Indian Institute of Technology Madras

NPTEL

National Programme on Technology Enhanced Learning

COMBUSTION

Lecture 29
Laminar Premixed Flames - Rigorous Analysis 1

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So today what we will do is try to actually make a mathematically rigorous analysis of the laminar pre-mixed flame problem what we have been doing so far is somewhat like a phenomenological construction of energy balance and mass balance they are looking at the preheat zone and the reaction zone separately in fact we went all the way up to even considering not only upstream diffusion of products in consequently a downstream production of reactant species concentration relative to the far upstream reactant concentration.

We also take into account the effect of non unity lowest number of the reactants in trying to find out how the reactant concentration profiles are going to change because of that but however when you are now doing the rigorous analysis.

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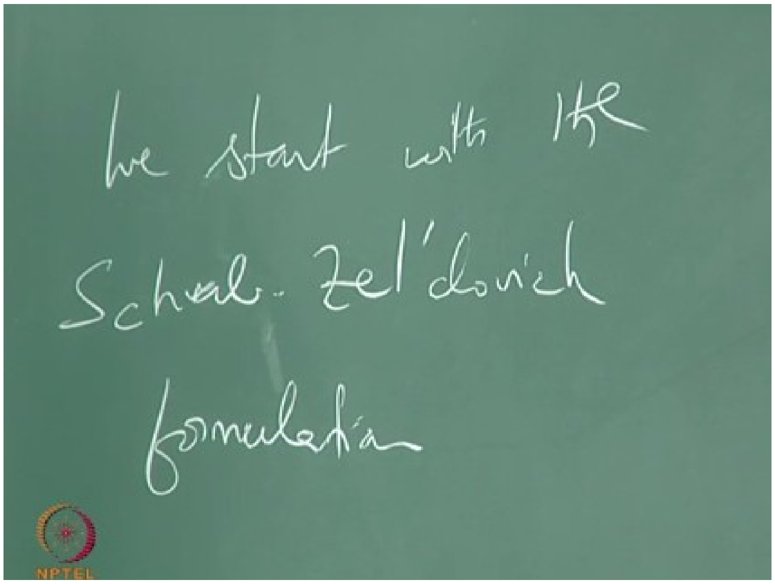

We start with the

Schvab-Zel'dovich

formulation

We start with the Schvab's Zeldovih formulation which means we have to go along with the 11 assumptions that that are attendant to the Schvab's Zeldovih formulation and then we make further assumptions hopefully only a few that is particular to this particular this problem.

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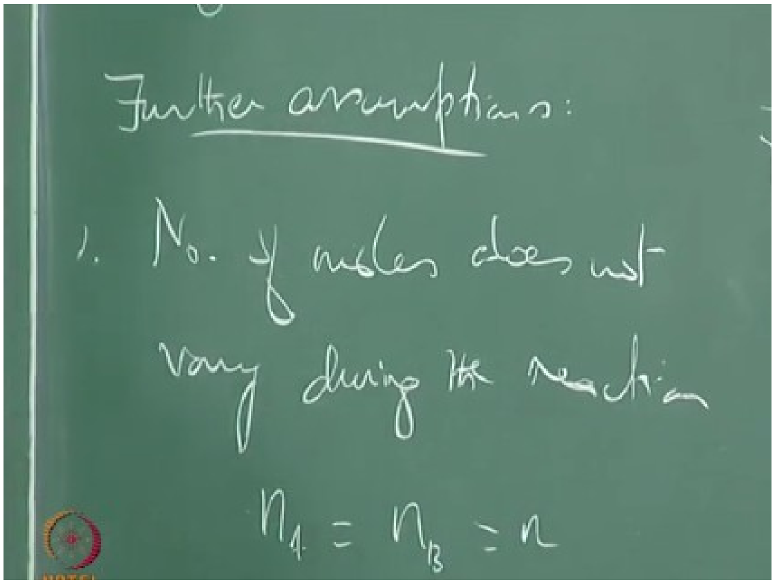

Further assumptions:

1. No. of moles does not vary during the reaction

$n_A = n_B = n$

So further assumptions the first thing that we make is say somewhat a peculiar simplifying assumption which could be relaxed number of moles does not vary does not vary during the reaction. So let us suppose that we have nA= nB = n.

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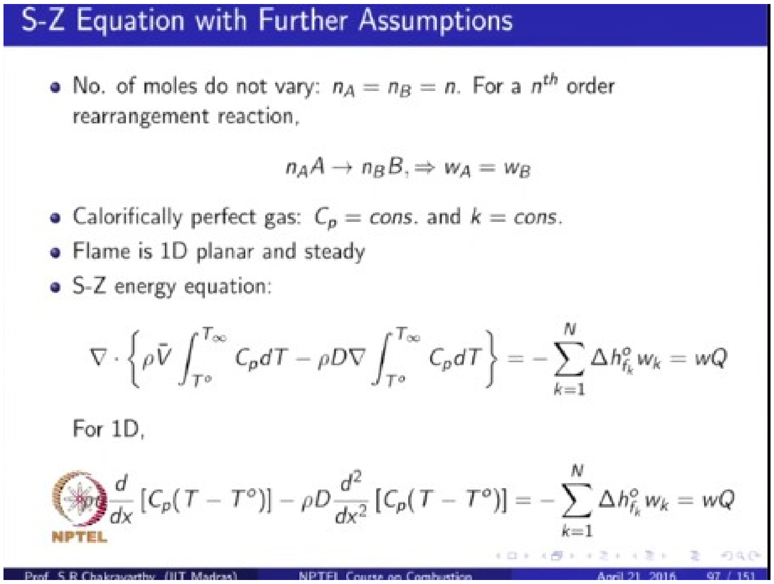

S-Z Equation with Further Assumptions

- No. of moles do not vary: $n_A = n_B = n$. For a n^{th} order rearrangement reaction,

$$n_A A \rightarrow n_B B, \Rightarrow w_A = w_B$$

- Calorifically perfect gas: $C_p = cons.$ and $k = cons.$
- Flame is 1D planar and steady
- S-Z energy equation:

$$\nabla \cdot \left\{ \rho \bar{V} \int_{T^o}^{T_\infty} C_p dT - \rho D \nabla \int_{T^o}^{T_\infty} C_p dT \right\} = -\sum_{k=1}^{N} \Delta h^o_{f_k} w_k = wQ$$

For 1D,

$$\frac{d}{dx}\left[C_p(T - T^o)\right] - \rho D \frac{d^2}{dx^2}\left[C_p(T - T^o)\right] = -\sum_{k=1}^{N} \Delta h^o_{f_k} w_k = wQ$$

And this corresponds to saying.

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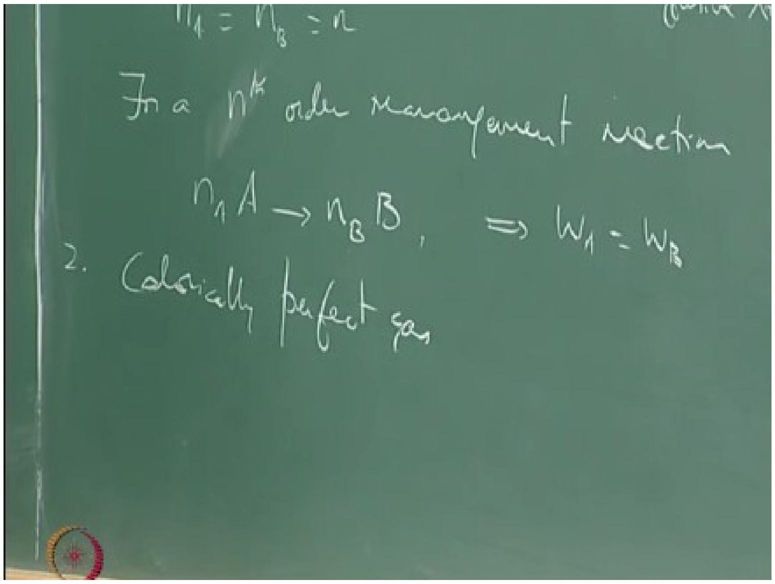

$n_A = n_B = n$

For a n^{th} order rearrangement reaction

$n_A A \rightarrow n_B B$, $\Rightarrow W_A = W_B$

2. Calorically perfect gas

So for example for a should say n for a let us say ν^{th} order or should say maybe we just go to n4 n^{th} order rearrangement reaction suggests like let us say nA = nB be when you now say nA = nB then this actually ⇒ $W_A = W_B$ so what basically happens is your mass fractions and mole fractions are you do not have to worry about a further as a further equation that relates to mass fractions in mole fractions second is a we assume calorically perfect gas now notice that

We did not make this assumption as part of the Schvab's Zeldovih formulation we could still deal with a ∫ T reference to TCPT CPCT in the case of Schvab's Zeldovih general that means you could assume a temperature varying specific heat but here we make a calorically perfect gas assumption that is CP =constant also essentially we are saying all thermal properties are constant k is a constant.

