Indian Institute of Technology Madras

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National Programme on Technology Enhanced Learning

COMBUSTION

Lecture 17
Multi-Component Momentum Equation

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So let us now look at something called a multi-component diffusion equation diffusion coefficient.

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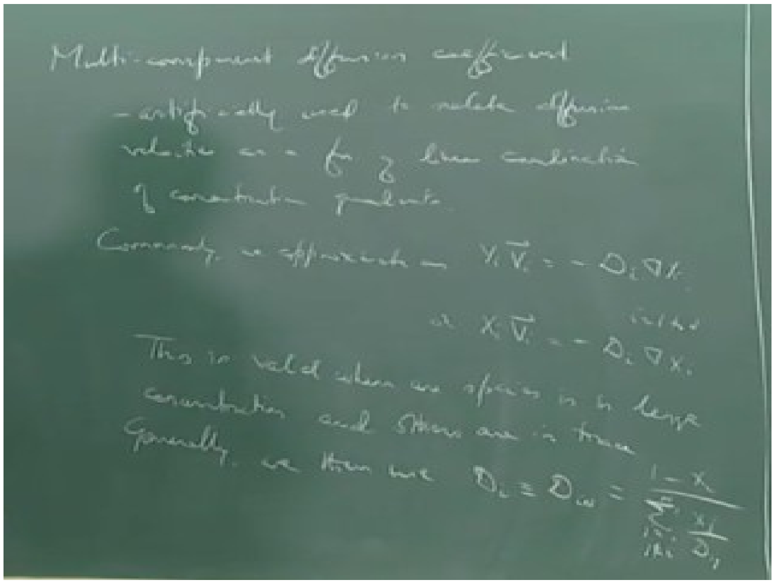

Multi-component diffusion coefficient

– artificially used to relate diffusion velocities as a fn of linear combination of concentration gradients

Commonly we approximate — $Y_i \vec{V}_i = -D_i \nabla Y_i$

or $X_i \vec{V}_i = -D_i \nabla X_i$

This is valid when one species is in large concentration and others are in trace

Generally, we then use $D_i \equiv D_{im} = \dfrac{1-X_i}{\sum_{j=1, j\neq i} \frac{X_j}{D_{ij}}}$

Right what we did originally was to get Fick's law for applied to a binary mixture where we notice that you have something called a binary diffusion coefficient that is a diffusion coefficient that is valid for a pair of species and then we now switched into a multi component mixture where we wanted to look at how the diffusion happens in a situation where we have lots of species and now he came up with a monstrous equation the other day which under some conditions might begin to look like ficks law.

Let us say for example again for a binary mixture but what we noticed was even in the even in the presence of multiple species the role of the boundary diffusion coefficient was still there that is to say we were still looking at mixing happening between a pair of species at a time right, so if you now had like three species ABC then we were looking at a DAB DBC and DCA okay so we are now three diffusion binary diffusion coefficients for each pair of species that is involved there.

But many times in you in looking at let us say tens or hundreds of species then we now have to worry about binary diffusion coefficients of each of the species which with each other species right so that that becomes a little bit more complicated and it is not quite amenable to for the simplification for example when we now that the multi-component diffusion equation we could reduce it to Fick's law for the case of a binary mixture okay.

By making lots of simplifications in trying to justify disregarding the three other terms and to look for only the first term that contributes to this and then we could see okay Fick's laws is retrieved but the question is if you had a truly multi-component a mixture could we even think about a something like a Fick's law something as simple as a Fick's law right and for this the binary diffusion coefficient is going to be a bit painful to deal with because we are now saying that you have to have a DAB times ∇ YADA DBC times ∇ YB and so on.

And these binary diffusion coefficients are not going to be the same right therefore many times it is worthwhile to think about what is called as a multi-component diffusion coefficient this is not the reality okay the binary diffusion coefficients are the reality but as we will now think about a multi-component diffusion coefficient artificially used artificially used to relate to relate division velocities as I is a function of linear combination of concentration gradient right.

So the way we want to do this is so commonly we approximate as let us say Yi Vi vector is equal to -Di Yi I = 1 to n course or we should also be able to say X_I V_I vector = Di ∇ Xi right we have seen this before for a binary mixture like for example if a now multiplied both sides by ρ the mixed up mixture density then I get $\rho_i V_i$ vector is = - $\rho D_i \nabla Y_i$ previously we have done this only for a binary mixture.

So if you were to apply this to spices A then we would have a D_{AB} so that has two subscripts to denote that this D is valid for a pair of species A and B but now we are looking at a multi component mixture that means you have more than two species but we are having only 1subscript that is to say we now want to tag the diffusion coefficient only to one species much like how it is CP or maybe thermal conductivity or any of those quantities would be that does not make sense honestly.

Because as we said if some species has to mix into another species you need to have other species involved in this process right so D_{AB} made sense because it was actually for a pair of species that way but D_i how can you have a diffusion coefficient that depends one day on one species the answer in this situation is as I said it is a bit artificial but it is not to artificial okay because what we could now think about is how is this species how is one particular species in a mixture mixing relative to let us say a species that abundantly present in the mixture.

Let us now begin to think about how each of the species in the mixture is going to mix with a one species that is the most abundant in the mixture right so typically in combustion applications we are looking at like fuel air mixture right in air you have a lot of nitrogen that is pretty much sitting there and that is like an abundant species around, so if it is now possible for us to think about how each of the species like the fuel or the intermediates the products the oxidizer air the oxygen that is and so on.

