Flight Dynamics – II

(Stability)

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Module No. # 07

Lateral Directional Static Stability and Control

Lecture No. # 22

Example Problems

So, yesterday I said that we will do some problems. So, let us try to look at some problems.

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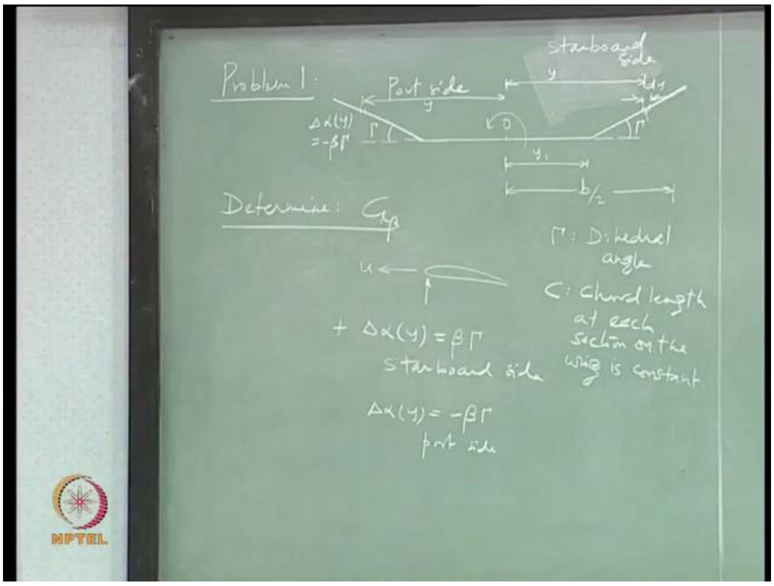

So, we have a straight wing, but having dihedral, only for this part.

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Span of the wing is b, and half span is b over 2. What I want to measure is, Cl beta, which we get from this kind of arrangement. So, you can assume that the dihedral angle is this Γ... same on both sides.

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Can assume it to be symmetric, about this (centerline), how do you go about solving this? I want to find out Cl beta. So, I will assume that there is a beta - positive beta coming from this side. What is this side called? The right side of the wing starboard; the right wing into the wind is the starboard…

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And this (left side of the wing) is the port side. So, positive beta is right wing going into the wind right. So, I want to find out this parameter Cl beta. Let us consider this small airfoil section on the wing, which is at a distance y from this center-line.

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And let us assume that this is a rectangular wing. So, c is not changing, it is not tapered wing. So, chord length of each section of the wing is same. We will assume that.

(No audio from 05:01 to 05:22)

..... so, if you remember we said that this section is going to see a wind, which is upward, is it not? So, you have a wind component which is normal to this section. That is going to be, that is going to give us an increase in angle of attack at that particular section, and that delta alpha y; so, this angle of attack that you will see on this section is a positive angle of attack right, because of the wind coming from this side, positive sideslip. Delta alpha at y, location y, is beta into the dihedral angle. Similarly, if you take the section at a distance y on the other side, that is going to see a decrease in angle of attack.

$\Delta\alpha(y) = +\beta\Gamma$ on starboard (right) side; $\Delta\alpha(y) = -\beta\Gamma$ on port (left) side

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So, this aircraft is flying level flight, lift on both the sides are balanced; so, that we do not have any roll, now there is a roll upset, because of which there is a sideslip angle introduced. I want to find out what this C l beta is for this particular kind of wing. So, this is for the starboard side of the wing, positive angle of attack. Delta alpha y is minus beta into dihedral angle on the portside. Because of this delta alpha, we are going to see a corresponding change in the lift.