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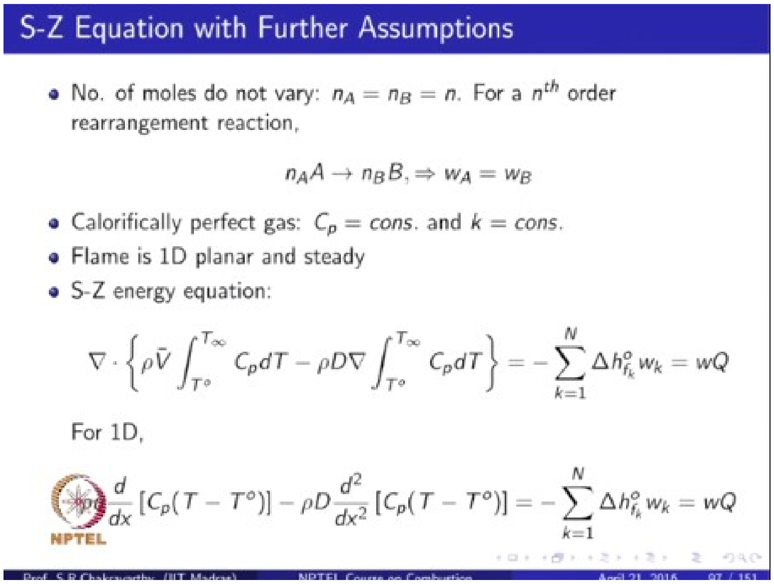

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So let us assume that the thermal conductivity is a constant as well.

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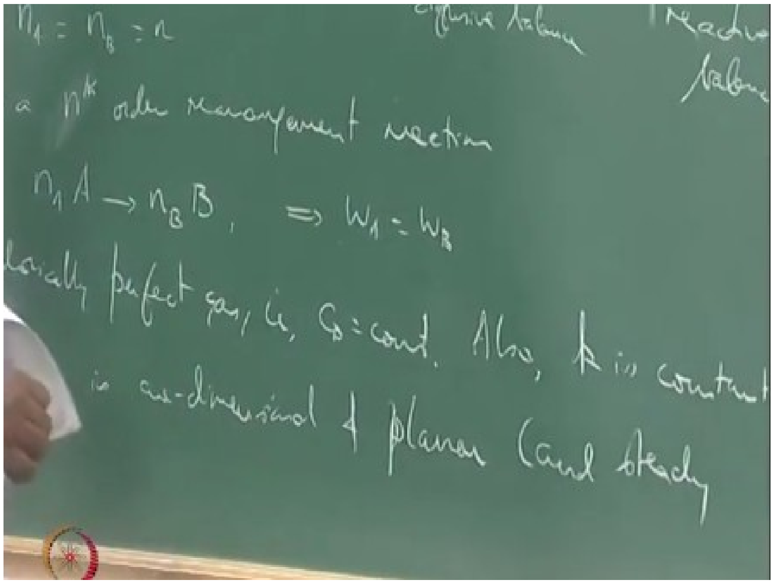

$n_A = n_B = n$ diffusive balance Reactive balance

a n^{th} order unimolecular reaction

$n_A A \rightarrow n_B B$, $\Rightarrow W_A = W_B$

locally perfect gas, i.e., c_p = const. Also, k is constant

is one-dimensional & planar (and steady

Number three frame is one-dimensional and planar and steady of course steady state is assumed in the Schvab's Zeldovih but just we want to point out here that we are looking at a steady flame which is one-dimensional and planar that is assumption which is not very unrealistic if you think about for example if you had a slotted Bunsen burner you could actually have a conical flame and locally along the side of the cone or since a conical like a tent shape flame and locally along the side of the tent like the shoulder of the flame is reasonably planar course it does not look one-dimensional but we will notice later on that we should be able to actually take a component of the flow perpendicular to the flame.

And then treat the flow field across the flame locally like the way we are doing here, so that this is this a very reason able assumption the other ones are obviously simplifying assumptions so let us now proceed with how we would do the Schvab's Zeldovih.

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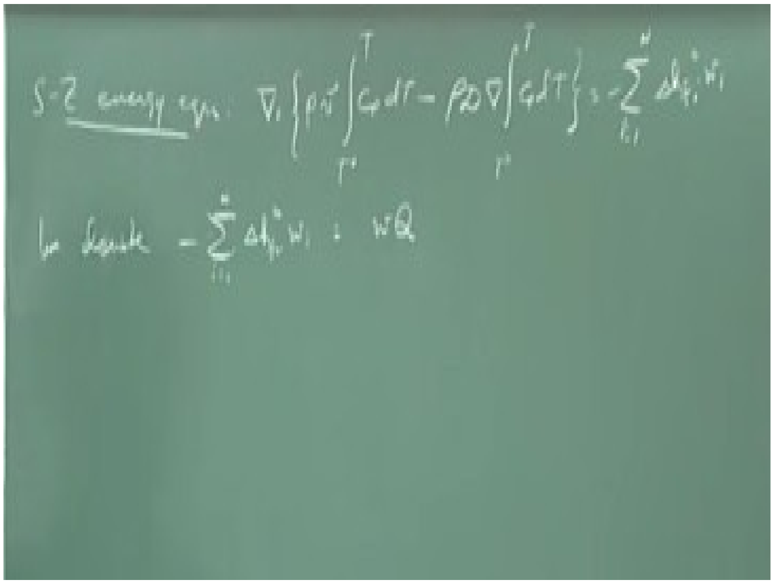

S-Z energy eqn. $\nabla\cdot\left\{\rho\vec{v}\int_{T^0}^{T} c_p\,dT - \rho D\nabla\int_{T^0}^{T} c_p\,dT\right\} = -\sum_{i=1}^{N}\Delta h_{f_i}^{0}\,w_i$

We denote $-\sum_{i=1}^{N}\Delta h_{f_i}^{0}\,w_i = w_Q$

So the Schvab's Zeldovih energy equation goes like take a divergence of ρV vector ∫T superscript not T CP DT - PD ∇ ∫T superscript 0T to CP DT = - Σ = 1 to n Δ HFi0 Wi is what we had we denote negative Σi = 1 to n Δ HFi0 Wi as let us say WQ all right.

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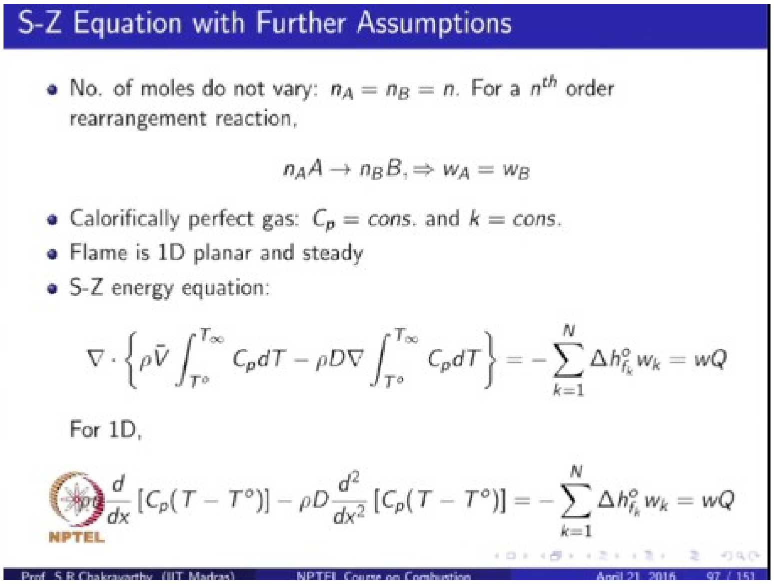

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For 1D,

$$m \frac{d}{dx}\left[C_p(T - T^o)\right] - \rho D \frac{d^2}{dx^2}\left[C_p(T - T^o)\right] = -\sum_{k=1}^{N} \Delta h_{f_k}^o w_k = wQ$$

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Now that is kind of like a Σ =like reaction rate times the heat of reaction of course we are assuming single step reaction is part of the Schvab's Zeldovih formulation of course this was not the final energy equation that we derived the Schvab's Zeldovih this is what we had actually termed as the Schvab's Zeldovih energy equation while we were doing the formulation so it is basically taken right from there we have nt form the coupling terms or the α and all those things at this stage. But let us suppose that we can just proceed from here and then apply the one dimensional apply this to one-dimensional situation.

