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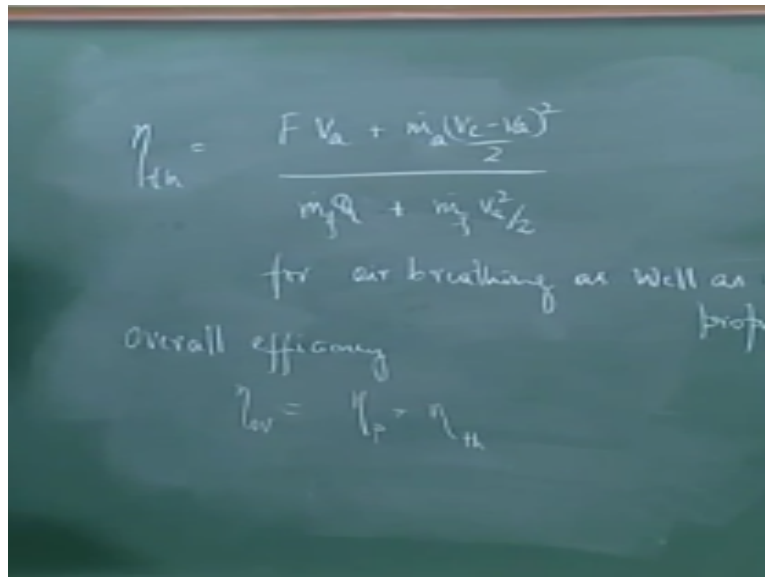
**Aerospace Propulsion
General Performance Parameters II**

Lecture - 9

**Prof. Ramakrishna P A
Department of Aerospace Engineering
Indian Institute of Technology Madras**

In the last class we had seen that how propulsive efficiency plays a role in determining what kind of systems we need to use under what conditions in this class let us look at what is the role of overall efficiencies okay.

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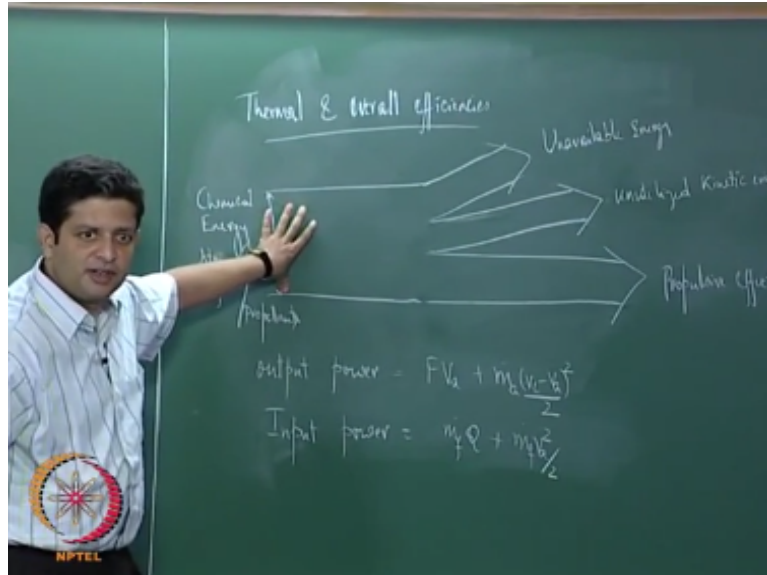

$$\eta_{th} = \frac{F V_a + m_a \frac{(V_e - V_a)^2}{2}}{m_f Q + m_f \frac{V_e^2}{2}}$$

for air breathing as well as
overall efficiency

$$\eta_{ov} = \eta_p \eta_{th}$$

Now towards the end of the last class we derived expression for the thermal efficiency of the system for both air-breathing as well as non air breathing systems and this is the expression that we had direct, now from this we can derive an expression for the overall efficiency what is the overall efficiency overall efficiency is this divided by this right now we have done this divided by this we have done this.

(Refer Slide Time: 01:10)



Divided by this so the overall efficiency we can express it in terms of propulsive efficiency and thermal efficiency we will see how to do that a little later as I can write this overall efficiency as it have propulsive into η thermal right.