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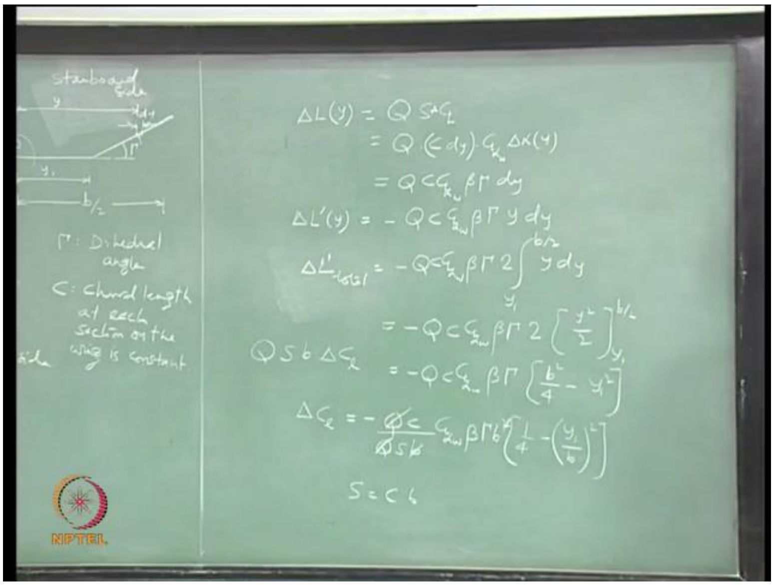

So, delta L y is Q into that area; so, c into dy ; chord length into that strip width dy (()).

$$\Delta L(y) = QS\Delta C_L = Q(c.dy)C_{L\alpha w}\Delta\alpha(y) = Q(c.dy)C_{L\alpha w}.|\beta\Gamma| \quad (1)$$

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C l alpha which is of the wing into delta alpha y. This is …

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$$\Delta L'(y) = -QcC_{L\alpha w}\beta\Gamma y dy \Rightarrow \Delta L'_{total} = -QcC_{L\alpha w}\beta\Gamma.2\int_{y1}^{b/2} y dy \quad (2)$$

And moment is what we want to find out. Moment, because of this force; so, that moment is in the negative direction. So, Z is going towards Y; so, that we are getting a negative roll. So, L prime is the sign we are using for the roll moment.

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And this I will get from the two sides of the wing, and both of them will add up; so, delta…(Refer Eq(2))

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Or if I want to get the total value…(Refer Eq(2))

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2 into ; so, plus delta L, minus delta L both of them are going to give the same negative rolling moment; so, I have to add them up. And this y is going to be starting from this point y1 (Refer Eq(2)).

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So, change in delta C l, the rolling moment co-efficient, that has to be now found out keeping the wing as reference condition; so, Q S b into delta C l, this is a change in rolling moment co-efficient.

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So, remember that we are measuring this beta from zero condition. So, you can consider this beta as also delta beta, change in the sideslip angle. You are flying level, and from, level condition is when your sideslip angle is zero. Now, we are talking about a disturbed situation when the sideslip angle is positive; so, we are measuring this beta from that zero beta condition. So, you can take this also as delta beta.

$$\Delta L'(y) = QSb\Delta C_l = -QcC_{L\alpha w}\beta\Gamma.\left[\frac{b^2}{4} - y_1^2\right] \Rightarrow \Delta C_l = -C_{L\alpha w}\beta\Gamma.\left[\frac{1}{4} - \left(\frac{y_1}{b}\right)^2\right]; S = b.c$$
$$C_{l\beta} = \frac{dC_l}{d\beta} \approx \frac{\Delta C_l}{\beta} = -C_{L\alpha w}\Gamma\left[\frac{1}{4} - \left(\frac{y_1}{b}\right)^2\right] \tag{3}$$

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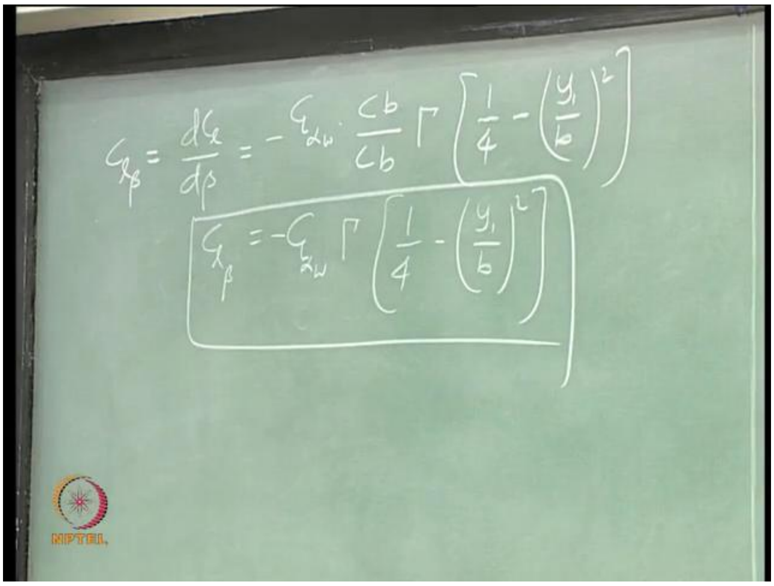

So, what you get here is ....(Refer Eq(3))

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And this S is, c into b. The wing plan form area…

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c into b over.