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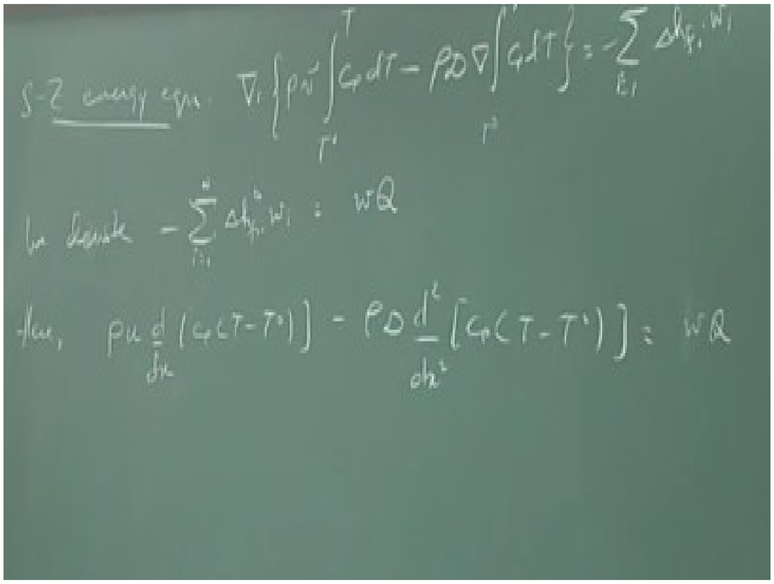

S-Z energy eqn. $\nabla \cdot \left\{ \rho \vec{v} \int_{T^0}^{T} c_p \, dT - \rho D \nabla \int_{T^0}^{T} c_p \, dT \right\} = -\sum_{i=1}^{N} \Delta h_{f,i}^{0} w_i$

We denote $-\sum_{i=1}^{N} \Delta h_{f,i}^{0} w_i = wQ$

Thus, $\rho u \frac{d}{dx} (c_p (T - T^0)) - \rho D \frac{d^2}{dx^2} [c_p (T - T^0)] = wQ$

Then here we can now write this as ρu d/ dx of $C_P C_T$ – T_0) ρd d ^{2}/ dx ^{2} CP T- T_0 = WQ okay.

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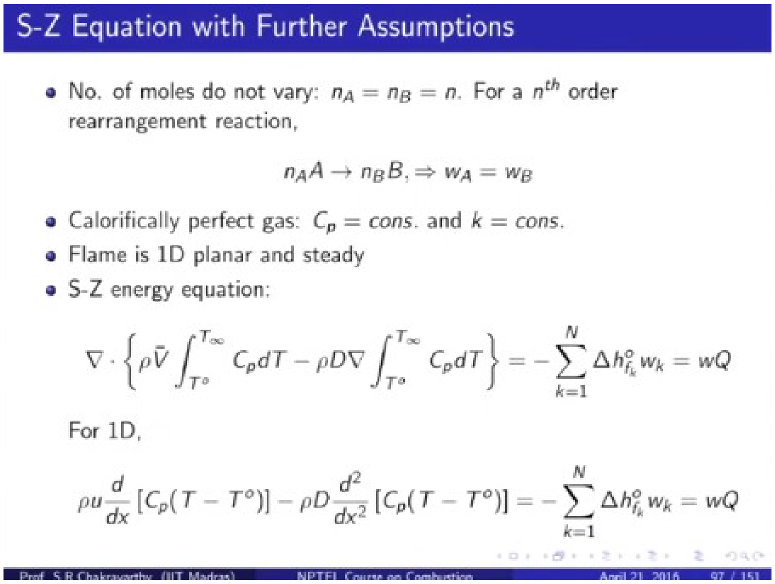

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For 1D,

$$\rho u \frac{d}{dx} [C_p (T - T^o)] - \rho D \frac{d^2}{dx^2} [C_p (T - T^o)] = -\sum_{k=1}^{N} \Delta h_{f_k}^o w_k = wQ$$

Essentially the divergence has become like a d / dx and for a one dimensional flow we know that ρ u is a constant therefore we can actually pull this out because ρ u is a constant together as a product we can pull it out of the derivative and then we evaluate the ∫ for a constant CP and we also do the same thing over here and since it is one-dimensional we can write it as a ordinary derivative instead of a partial derivative because things are varying only along X which is in this direction across the flame okay.

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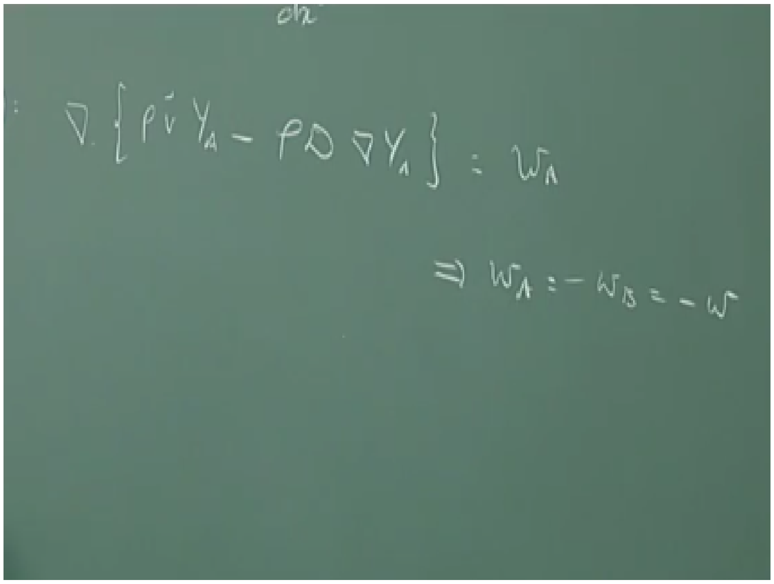

$$\nabla \cdot \left\{ \rho \bar{v} Y_A - \rho D \nabla Y_A \right\} = w_A$$

$$\Rightarrow w_A = -w_B = -w$$

Then we write the Schvab's Zeldovih which species equation for species A let us say let us say that is the reactant there and so we do we now write for the species now what you will find when you now try to apply the Schvab's Zeldovih formulation is we do not necessarily solve the entire n + 1 equations that we talked about where n is the total number of species involved okay although it would look like they were all coupled and we have to solve them simultaneously and so on.

What we are actually looking for is four equations to look identical, so if equations begin to look identical then it is sufficient for us to solve one of the equations that are that is part of the identical set instead of having to solve for all the equations simultaneously so this is how we simplify our lives instead of actually burdening ourselves with lots of equations to solve so here in the case of the prim explain we will see that for the moment it is actually sufficient for us to take the species A conservation alone.

We do not have to worry about lots of other species so here you have a divergence of $\rho V\ Y_A$ - PD ∇ YA= WA. Now since you have same number of moles everywhere we can we can point out that WA = - WB= - W right so WB is rate of production of B WA rate of depletion of A and so rate of depletion of a should have a negative sign to denote the depletion if you are now writing it in terms of the general reaction rate W on a mass basis. So here if you now try to unwrap the divergence and vector notation and so on fortunately we do not have an ∫ to evaluate in this case as in the energy.