(Refer Slide Time: 01:51)

$$\eta_{ov} = \frac{FV_a}{FV_a + \frac{m \dot{a} (v_e - v_a)^2}{2}} \times \frac{FV_a + \frac{m \dot{a} (v_e - v_a)^2}{2}}{m \dot{f} Q + \frac{m \dot{f} v_a^2}{2}}$$

$$= \frac{FV_a}{m \dot{f} Q + \frac{m \dot{f} v_a^2}{2}}$$

Now η propulsive is what is it a propulsive FV_a by a $V_a + m \dot{a} V_e - V_a$ whole square by 2 into η thermal $FV_a +$ right this is the expression that we get, so I can cancel these two out right and we will be left with this is nothing, but this quantity divided by this quantity right the propulsive power divided by the input power us what you have is the overall efficiency, now this expression is valid for both everything as well as non air-breathing propulsion now let us look at the first case that is everything propulsion.

(Refer Slide Time: 04:31)


For air breathing propulsion

$$\eta_w = \frac{\dot{m}_a (V_e - V_a) V_a}{\dot{m}_f Q + \dot{m}_j \frac{V_j^2}{2}}$$

$$= \frac{2(V_e - V_a) V_a}{2fQ + fV_e^2}$$

$f = \dot{m}_j / \dot{m}_a$

$$r = V_a / V_e$$

$$\eta_w = \frac{2(1-r)r}{\frac{2fQ}{V_e^2} + fR}$$


For air plating propulsion we can write this expression we as the overall is equal to $\dot{m} \cdot a \cdot V_e - V_a$ this is the thrust part into V divided by okay, now I can divide the denominator by numerator and denominator by $\dot{m} \cdot a$ I will get $V_e - V_a \cdot V_a$ where f is nothing, but I am not f by $\dot{m} \cdot a$ okay what is I will take again the definition of R is nothing but V_a by V_e if I use this definition of our and rewrite this expression I will get overall is equal to G_e^2 right, if I divide both the numerator and the denominator by V_e^2 square I will get this expression this is V_e .

(Refer Slide Time: 07:25)

$$E = \frac{fQ}{k^{3/2}} \quad \text{ranger between 4 - 10}$$

$$fr^2 \sim 0.01 - 0.04$$

$$fr^2 \ll E \quad \text{therefore neglecting } fr^2$$

$$\eta_{ov} = \frac{2r(1-r)}{E}$$

Let me call let me define a parameter called E which is nothing but F into Q divided by V_e^2 by 2 okay now this ranges between 4 to 10 for air breathing engines okay and FR^2 if you look at the other quantity that is fr^2 is typically between 0.01 to varies between 0.012 04, so this is very small compared to E, so therefore we will neglect this portion okay for everything proportion we will neglect FR^2 is very much less than E therefore neglecting, if R^2 we can write the expression for overall efficiency S to R into 1 - R divided by 8.

And this whole quantity here is nothing but this is nothing but E right, so we will get this expression for overall efficiency when is this Maxima again we need to differentiate this is very simple you will get $2R - 2R^2$.

(Refer Slide Time: 10:12)

$$\frac{d\eta_{ov}}{dr} = \frac{2 - 4r^2}{E}$$

$$r = \frac{1}{2} \rightarrow \eta_{ov} \text{ is a maxima.}$$

$$\eta_{ov} = \frac{1}{2E}$$

$$\eta = 5\% - 12.5\%$$

Right $r = \frac{1}{2}$ okay gives you the maximum value and what is the value of overall efficiency at this point are equal to half you put half here you will get 2 into $\frac{1}{2} \cdot 1 \cdot 1 - \frac{1}{2}$ that is 0.5, so that is 1 by $2E$, now E value we know ranges between 4 to 10 so overall efficiency what is the highest value $1 / 20$ is 5% so overall efficiency for everything propulsion varies between 5 to 5% to 12.5% which is a very, very low number okay, so please remember air-breathing propulsion air-breathing propulsion has very low efficiency is associated with it and it is typically in the range of maximum whether on 12.5% now let us do the same exercise for.