Mix with nitrogen okay and that is this present like a reservoir and it did not matter to nitrogen or the abundant species how well it mixed with individual species it is not going to get affected because it is available in abundance right when that is the case then it is possible for us to still think about the diffusion happening but for a pair of species as opposed to a tagging it to only one particular species but we achieve the goal of thinking about one species you simply do not worry about the abundant species right.

So the way to be typically do this is this is this is valid this is valid when one species is in large concentration large concentration and others are in trace now typically trace is a word that is used for something that is very small okay so it is sort of like an exaggeration that we are trying to do here in saying that when we are now having most species that are present in nominal quantities not negligible not an abundance we cannot release a trace but you get the idea that this is strictly speaking valid when one of them is in large concentrations other and trace.

But we will now go ahead and say well we will identify an abandoned species the other ones are not going to be in trace but we will still go with this idea right okay, so then what happens generally the we use then we then use Di is now considered identical to what is called as DiN okay that is the that is as if you are now looking at a binary diffusion coefficient of each of the species relative to the abundant species okay.

And this you can now say is generally written as 1 - Xi/ Σ j = 1 to n and j unequal to i Xj / Dij so that means we are actually using the Dij information for all the species okay in the in mixture in evaluating the what we call now as the multi-component diffusion coefficient okay strictly speaking this derivation can be possible only if you make an assumption that all the species are having equal velocities whereas we would ignore that if we have to apply this in a situation where that is not the case.

And that is in the in general not the case alright so that there is an assumption that goes with it with it which we normally while it in addition there is also one more kvh.

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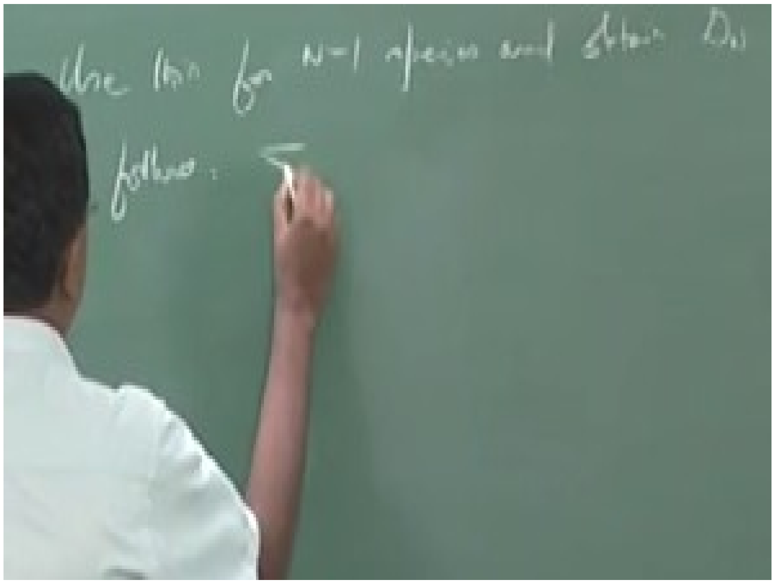

That we have to use which is a use this for N - 1 species okay and obtain DN okay as follows the point we do point the reason why we cannot use this for all the N species if you were to use something like this and therefore get something like this and then plug it in your species conservation equations and add strictly speaking we cannot set the summation o fall these diffusion fluxes do diffusion mass fluxes 0 okay.

So it is sort of like it sort of violates mass conservation therefore strictly speaking if you are going to use this kind of a multi-component diffusion coefficient and correspondingly your relationship between the diffusion velocity for the species and its concentration ∇ can be expressed as simply like a Fick's law okay.

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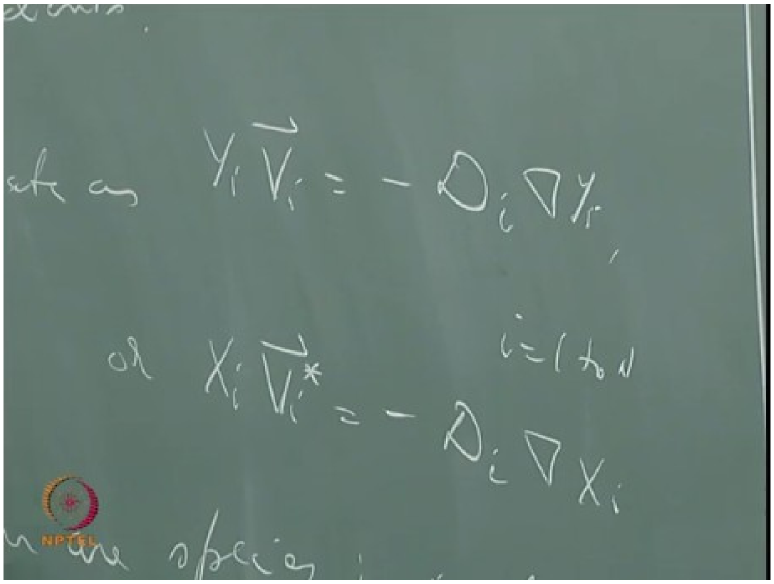

That is what we are trying to trying to do where we are trying to get this form for relating your Vi’s to ∇ Yi’s or Vi* I should say to ∇ Xi right so if you have to do this then do not do this for all the N species okay.