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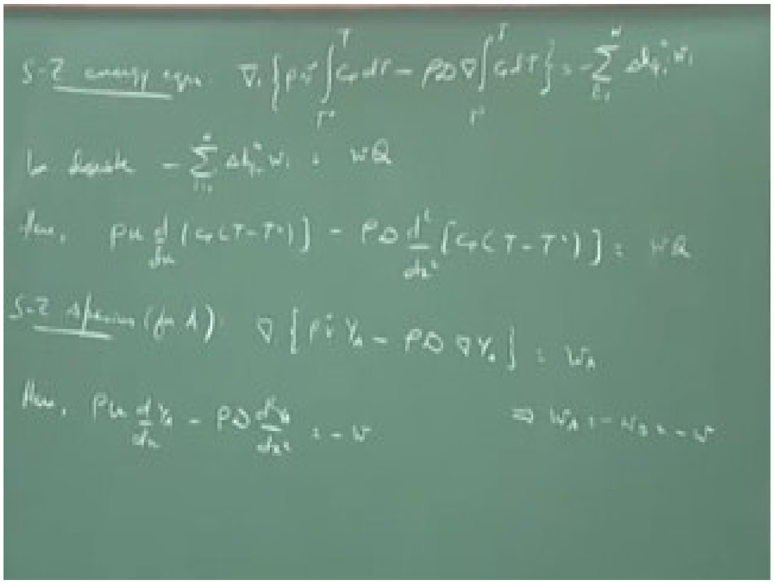

S-Z energy eqn. $\nabla\cdot\left\{\rho\vec{v}\int_{T^0}^{T} c_p\,dT - \rho D\nabla\int_{T^0}^{T} c_p\,dT\right\} = -\sum_{i=1}^{N}\Delta h_{f,i}^{0} w_i$

1-step $-\sum_{i=1}^{N}\Delta h_{f,i}^{0} w_i = wQ$

Here, $\rho u\frac{d}{dx}\left[c_p(T-T^0)\right] - \rho D\frac{d^2}{dx^2}\left[c_p(T-T^0)\right] = wQ$

S-Z species (for A) $\nabla\cdot\left[\rho\vec{v}Y_A - \rho D\nabla Y_A\right] = w_A$

Here, $\rho u\frac{dY_A}{dx} - \rho D\frac{d^2Y_A}{dx^2} = -w$ $\Rightarrow w_A = -w_B = -w$

So it should simplify as d/ dx of YA - ρD d ^{2}YA/ dx ^{2} = - W as a matter of fact I should point out that we are also assuming the ρD to be constant by the way.

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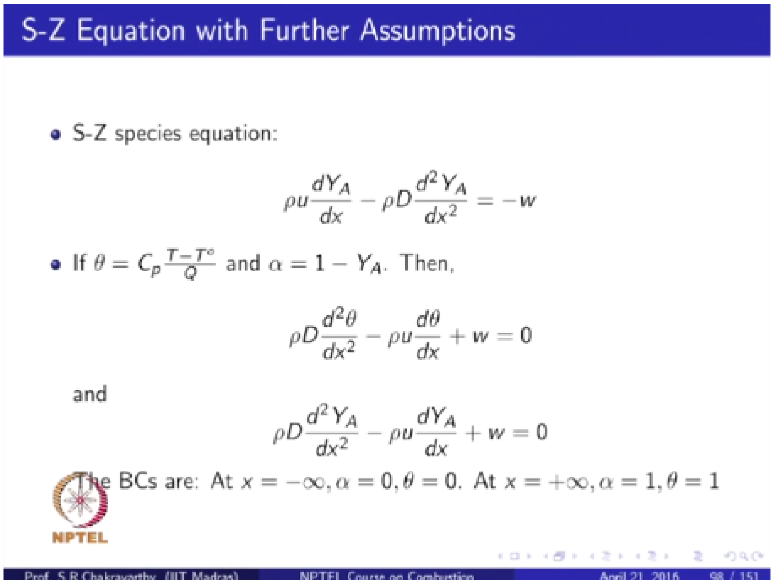

S-Z Equation with Further Assumptions

- S-Z species equation:

$$\rho u \frac{dY_A}{dx} - \rho D \frac{d^2 Y_A}{dx^2} = -w$$

- If $\theta = C_p \frac{T - T^o}{Q}$ and $\alpha = 1 - Y_A$. Then,

$$\rho D \frac{d^2 \theta}{dx^2} - \rho u \frac{d\theta}{dx} + w = 0$$

and

$$\rho D \frac{d^2 Y_A}{dx^2} - \rho u \frac{dY_A}{dx} + w = 0$$

- The BCs are: At $x = -\infty, \alpha = 0, \theta = 0$. At $x = +\infty, \alpha = 1, \theta = 1$

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That that is that is not that sort of implicitly stated here because we are assuming d to be a constant and this is like an incompressible flow where the mixture density does not change so therefore and then of course we are not even bothering about the mixture density changing the temperature all right therefore we would actually suggest that ρD is a constant and you can go through with the divergence all the way to the applying the ∇ and therefore you get the second derivative here all right.

Now we want to actually try to see if these two equations or looking alike in some way and this is what we also try to do when we did the Schvab's Zeldovih which formulation any way right.

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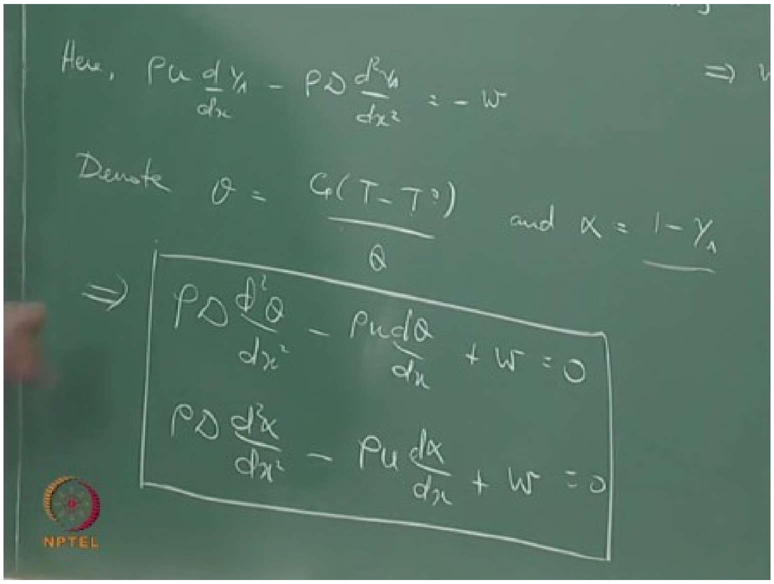

Here, $\rho u \frac{dY_A}{dx} - \rho D \frac{d^2 Y_A}{dx^2} = -w$ $\Rightarrow$

Denote $\theta = \frac{C_p(T-T^0)}{Q}$ and $\alpha = \underline{1-Y_A}$

$$\Rightarrow \boxed{\begin{array}{l} \rho D \frac{d^2\theta}{dx^2} - \rho u \frac{d\theta}{dx} + w = 0 \\ \rho D \frac{d^2\alpha}{dx^2} - \rho u \frac{d\alpha}{dx} + w = 0 \end{array}}$$

So here if you denote θ = CP T - T_0 / Q and α = 1- YA this is pretty interesting actually because for this still is kind of obvious we want to get the Q down here so we want to get the Q down here okay and then you have essentially T is the one that is varying CP is a constant T0 is constant but you have this T with all this paraphernalia like CP times T - T_0 together and then okay.

So you want now see that is like the variable now instead of the actual variable temperature and then we get this Q down here so that it will also have a W just there on the right-hand side just as well as a species equation but the species equation has a negative sign with for the W therefore if you want to have only a W there are all you need to do is to actually multiply this by a negative sign on the on the left-hand side that means we should have actually been saying α = - YA not 1 – YA.

So why are we looking at 1 - YA this is a trick okay that that is not very highlighted in textbooks and so on what we are basically looking for is something physical look at how we actually preceded in the phenomenological development of the structured of the pre-mixed flame that was the key we notice that the temperature profile goes like this and then we started asking questions about how the product profile look like right.

And then we came to the conclusion that if they put if most of the products are actually produced here we should expect anticipate that the product concentration should actually grow only we were here but once the products are actually formed they kind of look around and see hey I am actually a lot over here and I am getting conducted down there so I am also there the only place that I am not here not around this here therefore can I diffuse but I am getting conduct down.

So there is like a balance of how much up stream diffusion the products can do versus the downstream convection that they experience and that would actually give rise to a concentration profile that becomes pretty much identical to the temperature profile in the in the event of lowers number =1 which is assumed in the Schvab's Zeldovih formulation right.

So then we realize that the products p the product species also goes through a convective diffusive balance in what we thought was the preheat zone for only the thermal balance earlier and so we anticipate that the product concentration should look pretty much like the temperature concentrations if you normalized in certain ways right.

So how would you normalize like this looks like a normalized temperature in some sense not exactly T to the superscript not which is a standard temperature we should have actually put T to the subscript not for the initial temperature then this would have looked like a normalized temperature and I will come to that pretty soon.

But this one instead of just saying α is = - YA which would have done the job for us here to actually make this look similar to that with T replaced by θ through this transformation we are actually saying 1 -YA because α now begins to actually act like the product species constant concentration right.

