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Aerospace Propulsion

Hybrid Rocket Combustion

Lecture 41

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In the last class we were discussing about hybrid rockets or in this class let us look at how the burn rate relationship AGOX to the power of n is obtained so firstly to do that we need to understand how the burning takes place inside the hybrid rocket motor.

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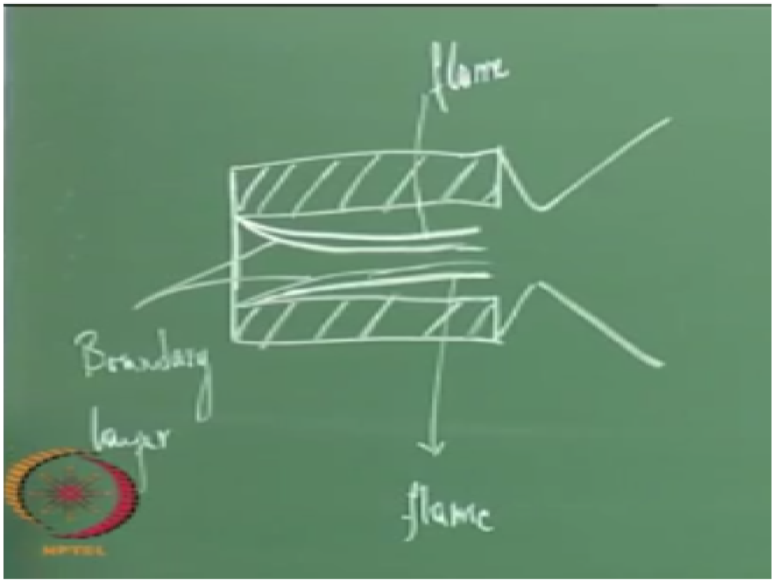

Now if this is the hybrid rocket motor then oxidizer is injected here right so a boundary layer develops and the combustion takes place inside the boundary layer okay so this is how the combustion takes place inside a hybrid rocket motor now if we look closely into this combustion zone then we will be able to look at what are all the processes that are taking place in that zone.

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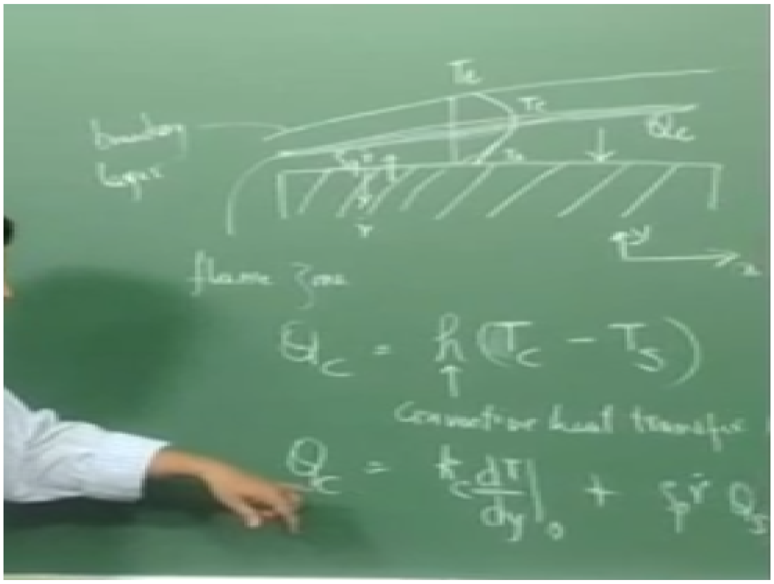

So let us say this is the solid fuel that we have then let us say we have taken somewhere in this region right where the boundary layer is already established so this is the boundary layers and let us say this is the flames own right now what is happening because of this boundary layer there is a heat transfer to the propellant which is convective in nature right and that heat transfer is being taken up partly for maintaining the temperature profile in the condensed phase and then partly to evaporate the fear.

So if we were to draw the temperature profile inside the boundary layer it would go something like this is Ts this is Tc and this is Te the combustion that is achieved here I mean the temperature that is a trade here is the highest simply because there is combustion and after which it is diluted by oxidizer right now the propellant burns in this direction and there is because of that there is a mass addition that is given by ρ Pr. okay mass is getting in and there is something known as blowing effect that we had discussed when we were discussing about erosive burning in solid Rockets.