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So, this is how you find out C l beta. So, any other condition also given, you should be able to determine this. So, this problem is there in Nelson's book (()). It is not the solved problem, the exercise problem.

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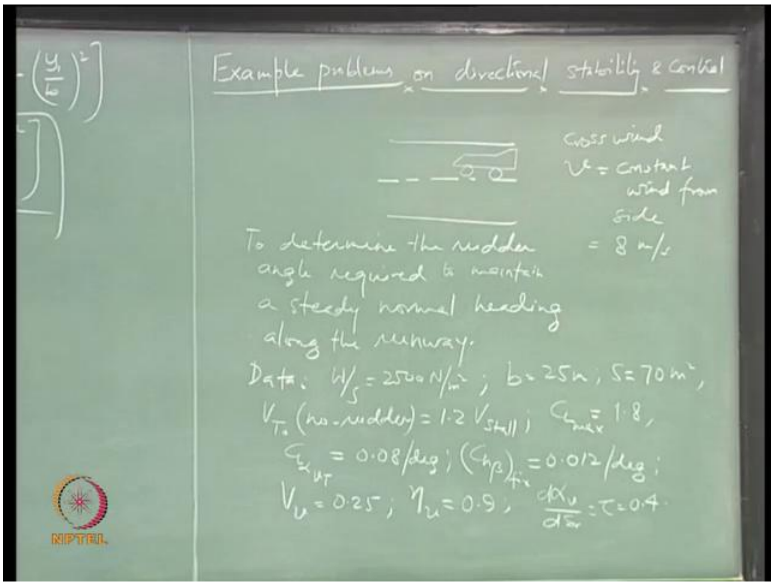

Now, let us look at some example problems.

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So, here is our runway, and airplane is about to take-off. So, it is already aligned with this centre-line; and suddenly there is a wind, you know, constant wind from the side. So…

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The magnitude of that is 8 meter per second. So, airplane is ready to takeoff, and it is detected that there is a wind of this magnitude coming from the side. You know, you should remember, when aircraft is taking off it has to take off; not that it has to wait for the wind to go away and then it will take off. Some of the airports like Chicago or you know LA, you will see that every single minute, there are some 2, 3 flights taking off. It is amazing scene, you know, just sit there and see; so, you will not see the airstrip void of airplanes, all the time going, one after another. So, that kind of traffic. You cannot just wait for the wind to go away and then you takeoff.

Because of this crosswind, now we should use the rudder to trim the aircraft; so, that it can go along this centre-line. So, the problem is to determine the rudder angle required … to maintain a steady, normal heading along the runway. And the data given is … so, wing loading is given; span b, span of the wing is 25 meters, wing planform area is 70 square meters. So, to take off with no rudder, velocity should be 1.2 times V stall.

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Lift curve slope of the vertical tail.

(No audio from 21:11 to 21:24)

C n beta fix. You know, we talked about, remember we talked about stick free stability, and stick fixed stability; in this case, this C n beta is for rudder, which is fixed.

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So, vertical tail volume ratio and the tail efficiency factor.

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And rudder effectiveness parameter. Can you see this, all of you? So, I want to determine the rudder angle required, so that I can fly along the runway, and there is a side slip, because of the crosswind. So, what is the magnitude of the sideslip angle? So, for that I need to know first the velocity. So, how do I go about finding that? Right, if you know CL max you can find out what is V stall?

