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**Module No. # 04 Longitudinal Static and Neutral Point Lecture No. # 10 Power Plant Contribution**

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So, today we are going to look at effect of power plants on longitudinal stability. We will talk about two different types of engine plants. One is Turboprop engines and the other one is Turbojet engines (No Audio from 00:50 to 1:10). So, let us look at what effects of a power plant, which is turbo prop engine, is going to have on the longitudinal stability.

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For many small aircraft you will find that the engine is this kind of engine, a turbo prop engine is at the nose, so it is in front of the wing. So, when the propeller is running, it will not only produce a thrust, it will also produce a force in the normal direction, which is Np, and the distance of this plane, which is the propeller disk plane, from the CG is lp, and the thrust-line is lying below CG at a distance h. So, you see here that the propeller is lying in the upwash region of the wing. If this is your remote wind direction, what is the effect of upwash? It is going to change the angle of attack, right, so, you have angle of attack which is increased because of the upwash at the propeller, and this is upwash angle, epsilon.

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- direct effect: because of<br>- indirect effect: because of<br>- the propeller slipstream

There are two effects which a running propeller can have on longitudinal stability: One is the direct effect, the other one is indirect effect. Direct effect is because of these forces that are being produced, right, and indirect effect is (because of the propeller stream) (No Audio from 04:38 to 04:58). So, flow is leaving this propeller and it is going to affect everything behind it, and that is going to have an effect, which is indirect effect on longitudinal stability. So, look at them one by one. So, let us talk about the direct effect first, today. So, what we want to find out? We want to find out contribution of the forces to the pitching moment.

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So, look at direct effect first. So, Mcg, the pitching moment about CG due to the propeller, running propeller, is, T into h it is positive, plus this Np into lp  $M_{cg,p} = T \times h + N_p \times l_p$ . You can write this thrust in terms of a non-dimensional coefficient, which is thrust coefficient, and it is called CT, CT into rho V square D square  $T = C_T \rho V^2 D^2$ . So, CT is  $C_T = \frac{I}{\rho V^2 D^2}$  $C_T = \frac{T}{T^2}$  $\rho$  $=\frac{1}{2}$  (No Audio from 07:04 to 07:28) and V is this, relative wind speed. Let us also define this Np. (No Audio from  $07:40$  to  $08:11$ )

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Ok, so, now, I can write everything in terms of these coefficients and we can find an expression which is this (No Audio from  $08:26$  to  $08:39$ ) into N, this N is the number of blades. (No Audio from 08:45 to 09:23)

$$
C_{mcgp} = C_T \frac{2D^2}{S_w} \cdot \frac{h}{\overline{c}} N + C_{Np} \frac{l_p}{S_w} \cdot \frac{S_p}{\overline{c}} N
$$

$$
\left(\frac{dC_m}{dC_L}\right)_p = \left(\frac{dC_T}{dC_L}\right)\frac{2D^2}{S_w}\cdot\frac{h}{\bar{c}}.N + \left(\frac{dC_{Np}}{dC_L}\right)\frac{l_p}{S_w}\cdot\frac{S_p}{\bar{c}}N
$$

Finally, I want to find out this quantity and see if it is negative or positive, right?. (No Audio from 09:37 to 10:13). You think  $C_T$  will depend upon  $C_L$ ? Yes or no?  $C_L$  is the lift coefficient of the aircraft and that will depend upon the angle of attack at the fuselage reference line, and the thrust produced is also going to depend upon this angle because you are seeing the flow at some angle, is it not? So, this *C<sup>T</sup>* is a function of propeller efficiency and the lift coefficient of the aircraft.  $C_T = K \eta_p C_L^{3/2}$ 

This *K* is a function of the power that an engine can produce, the density, because this aircraft is going to operate at different altitudes and density is going to change with altitudes and the wing loading. So, this  $K$  is a function of these three parameters (*BHP*,  $\rho$ , *W* / *S*) and you can write  $C_T$ :  $C_T = K \eta_p C_L^{3/2}$ . Remember this is only an estimate, if you want to find exact values of these quantities, then you have to put your aircraft in the wind tunnel and make measurements.