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For non-airbreathing Engines

$$\eta_{low} = \frac{FV_a}{\dot{m}Q + \dot{m}V_a^2/2}$$

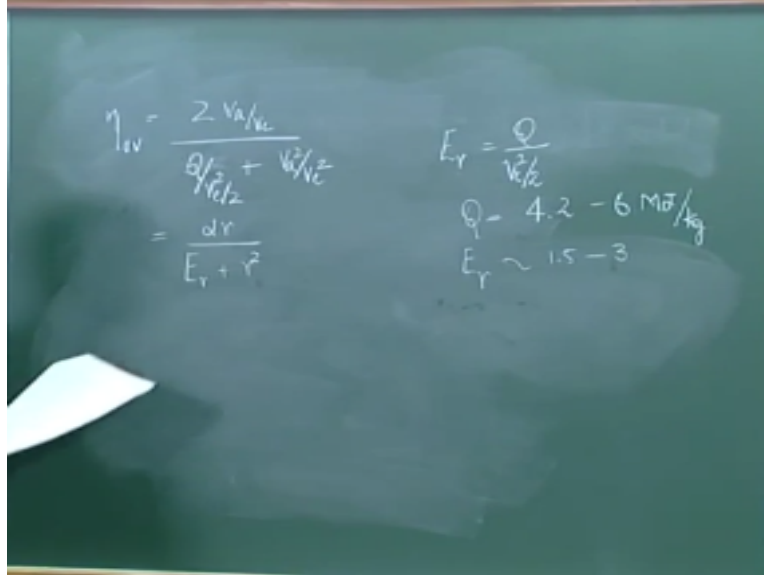
$$F = \dot{m}V_e$$

$$\eta_{ov} = \frac{2\dot{m}V_eV_a}{2\dot{m}Q + \dot{m}V_e^2}$$

$$\gamma = V_a/V_e$$


Rocket engines if we do the same exercise for non air breathing or rocket engines again overall efficiencies FVA divided by $\dot{m} \cdot f^3$ right here there is no $\dot{m} \cdot f^2$ right for rockets there is you need to carry both fuel and oxidizer, so this is $\dot{m} \cdot Q + \dot{m} \cdot V^2/2$ by 2, so we know that f is equal to $\dot{m} \cdot V_e$ right for rocket engines and if you substitute that you will get η overall is equal to $\dot{m} \cdot V_e$ okay fine I can cancel out \dot{m} . here and again using R is equal to V_a by V_e I get to r or if I divide both the numerator and denominator by V_e^2 I get overall efficiency is get the expression for overall efficiency.

(Refer Slide Time: 14:56)



Dividing by V_e^2 as and in the denominator I have now using the definition of our I can write this as $2r$ divided by I will call this quantity as E_r I will define E_r as equal to Q by $V_e^2 / 2r$ indicates here Rockets, so I will get this as $E_r + r^2$ now typical value of Q for the rocket the heat of formation or heat of reaction what is it Q for kerosene 42Mj what do you think will be the Q for Rockets higher than that lower than that what do you expect it to be higher sure actually the Q would be something between 4.2 to 6Mj/kg .

Slower than that why do you think we use such a bad propellant as rocket in rockets or is there a catch here if you look at Rockets need to carry both fuel and oxidizer right this is per kg, now this includes both fuel and oxidizer whereas if you looked at kerosene it was only for fuel you would not account for the oxidizer because here you are looking at both fuel and oxidizer this will have to come down right this will have to be lower than what you get with kerosene okay so Eq is something like this.

And therefore E_r will range between something like 1.5 to 3 okay so compared to that r^2 you cannot neglect if especially rockets can operate in a regime where r s greater than 1 , so you cannot neglect r^2 compared to E_r the thing that we did in terms of aircraft engines or air breathing engines is not valid yet, so again we have to find the maximum value we will take the derivative.

(Refer Slide Time: 18:46)