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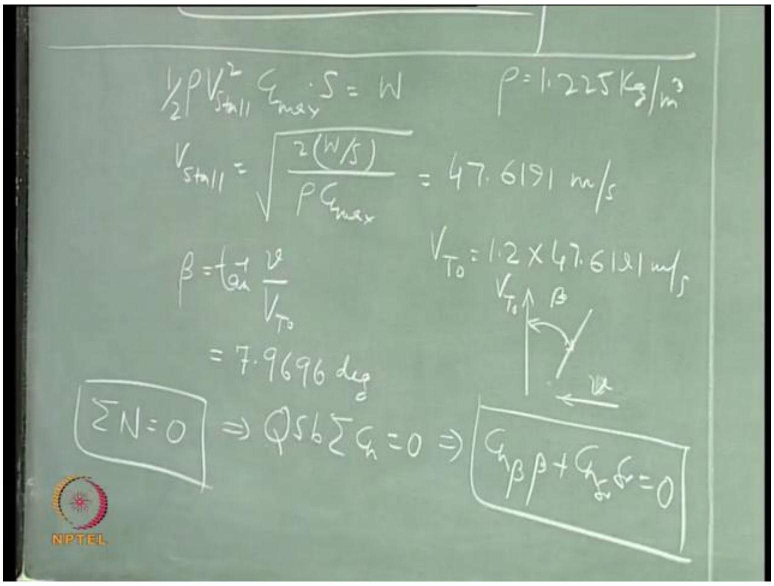

$$\tfrac{1}{2}\rho V_{stall}^2 C_{L\max}.S = W;\quad V_{stall} = \sqrt{\frac{2(W/S)}{\rho C_{L\max}}} = 47.6191 m/s;\quad V_{TO} = 1.2V_{stall}$$
$$\beta = \tan^{-1}\frac{v}{V_{TO}} = 7.9696\deg;\quad \sum N = 0 \Rightarrow C_{n\beta}\beta + C_{n\delta r}\delta r = 0 \qquad (4)$$

So, you can find out what is ... and assume it to be at sea level condition; so, rho can be taken as 1.225 kg per meter cube.

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So, you use the data, you will find that V stall is 47.6 meter per second. So, side slip angle can be calculated using this cross wind speed, and the V take off. V take off is …

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Beta is this …

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But you can find this.

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So, remember I want to kill the moment created because of the sideslip using rudder. This sigma N, sum of yawing moment from different places, different contributions has to be 0; what this means? This means …

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Right. So, Cn contribution from different sources, this gives me …(Refer Eq(4))

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C n beta is given; beta we have found. Delta r is to be found. So, I should know what is C n delta r, and C n delta r you can find out from the information given here for example, this rudder effectiveness, the lift curve slope of the vertical tail.

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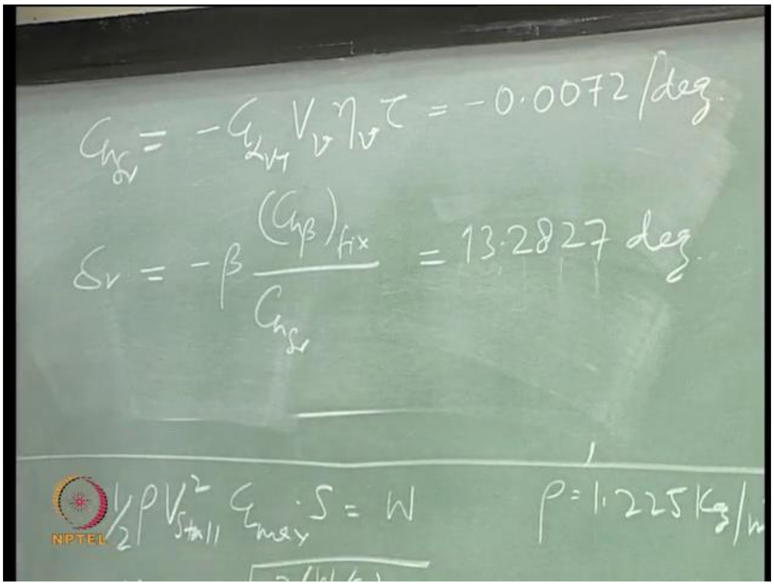

So, C n delta r is … (refer Eq(5))

$$C_{n\delta r} = -C_{L\alpha VT} V_v \eta_v \tau = -0.0072/\deg$$
$$\delta r = -\beta \frac{(C_{n\beta})_{fix}}{C_{n\delta r}} = 13.2827\deg \qquad (5)$$

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Here, I am not accounting for the side wash angle. That information is not given; so, I can straight away write this expression for C n delta r ..... (refer Eq(5))

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This is exactly same problem as when your aircraft will have asymmetric throttle condition. Let us say you are flying your aircraft and one engine fails. Then, how do you trim your aircraft to follow the same path. So, the other example is that, asymmetric power condition.