So, what is dCT over dCL?  $\frac{a_{\text{C}}}{a_{\text{C}}}=K\eta_{\text{n}}\frac{3}{2}C_l^{1/2}$ 2  $\frac{3}{p}$  $\cdot \frac{3}{2}C_L^1$ *L*  $\frac{T}{c} = K \eta_p \frac{3}{2} C$ *dC*  $\frac{dC_T}{dC} = K \eta_n \cdot \frac{3}{2} C_L^{1/2}$ . Everything here is positive-  $C_L$  has to be

positive, eta p  $\eta_p$  is positive, *K* is positive.

$$
\left(\frac{dC_m}{dC_L}\right)_p = \underbrace{\left(\frac{dC_T}{dC_L}\right)\frac{2D^2}{S_w}\cdot\frac{h}{\bar{c}}.N}_{1} + \underbrace{\left(\frac{dC_{Np}}{dC_L}\right)\frac{l_p}{S_w}\cdot\frac{S_p}{\bar{c}}.N}_{2}
$$

So, what effect this term is going to have on stability? Call this 1 and 2. '1' is destabilizing, but it will depend upon (sign of) h. If h is positive, means what? If this thrust line is lying above CG, it is going to have a stabilizing effect. For  $h > 0$  effect of '1'

is destabilizing (No Audio from  $13:07$  to  $13:33$ ). right. So, you can choose your h for getting more or less stability.

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It turns out that you can also approximate this. Now, due to thrust as this *c h dC dC L T*  $\frac{m}{\cdot}$  = 0.25  $\int$  $\setminus$  $\overline{\phantom{a}}$  $\setminus$ ſ , this for a fighter. So, you clearly understand its going to shift the, or how this h over c bar is going to shift the stability curve, is it not? Let us look at the second term '2' now. *L p p Np L Np dC d d dC dC*  $dC_{N_D}$   $dC_{N_D}$   $d\alpha$  $= \frac{d \epsilon_{Np}}{d \alpha}$ . J  $\setminus$  $\overline{\phantom{a}}$  $\setminus$ ſ . Do you think this term is going to change with CL? The angle of attack that the propeller is seeing is this, alpha p  $\alpha_p$ , and alpha p is alpha plus epsilon  $\alpha + \varepsilon$ .  $\frac{\alpha \alpha_p}{\alpha} = \frac{\alpha \alpha_p}{\alpha} \cdot \frac{1}{\alpha \alpha} = \frac{1}{\alpha} |1 + \frac{a \varepsilon}{\alpha} |$ J  $\left(1+\frac{d\varepsilon}{l}\right)$  $\setminus$  $=\frac{d\alpha_p}{d\alpha}\cdot\frac{1}{dC_t/d\alpha}=\frac{1}{a_{xx}}\left(1+\frac{d\varepsilon}{d\alpha}\right)$ ε  $\alpha$  aC, a $\alpha$  $\alpha$  a $\alpha$ *d d*  $d\alpha$   $dC_l/d\alpha$  *a d dC d*  $L \prime$  *w*  $u_w$ *p L*  $\frac{p}{p} = \frac{d\alpha_p}{1} \cdot \frac{1}{16 \cdot 10^{11}} = \frac{1}{16} \left( 1 \right)$ /  $\frac{1}{1-\epsilon} = \frac{1}{1+\epsilon} \left(1+\frac{d\epsilon}{d\epsilon}\right)$  (No Audio from 15:51  $\overline{\text{to 16:08}}$  And this is for the, I mean, we can approximate it to be that of the wing, because there is not much of lift produced on the tail as I said earlier. So this I can write as the lift curve slope of the wing into 1 plus over  $\frac{m}{\sqrt{2}} = \frac{m}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot 1 + \frac{ac}{\sqrt{2}}$ J  $\left(1+\frac{d\varepsilon}{l}\right)$  $\overline{\mathcal{L}}$  $=\frac{d\alpha_p}{d\alpha} \cdot \frac{1}{dC_t/d\alpha} = \frac{1}{a_{xx}} \left(1 + \frac{d\varepsilon}{d\alpha}\right)$ ε  $\alpha$  dC, d $\alpha$  $\alpha$   $a\alpha$ *d d*  $d\alpha$   $dC_l/d\alpha$  *a d dC d*  $L \prime$  *w*  $u_w$ *p L*  $\frac{p}{p} = \frac{d\alpha_p}{1} \cdot \frac{1}{16 \cdot 10^{11}} = \frac{1}{16} \left( 1 \right)$ /  $\frac{1}{10}$ right?