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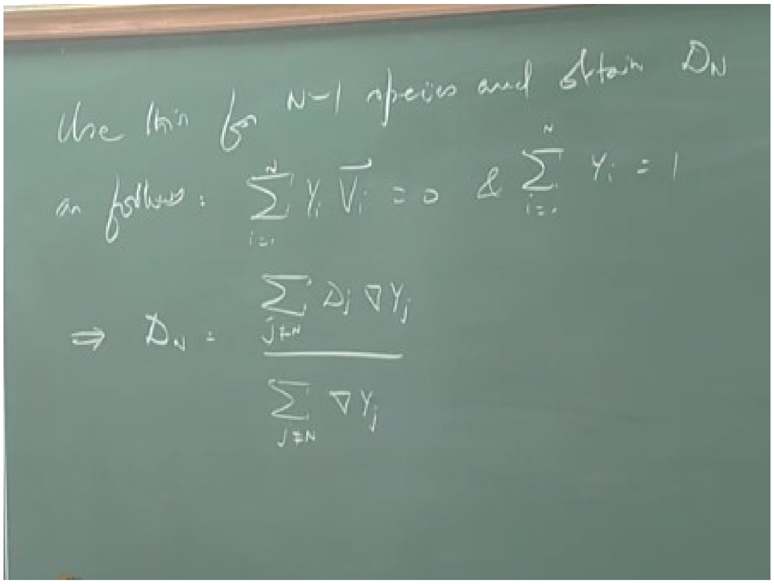

Use this for N−1 species and obtain D_N as follows: $\sum_{i=1}^{N} Y_i \vec{V_i} = 0$ & $\sum_{i=1}^{N} Y_i = 1$

$$\Rightarrow D_N = \frac{\sum_{j \neq N} D_j \nabla Y_j}{\sum_{j \neq N} \nabla Y_j}$$

The N species has to be obtained from adopting Σ Yi Vi vector is equal to 0 i = 1 to N and Σi = 1 to N Yi = 1 okay use these to obtain your DN Σi unequal to N Dj ∇ Yj/ Σj unequal to N DJ∇ Yj but many times again.

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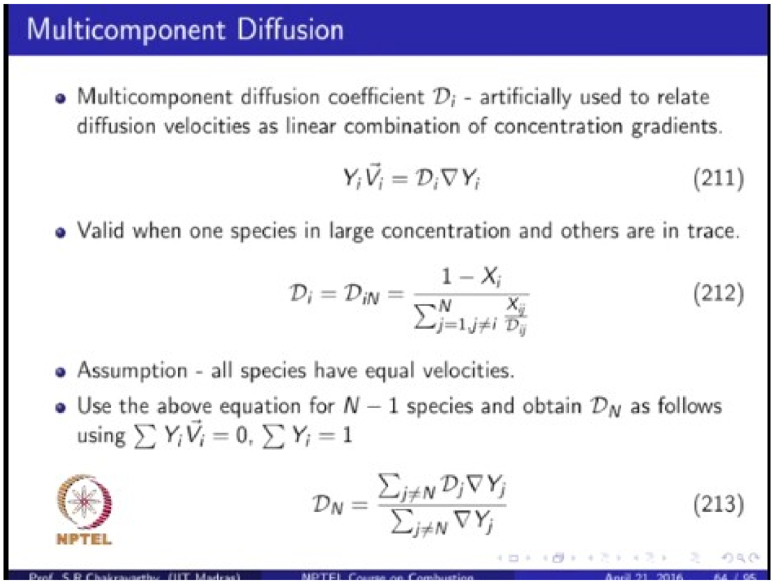

Multicomponent Diffusion

- Multicomponent diffusion coefficient $\mathcal{D}_i$ - artificially used to relate diffusion velocities as linear combination of concentration gradients.

$$Y_i \vec{V}_i = \mathcal{D}_i \nabla Y_i \qquad (211)$$

- Valid when one species in large concentration and others are in trace.

$$\mathcal{D}_i = \mathcal{D}_{iN} = \frac{1 - X_i}{\sum_{j=1, j \neq i}^{N} \frac{X_{ij}}{\mathcal{D}_{ij}}} \qquad (212)$$

- Assumption - all species have equal velocities.
- Use the above equation for $N-1$ species and obtain $\mathcal{D}_N$ as follows using $\sum Y_i \vec{V}_i = 0$, $\sum Y_i = 1$

$$\mathcal{D}_N = \frac{\sum_{j \neq N} \mathcal{D}_j \nabla Y_j}{\sum_{j \neq N} \nabla Y_j} \qquad (213)$$

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Just as we ignore the assumption that is involved here in saying that the velocities of all the species or equal we also many times ignore this that we just say let us not worry about the n th multi-component diffusion coefficient being obtained this way provided all the N-1 diffusion coefficients are obtained this way okay you see that these are actually the multi-component diffusion coefficients they are obtained okay.

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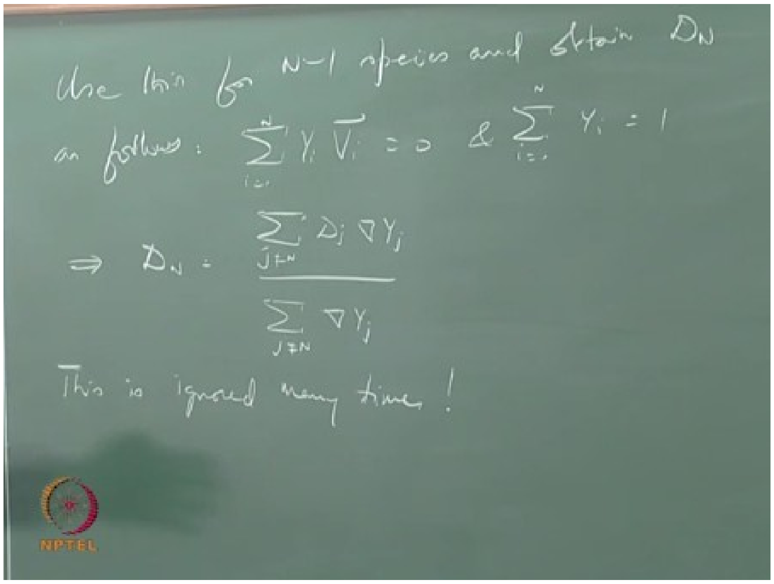