So it is essentially saying whatever so if I had only to react to two species let us say A and B and then you can clearly see that YB would be like 1 - YA right if you had multiple species will not it finally you are going to actually have something that the order which your subtracting YA so 1 - YA will do the trick for you to actually have α go move like the product C so that say that is essentially the idea here.

So we are actually looking for the equations to look similar by choosing a if you want to call it a transformation by choosing a transformation for the species reactant species concentration that will mimic the product variation because that is how we actually develop the film and logical understanding on how off-screen product diffusion causes a downstream reactant depletion this route through diffusion,

So yes so that that say that the effect of that is going to be felt in the flow okay so what will happen is if you if you really have a temperature so the density that depends on temperature then your mass balance is going to get affected so your when you say that ρu = a constant of the ρ changes with temperature your U is going to accelerate right there is something that we noticed in the Rankin hugoniot formulation that across the deflagration the flow accelerates right.

So 1 / ρu increases ρ decreases and therefore T should increase anyway and then you will also increase all that stuff is correct but in the in the Schvab's Zeldovih formulation we are actually adopting a given flow field and the given flow field here is a uniform flow field that means no matter what they constitute constituents of the mixture or what is the composition of the mixture is as we go from the reactants to the products through the flame the mixture as a whole is having only the same velocity.

We are ignoring the fact that the flow is going to accelerate it is as if like we are not interested in it we are not worried about it okay for correspondingly then we can actually go with a constant density situation or we do not explicitly state that we say ρD is a constant and then I start thinking about what should D vary with ρ in a way that ρD is constant and so on that is not exactly the way it should it is justified okay.

So the way it is justified as we are basically thinking about like an incompressible mixture and we are violating the gas law or the perfect gas law but that violation is more felt downstream of the flame when our business is over so we do not do not worry about it okay so this is something that I pointed out that right at the beginning when we are doing the Schvab's Zeldovih or when we actually did the one-dimensional momentum to show that it will it will reduce to something like pressure is approximately = a constant.

The point there being if you want to take into account the velocity variation in the flow then only then you have to worry about the density variation with respect to temperature and then the flow problem on the combustion problem will get coupled here we are actually looking at a combustion problem that is decoupled from the flow problem for which we have to make this assumption okay.

So if you now go back and plug θ and α in the energy and the species equations respectively you now get equations that look like $\rho D d^2 \theta / dx^2 - \rho u d \theta / dx + W = 0$ and $\rho D d^2 \alpha / dx^2 - \rho u D\alpha / dx + W = 0$ hey those two equations look exactly the same except that one of them has a θ the other one has an α to solve for right now if you are Greek you might have a preference of θ / α but I am not so I am not going to really worry about which equation I am going to solve is that okay is it okay to say now it is sufficient for us to solve only one equation.

I do not have solved both the equations or what is the catch would θ and α look exactly the same as you solve for one and then expect the other one to behave exactly the same is it okay fine whenever you are confronted with two differential equations that look identical except for the D the symbol that is used for the independent variable also the dependent variable I am sorry or as a matter of okay.

So all the two equations go and actually give the same result or what are we what are we missing there is one more thing that we have to worry about all differential equations have the burden of having to satisfy boundary conditions okay, so at this stage we can say that θ is going to be related to α if it is it and it is sufficient for me to solve one equation and get and then satisfy it is boundary conditions and obtain one of them let us say θ then I should be able to say that θ is = $A\alpha + B$ all right.

And why would I say Aα + B because I am expecting to have two boundary conditions for α as well with which I can actually evaluate A and B instead of having to freshly solve this equation for α and then apply its boundary condition so that would actually give me room for applying boundary conditions for α or before we proceed in that direction where we are probably better off looking at what are the boundary conditions for θ and α and see if they are also identical right.

If they are identical then what do we say right so let us see and we and it is not terribly unrealistic to expect them to have nearly identical boundary conditions because as I said this is mimicking the product and in the in the end in the case of low unity lowest number we expect that these profiles should be the same right if these things have been normalized right all right so a whole your breath okay hold on to your seats we are going to have some fun here right.

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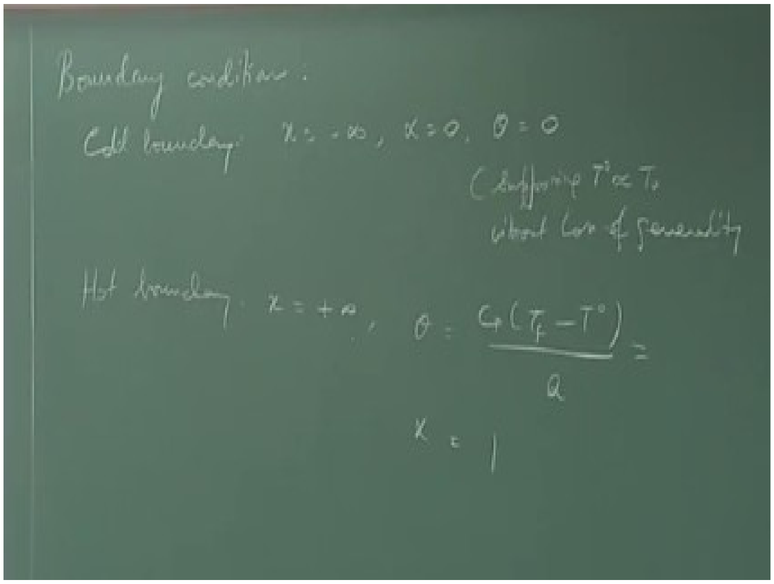

Boundary conditions:

Cold boundary: $x = -\infty$, $\alpha = 0$, $\theta = 0$

(supposing $T^{\circ} \propto T_0$

without loss of generality

Hot boundary: $x = +\infty$, $\theta = \frac{c_p (T_f - T^{\circ})}{Q} =$

$\alpha = 1$

So let us look at the boundary conditions whole boundary that x = - ∞ α = 0 θ = 0 assuming supposing T superscript T_0 without loss of generality because I could have actually written this as T- T subscript 0 – of sorry T - subscript T subscript 0+ T subscript 0 – two superscript 0 so I could have I could have added and subtracted my actual temperature initially that is essentially saying I have a sensible enthalpy initially above the reference value for the for the reactants that is now going to be added up over here which is a constant.

And therefore it is not going to really affect me at all so I could actually say that effectively T is whatever it is and then I and then say $\theta = 0$ and α is also=0 because I am assuming that I am going to have all of them as reactants upstream and then α Y should be =1 and the hot boundary $x = + \infty$ right now θ will now you give you CP TF- T_0 / Q and α should be =YA goes to 0 all the reactants are getting consumed in the flame and therefore you are going to αB =1 but what is θ.

You get this funny expression there can we evaluate it that=1 okay that is because of the adiabatic flame temperature in the case of an adiabatic flame temperature this is the heat that is released in the chemical reaction and all the heat there is released in the chemical reaction goes to increasing the sensible enthalpy of the mixture that is what an adiabatic reaction the flame is all about okay.

So CP TF - T_0 is a sensible enthalpy rice of the mixture with it which is primarily coming from Q and therefore this is also =1 lo and behold the boundary conditions are also identical wow we meet such a big fuss about you know 4, 5 n + 6 reactions in the combustion problem and then we you know belabor a lot to get the Schvab's Zeldovih formulation going and we still had to start get stuck with n + 1 reactions are n of which were fortunately homogeneous.

And all those kinds of things and finally we are able to actually get to two reactions that we just pick out of them and then we find that both of them are identical and the boundary conditions they are also identical therefore it is sufficient for us to solve only one equation great progress right.