Now, each of these derivations are very important, you know, you can miss out something and your equilibrium and stability of the aircraft will be disturbed; we are trying to find out the moment also to determine correct equilibrium condition and the stability. So, dCNP over dCL is  $\left| \frac{x-y_p}{\sqrt{2}} \right| = \frac{aC_N}{l} \left| \frac{1}{\sqrt{2}} \right| + \frac{ac}{l}$ J  $\left(1+\frac{d\varepsilon}{l}\right)$  $\setminus$  $\Big)$   $\frac{1}{-}$   $\Big(1 +$ J  $\left(\frac{dC_N}{d}\right)$  $\setminus$  $=$ J  $\backslash$  $\overline{\phantom{a}}$  $\setminus$ ſ  $\alpha$ ε  $\alpha$ ,  $a_w$   $d$ *d*  $d\alpha$   $\int_{a} a$ *dC dC dC p w N L*  $\left(\frac{Np}{N}\right) = \left(\frac{dC_N}{N}\right)$ .  $\frac{1}{N}\left(1 + \frac{d\varepsilon}{N}\right)$ , and the contribution to the final dCM over dCL, to the airplane dCM over dCL, is this *N c l S S d d*  $d\alpha$ ,  $\int_{a}$  *a dC dC*  $dC_m$   $\Big\{$   $dC_N$   $\Big\}$   $1 \Big\{$   $d\varepsilon$   $\Big\}$   $S_p$   $l_p$ *w p p w N L p*  $\left(\frac{dC_N}{d}\right) = \left(\frac{dC_N}{d}\right) \cdot \frac{1}{d\theta} \left(1 + \frac{d\varepsilon}{d\theta}\right) \cdot \frac{S_p}{S} \cdot \frac{l_p}{l}$  $\bigg)$  $\left(1+\frac{d\varepsilon}{l}\right)$  $\setminus$  $\Big)$   $\frac{1}{-}$   $\Big($  1+ J  $\left(\frac{dC_N}{d}\right)$  $\setminus$  $=$ J  $\backslash$  $\overline{\phantom{a}}$  $\setminus$ ſ  $\alpha$ ε  $\left(\frac{\partial N}{\partial \alpha}\right)$ .  $\frac{1}{a_{\alpha}}\left(1+\frac{ac}{d\alpha}\right)\frac{\partial p}{\partial s}$ . You have to multiply this thing by, you know, this term here. Let us write that  $(No<sub>1</sub>Audio from 18:02 to 18:42)$ . Now, there is some information available for say, different number of blades what is this *p N d*  $\frac{dC_N}{d}$  $\big)$  $\left(\frac{dC_N}{d}\right)$  $\setminus$ ſ  $\alpha$ quantity? There are people have done lot of work on this aspect and it is a constant, it is a positive constant.

So, for example, when the, when there is no thrust being produced and you are just wind milling- you can be running your propeller, but that is not producing lift, so, that is called wind milling- and for that condition dCN over dalpha is given. So, there is a person called Ribner and the chart which gives you this is called Ribner's chart. So, there are some information available and we can directly use them only for the initial designing phase.

