Flight Dynamics II (Stability)

Prof. Nandan Kumar Sinha
Department of Aerospace Engineering
Indian Institute of Technology, Madras

Module No.# 05

Longitudinal Control

Lecture No. # 13

Stick Free Stability, Most Fwd CG Location

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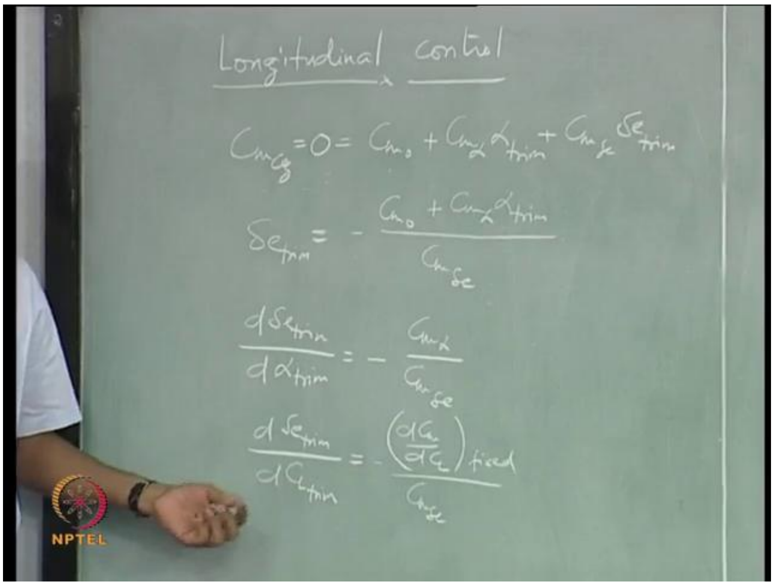

$$C_{mCG} = 0 = C_{m0} + C_{m\alpha}\alpha_{trim} + C_{m\delta e}\delta e_{trim} \Rightarrow \delta e_{trim} = -\frac{C_{m0} + C_{m\alpha}\alpha_{trim}}{C_{m\delta e}}$$

$$\frac{d\delta e_{trim}}{d\alpha_{trim}} = -\frac{C_{m\alpha}}{C_{m\delta e}}; \quad \frac{d\delta e_{trim}}{dC_{Ltrim}} = -\frac{\left(\frac{dC_m}{dC_L}\right)_{fixed}}{C_{m\delta e}} \qquad (1)$$

So remember, we said that to trim the aircraft at different speeds you need to use elevator right. So, trimming is.. (Refer Eq(1)), besides forces that you have to balance, the moment also has to be balanced, right.

Right. So, δe_{trim} is (Refer Eq(1)),... right. So, let us try to look at how much elevator you will need for change in angle of attack ... per unit deflection, per unit change in angle of attack, and this is (Refer Eq(1))..... right.

You can write this expression also as (Refer Eq(1))…... So, clearly .. you see here that to get a change in $C_{L\text{trim}}$, you need to deflect elevator, and that is going to depend upon this quantity. Is it not? And what is dC_m/dC_L, its the measure of stick fixed stability, static margin, is it not?

So, clearly the amount of static margin that you have or the pitch stability that you have is also going to determine, how much force a pilot is going to feel ... when he is trying to change the elevator. Pilot is the one who is going to pull the stick or push it and that force he has to apply is depending on this $(dC_m/dC_L)_{\text{fixed}}$. In another words, depending on the stability that aircraft has, its going to decide how much force a pilot is going to feel, right.