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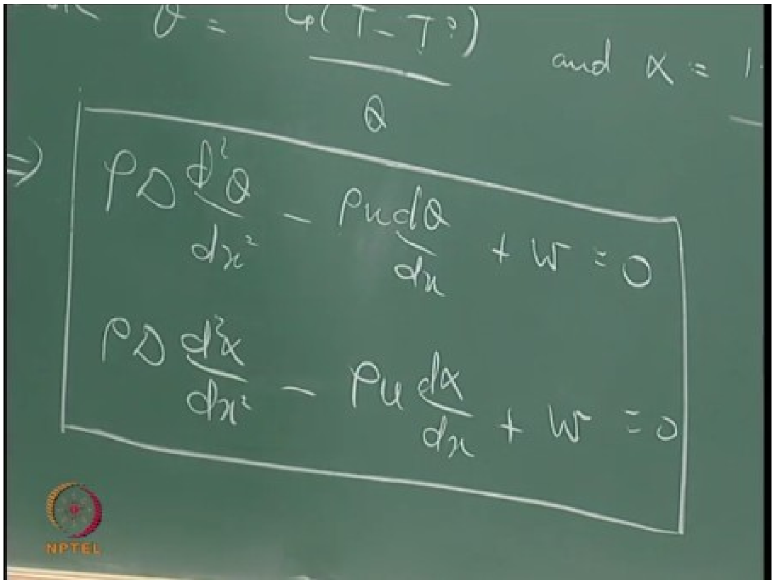

$$\theta = \frac{c_p(T - T^0)}{Q} \quad \text{and } \alpha = 1$$

$$\Rightarrow \boxed{\begin{aligned} \rho D \frac{d^2\theta}{dx^2} - \rho u \frac{d\theta}{dx} + w &= 0 \\ \rho D \frac{d^2\alpha}{dx^2} - \rho u \frac{d\alpha}{dx} + w &= 0 \end{aligned}}$$

So therefore considering one equation is sufficient if you are still persist and say well θ should be = $A\alpha + B$ and you can substitute the boundary conditions for α and then plug in values of θ and α there you will find that $A = 1$ and $B = 0$ all right, so and then you will be able to show that θ = so that is mathematically showing things okay.

So what we want to do now is get confronted by that one and only equation let us say we take the one that is written first right and then we want to deal with this we want to see how to solve this right for the first time in our lives as combustion students we are going to solve something so far we have been only posing problems okay oh my god how do we solve this what is the problem that is a second order ordinary differential equation with constant coefficients as a matter of fact okay.

ρD is a constant ρu is a constant okay and then you have a it says it is in homogeneous equation all right okay what is the in homogeneity you have the W that is the chemical reaction rate and you have the θ through the temperature sitting on top of an exponent with a negative sign but sitting in the denominator on top of an exponential is negative sign so that is the ugly looking expression that is been camouflaged by a simple W there you see and that is been the one that we have been really feeling about all these years and there we are we have to solve.

So how do you do this check out of the problem say well can I look at the flow where you do not have a W yeah sure of course fear you say you get into the flame you know you know with a lot of trepidation you get into the flame and then still do w over sure fine this looks great it is like good get in there and then no W at all for quite some time so I can solve that can I so what we would do is we now begin to notice the wavy.

They are talking about things earlier that you have a flame with a preheat zone with hardly any W where I could actually go ahead and jolly well solve this and then I have to look at this region where I have to confront the W and then point out that you pretty much have nearly the same enthalpy for the incoming species and outgoing species so you do not really have a convective enthalpy flux to worry about and therefore I am going to get rid of this right and I am going to look at a reactive diffusive balance and then try to integrate that right.

Why I mean we still have to solve check this w into account ultimately so what is a big deal the answer is what is it that we are actually looking for out of this equation okay can we now find out what exactly our problem is what is what is that one single quantity that we are trying to solve for do we know what the problem is you know lives we do not know the flow and speed okay.

So although this is like a very innocent-looking constant that is been pulled out we do not know what the U is right, so what this actually allows us to do is we can say while I am actually in the convective diffusive balance I keep the U which is what I do not know right and then it turns out to be like an Eigen value problem, so an Eigen value problem is where you have one of the coefficients that is an unknown and you supply the boundary conditions and then you turnout that you the boundary conditions are not sufficient enough for you to actually evaluate the Eigen value right.

So in this case what you do is you retain this in one of the regions and you get to the fit in the other region where you have to confront the W so you are kind of breaking your problem into two power divided and rule right this is my problem this was my unknown keep the unknown get rid of the problem in one and keep the problematic thing and get it if the unknown and the other I should be able to handle this but what is a catch I solve for the this is going to have this is going to still need two boundary conditions because it is second order right.

And my zone is only from here to here right and strictly speaking I do not know exactly where to draw this line all right it is sort of like as I keep on entering into this region and then the chemical reactions are happening somewhere I suddenly feel tube too hot and then say I think I have got into the reaction zone it is a feel right I do not know exactly where it is and I need a boundary condition there well I do not even know where the boundaries okay.

And look at this part right if you do not want to keep this and that I still am not throwing out the leading order term which means I need to have it is still second order equation and I need two boundary conditions and the boundaries are this fuzzy boundary center here and the far down stream boundary okay but all we have stated so far as I knew what the boundary condition is here I knew what the boundary condition is there.

So what do we have to do here we have to now come up with two boundary conditions because it is second order which are shared by these two so that is what is called as interface conditions so that is an interface between two zones we have to supply two conditions that are shared by these two okay and whenever we are looking for two conditions it is because it is second order and if it is second order then we are looking for two conditions one in the value in the other word in the first derivative.

And in fluid mechanics the first derivative mostly has some significance in the sense the first derivatives going to actually deal oh the heat flux the value itself is going to about is going to be on the temperature okay so we will go through these steps rather quickly in mathematical way.

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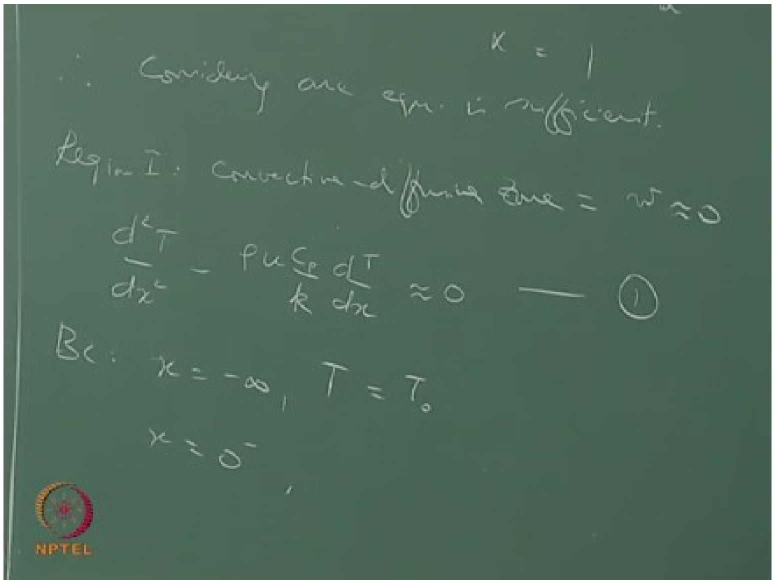

$x = 1$

∴ Considering one eqn. is sufficient.

Region I: convective-diffusive zone ⇒ $\dot{w} \approx 0$

$$\frac{d^2T}{dx^2} - \frac{\rho u C_p}{k}\frac{dT}{dx} \approx 0 \quad \text{———} \quad ①$$

BC: $x = -\infty, \; T = T_0$

$x = 0^-,$

So region one is our convective diffusive balance right convective diffusive zone which implies that W is approximately = 0 and then we can write d^2 we can go back to T do not worry about θ anymore that was just to show that θ and α are the same you can go back to an equation that is over there so d^2 T / dx^2d - ρuCp /k dt/ dx is approximately = 0 now when you say this is approximately = 0 this equation is what is called as the 0th order equation.

So what we are actually doing is a part of a larger approach called asymptotic matching okay or what is called as matched asymptotic expansions strictly speaking what we should say is the temperature profile can be expressed as an asymptotic expansion alright and we now try to take the what is called as a 0thorder wear whatever is actually a small quantity in the equation is set identically = 0 it is not exactly 0 right.

You do have the temperature rise and you are strictly speaking have a reaction rate that is beginning to increase fortunately for us if the E is large it is going to get confined to a smaller region okay so that is what we are really counting on but it is approximate so to do leading order the 0th order we now set it identically = 0 get rid of it from the equation in the first order we have to refine the equation with keeping that small value you see we would not do that fortunately okay.

We would not do that we will just do only the 0th order matching so let us let us call this equation 1 and the boundary conditions here as we talked about we want to retain the cold boundary conditions which is T = T $_0$ and x= 0 - so we are now locating our x = 0 at the edge of the preheat zone rare reaction zone match.