Now so there is heat that is being transferred let me call it as Qc okay that is heat that is being converted into the propellant surface now this Qc I can write it as h into Tc - Ts okay this is the convective heat transfer coefficient and this is the adiabatic flame temperature or the combustion temperature and this is the surface temperature now this heat as I said is a part of it goes to maintaining a temperature profile and part of it is used for evaporation so Qc must be equal to let me call this direction as X and this direction is y.

So okay this is the temperature there is the part of the heat that is taken up to maintain the temperature profile and this is the Qs get seen earlier is the heat of evaporation at the surface so this we are seeing with regards to solid propellant that we can rewrite this part as ρP r . $C_P T_S$ - T∫ let me call the temperature here as T∞.

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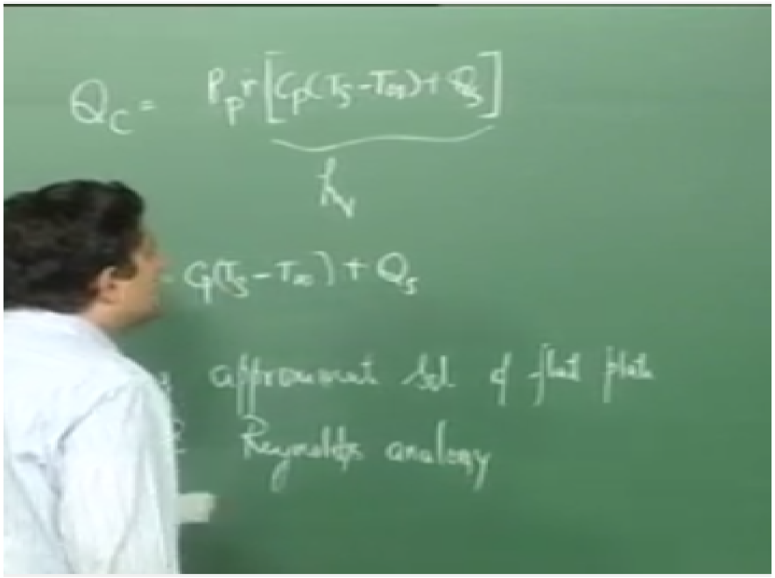

So I can rewrite this as Qc must be equal to ρpr. okay now for ease of our terminology and other things to follow let me call this part as hv hv is nothing but Cp TS - T∞ + Qs now we know that this problem is a boundary layer driven problem okay so what we are going to use is something known as Reynolds analogy what it says is there is a similarity between the momentum boundary layer and the thermal boundary layer right and we use that it is easier to calculate the momentum boundary layer there are correlations for it and we use that to calculate the thermal boundary layer.

This can happen when only what Prandtl number is close to 1 which in carry gases it is always close to 1 so we can make this assumption so using approximate Reynolds analogy.

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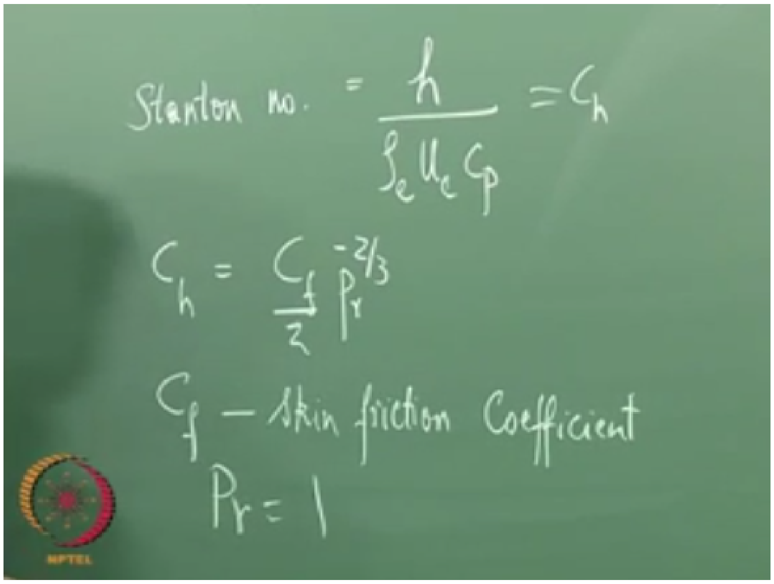