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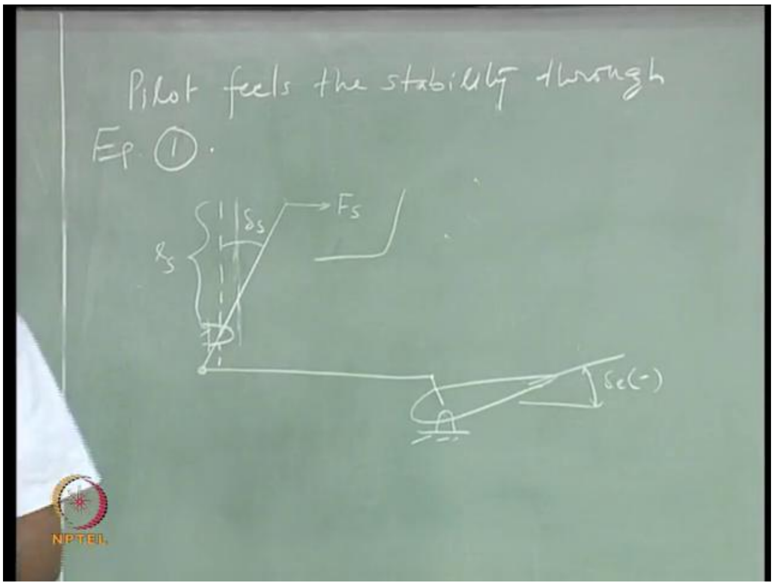

So, .. Pilot will feel the stability through this equation one. So, let us try to look at the mechanism, you know, by which you will change the elevator deflection, and this is roughly how it should look like.

Fs is the stick force, and this (deflection towards pilot) is a pull, right. So, pilot is sitting somewhere here, and he is pulling the stick towards himself. OK, may be this line can be shifted here, and this is the arm that he has, right.

So, this is going to.....So, δe_{up} is negative ... that we have said yesterday and.....

So, there is a hinge line about which this elevator is moving. So, you apply this force, pull it towards yourself, and you will see a elevator deflection which is upward, and that is negative. If I want to find out what this force should be ... I have to talk about the moment that it has to overcome. Is it not? Elevator is in the flow field. So there, there are forces acting on the elevator, you want to overcome that force when you are trying to change the elevator deflection right.

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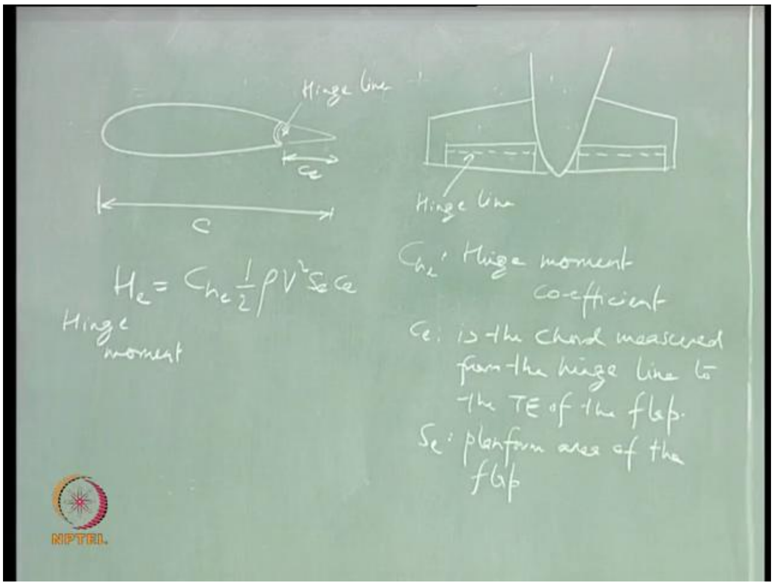

So, this (Refer sketch on the slide) is your elevator, hinged at this point to the horizontal tail. So, let us say this...... is the, some chord length, which is measured from the hinge line, and this c is the chord length of this tail, or any, any surface actually, any control surface, because similar things we will also have to repeat for rudder or aileron ... whatever we are talking about here, similar things, know, similar explanations, we can also give for other control surfaces. So, hinge moment is the moment about this hinge line that the pilot has to overcome when he is applying this stick force. So, the expressions for this hinge moment $H_e = C_{he} \frac{1}{2} \rho V^2 S_e c_e$ ... it will be all along the tail. So, that is why I am writing this, hinge line, and this is equal to .... this, C_{he} is hinge moment coefficient. Let me draw this picture, which will make things clear.