Use this for N-1 species and obtain D_N as follows: $\sum_{i=1}^{N} Y_i \vec{V_i} = 0$ & $\sum_{i=1}^{N} Y_i = 1$

$$\Rightarrow D_N = \frac{\sum_{j \neq N} D_j \nabla Y_j}{\sum_{j \neq N} \nabla Y_j}$$

This is ignored many times!

So this is this is ignore many times okay let us now move on what we have done so far in trying to look at conservation equations in general for a reacting flow system of surely multi component mixture is to derive the species conservation equation okay we did this for a binary mixture we now expanded it for a multi component mixture then we noticed that we need to have a closure for capital Vi vector for which we wrote the multi-component diffusion equation and then looked at what are all the situations by which you could simplify this and we have got to this point okay.

And we also noticed that as you now add up the species conservation equation all the species conservation equations you will now get your overall continued equation for the mixture as if it was anon reacting single species case right we have not shown this in these lectures for the multi-component species conservation equation but that I would like to leave it as an exercise for you to go back look at the multi-component species conservation equation equations and then add them up to show that even for a truly multi-component system you should get the overall continually equation to be like a single species non reacting situation all right.

So that means we have in our TT the species conservation equation and the overall continued equation these are the two things that we have okay and of course we also have the multi-component diffusion equation so far we could we could now count the number of unknown sin our in our minds density for example density of the mixture is one of the unknowns the corresponding equation that you can think about is like the overall continued equation right.

And then you have a mixture velocity the mass average to mix the velocity that is showing up in the overall continued equation that is an unknown for which typically we resolved the momentum equation for the mixture right which is what we would now go ahead and do and then we have the mass fraction as an unknown Yi for i = 1 to N for which we now have species conservation equations 1 to N and N species conservation equations which in turn shows up capital Vi for which we had the multi-component diffusion equation okay or its simplifications or many derivatives which ultimately leads something like a ficks law.

But if you do not want to do the simplifications you want to deal with the multi-component equation in all its complexity then keep in mind that you have Xi showing up there is an equation for grad Xi rather than for Yi which is your unknown in your in your list therefore you need to now use N equations that rate Xi and Yi okay so as and when you now come across unknowns you have to start thinking about corresponding equations to solve okay.

So in this list so far of course WI contains temperature for which we will have to resort to the energy balance okay and then we will find that the momentum equation involves pressure for which we also have to look what the equation of state so that will completely closed the system of equations so in what we have looked for so far we have not listed the mixture momentum mixture energy and mixture equation of state.

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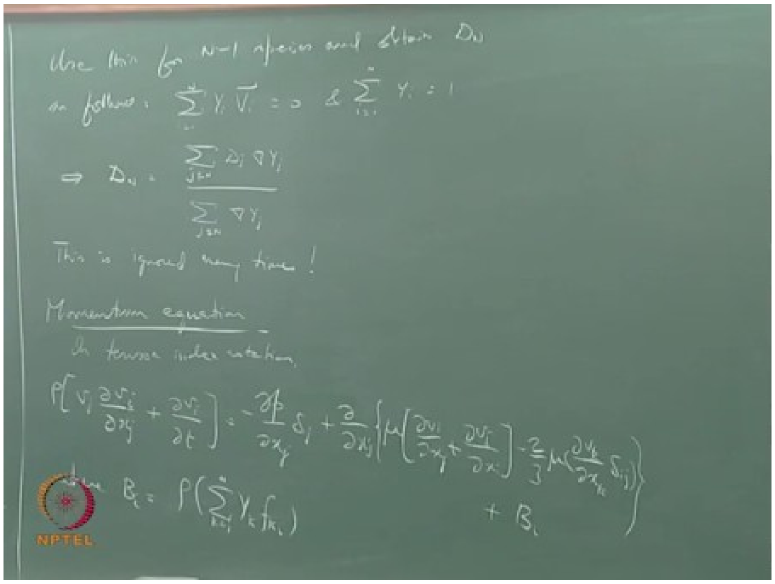

Use this for N−1 species and obtain D_N

as follows: $\sum_{i=1}^{N} Y_i \vec{V}_i = 0$ & $\sum_{i=1}^{N} Y_i = 1$

$$\Rightarrow D_N = \frac{\sum_{j\neq N} D_j \nabla Y_j}{\sum_{j\neq N} \nabla Y_j}$$

This is ignored many times!