$$\text{Stanton no.} = \frac{h}{\rho_e U_e C_p} = C_h$$

$$C_h = \frac{C_f}{2} P_r^{-2/3}$$

C_f – Skin friction Coefficient

$P_r = 1$

That is there is similarity between momentum and heat or momentum and energy so we can define something known as a stanton number ue is nothing but the velocity here okay so h is the heat transfer coefficient ρe is the density at the beyond the boundary layer and Ue is the velocity beyond the boundary layer CP is the specific heat this is a non-dimensional number it is also known as diamond dimensionless heat transfer coefficient okay now if we use the Reynolds analogy what it says is that this stanton number indicated by Ch = Cf/2 where Cf is nothing but skin friction coefficient okay.

And we know that rundle number is close to 1 for gases so this term becomes merely Ch – Cf/2 now we have defined what a stanton numbers and we have found out that by Reynolds analogy we can equate it to this skin friction coefficient if we multiply the numerator and denominator of the stanton number by a Δ T that is given by Ts - Tc - Ts okay we will do that and see what we will get.

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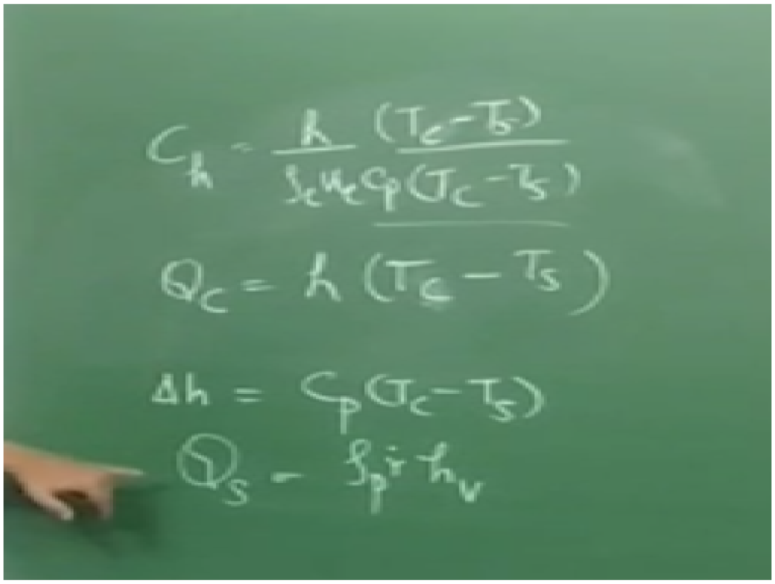

$$C_h = \frac{h\,(T_c - T_s)}{\rho_e u_e C_p (T_c - T_s)}$$

$$Q_c = h\,(T_c - T_s)$$

$$\Delta h = C_p (T_c - T_s)$$

$$Q_s - \rho_p \dot{r}\, h_v$$

So we had stanton number Cf = h/ρe ue Cp so we will multiply both numerator and denominator by Tc - Ts so it does not change anything to the left hand side but what is H into TC - TS that is the convective heat transfer right so Qs = h into Tc - Ts so what we will end up getting is sorry this is not Cf Ch Ch is equal to this which one oh okay Q_C fine now what we need to this part is the Q_C so what is this part we can define it as some change in enthalpy so we will call the $\Delta h = C_P\ T_C - T_S$ so what we will get this we also had the other term that is the heat.

That was taken up by the solid let me call that as q s that was is equal to ρp r. hv right but we know that these two must be equal right that was the heat balance at the surface for the heat balance at the surface these two must be equal.