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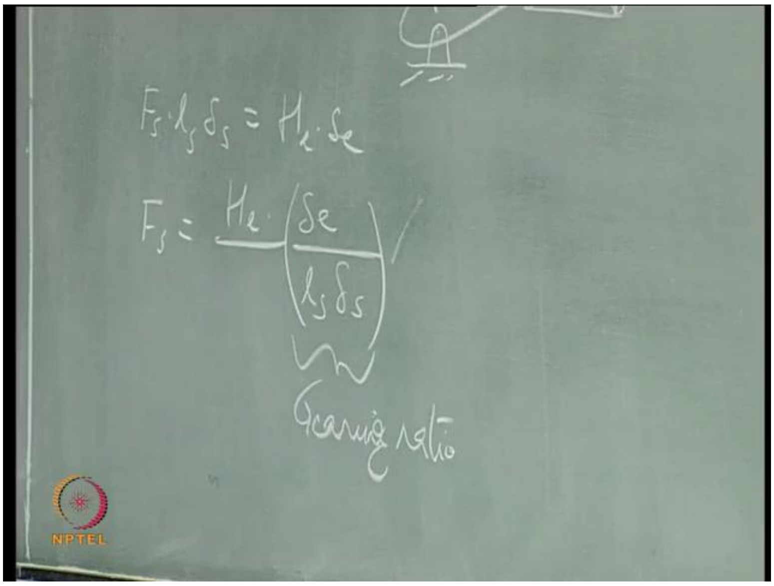

$$F_S.l_S\delta_S = H_e.\delta e \quad \Rightarrow \quad F_S = H_e.\underbrace{\left(\frac{\delta e}{l_S\delta_S}\right)}_{Gearing\ Ratio} \qquad (2)$$

So, this is the elevator, ... and hinge line is this. ... So, let us try to balance out the work, work that this force, stick force, applied by the ... applied by the pilot does (Refer Eq(2)).

(()) Tail, it will be slightly changed, right.... So, actually again, I am going back to what I said, these are some estimates, and no analytical expression actually can be correct, you have to put everything in the wind tunnel and measure the moment and forces, ok, if you want to get an exact answer to any of these quantities. So, this pull is a positive force, when the pilot is trying to pull the stick towards himself, that is the positive stick force, and that gives me δe upward, which is negative. And, let us try to find out this force; this force know, if the pilot is not applying, someone else, or some other mechanism has to, to do this job, right.

For example, if you want your aircraft with control systems mounted, then this force will not be directly applied by the pilot but automatic control system can do this. Is it not? So, you press something, and that you know at the press of a button, this should happen, elevator should deflect, right. So, actually they will not even have to do this for bigger aircraft, where you have strong control systems in place.

You, you will only define the flying condition, and then everything will be taken care off. So, what is the work done by this force in deflecting this stick by this angle, f into, right. Fs into ls delta s, (Refer Eq(2))... delta s is a small angle, and that has to be equal to the work, that has to be done, right, by this ...... (Refer Eq(2)) which is He into delta e, right.

So, what is Fs? Fs is He into delta e over ls delta s, right (Refer Eq(2)). This is called, this quantity in the bracket, is called gearing ratio. So, let us try to analyze one situation where elevator is free, it is not changing, and the pilot wants to just leave the stick, then what will happen? So, remember that, I am, what I am saying is that, when you are designing a control surface, you have to remember three things.

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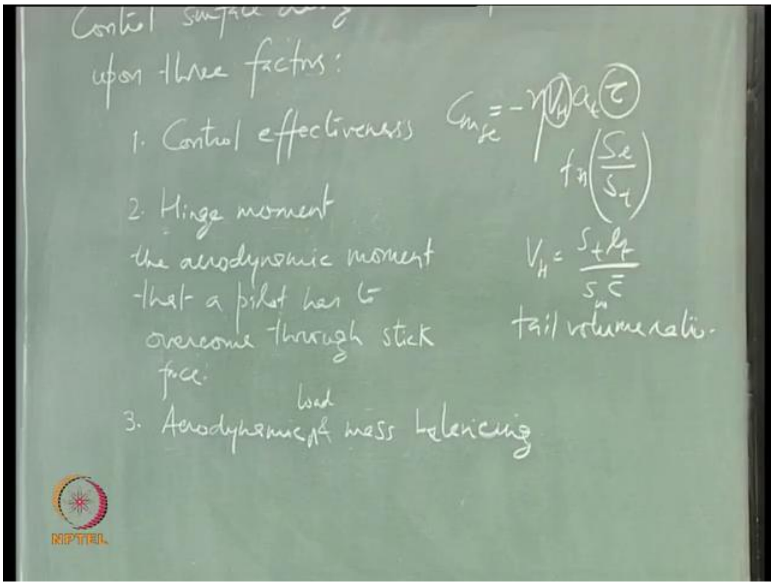