Momentum equation

In tensor index notation

$$\rho\left[v_j \frac{\partial v_i}{\partial x_j} + \frac{\partial v_i}{\partial t}\right] = -\frac{\partial p}{\partial x_j}\delta_{ij} + \frac{\partial}{\partial x_j}\left\{\mu\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) - \frac{2}{3}\mu\left(\frac{\partial v_k}{\partial x_k}\delta_{ij}\right)\right\} + B_i$$

where $B_i = \rho\left(\sum_{k=1}^{N} Y_k f_{k_i}\right)$

These three or yet to be done so let us now do that so momentum equation right I am just going to write it out I am not going to do anything to derive this and I am going to adopt the tensor index notation to do this in tensor index notation ρ times Vj ∂ Vj/ ∂ x sorry ∂ Vi / ∂ Xj + ∂ Vi / ∂t = - ∂p / ∂Xi, Xj Δ ij +∂/ ∂Xj of μ ∂Vi / ∂Xj + ∂Vj/ ∂Xi – 2/3 μ∂Vk / ∂Xk Δij close the brackets + Bi where Bi = ρ times Σ k = 1 to N.

Yk fki I am going to say a lot of things now just say I am going to write it out okay first in terms of index notation if you now have 1 subscript that means we are looking at a vector okay so for example we now say Bi that is a vector it is I goes from 1 to 3 1, 2, 3.

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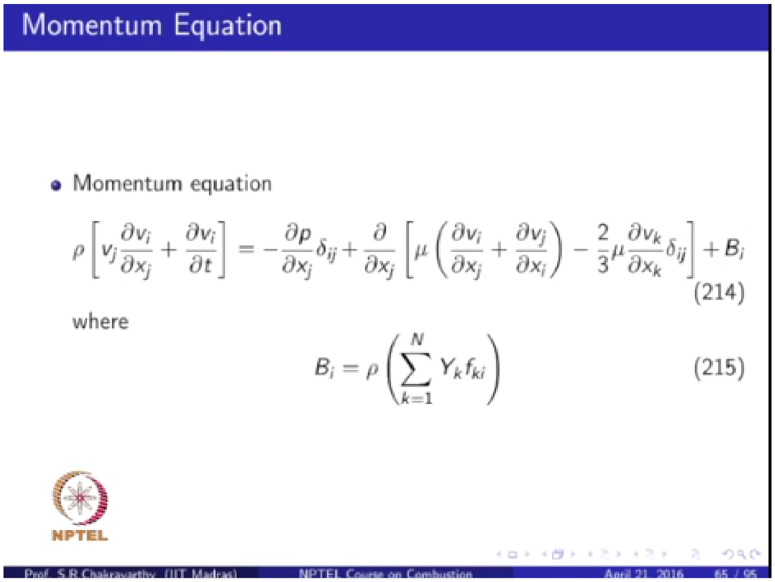

Momentum Equation

- Momentum equation

$$\rho\left[v_j\frac{\partial v_i}{\partial x_j}+\frac{\partial v_i}{\partial t}\right]=-\frac{\partial p}{\partial x_j}\delta_{ij}+\frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right)-\frac{2}{3}\mu\frac{\partial v_k}{\partial x_k}\delta_{ij}\right]+B_i \tag{214}$$

where

$$B_i=\rho\left(\sum_{k=1}^{N}Y_kf_{ki}\right) \tag{215}$$

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It will take index values 1, 2, 3 for 3 components in a three-dimensional space okay we can probably go a little bit backwards Bi is actually ρ times Σk = 1 to N Yk fki alright and then of course ρ does not depend on the summation it could be taken in and if you now were to think about ρYk as ρk, ρk fki fki obviously is a vector but k-means it depends on species right so when are we seen a f corresponding a small f corresponding to a species before we did not do we did this not too far back he probably did it like last class or yeah.

This is nothing but the body force per unit mass okay so in this case it is going to be like the i th component of the KH species body force so IH component of the body force per unit mass of the species k here so it sort of like acceleration we went through went through this sometime back times it is density it is going to mean that this is actually done per unit volume okay this is a volumetric equation this equation is now gathered at a particular point on a per unit volume basis that the reason why we are multiplying by density because this is actually per unit mass times density gives you per unit volume all right.

So fki is basically body force per unit wall unit mass and there therefore we now want to do this to plug it into this equation, so Bi then is like the net body force on the mixture all right this keeps in mind that the body force could be different for different species okay and this is because you're looking at like an electric field or a magnetic field that could be acting on different species differently and so on okay.

Then I want to also point out that we have a summation will repeated indices in place okay so if you now have like a Vj and Vj and Xj then we have to sum over the repeated index the remaining index will be I which is the prevalent index the entire equation so this is for the if' component of the vector equation as in this case for example you now see just that the summation is in place where were here as well where Δ ij kronecker Δ and Kronecker Δ stands for a value of one when I = j and 0 when I is not equal to j.

Therefore this basically will give you a ∇p so what we use to write in vector notation is grad p needs to go through a complicated route over here and this is first of all assuming that we are looking at a Newtonian fluid for the mixture alright and or rather we should say strokes you in fluid okay because it also has in place the Stokes hypothesis about the relationship between the linear viscosity and the bulk viscosity and here ∂ Vk/ ∂xk again has a repeated index summation over it.

So that actually indicates the divergence of velocity alright, so the divergence indicates a dilatation and the - 2 / 3 μ is the coefficient that comes out because of the bulk viscosity so that is acting on the dilatation of the fluid and of course for an incompressible fluid ∂Vk / ∂xk will be = 0 the divergence is 0 that is 0 that is actually coming from the overall continued equation if you sum up your overall continuity equation and and recognize that the mixture is incompressible than the density does not change with time and your and the divergence of velocity will be equal to 0.

So you should be able to get this right so in all these things we now all are having to reckon with two more new things that that have that we have not seen before the first one is pressure okay and the second one is μ everything else is supposed to be counted as either an unknown or given unknowns that we have already counted a ρ in the overall continued equation and Vi or Vj whatever it is in the overall convenient equation again Yk is an unknown that is counted in the species conservation equation fki is supposed to be given to us.