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When you are trying to determine stability with respect to each of these components (No Audio from 20:05 to 20:15) At T equal to 0, so, propellers are running, but thrust is not being produced, so, it is wind milling. (No Audio from  $20:35$  to  $20:50$ ). To start with, they will use this value for the initial phase and they will multiply it by a factor to account for the case when thrust is being produced by the running propellers. Some data is available and this is from  $(N_0 \text{ Audio from } 21:08 \text{ to } 22:25)$ . 0.00165 for 2 blade propellers

$$
\left(\frac{dC_N}{d\alpha}\right)_{p,T=0} = \frac{0.00235 \text{ for 3 blade propellers}}{0.00296 \text{ for } 4....} \text{ So, this is for two propellers and they are}
$$
  
0.00510 *for* 6....

rotating in the opposite direction . Now, what kind of effect this is going to have, where I am saying that this is positive constant for different number of blades, so, this is also going to be a function of blade height and blade angle, so, what effect this is going to have?

It depends upon what this  $\ln l_p$  is. If  $\ln \ln p$  is positive, you know, that is how I have drawn it here, then the effect is destabilizing, but if the CG is ahead of the propeller, then the lp is going to be negative. Do you know of any aircraft where this is the case? So, it is a pusher type of airplane, you have any idea of any such airplane? SARAS.

SARAS is one such pusher type airplane. So, where lp is negative again, it depends upon where the CG is. So, this kind of arrangement of power plants on the airplane can change the stability characteristic. And it can change the stability characteristic to a large extent, so, it clearly depends upon what lp over c bar  $(l_p/\bar{c})$  is. It can have a large effect. This is, the direct effect. Indirect effect is: so, if I want to write the complete, you know, pitching moment about the aircraft CG due to every component, then it is something like this

$$
C_{mcg} = C_{Lw} \left( \frac{X_{CG} - X_{ACw}}{\overline{c}} \right) + C_{macw} + C_{m\text{fix}} - \eta V_H C_L + C_T \frac{2D^2}{S_w} \cdot \frac{h}{\overline{c}} .N + C_{Np} \frac{l_p S_p}{S_w \overline{c}} .N
$$

(No Audio from  $25:17$  to  $26:10$ ) - the contribution coming from the wing, fuselage, tail, this direct thrust contribution, and this is the normal force on the, contribution coming from the normal force on the propeller.

So, in general the flow field behind this  $(Audio not clear from 26:37 to 26:44)$  This is some unbalanced force  $N_p$ , I cannot tell you what that force may be, but this has been observed, that this force will be there, even when you are wind milling, you know, you are not producing any thrust, this is there.  $(Audio not clear from 27:00 to 27:07)$  What is that? (Audio not clear from 27:08 to 27:18) Not really, not really, I do not think. See, look at this, flow is coming at some angle, so of course, the forces in the, you know, in the vertical direction is not going to be, it is not going to compensate for the one acting upward and one acting downward, so, some unbalanced force, and it is there, when you are (Audio not clear from 27:53 to 28:07)

This will have an effect on the lift, but not significant because lying, wing is lying behind this, so slipstream is going to have an effect on the lift coming from the wing. So, all of these effects are going to be there. But they are not so significant, they are very small contributions. The major contribution actually comes from this quantity. So, tail, the flow field that the tail is seeing is now different because of the propeller slip stream.

So, let us look at that effect and that is a major effect. So, this contribution (No Audio from 29:06 to 29:23) and all of these are small contributions. Remember, we are talking about the effect of propeller slipstream on these quantities and that is small, but this changes significantly, this is what we are going to look at under the indirect effects, this is not a direct effect.

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Effect of increased dynamic pressure a

(No Audio from 29:58 to 30:50) Propeller slipstream effect on wing lift coefficient. (No Audio from 31:07 to 31:15) Downwash at the tail is slightly modified because of the propeller slipstream (No Audio from 31:21 to 32:09). Ok so, I said these two effects are not so significant, so we will talk about the other two effects.

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This contribution is now slightly modified and this quantity which is the ratio of the dynamic pressures, that is, that is going to change and this *v<sup>s</sup>* over *V* squared, right. This is a function of the thrust coefficient. So, this quantity is  $($  No Audio from 33:12 to  $33:23$  of course, when  $C_T$  is 0, when there is no thrust produced by the propeller then this is equal to 1, right. But if there is thrust being produced then you know that this  $C_T$  is also going to be a function of *CL*.