You know it will depend upon three factors: First is control effectiveness, ... and this is also the control power, that related to the control power, right. So, if you remember, we wrote this expression for $C_{m\delta e}$, which is change in pitching moment that you can get by changing the elevator deflection, and that, the expression for that was $C_{m\delta e} = -\eta V_H a_t \tau$ ..... this, right.

So, V_H and this τ, τ is a function of the area of the planform of this flap, and the areas of ... planform of this tail (S_e/S_t), right.

So, this parameter was a function of this... ratio of the areas. V_H is what? VH is tail volume ratio, $V_H = l_t S_t / S_w \bar{c}$. So, you have some parameters which you can adjust, but you remember, you also, when we are talking about stability, there also we said that we can use these parameters to change the stability characteristics, right.

So, now stability characteristics, while changing, you are also actually changing this control power, right, if you are doing it through this quantity, tail volume ratio. Second factor is hinge moment; hinge moment is nothing but the aerodynamic moment that a pilot has to overcome… right. Now, the third factor is, can we reduce this moment by doing something, can we reduce the hinge moment somehow.

So, how you can change the hinge moment, know, the moment that a pilot has to overcome, that is a third factor, that, one has to remember while designing the control surface. So, let us look at a case when the pilot has left the elevator to rotate freely, he is not applying any force. What will happen in such a case? What will happen to the hinge moment? If a surface is, .. if something is allowed to rotate freely, what should happen to the hinge moment, zero, right.

So, let us look at what happens in such a case, when elevator is not fixed, but is free to rotate, ... you are not holding the elevator somehow....

So, let us look at…

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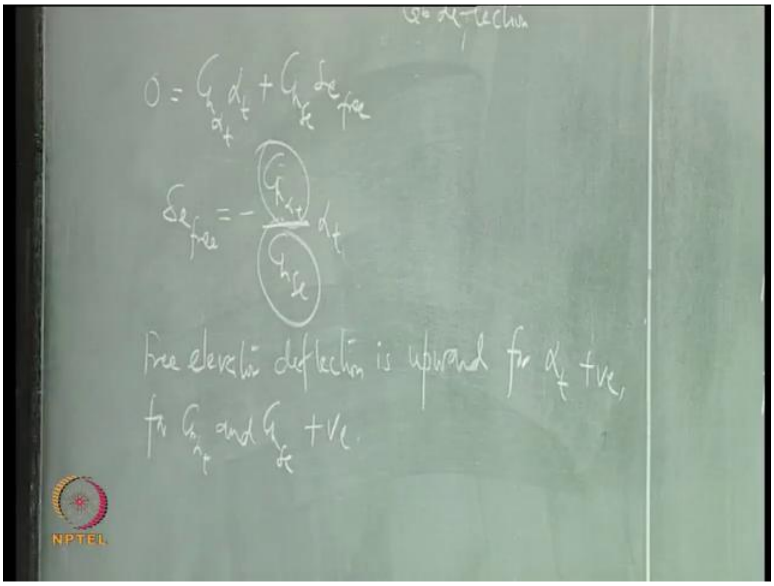

$$C_h = 0 = \underbrace{C_{h0}}_{0} + C_{h\alpha t}\alpha_t + C_{h\delta e}\delta e_{free} \Rightarrow \delta e_{free} = -\frac{C_{h\alpha t}}{C_{h\delta e}}\alpha_t \quad (3)$$

So far we have looked at stick fixed stability, right. Now, talking about one case, and that stick fixed was this. So, you have fixed the elevator; elevator is not allowed to move from that fixed position, right. Here is one case where elevator is left free, that, there can be situation like that.