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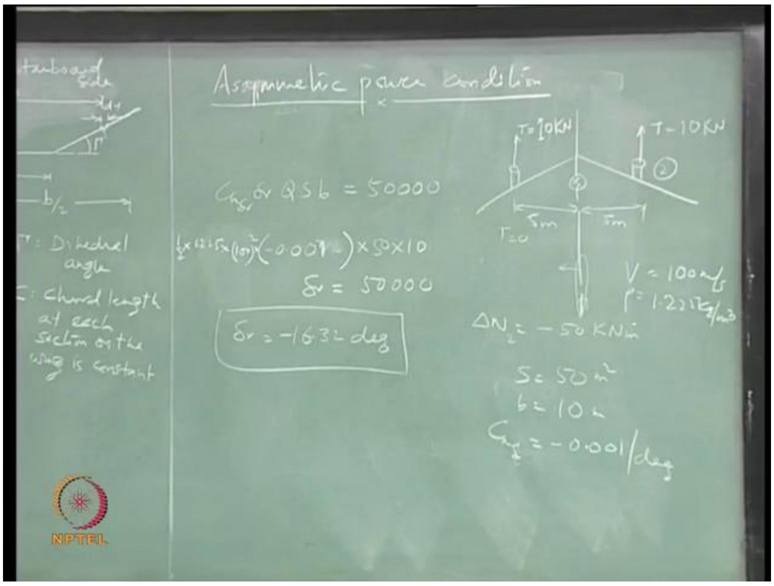

So, each engine is producing thrust which is 10 kilo Newton, and the engine is located at a distance 5 meters from the centre line.

$$
\begin{aligned}
\Delta N &= QSb\Delta C_n = QSb\frac{dC_n}{d\delta r}\delta r = QSbC_{n\delta r}\delta r \\
&= \tfrac{1}{2}\times 1.225\times 100^2\times -0.001\times 50\times 10\times \delta r = 50000 \Rightarrow \delta r = -16.32\,\text{deg}
\end{aligned} \qquad (6)
$$

(No audio from 30:47 to 30:57)

So, one engine runs out; that means, you have an unbalanced yawing moment. So, if this runs out, I will have a positive or a negative yaw moment. So, if this (right, starboardside) engine, let us say here the thrust becomes 0. Then, delta N due to '2'; you know, cg is located somewhere here, is minus 50 kilo Newton meter $\Delta N = -50000 Nm$; and this I have to kill using rudder. Rudder should be powerful enough to kill this yawing moment or balance this yawing moment. So, I am using the, this is the equation that you have to use. So, rudder deflection should be in which direction, should be your, to your right, or left. So, if I, I am having this vertical. So, rudder deflection should be which side, this side or this side, this side because you have to create a lift on this side or side-force.

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And that deflection of rudder is what, negative or positive? All control derivatives are negative; so, positive rudder deflection should give you a negative yawing moment. So, here I am deflecting rudder to this side, and it is giving me a positive yawing moment; so, this deflection on this side is negative.

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Let us use the same data; so, C n delta r is minus ....(Refer Eq(6))

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So, actually this plus this should have been equal to 0. So, I have already taken care of that, when I am taking it on my right hand side, I am writing this as positive.

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So, here the S given is ..... think, C n delta r is not that, but C n delta r is minus 0.001 per degree.

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The velocity at which it is flying level, when it runs out of the engine is 100 meter per second, and it is at sea-level conditions; so, you can take this rho as 1.225. So, this is half into … into delta r. (Refer Eq(6))

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So, now I am going to stop at this stability and control, static stability and control; and then we are going to start with dynamics. So, it is very appropriate time; so, I will just give you a brief outline, what we are going to see in the next part of the course.