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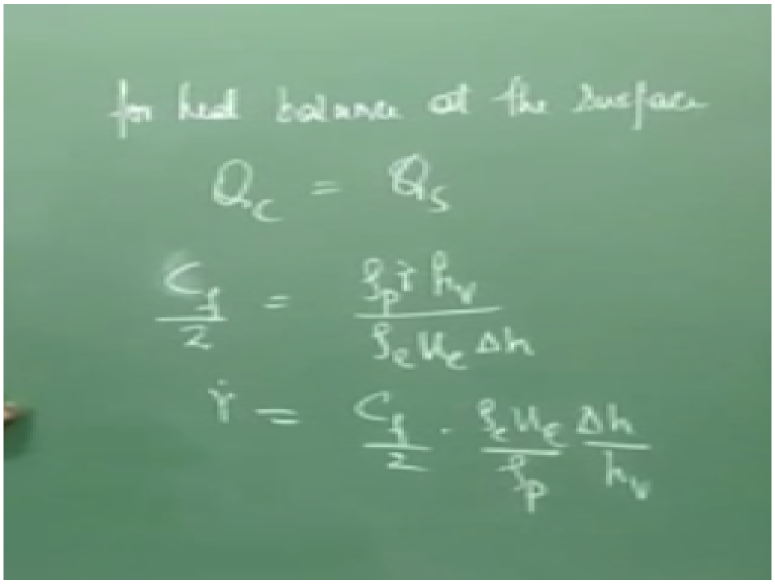

for heat balance at the surface

$$Q_c = Q_s$$

$$\frac{C_f}{2} = \frac{\rho_p \dot{r} h_v}{\rho_e u_e \Delta h}$$

$$\dot{r} = \frac{C_f}{2} \cdot \frac{\rho_e u_e}{\rho_p} \frac{\Delta h}{h_v}$$

So I can now replace in my stanton number definition this here right so and stanton number this also I know is nothing but Cf / 2 by two right so I will use that so I will get ρp or Cf / = ρp r . hv right or in other words I can rewrite a expression for r . as r . = Cf / 2 x ρe ue Δh/ hv / ρP now we have been able to get this expression for r. in terms of ρV x ue is nothing but in some sense mass flux and Cf /2 what we need to further determine is what is this Cf for the particular geometry that we have considered.

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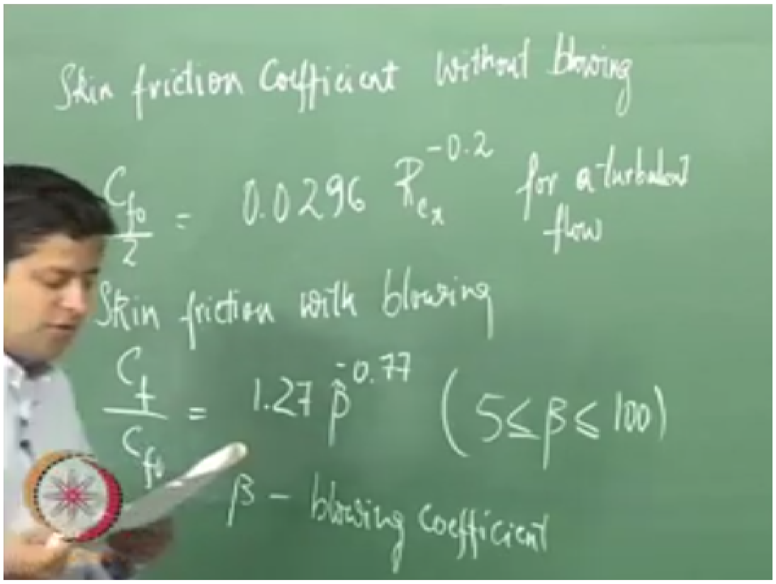

Skin friction Coefficient without blowing

$$\frac{C_{f0}}{2} = 0.0296 \, R_{e_x}^{-0.2}$$ for a turbulent flow

Skin friction with blowing

$$\frac{C_f}{C_{f0}} = 1.27 \, \beta^{-0.77} \quad (5 \leq \beta \leq 100)$$

β – blowing coefficient

So skin friction coefficient are without blowing we can write it as Cf0/ 2 is equal to in terms of Reynolds number this x indicates the location along the axis this is for a turbulent flow now what happens because of blowing is the boundary layer in essence tends to get thicken okay so the skin friction with blowing will be different and it can be given as Cf /Cf0 where β varies from π less than 1β is nothing but the blowing coefficient.