$$\frac{d\eta_{ov}}{dr} = \frac{2(E_r + r^2) - 2r(0 + 2r)}{(E_r + r^2)^2}$$

$$= \frac{2E_r - 2r^2}{(E_r + r^2)^2}$$

$\frac{d\eta_{ov}}{dr}$ is zero when $2E_r - 2r^2 = 0$
 or $r = \sqrt{E_r}$

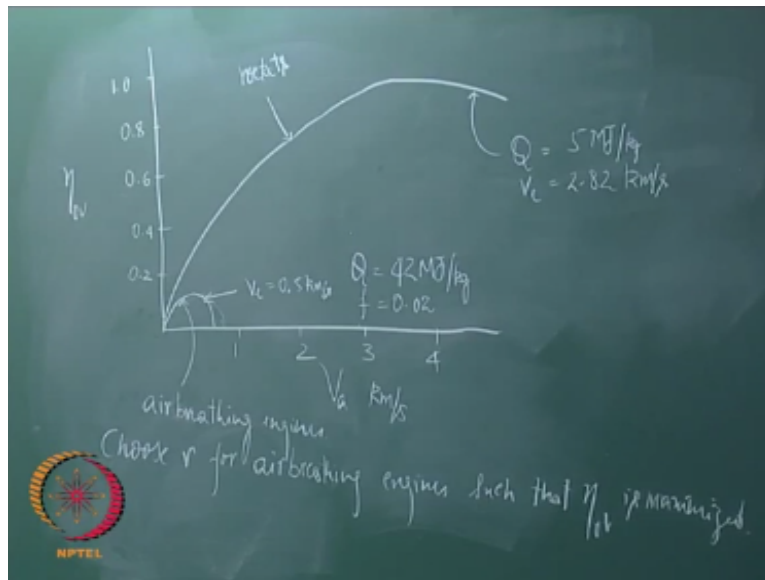
$\eta_{ov} = \frac{1}{\sqrt{E_r}}$

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So taking the derivative of overall efficiency with respect to our here I get to $E_r + r^2 - 2r + r^2$ square triple square right and this simplifies to $2E_r - 2r^2$ divided by $E_r + r^2$ the whole square, so when well the derivative go to 0 in this case either when this goes to ∞ or when the numerator goes to 0 in this case the denominator cannot go to ∞ , so the numerator has to go to 0 that is or R is equal to okay right, so you get the maximum value when r is under root of E_r and so r will be typically greater than one because E_r ranges from 1.523 so R will be greater than 1 in this case.

And η overall maximum would be if you substitute this into this expression here so you get under root 2 under rooter to E_r^2 so you get 1 by under root E_r okay so if it is 1.5 efficiencies can go very high actually in 4 rockets right and let us plot this.

(Refer Slide Time: 22:14)



If we plot overall efficiency for both everything as well as non air breathing engines and on x-axis we have V_e in kilometers per second this curve is for everything engines and this is for rockets or one air-breathing engines, you see that overall efficiency is much higher for rockets compared to aircraft engines aircraft engines this was around 12.5% right, whereas rockets it can go up to 85% right now before we go there firstly I forgot to tell you that if you look at in this case also there is an optimum value for V_e with respect to overall efficiency for air breathing engines one value.

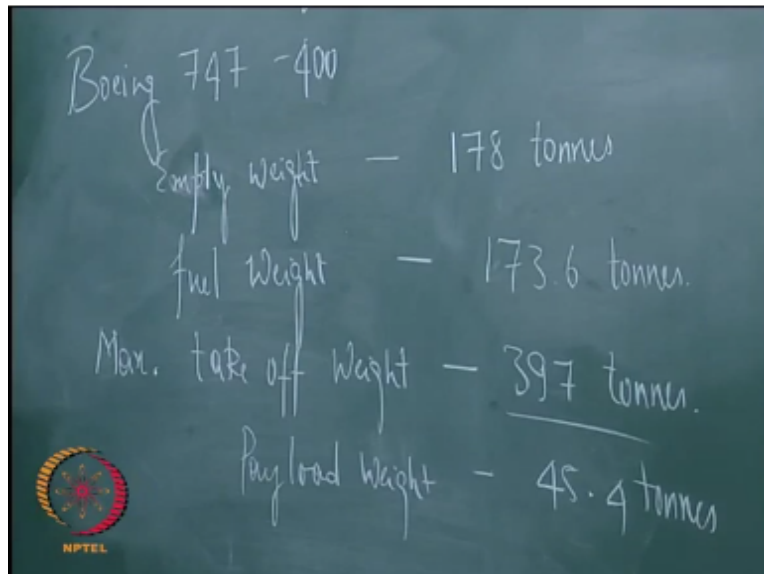
And there is another value that you can choose in with regards to propulsive efficiency okay there are two efficiencies propulsive and overall efficiency for air breathing engines you usually the value of r that gives you maximum overall efficiency it does not give you higher propulsive efficiency but you go for the maximum overall efficiency, so you choose r such that your overall efficiency is maximum for everything okay for everything engines such that overall efficiency is maximum maximized right.