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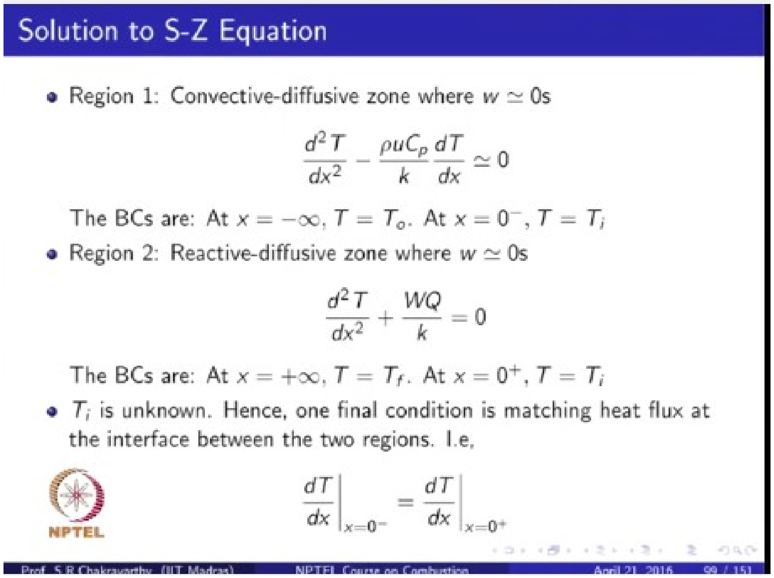

Solution to S-Z Equation

- Region 1: Convective-diffusive zone where $w \simeq 0$s

$$\frac{d^2T}{dx^2} - \frac{\rho u C_p}{k}\frac{dT}{dx} \simeq 0$$

The BCs are: At $x = -\infty$, $T = T_o$. At $x = 0^-$, $T = T_i$

- Region 2: Reactive-diffusive zone where $w \simeq 0$s

$$\frac{d^2T}{dx^2} + \frac{WQ}{k} = 0$$

The BCs are: At $x = +\infty$, $T = T_f$. At $x = 0^+$, $T = T_i$

- T_i is unknown. Hence, one final condition is matching heat flux at the interface between the two regions. I.e,

$$\left.\frac{dT}{dx}\right|_{x=0^-} = \left.\frac{dT}{dx}\right|_{x=0^+}$$

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And we want to call this x =0 - T = Ti.

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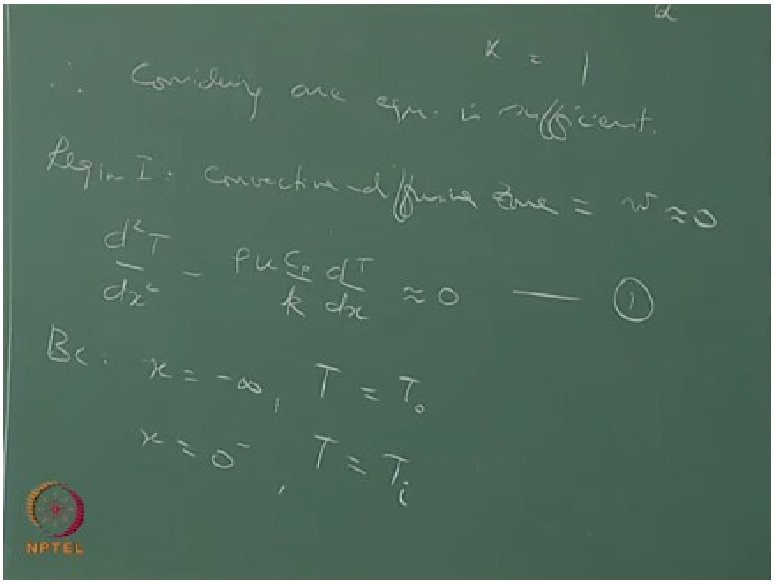

$\kappa = 1$

∴ Considering one eqn. is sufficient.

Region I: Convection-diffusion zone = $\dot{w} \approx 0$

$$\frac{d^2T}{dx^2} - \frac{\rho u c_p}{k}\frac{dT}{dx} \approx 0 \quad \text{——} \quad ①$$

BC: $x = -\infty, \; T = T_0$

$x = 0^-, \; T = T_i$

Many times we attempted to think that i stands for ignition temperature okay and then we were always looking for what is called as an ignition temperature that is like a popular notion and it is a popular engineering notion to think about like an ignition temperature mathematically speaking we have to confront a cold boundary difficulty that you have reactions happening even there it is just then it is just simply not significant enough as far as here Ti is unknown actually it is not like we are going to plug in a value there that is about this is somewhere in between the room temperature and flame temperature that is not what you are going to do.

It is an unknown it is a it is a temperature that is going to be shared as the upstream boundary condition for the reaction zone all right and it is it is an unknown.

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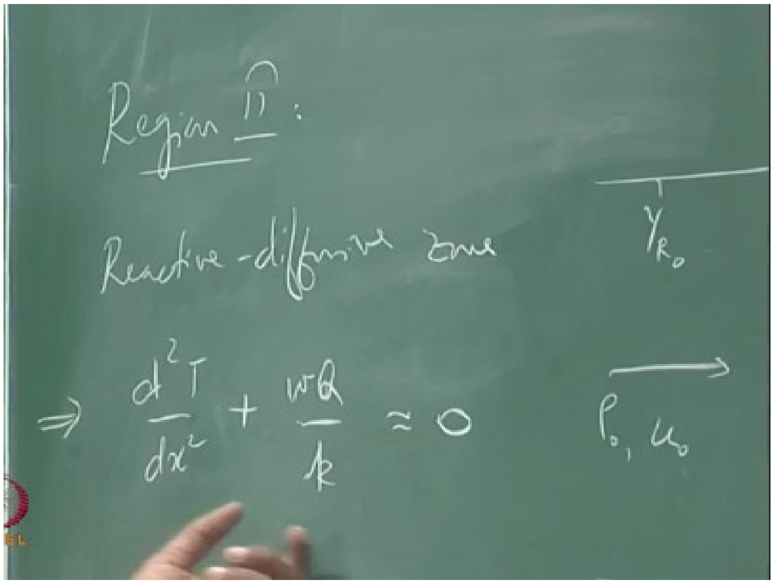

So that is region 1 for you region 2 is a reactive diffusive zone here what we then say for the 0th order reaction is $d^2 T/dx^2$ + WQ / k is approximately = 0 means this is the diffusive part this is the reactive part the balance is essentially that you hardly have any convective flux that is going on that is changing so you do not have a essentially what that really means is you do not have a DT / dx significantly okay small quantity.

So the boundary condition here we can easily write the hot boundary condition which is $X = \infty$ you can say $T = T_F$ that is like what we had started with but then the interface condition is x = 0 + we have to write the same $T = T_i$ which is an unknown right now if you are really bit a little bit more careful you would probably want to write this equation for the region one as $d\,^2T$ 1/ $dx^{2\,-}$ - ρu CP / KDT1 / DX and then write these temp these are T1 =T_0 andT1 =Ti you can say here you can plug in a T2 and then T2, T2 and then finally say the interface T1 should be = T2.

And so on okay but we understand that ultimately it is going to be a single temperature profiles of you're interested in and then we said that we wanted to have one more boundary condition at the interface that needs to be shared and since it is a second-order equation we can go up to the first order the first derivative is signifying heat flux and therefore what this really means is matching heat flux heat flux at the interface between the two regions.

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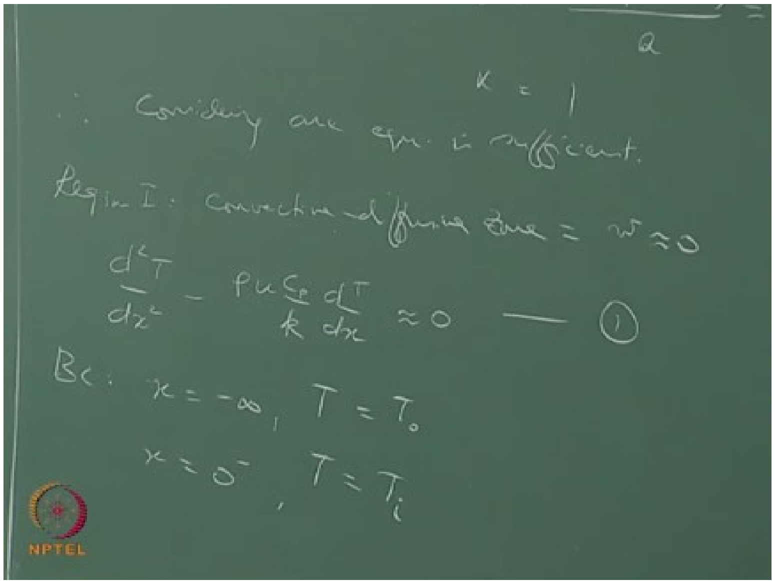

a

K = 1

∴ Considering one eqn. is sufficient.