So, let us look at this little more carefully. What is  $C_{Lt}$ ?  $C_{Lt}$  is  $C_L$  alpha tail into the angle of attack at the tail, which is (No Audio from  $34:04$  to  $34:11$ ). Now, this epsilon is from the wing  $(\varepsilon_w)$ , there will be another angle **(No Audio from 34:19 to 34:24)** another contribution to this downwash angle from the propeller slipstream  $(\varepsilon_p)$ . (No Audio from 34:29 to 34:50) Now, if I want to take the derivative of this with respect to the *CL*, that is what we are looking at, that will tell you about the stability.

$$
\left(\frac{dC_m}{dC_L}\right)_t = -\frac{a_t}{a_w} V_H \left(1 - \frac{d\varepsilon_w}{d\alpha} - \frac{d\varepsilon_p}{d\alpha}\right) \left(\frac{v_s}{V}\right)^2 - V_H C_L \frac{d(v_s/V)^2}{dC_L}
$$

(No Audio from 35:00 to 35:15) Now, we are doing all kind of maths here. By now you should have adjusted to finding the expressions, so, I am not going to write it in detail(No Audio from  $35:28$  to  $36:03$ ). So, this is the total contribution of tail to pitch stability (No Audio from  $36:10$  to  $36:45$ ).

Propeller indirect effect. So, what do you think this effect is, stabilizing or destabilizing? Now, we are talking about two terms, and clearly this is greater than 1 when the propeller is running, 1 plus 8 CT over pi  $1 + \frac{8C_T}{\pi}$ . But if I only want to look at the effect of this downwash angle (from the propeller slipstream) , what is it? so, minus into minus, plus: 2  $\frac{r}{\sqrt{2}}$ J  $\left(\frac{v_s}{11}\right)$ Y ſ *V v d d V a*  $\frac{a_t}{N_H}V_H \frac{d\varepsilon_p}{d\varepsilon} \left(\frac{v_s}{V} \right)$ *w t* α  $\frac{\varepsilon_p}{\varepsilon} \left( \frac{v_s}{v_s} \right)^2$ . So, it is going to have a destabilizing effect, if I only want to look at the effect of the downwash because of the propeller.

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\* Effect of downward inhoduced by<br>propeller slipsteam at the tail

(No Audio from 37:45 to 38:25) What else do you see there? This is one effect and this effect is destabilizing. If you combine these two together, actually, it can make an aircraft unstable, you were expecting horizontal tail to provide you stability, that is how you added the tail. But this is going to have a, if they come together, then they are going to have a, they can have destabilizing effect also, depends on the magnitude of these two quantities. What else do you see there?

$$
\left(\frac{dC_m}{dC_L}\right)_t = -\frac{a_t}{a_w}V_H \left(1 - \frac{d\varepsilon_w}{d\alpha} - \frac{d\varepsilon_p}{d\alpha}\right)\left(\frac{v_s}{V}\right)^2 - V_H C_L \frac{d(v_s/V)^2}{dC_L}
$$

(Refer Slide Time: 32:29) So, in general it is this term is 1 plus 8 CT over pi π  $\left(\frac{s}{f}\right)^2 - 1 + \frac{8C_T}{f}$ *V*  $\left(\frac{v_s}{\sigma}\right)^2 = 1 + \frac{8}{\sigma^2}$ 2  $=1+$  $\bigg)$  $\left(\frac{v_s}{v} \right)$  $\setminus$  $\left(\frac{v_s}{v_s}\right)^2 = 1 + \frac{8C_T}{v_s}$ , so, this is going to provide stability- is it not?- we are multiplying this quantity by something which is more than 1, though I said that if you put the tail behind the wing, then the tail efficiency factor will go down because it is lying in the wake of the wing, but here you see that because of the propeller slipstream this quantity has become greater than 1 even though the tail is behind the wing.