Now if you see this graph you will find that I have to say what kind of fuel and other things $Q = 5 \text{ MJ/kg}$ he is 2.82 km/s and there for everything Q is equal to and f pure a ratio is somewhere around 0.02 okay so if you take a look at this graph it feels that overall efficiency of rockets is way better than overall efficiency of everything engines notice that V_e is 2.82 kilometers in the Maxima occurs somewhere here which is what I had said earlier r s greater than 1 for maximum value it goes as under root here.

So why not use rocket engines everywhere so overall efficiencies are very high, so we should be using rocket engines instead of air-breathing engines everywhere is that a valid statement hey efficiencies, if you look at it says they are very efficient machines so why not we use rocket engines everywhere what is the heat is P in essence yes you are saying something, but if you look at aircraft engines and rocket engines or everything and non air breathing engines there is an important thing that distinguishes them that is a rocket engine or a non air breathing engine will have to carry.

Both fuel and oxidizer on board right because they carry fuel and oxidizer on board and two rocket engines typically operate at very, very high pressures okay, so they add heat at very high pressures therefore they can expand to low pressures there is greater availability and hence they are more efficient in that sense, but efficiency per se does not have any meaning here because you have to carry a lot of propellants in order to achieve what you want achieve to give you an example.

(Refer Slide Time: 29:11)



If you take PSLV weight at lift off it is something like 294T its payload for to low-earth orbit would be something like 3,250 kgs almost 1/10 of its overall weight okay, so the useful thing that you can carry in a rocket mode rocket propulsion is very small compared to the overall weight although it might be efficient you end up having to carry more of fuel and oxidizer there because you have to carry both and the useful payload that you can carry is very, very small typically what is this 100th is it 300 yeah 100th of the overall weight right.

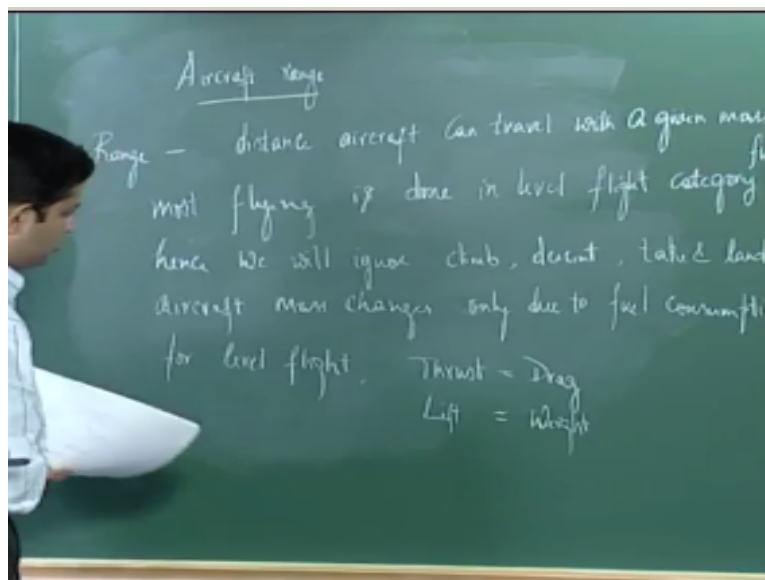
Now let us take a look at what are similar numbers for Boeing 747 it is empty wait this around 178 tons yeah it is fuel weight they are laying around 173 tonnes and it is maximum takeoff weight that is what is the weight of the aircraft maximum weight of the aircraft at which it can take off 397 tones, so the payload weight should be the difference of this - these two put together right, so the payload weight comes to be something like okay if you come correct with the overall weight or the maximum takeoff weight.

It has increased from one by 100 to something like 1 by 10 which is far better right, so if you have to have a mission wherein you are looking at going beyond the sensible atmosphere then you have to use rocket motors there is no other go, but if you have to look at a mission where in you traveling within the sensible atmosphere then it makes sense to use everything propulsions again there is a restriction, if you are looking at a very fast response system then again you need to go in for rocket engines even within the atmosphere okay the other point is what you made

earlier that is for rocket engines it is more meaningful to look at what is the ISP, ISP tells you what is it that you need to carry what is the propellant.