Region I : Convective-diffusive zone ≡ $\dot{w} \approx 0$

$$\frac{d^2T}{dx^2} - \frac{\rho u C_p}{k}\frac{dT}{dx} \approx 0 \quad \text{——} \quad ①$$

BC : $x = -\infty$, $T = T_0$

$x = 0^-$, $T = T_i$

Two regions we get dT / dx x = 0 – should be =dT / dx x = 0 + well strictly speaking we should say KdT / dx but since it is the same species and as the as you now go to 0 from both sides it is essentially the same location the case are the same so we do not have to worry about it but these ∇s are coming out of two different solutions and we are actually trying to force this is going to say that we are going to force these two solutions to have the same ∇ at this point what that really means for you is keep in mind.

What we were talking about for the length scales characteristic length scales for thermal balance in the in the preheat zone versus the species mass within in the in the in the in this zone when we were looking at non-energy some other day we pointed out that if you were to just bother about the Schvab's Zeldovih only here and then take a equation that looks like this with a 0 on the right-hand side you can actually solve.

This but what you will find this you will get a solution that kind of goals like this and then goes exponentially there for x > 0 which is not the domain of validity it is outside the domain of validity of this equation if you were to extend the solution that you are going to get from there for your T1 of x you're going to get a solution that goes exponentially right that is what a second order request second order homogeneous equation is going to look like you try to solve this you will get exponents right.

So you have this on the other hand what we are talking about in this equation this is also something that you can solve what you will find this is going to actually give rise to something like that and what we are saying is a displace we want to have the temperatures as well as the slopes match and that is where we want to position out X =0 somewhere there right so this is typically how we want to construct the composite temperature profile in a composite manner by having this asymptotic matching what you are doing here is a 0th-order matching strictly speaking because we are identically sitting whatever is a small quantity in the equation to 0 and solving for it.

So this is the problem and out of this we are trying to get the velocity u we will do the first region and then hold on till next week for the second region with bated breath I hope right we cannot wait to get there so of course we say that this equation is too and we what we want to do is integrating one.

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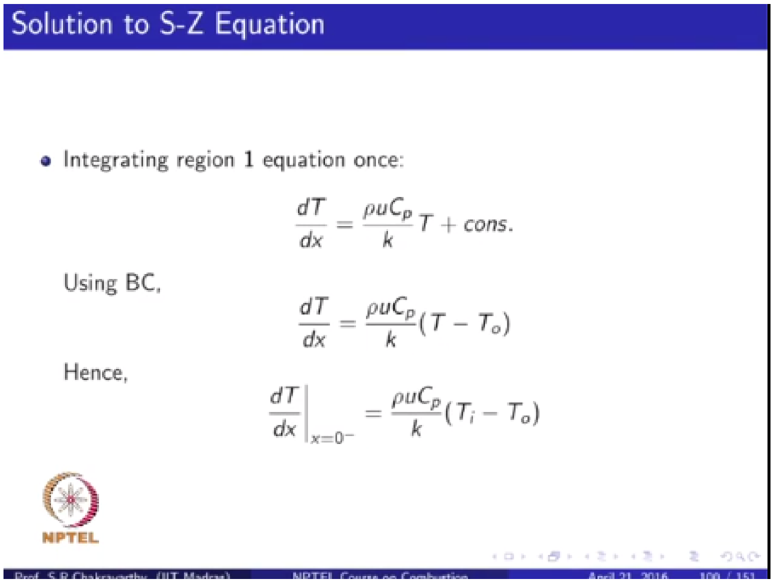

Solution to S-Z Equation

- Integrating region 1 equation once:

$$\frac{dT}{dx} = \frac{\rho u C_p}{k} T + cons.$$

Using BC,

$$\frac{dT}{dx} = \frac{\rho u C_p}{k} (T - T_o)$$

Hence,

$$\left.\frac{dT}{dx}\right|_{x=0^-} = \frac{\rho u C_p}{k} (T_i - T_o)$$

Once right we say dT / dX = ρu / k CP t + a constant.

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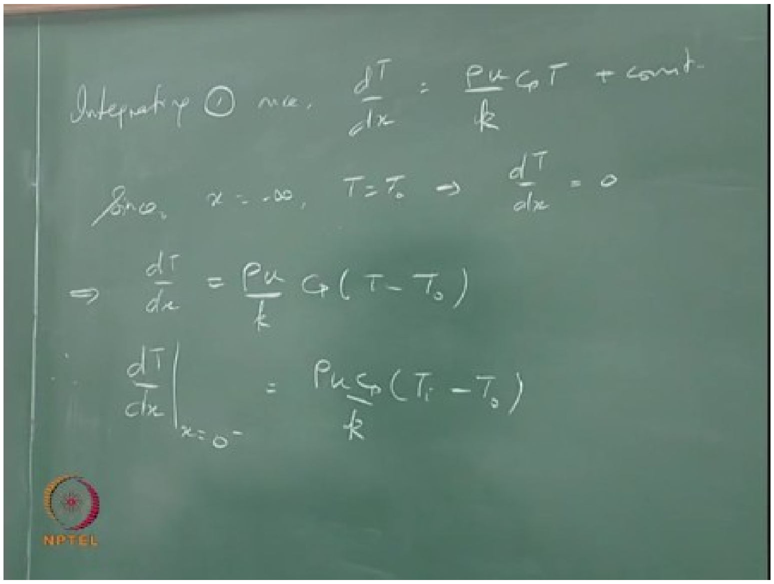

Integrating ① once, $\frac{dT}{dx} = \frac{\rho u}{k} C_p T$ + const.

Since, $x = -\infty$, $T = T_0 \Rightarrow \frac{dT}{dx} = 0$

$$\Rightarrow \frac{dT}{dx} = \frac{\rho u}{k} C_p (T - T_0)$$

$$\therefore \left.\frac{dT}{dx}\right|_{x=0^-} = \frac{\rho u C_p}{k} (T_i - T_0)$$

And we have to supply a constant of the boundary condition to satisfy to evaluate this constant of integration one way of doing this is to actually integrate it twice and then you now deal with two constants and then apply the boundary conditions right but I am not going to do that what I am going to look for is I am actually interested in dT /dx at 0 – okay so I am not really interested in going ahead with integrating this twice to look at actually the temperature profile this is a point where the analysis sort of cheats you okay.

So until now we have been talking about these profiles matching and all those things we are actually going to hear we are going to think that we will we will get this matching and get this composite profile but typically at this stage the analysis does not abandon sits care about the temperature profile it now starts bothering about the velocity which is Eigen value right that is what we are looking for.

So we do not really want to care about getting this too and I am stating this explicitly because textbooks do not states is explicitly and then when you go back to laminar flame analysis to actually look for the temperature profile you will not find it there okay and then they but they are like let us just go to the go through go through this and then get the velocity put a box around then we got laminar flame speed let us talk about the laminar flame speeding this keep going on okay.

So you never really get unless you go dig up literature further you cannot get this temperature profile so that is what that is a kind of attitude that you are going to get so since that means we now have to actually evaluate what the dT/ dx is far upstream so we since we are saying that at far upstream in this region 1 $T = T_0$ at $x = -\infty$ then $T = T_0$ for quite some distance they are there for the dT/ dx going to be $= 0$ $x = -\infty$ $T = P_0$ $dT / dx = 0$ this is asymptotically approaching that the reason why we are able to actually reduce this otherwise we cannot okay.

So that means $dT / dx = \rho u / kC_P\, T - T0$ right now if you think about it this is the first thing that we wrote in the phenomenological balance we said that $\rho u CP$ is essentially your $\rho u CP\, T - T_0$ is actually the enthalpy rice that is happening and that is because of the heat conduction $k\, dT/ dx$ and we approximated it as $k\, T - Tf - T_0$ divided by Δ that is what we did earlier okay so whatever we could actually write by just looking at things if you think about it takes about an hour for you to get.

If you go through the mathematics all right and there we are so that is what we got now therefore if you want to now apply this to the other boundary we now say at $x= 0$ - this is $= \rho u CP /k\, Ti - T0$ here we are imposing the next boundary condition and then what you are going to say here and or maybe this is a good point to stop okay so we will just say this we keep in mind we do not know what Ti is that is an additional problem that we have to worry about and we will see how to handle this next class.

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