So, if a pilot leaves the stick, does not do anything, what will happen to the stability of the aircraft, that is what we are going to see through this. So, this hinge moment coefficient is going to depend on, hinge moment is the aerodynamic moment, that is coming over to this small flap, right, that has to be overcome, right.

What is it going to depend upon? Angle of deflection, angle of attack, right. So Ch alpha t, alpha t, angle of attack at the tail, (Refer Eq(3))... so also going to depend upon the elevator deflection. If you have a tab, which we will talk about later, if you have a tab which is a surface ... attached to the elevator, we will talk about this little later. So, its also going to be a function of the tab deflection, ok, plus there can be a residual moment which is there forever. So, you have to overcome that, and this C_{h0}, right.

Now, I am, what I am saying is, elevator is free to rotate, and let us try to make things simpler, just drop this for now, and C_{h0} is also zero, assume that. So, we have, ... so delta e free (Refer Eq(3)). So, you are on the ground, and you are not applying any force, and let us say there is a flow, what position the elevator is going to take; that is what we are trying to find out from here, right.

Right. So, this is the free position of the elevator (Refer Eq(3)), and these quantities are actually positive; both of them are positive. For alpha t α_t which is positive, delta e δe is going to be upward, right. So, free elevator deflection ....., and they are positive, there people have done some measurements and they have found that; these two parameters ($C_{h\alpha t}$ and $C_{h\delta e}$) are positive. So, automatically elevator should show an upward deflection, without doing anything, whenever there is flow over the airfoil.

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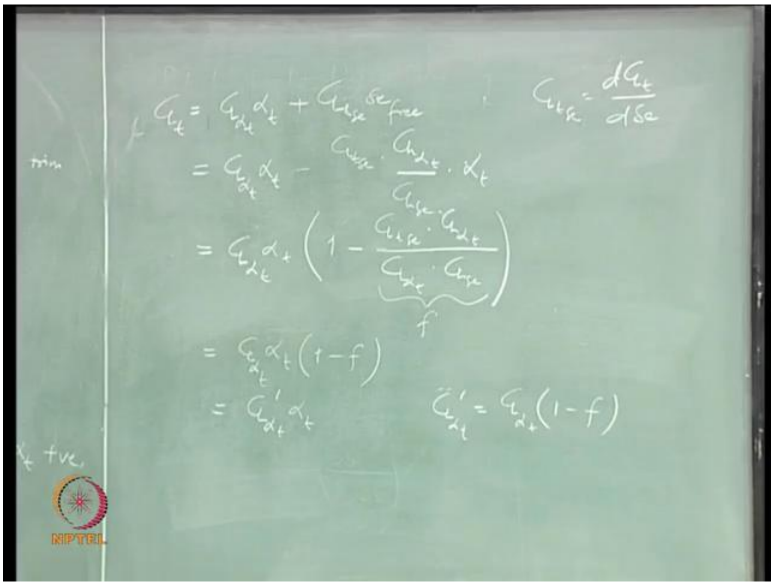

$$C_{Lt} = C_{L\alpha t}\alpha_t + C_{Lt\delta e}\delta e_{free} = C_{L\alpha t}\alpha_t - \underbrace{C_{Lt\delta e}}_{(dC_{Lt}/d\delta e)} . \frac{C_{h\alpha t}}{C_{h\delta e}} . \alpha_t$$
$$= C_{L\alpha t}\alpha_t \left(1 - \underbrace{\frac{C_{Lt\delta e} . C_{h\alpha t}}{C_{L\alpha t} . C_{h\delta e}}}_{f} \right) = C_{L\alpha t}(1-f)\alpha_t = C'_{L\alpha t}\alpha_t \qquad (4)$$

Let us look at what happens to the lift coefficient of the tail in such a situation. So, CL alpha t is going to be (Refer Eq(4))...... right. So, automatically you see this component ... and that is ..... (Refer Eq(4)) know this CL t delta e is this ..... into .... and call this some f (Refer Eq(4)).

So, this total quantity becomes .... yeah.

Student: (( )).

C g of the, moment is about C g of the aircraft.

Student: (( )).

Yeah.