If larger the ISP then smaller is the propellant weight that you need to carry onboard right but if ISP becomes smaller then you need to carry more propellant on board okay, so ISP is a better indicator of performance and rockets and not overall efficiency whereas for everything if propulsion it is overall efficiency is a very good indicator of what is the performance okay now let us look at how the overall efficiency impacts the range of the aircraft.

(Refer Slide Time: 35:11)



Now before we go that we need to make a certain set of assumptions in order to derive this expression for range of an aircraft and how overall efficiency impacts it okay what is range firstly range is the distance that the aircraft can travel without refueling right, so that is if you are given a particular mass fuel what is the distance that the aircraft can tow now if you look at long-range aircraft typically long-range aircrafts there is firstly the aircraft is need to taxi to the runway then take off right line to a particular altitude typically around 11 kilometers and then it will cruise at that altitude right.

And further upon reaching the destination or getting close to the destination it has to climb down descend and then land and then taxi to its docking position right all these operations are there but most of the flying is done in the cruise range okay, so we will only consider that part when we are looking at what is the range that it can do okay, so most flying is done in level flight category


hence we ignore climb descent takeoff and landing we also need to make another assumption which is quite valid for civilian aircrafts.

That is the mass of the aircraft changes only because of you are expelling out fuel okay fine which is not true in case of military aircrafts especially they have to drop bombs or drop fuel tanks and things like that or fire missiles then the mass of the aircraft changes because of other things but in case of civilian aircraft the entire mass of the aircraft only changes because of fuel being expelled out.

To aircraft change is only due to okay, now so we made these assumptions now let us look at what is a level flight what are the things that are true in level flight thrust must be equal to drag and lift must be equal to weight, so for level flight.

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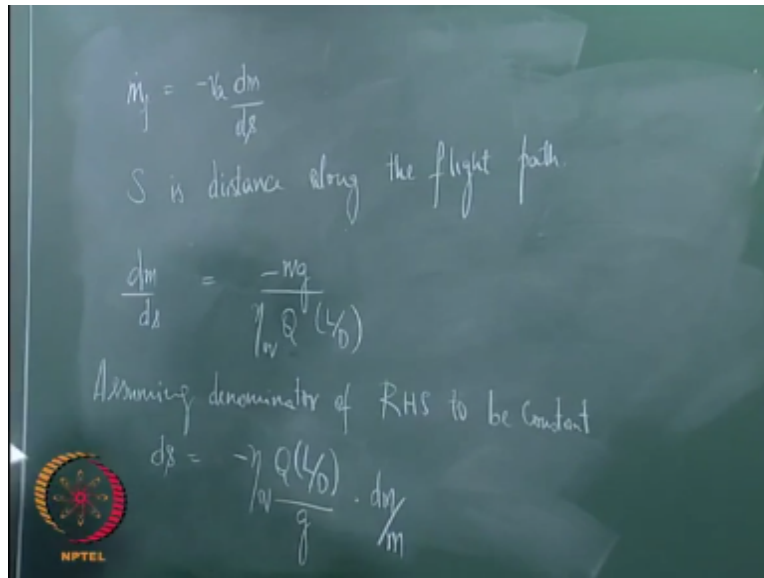
$T = D, L = W$
 $M \rightarrow$ instantaneous mass of the aircraft
 $T = L(D/L) = \frac{mg}{(L/D)} \rightarrow$ aerodynamic parameter
for Boeing 747 - 17 during cruise
Concorde - 7 during cruise $M=2$
Thrust power, $TV = \frac{mgV}{L/D}$



So T is equal to D and lift is equal to weight and let me call E_m as the instantaneous mass of the aircraft then I can write this expression for thrusters thrust must be equal to lift into D / H okay and I can again rewrite this is lift is nothing, but weight rate as M into g_m is the instantaneous mass of the aircraft divided by L / D is a aerodynamic parameter, okay and you can be defined as such for a overall aircraft the idea what are the typical values for L / D for an aircraft L / D for aircrafts righty it is something like for Boeing 747.

It is somewhere around 17 during cruise and one code it is somewhere around 7 during its cruise at Mach number 2 for Concorde it is lower because drag is higher because it is going at supersonic speeds right, so which is better anybody of a larger value of L / D is better or a smaller value of L by D is preferable which is preferable larger or smaller value of L by D is preferable because then you would have to spend less fuel okay, so which is why con codes are very expensive right L / D is small L by D is large here.