Okay on how individual species is being acted upon by body forces per unit mass of them therefore the thing that we do not know here is P that is an that is reckoned as an unknown okay whereas μ is a parameter in the problem it is reckoned as a fluid property the question is we now have a mixture of fluids okay what μ do we take alright so keep that in mind we have a little problem here a keep that in mind and we will proceed.

In fact we are now continuing to our search for more equations because the species conservation equation had a Wi on the right hand side which high light temperature in the rainiest term okay so we want to now closed for temperature and for this we want to now look for the energy equation. So the course we now also have a pressure and we will now have to close for that with the equation of state we will do that next but the next thing that you're going to do right now is the energy equation.

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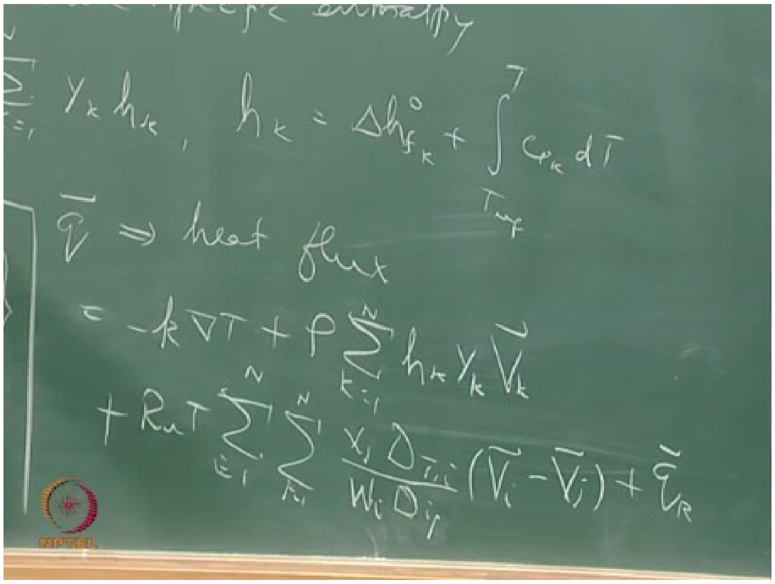

specific enthalpy

$$\sum_{=1} Y_k h_k, \quad h_k = \Delta h^0_{f_k} + \int_{T_{ref}}^{T} c_{p_k}\, dT$$

$\vec{q} \Rightarrow$ heat flux

$$= -k\nabla T + \rho \sum_{k=1}^{N} h_k Y_k \vec{V}_k + R_u T \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{X_j D_{T,i}}{W_i D_{ij}} (\vec{V}_i - \vec{V}_j) + \vec{q}_R$$

The energy equation just like the momentum equation is also written for the mixture the means we are not worried about energy of this wave of a particular species and so on the only thing that we do that a species-specific is the mass conservation for a particular species okay but rest of the things we do for the mixture as a whole, so the energy equation is also applied to the mixture now there are several ways in which you could write the energy equation.

So one of the maybe the mother of all the equations would be to write out an equation for the stagnation enthalpy because the stagnation enthalpy will take into account the kinetic energy involved in the motion of the of the mixture and it will also involve the pressure work because we will now say $H = E + P / \rho$ okay where E is the internal energy and P / ρ is the pressure and density.

You now look at a change in enthalpy it also taken takes into account not only the internal energy change but also the pressure work term and that is for the H the enthalpy alone okay if you are now looking at H0 which is $H+ V^2$ plus then it also takes into account kinetic energy changes, so the mother of all equations probably should be to write in terms of the stagnation enthalpy.

Now what I would like to point out here is that is who were doing things a little bit the reason is if you were to write your energy equation in terms of the stagnation enthalpy keep with you and then you take a dot product of your momentum equation with velocity vector which is of course we are now looking at the mixture velocity that is the mass averaged mixed velocity of the mixture you take a dot product of this is a vector equation okay.

So you know take a dot product of this equation with velocity you now get this in dimensions of energy okay and that would be what is called as the mechanical energy equation that means this equation then will only deal with the mechanical energy this is the reason why in many PhD qualifying exams people may ask you something that you learnt in your in your undergraduate fluid mechanics is the Bernoulli's equation a statement of energy conservation or is it a statement of momentum conservation.

The answer is both right you could conserve momentum or you could conserve mechanical energy it is a matter of just taking a dot product at the momentum equation to get your mechanical energy right so in other words if you are conserving momentum it is amounting to conserving mechanical energy what it does not do is to conserve the thermal energy okay.

So if you now obtained your mechanical energy and subtracted it from the energy equation written in terms of the stagnation enthalpy then you get an energy equation that takes into account only the thermal energy conservation okay that is what we would do we would just write out the equation for the thermal energy conservation we will not worry about the stagnation enthalpy at all right.

So here we are ρ Dh/ Dt = - ∇q vector + Q. + ρ dot Σ K = 1 to N Yk fk vector dot Vk vector + ϕ +Dp / Dt + now I told you anytime you now look at an equation that keeps going on and on okay do not worry about it you start looking at a term by term and then start making sense out of this D/ D DT D / Dt refers to the substantial derivative or material derivative or total derivative right which in turn would be like ∂/∂t + V. ∂ that is operator that we have to replace for this to look at the this is this is essentially the inertial term in general which can now be looked at in an all Aryan frame of reference as a unsteady term and a convective term together right.