It will be small and (( )), ok, I will come back to your question later, because, yes, aerodynamic forces are large.

But, we will, .. we will see if ... it falls into this third factor, we have written there aerodynamic load and mass balancing.

Student: (( ))

Yes.

Student: (( )).

Can I come back to this later, maybe I will also have to think a bit on this. So, this CL alpha that we see is modified, right.

So, if you look at $C_{L\alpha}$ - the new $C_{L\alpha}$, because of the free rotation of the elevator is .... (Refer Eq(4)) right. Now, let us go back and see what happens to our C_{m0} and $C_{m\alpha}$. So, you see, how many derivatives one has to worry about when you are designing an accident proof aircraft, you know, you can design your MAV may be a small MAV (( )) and let it have n number of accidents you do not care about, but when people are designing a full passenger aircraft, ... four hundred people .... people sitting inside, then you have to worry about each one of these terms.

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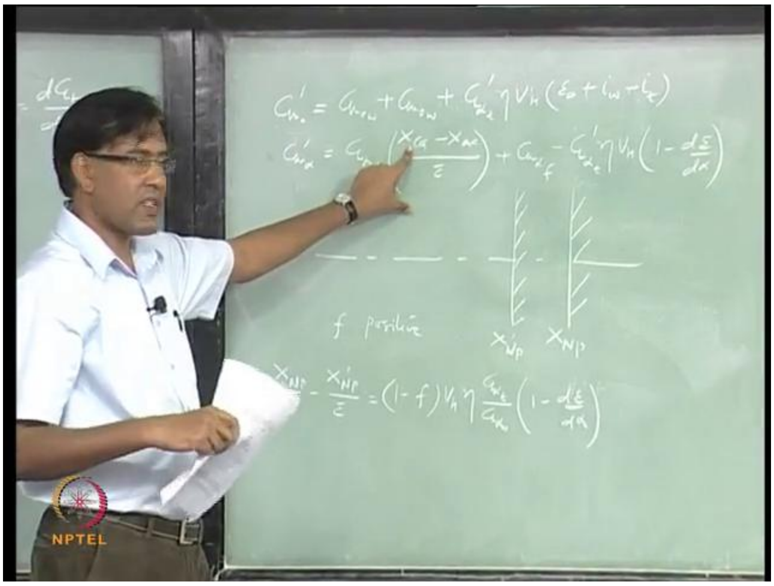

$$C'_{m0} = C_{m0} + C_{m0w} + C'_{L\alpha t}\eta V_H(\varepsilon_0 + i_w - i_t)$$
$$C'_{m\alpha} = C_{L\alpha w}\left(\frac{X_{CG} - X_{AC}}{\bar{c}}\right) + C_{m\alpha,f} - C'_{L\alpha t}\eta V_H\left(1 - \frac{d\varepsilon}{d\alpha}\right) \quad (5)$$

So, C_{m0} new is ...... $C_{m\alpha}$ is also going to change, right, because the lift curve slope of the tail is changing ... for free elevator deflection. So, clearly something has changed, right. $C_{m\alpha}$ have changed ... and this is going to depend upon this new slope of the lift curve for the tail, right. So, I am going to see a change in what, something which is changing know, right, what is that?

$C_{m\alpha}$ is changing, so static margin is changing, ... stability margin is changing, ... and that automatically implies that the neutral point location is changing, right. Which way, it will depend upon what this f is, right. If f is positive then, then XNp prime is going to be….. (Refer Eq(5))

Student: (( ))

Will reduce, if f is negative this .... right. So, it will be… going back.

Going back right. So, if I say this is my XNp; the new XNp is going to be depending upon what this f is… If f is less than, f is positive, then this one minus f is positive and lesser than one ... then you are going to see a reduced stability margin, right .

So, now I will write down the difference of the two, you have to remember that all the measurements are from the wing leading edge.

(( )) if we subtract Cm alpha from, C m, C m dash alpha (prime) then (( ))

You do not have one minus f.

Student: (( ))

Then I have to put a prime here.

(( )) if we expand CL alpha t dash, It will be CL alpha t minus f into C L alpha t; C L alpha t will get canceled when we subtract.