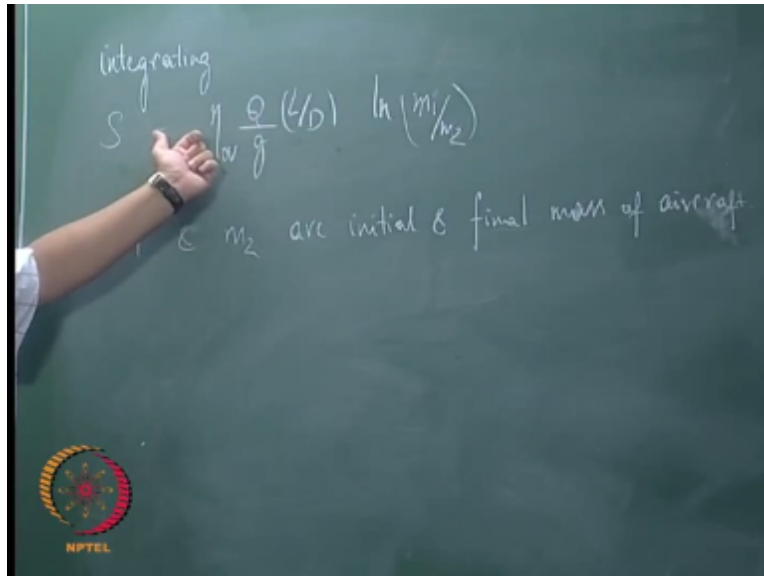
So you can afford it and you something like house pero one that flutters very quickly and flies around I mean it is a short distance flights mostly the L / D for that is around four whereas albatross which is more of a long distance flier and mostly sails through has a L / D of around 20 so L / D also indicates what you can do if you have a smaller L / D then you are mostly restricted to short flights if you have a large L by D then you can think of longer flights okay coming back here so from here we can define thrust power as T into V_a and T into V_a is given by in this case energy $V / L / D$ and we know from our efficiency calculations and other things what is thrust into V_a we had called it F into V_a .
(Refer Slide Time: 46:13)



In that it is nothing but it have overall into $M_0 F$ into Q right okay, so now we combine these two we can write $m \dot{F}$ is equal to $mg V_a$ into Q okay, now this is mass flow rate of fuel this is a part of the entire mass of the airplane right, so using that fuel to be part of aircraft weight I can write $m \dot{F}$ is nothing but dm / dt and because this is going to decrease right it should be with a minus sign okay, now I will change this variables from dm / dt to ds because we know that $V dt$ is nothing but ds is the incremental range.

So I get from this dt is nothing but ds / V_a and if you substitute for that here you will get $m \dot{F}$ by ds okay, now s is nothing with the distance along the flight path S is okay, so using this and this write $m \dot{F}$ I know that expression so I can write an expression for dm/dS as $-mg /$ divided by η overall Q into L / D , now in this case if we assume the overall efficiency to be constant and Q to be constant and L / D to be constant we can integrate this expression assuming to be constant then $D s$ is nothing but how we can integrate this.

(Refer Slide Time: 51:10)



Integrating I will get S is equal to τ overall into $\log m_1 / m_2$ m_1 is the or I will call it m_i & m_f now I will retain it as m_1 m_2 m_1 and m_2 are initial and final mass of aircraft okay, so in this expression you see that if the overall efficiency is high you get a longer range or also if your L by D is better you get a longer range and we will stop here and look at in the next class on the cycle analysis for various air breathing engines thank you.

Online Video Editing/Post Production

K.R. Mahendra Babu

Soju Francis

S. Pradeepa

S. Subash

Camera

Selvam

Robert Joseph

Karthikeyan

Ramkumar

Ramganesha

Sathiaraj

Studio Assistants

Krishnakumar

Linuselvan

Saranraj

Animations

Anushree Santhosh
Pradeep Valan .S.L

NPTEL Web & Faculty Assistant Team

Allen Jacob Dinesh
Bharathi Balaji
Deepa Venkatraman
Dianis Bertin
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Kamala Ramakrishnan
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Muthu Kumaran
Naveen Kumar
Palani
Salomi
Sridharan
Suriyakumari

Administrative Assistant

Janakiraman. K.S

Video Producers

K.R. Ravindranath
Kannan Krishnamurthy

IIT Madras Production

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