Now if you want to if you want to derive this for yourself you would understand that you got the divergence because this was actually a surface term in a arbitrary control volume this is the way we have actually got our species conservation equation derivation okay we had a surface mass flux across the control surface okay and then we applied the Gauss's divergence theorem to get a divergence.

So whenever you have a divergence you had immediately think that this is actually coming from like a surface or from the sides okay and we will expand on this Q. is actually the what do you think is Q. that let us let us just tease ourselves a little bit for a minute what do you think is Q. sorry energy supplied by what external source damn you knew the answer what else could it be a what are we looking for here what are we looking for why are we here heat put into the reactor did he say puts you put into reactor I put into the reactor I am a combustion person right I want to get out something right.

I put in fuel and I want to get heat so how many of you think that this Q. is actually the heat that is coming under the reactions like half hands up why oh is the heat from the reactions hidden in here anywhere this is the test of a combustion scientist or a student I should say right where is the heat released from chemical reactions present in this we should probably ask this question at the end of explaining all the terms but I cannot wait I mean that is what I am really looking for forget about everything else in fact I will forget about lots of those right.

So some of these are basically written once and then we will say well forget about this but we cannot forget the heat released from chemical reactions because that is out that that is our essence but where is it in this is it the Q dot right the Q dot is essentially an external energy release rate term so this is a is the heat release due to chemical reactions contained within your system or is it or is it to be externally put in right.

So you now take a like a reactor or a combustor or whatever it is and then you now put in your air and you put in your fuel okay and do we have to put in heat know right where was the heat contained that is now released the H where is where is it in the H standard what the standard heats of formation of what the parts of this on the reactants right so this H is the mixture enthalpy mixture specific enthalpy okay.

So what is going to happen here is H is the mixture specific the reason why we are multiplying by ρ specific means per unit mass okay specific enthalpy and that is given by = Σ K = 1 to N Yk hk where yk is the mass fraction still unknown okay in the system of equations but hk is the specific enthalpy of species k okay by the way notice that we have shifted from A and B or I and J and all those things for species to a capital K because if you now did this with a tensor index notation were i’s and j's competing with us okay.

So we now use K for species from now on yeah so hk is the specific enthalpy of species k that still does not show you the heat during the chemical reactions because you know further have to identify that hk is = Δhf0k + ∫ t ref to t cp k dt cpk dt there are two things that I would like to point out in this is what is contributing to your heat release this is not the full heat release of course this is only the standard heat of formation of the kh species if you now plug this in here and I am going to get X summation yk Δ hfk.

That is like saying I want to add up all the heats of formation of all the species that are involved in this mixture weighted by their mass fraction okay and keep in mind it is a products that are going to have very high negative heats of formation the reactants are not going to have that high negative heats of formation they could have like small negative heats of formation or zero or close to zero or a small positive or positively or more positive heats of formation all that stuff that that algebraic sum is like the net heat.

That we are going to get by thinking about a hypothetical process by which we broke down the reactants in to reference elements by putting in some heat and then got the reference elements to form the products right and then get lot more heat than what we put in right that is what is the heat lease from the chemical reaction since hidden in there okay the other thing is we were actually writing this equation searching for temperature as an unknown and look at where it is.

That is in the sensible enthalpy term on the top of this integral nothing you are known right so okay let us look at a few other terms let us look at grad θ divergence Q or what is Q first of all so I told you it is coming from this surface so this is heat flux okay so this is heat flux this is the energy rate equation okay so this is actually per unit time whatever we are writing here is actually per unit time things are happening as they happening you're looking at what is happening for you instead per unit time right.

And this is like heat flux that means what is happening per unit time per square meter per unit area right, so what is this heat flux this is now going to be like equal to - k grad T + ρ Σ K = 1 to N hk Yk VK vector + RuT Σ i = 1 to N Σ j = 1 to N Xj DT, i/Wi Dij times Di vector – Dj vector + qR vector clearly heat flux is a vector okay which is now going at an arbitrary direction say arbitrary it is going at a direction not exactly normal to a surface it is having like an N vector u unit vector unit normal vector that it that we have to take a component with respect to and that is how you would get the gausses divergence theorem applied over there.

And then there are about 1, 2, 3, 4, term yeah 4 terms okay how do you get four terms the first term – k ∇ T what is it that is a heat conduction the heat conduction is actually coming from this from the sides at from a particular point because you have a temperature ∇ with respect to this point and the adjacent point right so if you if you had a harder region on the side you are not going to have heat into this point if you are going to have a cooler region on this side you are going to have heat away from this point right.

So that's the familiar Fourier law of heat conduction so effectively we are assuming that we are looking at a Fourier fluid.

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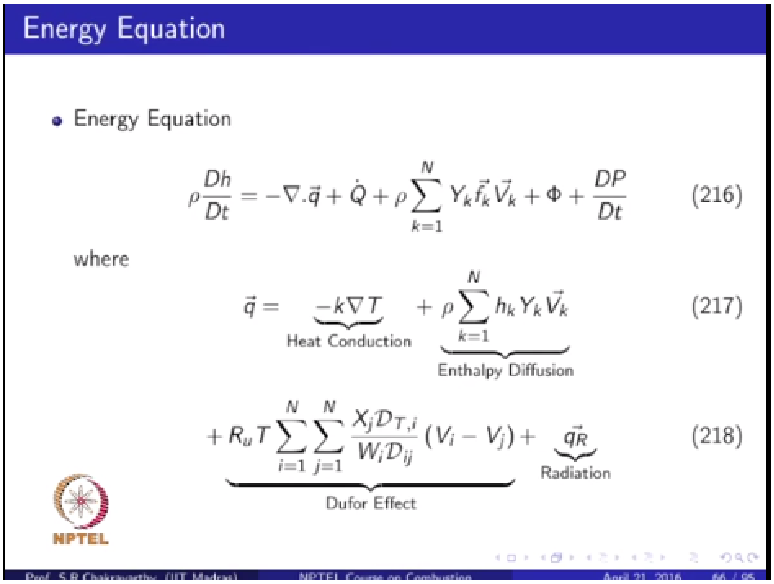