Do not have to go that far, you set this to zero, then this Xcg is XNp prime, right. In the other case…

When .... we subtracting the one (( )).

$$\begin{aligned}
0 &= \frac{X_{NP} - X_{AC}}{\bar{c}} + \frac{C_{m\alpha,f}}{C_{L\alpha w}} - C_{L\alpha t}\eta \frac{V_H}{C_{L\alpha w}}\left(1 - \frac{d\varepsilon}{d\alpha}\right) \\
0 &= \frac{X'_{NP} - X_{AC}}{\bar{c}} + \frac{C_{m\alpha,f}}{C_{L\alpha w}} - C'_{L\alpha t}\eta \frac{V_H}{C_{L\alpha w}}\left(1 - \frac{d\varepsilon}{d\alpha}\right) \qquad (6) \\
\frac{X'_{NP} - X_{NP}}{\bar{c}} &= (C'_{L\alpha t} - C_{L\alpha t})\eta \frac{V_H}{C_{L\alpha w}}\left(1 - \frac{d\varepsilon}{d\alpha}\right) = -fC_{L\alpha t}\eta \frac{V_H}{C_{L\alpha w}}\left(1 - \frac{d\varepsilon}{d\alpha}\right)
\end{aligned}$$

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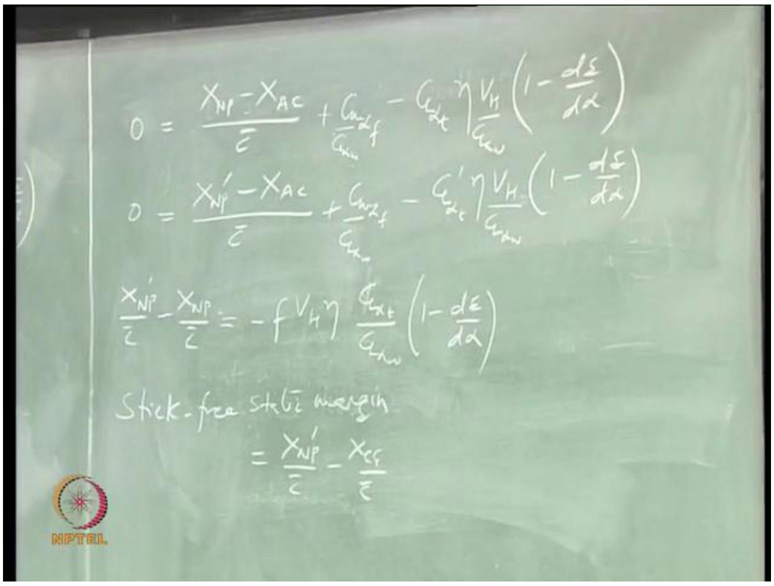

Let us try to do that. So, think you right.

One by CL alpha.

Yeah. So, XNp prime.

Student: (( ))

CL alpha t minus.

It will be minus.

It will be.

Minus f into VH eta C L alpha t (( ))

So, ... stick free static margin is...... So, I go back to the, the old question, that I asked, right.

How far ahead the CG can be, right, this question I asked sometime back, and did we also find an answer to that.

Student: (( ))

No neutral point is the most aft.

Aft (( ))

Most aft, ... now it is going to depend upon this also, if the elevator is free, then XNp prime is the most aft limit of the CG, right.

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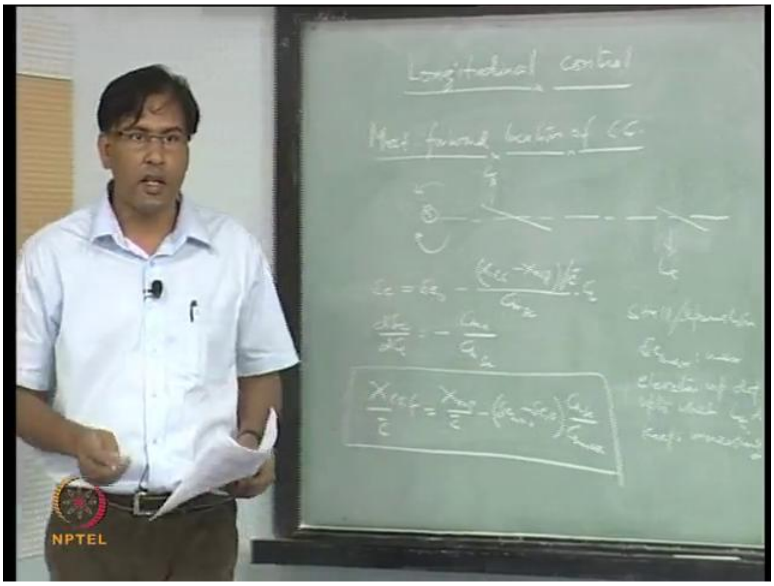