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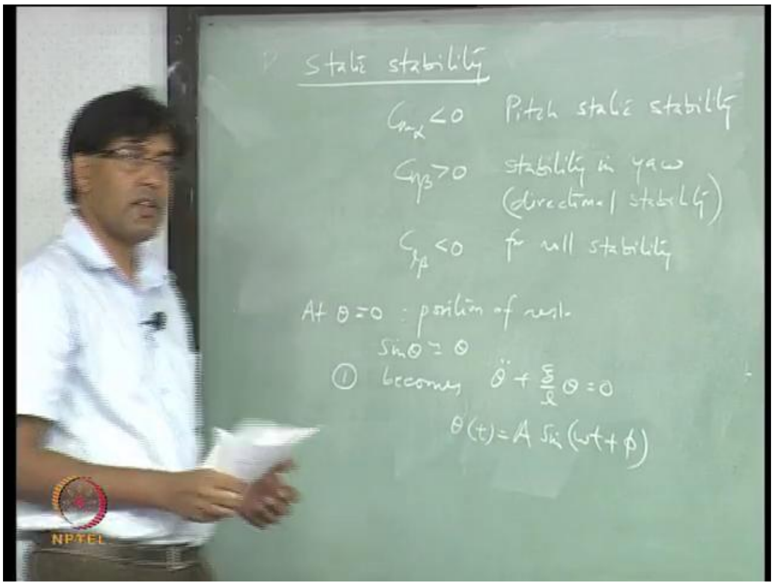

Static stability criteria, Cm alpha should be less than 0 for pitch, static stability…

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C n beta positive for directional stability.

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Now, let us say we met all the criteria. So, should we presume that the aircraft is going to fly stable, in all flying conditions. What we have looked at so far is the aircraft tendency to mitigate the disturbance, that is what we have looked at so far. But we cannot say if the disturbance is really decaying. So, you have a level flying condition and then your aircraft is disturbed so that alpha is changed. An aircraft has a natural tendency, because of this term ($C_{m\alpha} < 0$), to kill that alpha. But is that really happening? That we will be able to see only when we look at the evolution of that disturbed quantity in time. So, we have to look at the time response of that perturbed variable. So, dynamic stability is related to the time response.

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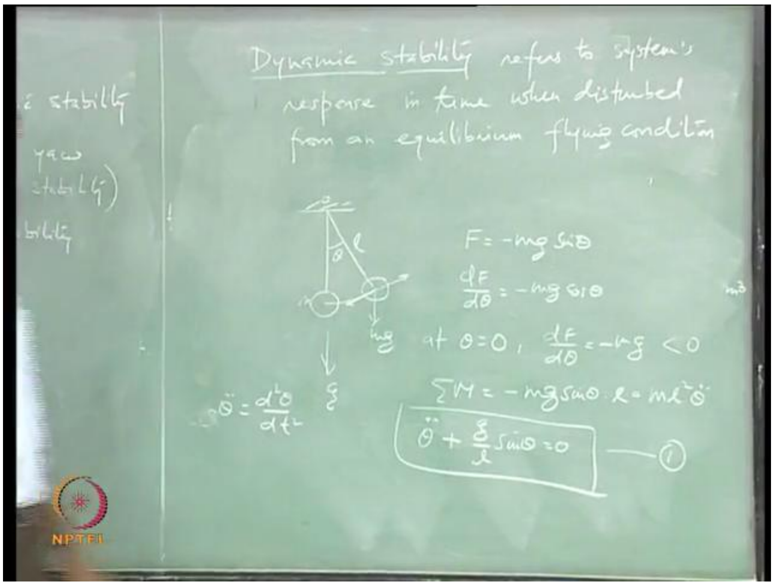

$$F = -mg\sin\theta; \frac{dF}{d\theta} = -mg\cos\theta; \left.\frac{dF}{d\theta}\right|_{\theta=0} = -mg < 0$$
$$\sum M = -mg\sin\theta . l = ml^2\frac{d^2\theta}{dt^2} \Rightarrow \frac{d^2\theta}{dt^2} + \frac{g}{l}\sin\theta = 0 \qquad (7)$$

So, our system here is aircraft, response in time when disturbed from a, from an equilibrium flying condition. And that flying condition itself has to be stable to fly with, you know, is it not? Otherwise you cannot fly. So, we are looking at stability from an equilibrium flying condition. What we want to look at, when the aircraft is disturbed in flight; all the variables are actually going to be disturbed. It is not only angle of attack, because you know when the wind is coming, wind is not going to come only exactly from the vertical direction; it can come from any direction. So, it can disturb all the variables, and we want to see, if those variables are, disturbed quantities, are decaying in time or not.

Static stability does not actually ensure that. So, static stability only tells you that the aircraft has a tendency to kill that disturbance. Simplest example is this, this pendulum, and let us say you disturbed it; so, it acquired some height or the amplitude. And it also sees a force which is trying to pull it back because of gravity, is it not? So, it is in the gravitational field; so, whenever there is a displacement; so, this position is the position of the rest. So, whenever this pendulum is disturbed, from this position of rest, there is a force developed on this mass m, which is, so if I take force as the positive along theta, then this force is minus mg sine theta.