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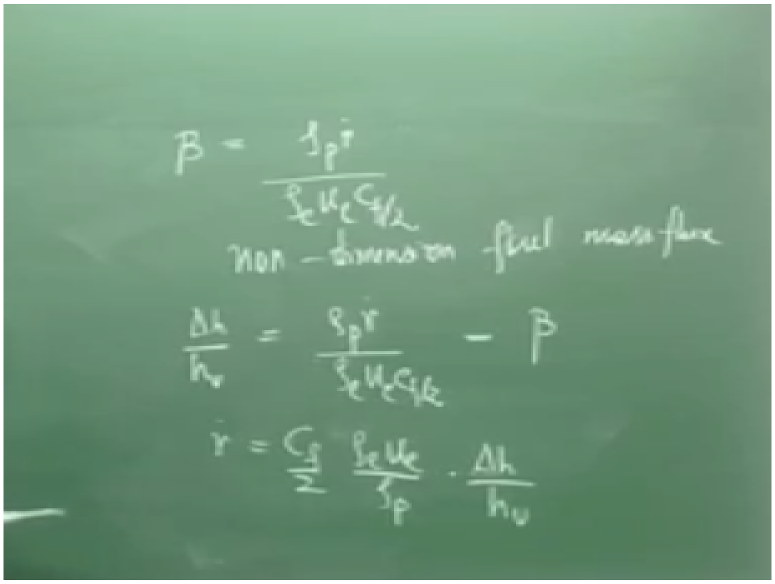

$$\beta = \frac{\rho_p \dot{r}}{\rho_e u_e C_f/2}$$

non-dimension fuel mass flux

$$\frac{\Delta h}{h_v} = \frac{\rho_p \dot{r}}{\rho_e u_e C_f/2} - \beta$$

$$\dot{r} = \frac{C_f}{2} \frac{\rho_e u_e}{\rho_p} \cdot \frac{\Delta h}{h_v}$$

And we can define β as follows β is defined as ρp r. that is the mass flux that is coming in from the surface to the mass that is going through the port okay. It is also known as non-dimensional so we have defined Cf in terms of Cf0 x β and we have β so we can now get the expression that we were looking for that is we had said that Ch is equal to this is nothing but ρp r . hv / ue that is this expression right.

So we will treat in use this expression if you look at this expression and the expression for β right there is some similarity between the two and we can write Δh \hv I it is nothing but ρpr .that is equal to β right so now I have this expression for r .here wherein I have defined Cf I have defined all other terms so I can get plug them in and get my expression for r .which we will do r. = Cf / 2ρe ue / ρp right now we know that Cf is nothing but Cf 0 x β so we will use that here and get our expression for r.

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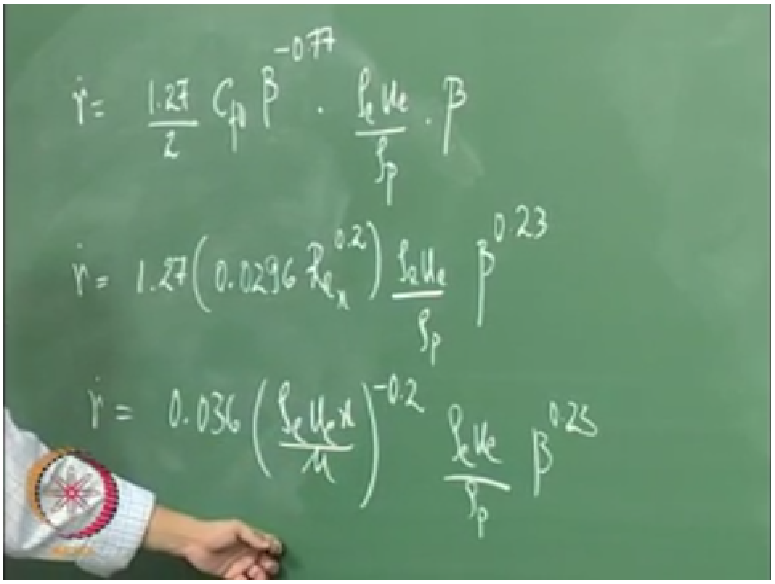

$$\dot{r} = \frac{1.27}{2} C_{f0} \beta^{-0.77} \cdot \frac{\rho_e u_e}{\rho_p} \cdot \beta$$

$$\dot{r} = 1.27 \left(0.0296 \, Re_x^{0.2}\right) \frac{\rho_e u_e}{\rho_p} \beta^{0.23}$$

$$\dot{r} = 0.036 \left(\frac{\rho_e u_e x}{\mu}\right)^{-0.2} \frac{\rho_e u_e}{\rho_p} \beta^{0.23}$$

So r. =1.27 /2 xCf0 x $\beta^{0.77}$ x ρe ue / ρa x β right because we know that Δ h/h y is nothing but β so Cf0 also we know in terms of Reynolds number as this expression here so if we substitute that we will get r . = β is – 0.77 and one so I can write it as $\beta^{0.23}$ Reynolds number is nothing but ρe ue x x/ μ so I get r.= so we have ρe ue here and here so we can Club those two.