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$$
\left(\frac{dG_{n}}{d\zeta}\right) = -\frac{a_{\zeta}}{a_{\zeta}}V_{n}\left(1-\frac{d\zeta_{\omega}}{d\zeta} - \frac{d\zeta}{d\zeta}\right)\left(\frac{s}{\zeta}\right)^{2}
$$
\n
$$
\frac{dG_{n}}{d\zeta}\left(1-\frac{d\zeta_{\omega}}{d\zeta} - \frac{d\zeta_{\omega}}{d\zeta}\right)\left(\frac{s}{\zeta}\right)^{2}
$$
\n
$$
\frac{dG_{n}}{d\zeta}|_{\zeta_{n}} = -V_{n}\zeta_{\zeta_{n}}\frac{d\left(\frac{V_{n}}{V_{n}}\right)}{d\zeta}\frac{1}{d\zeta}
$$
\n
$$
\frac{dG_{n}}{d\zeta}|_{\zeta_{n}} = \frac{d\left(\frac{V_{n}}{V_{n}}\right)^{2}}{d\zeta}
$$
\n
$$
\frac{dG_{n}}{d\zeta}|_{\zeta_{n}} = \frac{d\left(\frac{V_{n}}{V_{n}}\right)^{2}}{d\zeta}
$$

So, it is actually adding to stability through this term and decreasing stability through this term. What effect this term is going to have? Now, this derivative is, again you have to take the derivative of this with respect to CL, so this is going be positive, so, K  $(No)$ 

**Audio from 40:35 to 40:43.** 
$$
\frac{3}{2} K \eta_p C_L^{(3/2)-1} = \frac{d(v_s/V)^2}{dC_L}
$$
. This is what? **No Audio from 40:44 to 40:52)** But this is when the propeller is running and we are getting some thrust,

if the propeller is not producing any thrust, then this becomes 0 and the derivative, this derivative become 0. So, there is no contribution coming from this term when there is no thrust being produced by the running propeller. What other effect you see? So, everybody understands what I am saying through this, what other effect you are going to see because of this term (Term 2)? It depends upon, also upon this, *CLt*. If *CLt* is negative, then it is going to have a destabilizing effect.

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Let us think about a situation when you are trying to move the CG. So, your CG is here and tail is lying somewhere here. I am moving the CG, now, if you move the CG in this direction, what will happen? There will be destabilizing effect, more destabilizing effect from the wing because you are moving this CG towards the tail, away from the aerodynamic center of the wing. So, this  $l_t$  is going down (decreasing), but you want to trim the aircraft. If you want to trim the aircraft, then you have to produce more lift- is it not? You have to have *L<sup>t</sup>* which is higher. If I am moving CG aft towards the tail, then this  $l_t$  is going down. Now, to trim the aircraft you should have  $L_t$ , which has to be going up- do you see that, everybody sees that? Now, how do you, how do you balance out the moment? See if you talk about only the wing contribution and the tail contribution, then it is  $L<sub>w</sub>$  into this distance, and this is a destabilizing effect, we know that.

The stabilizing effect is coming because of that lift produced at the tail, so, that is stability. What about trim? When we are moving the CG backward, you know, or towards the tail, then this arm length is going to increase, so more destabilizing effect from the wing. But to trim the aircraft, what will you do? You have to cancel out this moment, so, you have to increase the lift at the tail- is that clear? -because this arm length goes down. So, when I am increasing the, this lift at the tail to trim the aircraft then, actually I am adding to the stability through this term ('2'), so, it is compensating for the loss of stability which is coming because of the wing- so, it is actually adjusting the stability. See, what happens when you are moving CG forward, towards the wing, then, wing will have less destabilizing effect, and to trim the aircraft you will have to produce lesser lift on the tail (more arm length  $l_t$  now) and lesser lift at the tail means destabilizing effect, you know, from the tail. So, it actually compensates for the change in stability when you move the CG backward and forward, so, effect of this term ('2') is that. (No Audio from  $45:53$  to  $46:46$ )

Is this clear, any question? So, we will see, how it is going to have an effect on the neutral point location- you know, we have not yet talked about the effect of fuselage contribution on the neutral point location and you know this part. So, in the next class we will talk about the effect of turbo jet engine plant on longitudinal stability. So, we can, we stop for today.