$$\delta e = \delta e_0 - \frac{(X_{CG} - X_{NP})/\bar{c}}{C_{m\delta e}} C_L$$

$$\frac{X_{CG,fwd}}{\bar{c}} = \frac{X_{NP}}{\bar{c}} - (\delta e_{\max} - \delta e_0)\frac{C_{m\delta e}}{C_{L\max}} \tag{7}$$

I want to find out the most forward location of CG. Conventional situation ok, wing camber is positive, and now we are looking at how far we can move the CG in the forward direction, not in the backward, right.

So, aft is moving towards the .... tail, forward is moving towards the leading edge, right, or the nose of the airplane. Now, I want to find out (maximum forward CG location), that will give you the maximum stability, right, CG most forward will give you the maximum stability margin, ok.

So, any answer to that? See what is happening, positive wing camber, we are trying to create a negative moment through the wing, now I have to adjust that moment through tail.

So, I start changing the elevator up, right. So, that you have force down on the elevator or the horizontal tail and its going to balance that moment which is coming because of the wing, right. So, CG definitely is now ahead of the wing, is it not? This is the only way I can maintain C_{mCG} equal to zero, right, how far ahead.

Balancing the moment from the wing through the moment from the tail.

That is for maintaining that trim, maintaining the trim, right, C_m equal to, C_{mCG} equal to zero.

So, CG ......; CG is now here, right. Now, so definitely I have to produce a lift at the tail, which is downward, and this is going to create a moment about CG like this ..., and this ... opposite. What will .. it depend upon. It will depend upon the maximum elevator deflection, right, maximum positive elevator, or elevator up deflection, which is, elevator you cannot just keep on increasing, right.

Student: (( ))

After some point there will be stall, separation of flow, and the lift will start decreasing right. So, that is the delta e max that .. you can get positive, ... and added lift or advantage of lift, right.

Once you start going beyond that delta e max, there will be stall, and separation of flow and this Lt will start decreasing, right.

So, that delta e max, upto which you keep on seeing this Lt down increasing, right. So…

Student: (( )).

Wing has to produce additional lift.

Because there is component of lift downward by which we have to balance the weight.

Yeah you have to balance the weight.

(( )) stability versus wing (( ))

So, let us try to write an expression for that. So, remember we already found this d delta e over d C L, and that was minus of C m alpha, right, over....... $\frac{d\delta e}{dC_L} = -\frac{C_{m\alpha}}{C_{m\delta e}} \cdot \frac{1}{C_{L\alpha}}$ from trim condition. $\frac{\delta e - \delta e_0}{C_L - 0} = -\frac{C_{m\alpha}}{C_{m\delta e}} \cdot \frac{1}{C_{L\alpha}} = -\frac{(X_{CG} - X_{NP})/\bar{c}}{C_{m\delta e}}$

And from this you can find out, this is going depend upon the static margin, right, C m alpha is going to give you the static margin. So, I am now writing down the, f is for the forward -forward C g location is...... CL max will be corresponding to the maximum Lt that you can get, corresponding to this delta e max, right. So, any question. So, we can stop now.

$$\delta e = \delta e_0 - \frac{(X_{CG} - X_{NP})/\bar{c}}{C_{m\delta e}} C_L$$

$$\frac{X_{CG,fwd}}{\bar{c}} = \frac{X_{NP}}{\bar{c}} - (\delta e_{\max} - \delta e_0)\frac{C_{m\delta e}}{C_{L\max}}$$