Energy Equation

- Energy Equation

$$\rho\frac{Dh}{Dt} = -\nabla.\vec{q} + \dot{Q} + \rho\sum_{k=1}^{N} Y_k\vec{f_k}\vec{V_k} + \Phi + \frac{DP}{Dt} \qquad (216)$$

where

$$\vec{q} = \underbrace{-k\nabla T}_{\text{Heat Conduction}} + \underbrace{\rho\sum_{k=1}^{N} h_k Y_k \vec{V_k}}_{\text{Enthalpy Diffusion}} \qquad (217)$$

$$\underbrace{+ R_u T\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{X_j\mathcal{D}_{T,i}}{W_i\mathcal{D}_{ij}}(V_i - V_j)}_{\text{Dufor Effect}} + \underbrace{\vec{q_R}}_{\text{Radiation}} \qquad (218)$$

That means a fluid that a heat-conducting fluid that satisfies Fourier law all right the fluid is actually a mixture of lots of species okay so what is this k it is a conductivity of the mixture right so similar to this μ being viscosity of the mixture so we need to have an idea of how to deduce the conductivity of the mixture and the viscosity of the mixture for a mixture whose composition is it to be solved for okay.

So actually we do not know this and this is changing from point to point and time to time okay in this mixture because the composition is changing as we go from fuel air over here to lots of stuff and then finally some products and so on okay so I am not going to tell you how the μ on k depends on the species concentration instantaneously I am going to put this out in a website with that that the that this video will go with it has to go to kinetic theory okay.

So we now we cannot do this in the continuum framework we will write out equations that give you this later on what is this term can we carefully look at this okay ρ is allowed you can take this in and then make it make this ρ Yk as ρ K hk alright Vk as a matter of fact so long as we had Yk hk Σ you would get back your h okay but then you had a Vk that prevents you from summing over only Yk hk right Yk is waiting the Σ.

So Vk is fighting the summation what is Vk that is the diffusion velocity of species k right how did we get this term the answer is you see when we were looking at this we had a $\partial / \partial t + V. \partial$ h the V there is the mixture of is a mass average velocity of the mixture okay and then keep in mind that h is Σ Yk hk the mass average to velocity of the mixture is not species dependent it is now a mixture property.

So you could have pulled it out of a summation and kept your sigma yk hk as h that is how you got your h okay but in reality how is the enthalpy flowing into your point and getting out of the point this looks like it is convection right in convection oven that hi but this is like convection of enthalpy of the mixture okay but in reality whenever any matter moves it carries with it mass momentum.

And energy that is the reason why we are able to do mass conservation momentum conservation and energy conservation here we now have a bunch of species each of which is carrying some ass for which we did a call a species conservation equation at any particular point okay and then we had a mixture momentum conservation all right but if you are now thinking about the energy or the enthalpy that the species carries with it okay now it carries with it enthalpy at its velocity but what we have accounted for is like a overall enthalpy that the mixture carries at the mixed a velocity.

That is what this accounts for then there is a deficit there is still enthalpy carried by individual species at its diffusion velocity with respect to a mixed a fixed coordinate system right in reality it is carrying it is enthalpy that it is velocity but then now we have to take like a mixed or average mass average velocity if the mixture okay and then that does not depend on species anymore so we should now be able to add up a contribution of all the species put together in terms of identity to see how the enthalpy of the mixture gets convicted right.

But then there was one more contribution that we forgot there is like a diffusion velocity with which the enthalpy is still going all right that is coming up if you now have a mixed-effects coordinate system that is what this is so you know having Vk time hk ρ k okay so that is actually the enthalpy of the mixture that is due to diffusion okay so you have to keep that in mind and then this is pretty interesting this is what you would call as do for effect okay where this is the opposite of the so ret effect that we saw the other day okay were there what we saw was if you now had a temperature ∇ it gave rise to a contribution to a concentration ∇.

Here if you now have diffusion velocities okay which in turn are supposed to be functions of concentration ∇s it gives rise to a heat flux okay, so why you had a temperature ∇ giving rise to a concentration ∇ is called solid effect where you have a concentration variant giving rise to a heat flux is a is called the dufour effect okay and you can clearly see that you have Dt, I/ showing up here as well alright so that is what is called as the dufour effect and finally QR is a radiation heat flux.

So what we have had so far here is only the conductive heat flux therefore you also have to add a radiation heat flux the radiation heat flux will have to be model based on Stefan's law and if you have an enclose or something of that sort you have to have these shape factors coming in and so on absorptive and emissivity’s and all those things that is something that you want to do separately but at the moment you just try to plug in a but keep in mind that it is going to depend on temperature okay.

So it has a t to the full expression and that could be an unknown because temperature is an unknown okay, so you could plug in an expression for that if you want we will stop here for now and then pick it up from here next week we still have few more terms to go we want to know what this is what this is and so on okay we the energy equation takes some amount of energy.

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