Now, let us look at, how this force is changing with respect to this angle theta. So, you have to look at the derivative of this force with respect to theta. I am checking stability of an equilibrium condition. So, whenever we talk about stability, we are talking about stability about an equilibrium point. So, my 1 equilibrium point is theta equal to 0. That is the position of rest so I want to find out what is happening to this derivative at that position of rest. So, at theta equal to 0, this derivative is …(Refer Eq(7)) . So, the restoring force is trying to kill that theta, that is what it means? Static stability, you only want to see how the force is developing over the body, when the body is displaced from its equilibrium condition.

So, we said about static stability by looking at this derivative. Now, what can we say about the dynamic stability of this system, at equilibrium condition, which is 0. So, I have to write down the equation of motion. And equation of motion for this. So, if you have this length of the wire is l. So, this force is creating a moment about this point of suspension. So, which is … (Refer Eq(7)) is equal to the inertia force. So, the equation of motion is actually this (Refer Eq(7)).

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Now, what do you say about stability? This force, or the derivative, is appearing here, you know, in some fashion, but what about this…

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Now, I have to look at the evolution of this theta in time, and what is the solution for this? Analytical solution, is it easy or difficult to find? We are interested only in looking at the motion around this equilibrium condition. And the, that motion, is the perturbed motion. So, we can think about a linear model where I can approximate this sine theta as theta.

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So, one …

$$\frac{d^2\Delta\theta}{dt^2}+\frac{g}{l}\Delta\theta=0;\sin(\theta^*+\Delta\theta)=\sin(0+\Delta\theta)\approx\Delta\theta;\omega=\sqrt{g/l} \qquad (8)$$

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What is the solution for this (nonlinear Eq(7)) ? This you know. This was slightly difficult, but this, solution of this (linear equivalent equation (8); perturbation model around theta 0 Equilibrium position) you know, what is the solution? A is the amplitude, Omega is the frequency; so, this is the solution of (1). And, what is it telling you? It is telling you that when you hit this ball or the bob (()) from its position of rest which is at theta equal to 0. This is how theta is going to be in time, which means theta is always, you know, going back and forth around this theta equal to 0 position. So, what can we say about stability- dynamic stability? We can say nothing, about the dynamic stability. Static stability yes, know, because it is at least, it is going through this, passing through this 0; so, static stability does not really indicate the dynamic stability. Is that clear?

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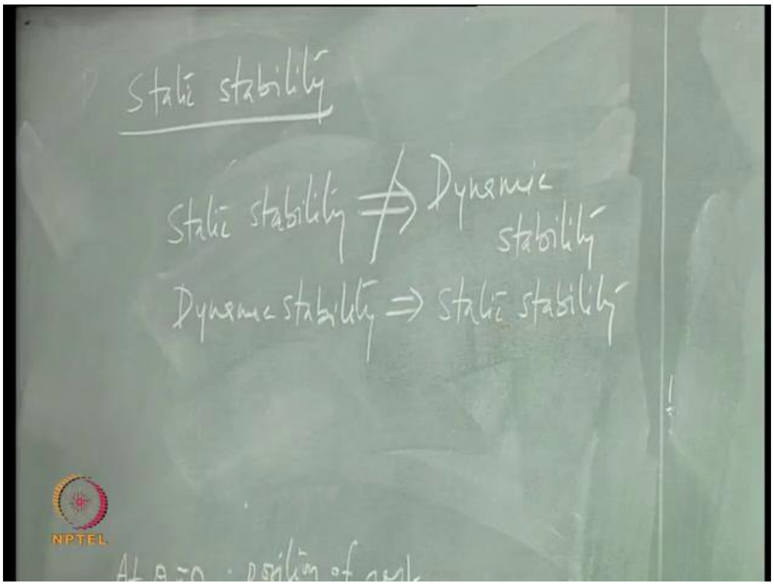

Does not imply.

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If the dynamic, if the system is dynamically stable, will it automatically imply static stability; yes, right.

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So, there is a conflict; so, we have to actually start with the equations of motion, try to linearize it around equilibrium points, and then look at the response to determine the stability; and that is what we are going to start now. So, in the next class, we will write down the aircrafts equations of motions.