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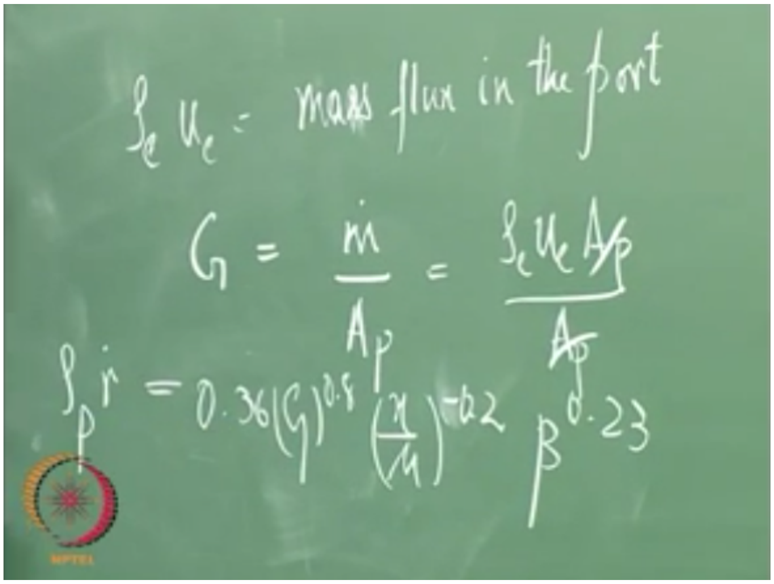

$\rho_e u_e$ = mass flux in the port

$$G = \frac{\dot{m}}{A_p} = \frac{\rho_e u_e \cancel{A_p}}{\cancel{A_p}}$$

$$\rho_p \dot{r} = 0.36 (G)^{0.8} \left(\frac{x}{\mu}\right)^{-0.2} \beta^{0.23}$$

And rewrite our expression as r. = and I can take the ρp that is here this ρ p I can multiply it back with the r. so I will get ρp r . is equal to what is ρ ue is nothing but the mass flux in the put right so mass flux let us define G as m . /Ap so that will be nothing but ρe ue so you have we can replace this with G right and we will get - s thank you okay so we come to this expression there is 0 here.

So we get this expression where in the burn rate is related to the mass flux through the port right and if you look at when we do experiments it is easier to characterize this in terms of the oxidizer mass flux itself because that is something that we have control of and we know what it is so which is why you will find that if you take a look at any hybrid related literature you will find that the burn rate is expressed as a function of G ox okay.

So we can recast this in terms of zero X and that is what you will find in most literature as a dot is a function of 0 X to the power of n and if you look at this expression you will find that it depends very weakly on X the axial location and beta but it is a very strong function of the mass flux right now let us look at the problem that we had said yesterday with regards to what happens to burn rate as we proceed in from the head end to the nozzle length.

If you look at this expression here it looks like what should happen to the burn rate as you increase X it should decrease right set the opposite yesterday right is that correct order is that wrong if you look at this yes you are saying but you are forgetting that there is a greater dependence on G if you look at what is happening as you move from the head end of the port to the nozzle end there is continuous mass addition and this is increasing as you go from heading to the nozzle end and this is raised to a power of 0. 8.

