{\partial u} \quad (5)$$
$$\left.\frac{\partial D}{\partial u}\right|_0 = \frac{1}{2}\rho 2u_0 SC_{D0} + \frac{1}{2}\rho u_0^2 SC_{Du}$$

So, I am talking about small angles. So, I can, you know, take this approximation. So, what you have is this .....

$$X_u = \left.\frac{\partial T}{\partial u}\right|_0 + \left.\frac{\partial L}{\partial u}\right|_0 \underbrace{\sin\alpha_0}_{0} + L_0\cos\alpha_0\left.\frac{\partial \alpha}{\partial u}\right|_0 - \left.\frac{\partial D}{\partial u}\right|_0 \cos\alpha_0 + D_0\underbrace{\sin\alpha_0}_{0}\left.\frac{\partial \alpha}{\partial u}\right|_0 \quad (6)$$
$$\Rightarrow X_u = -\frac{\frac{1}{2}\rho u_0^2 S(2C_{D0}+C_{Du})}{mu_0}; C_{Tu} = 0$$

(No audio from 41:28 to 42:18)

Now, as I said, we have to evaluate the derivative at the equilibrium flying condition, and actually that is how you are going to keep your model in the wind tunnel. Is it not? You are not going to give a motion to the model. You are just keeping the model, static model in the wind tunnel, and you are trying to measure the forces and moments. So, now I am going to look at my reference flying condition, and try to simplify this, ... this subscript is referring to the ...(Refer Eq(5,6)).

(No audio from 43:33 to 44:11)

So, you can see this complete form. So, what we will do is, we will assume that alpha naught is 0(Refer Eq(5,6)). So, that is one such condition if you want you can retain it in this form.

I am only trying to give an idea of what this force is with respect to change in velocity, forward speed. If you want, you can retain these terms like sine alpha naught cos alpha naught and so on.

So, let us try to .... (Refer Eq(5,6)) (No audio from 44:50 to 45:23) So, my V naught squared is u naught squared, because other two terms are zero. We have second or third term which is having the derivative of this, and if I want to evaluate it at the equilibrium flying condition, because of this term here, it will be zero(Refer Eq(5,6)).

So, I am going to write zero for that and …

(No audio from 46:15 to 46:53)

This term is again 0(Refer Eq(5,6)), because, look at this derivative. So if I evaluate this derivative at the flying condition when w naught is 0 and this becomes zero. So, the last term there is also 0. Now, if I say, this alpha naught is 0, if you want you can keep it right. If you want you can keep it for non 0 alpha flying condition. ... (Refer Eq(5,6)).

48:18

Let me not rewrite this. What I want to point out here is, this C_L is a function of this forward speed, and this term is going to be significant, when? Whenever we have to think about the compressible flows. At higher speeds. At subsonic speeds, this is going to be very small or zero probably, but at higher speeds, this is not going to be a small term. In that case, you have to include this also in your model, or the equations of motion.

So, let me quickly tell you what this derivative is when your flying condition is this. That is what we are having here. .... (Refer Eq(5,6)).

(No audio from 49:37 to 50:17)

You can check this. Here, I have assumed that there is no change in thrust with respect to the forward speed. So, we can stop for today.