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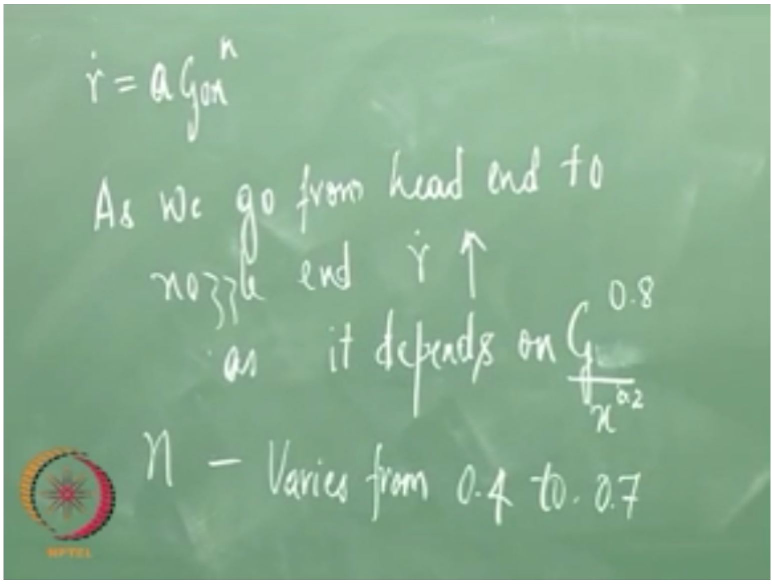

$\dot{r} = a G_{ox}^{n}$

As we go from head end to nozzle end $\dot{r}$ ↑

as it depends on $\frac{G^{0.8}}{x^{0.2}}$

n – Varies from 0.4 to 0.7

So therefore you will find that the burn rate as you go from heading to the nozzle length will always increase okay now people have done people have conducted experiments and found out that if you take the oxidizer mass flux the dependence of burn rate varies this n varies from 0.42 to 0.7 one of the things that we have kind of made an assumption right at the beginning was we had assumed something like a flat plate if we assume a pipe flow kind of a situation and redo the calculations.

We will probably get closer to what we what is observed experimentally, experimentally what we can get is in terms of the oxidizer mass flux and not the overall flux k overall flux involves that you take across-section and find out what is the fuel plus oxidizer that is coming which is a lot more difficult to do if you are doing an experiment, experiment you are better off knowing what is your mass flow rate of oxidizer and then calculating the oxidizer mass flux there was another problem that I said hybrid Rockets suffer from because of this n.

That is if you look at holding even in even when you hold the oxidizer mass flux constant we find that the fuel mass flow rate was changing now let us look at is there any condition at which the oxidizer mass flux if you hold constant or if you fold the oxidizer mass flow rate constant when the fluid flow rate also become constant.

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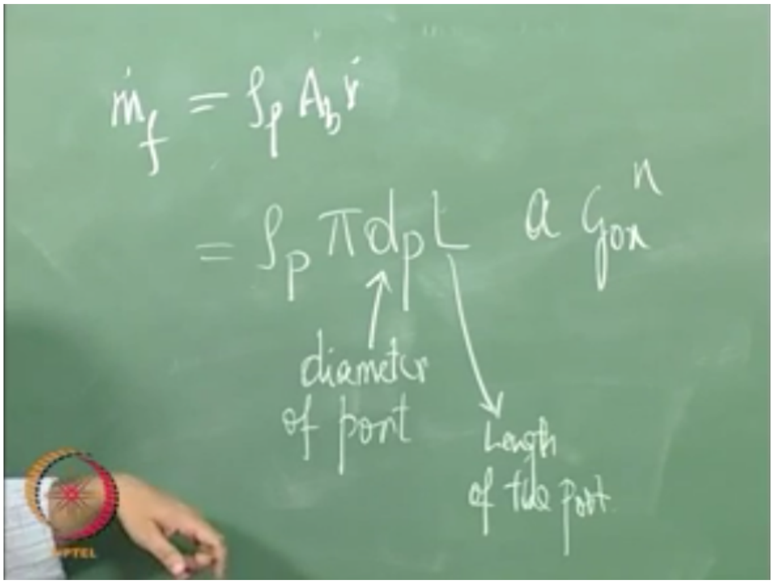

We know that the mass flow rate of fuel is given ρ_p A_b*I .so if you are taking a port configuration ρ_p into this is the diameter of the port and this is the length of the port this is the burning surface area into our dot is nothing but A into okay now let us determine what value of n will give us something that when you hold the oxidizer mass flow rate constant you can also get a few mass flow rate constant if you have to engineer such a hybrid then that is a very good situation.

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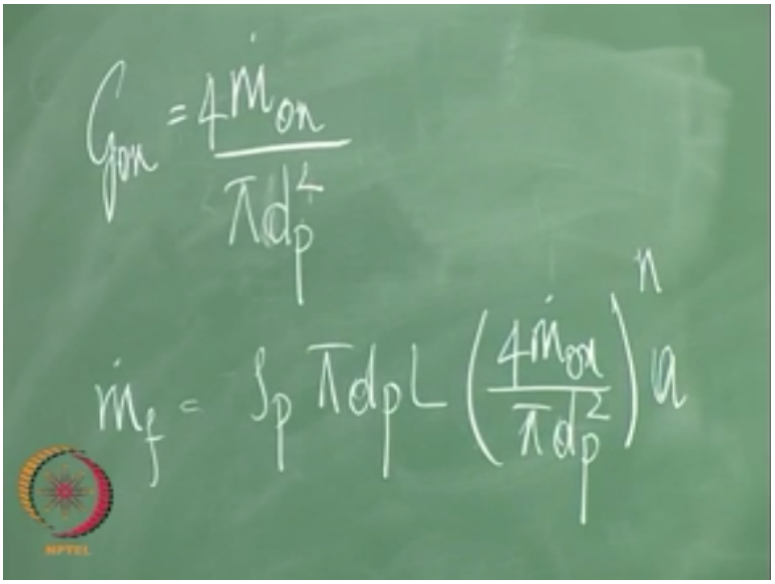

$$G_{on} = \frac{4\dot{m}_{on}}{\pi d_p^4}$$

$$\dot{m}_f = \rho_p \pi d_p L \left(\frac{4\dot{m}_{ox}}{\pi d_p^2}\right)^n a$$

Because you can operate it at a set specific impulse even though the diameters are thinking so Gio X we know is nothing but fine this is geo X so if we plug it into this equation we will get m dot Fo know this is you have 2 into n right in the denominator and if you look at this equation you have D to to the part of one end d to the power of 2n as very obvious that if you have a value of n that is equal to 0.5 for n equal to 0.5 m dot F becomes independent of the pore diameter so therefore you can operate it at a set o by F ratio so that you can choose that over F ratio such that it gives the maximum is B.

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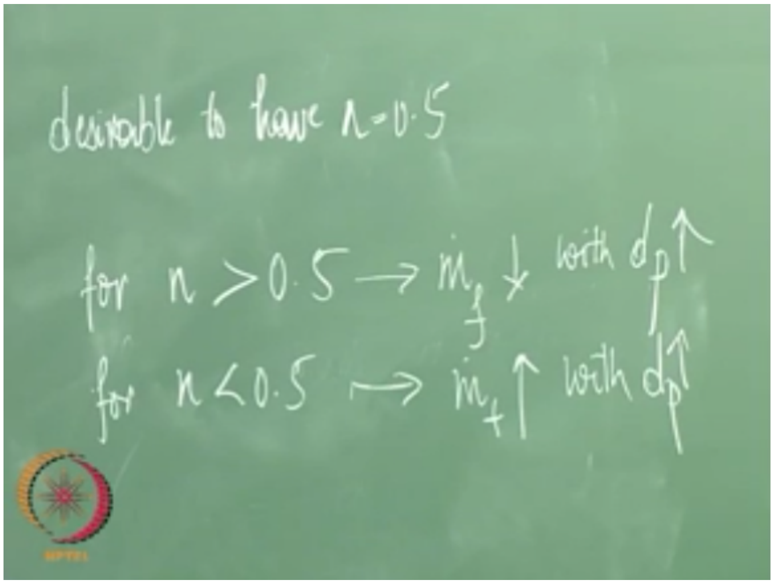

desirable to have $n = 0.5$

for $n > 0.5 \rightarrow \dot{m}_f \downarrow$ with $d_p \uparrow$

for $n < 0.5 \rightarrow \dot{m}_f \uparrow$ with $d_p \uparrow$

Now what happens if n is greater than 0.5 so it is desirable to have in equal 2.5 now for n greater than .5 what will happen to the mass of here as diameter increases it will reduce because you will have 2 N and you will have DP here so the denominator will be greater than the numerator so therefore as diameters increased mass flow rate of here will decrease and similarly or for the other case that is for n less than .5 okay this finishes our discussions on hybrid rockets so in a sense we have learnt three, three kinds of rocket engines in this course that is one is solid propellant the other one is liquid propellant and then lastly the hybrid propellant okay thank you.

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