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Aerospace Propulsion
Propellant Combustion – Combustion Modeling

Lecture 30

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In the last few classes we have looked at how solid propellants behave we had looked at how the solid propellants burn rate vary with pressure and initial temperature. If you remember in one of the earlier classes we had said.

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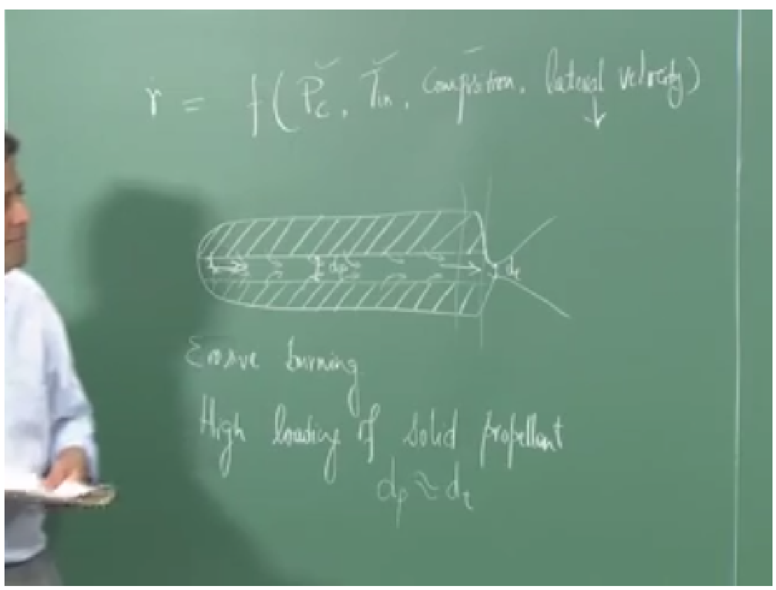

Burn rate of a solid propellant is a function of chamber pressure, initial temperature, composition and we had said lateral velocity these were the four things that are depended on we have till now seen all these three in this class let us look at how the burn rate varies with lateral velocity. What do we mean by that is something as follows, let us say this is the solid rocket motor right, this is the solid rocket motor if the burn rate from this point to this point varies because the flow rate in this direction is different from here to here.

Then we say it depends on lateral velocity and this particular phenomena is called as erosive burn, when does erosive burning take place. See we understand that a rocket motor there is a particular volume, now we always want to maximize the volume that is there in the motor we want to fill it with propellant when we do that we end up become making let me call this as dp pole diameter to this as dt throat diameter.

When we want to load more invariably the pore diameter becomes equal to the throat diameter dp, becomes nearly equal to dt then erosive burning effects are felt more. Now why does this happen is if you look at what is happening from the head end to the nozzle end there is continuous mass addition because of burning right.

So mass is getting added here and whatever mass gets added is also flowing through the foot and if you look at somewhere in this region, this region the flow rate is very similar to what is going to the tote so the flow rates are very, very high when the flow rates are very high there is additional heat transfer because of convection and due to which the burn rates in these portions could be higher and that is known as erosive burner.

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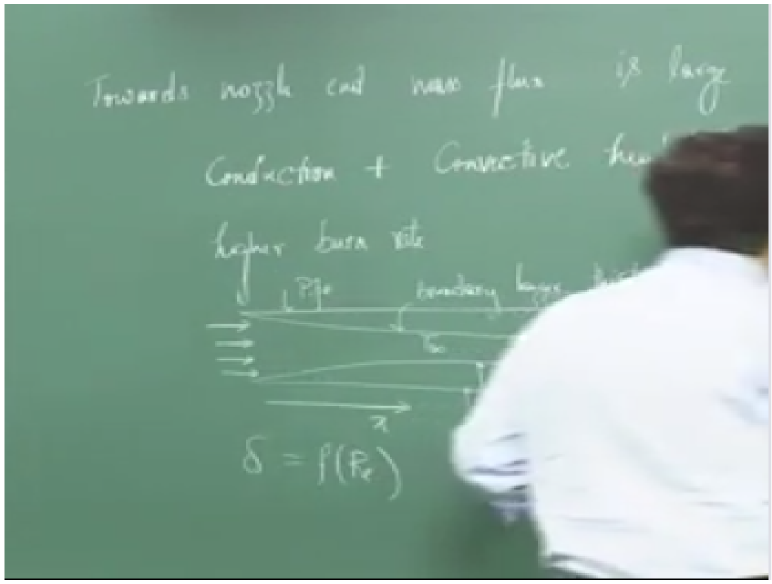

Towards nozzle end mass flux that is mass flow rate per unit area is large and due to which there is in addition to conduction heat transfer you will also have convective heat transfer leading to higher heat transfer to the propellant and therefore higher burn rate, right. What is the difference between conduction and convection here we are looking at there is a flow in this direction and heat being transferred perpendicular to the flow direction.

Why do we still call it convection because if you look at it there is conduction that we had already seen earlier, in the earlier classes that conduction heat, heat transfer is the reason for the propellant to burn. Now in addition to that we are saying there is convection, now the flow is in this direction but there is an additional heat transfer because of the flow to the propellant ,why does this happen if you look at any pipe flow problem there is something called as boundary layer.

And boundary layer thickness changes along the x direction, now there is flow from left to right like this and because of the flow there is a boundary layer that gets established and depending on the thickness of this boundary layer you will have if the boundary layer thickness is small you will have more to heat transfer, the boundary layer thickness is large you will have less heat transfer by convection, okay. This thickness of the boundary layer is determining the heat transfer which is why it is called as convective heat transfer because outside the boundary layer it is $T\infty$ and this is t wall if you have lesser thickness of the boundary layer this gradient of $T\infty$ -T wall/∂ will increase and you will have higher heat transfer.

Now if you come back here and look at this figure what is happening because of mass addition if I take sections like A,B and let me call this as C if I look at C the mass added mass flowing through this section is only this part of the propellant is contributed by this part of the problem. So there is very small mass flux through this region but when you come to here there is a much larger mass flux and when you come here is an even larger mass flux, okay.

So the mass flow rate from head in to nozzle end or the mass flux is increasing right, now this boundary layer thickness ∂ is a function of Reynolds number ∂ goes as or is a function of Reynolds number and if you look at what Reynolds number is, Reynolds number we can write it as.

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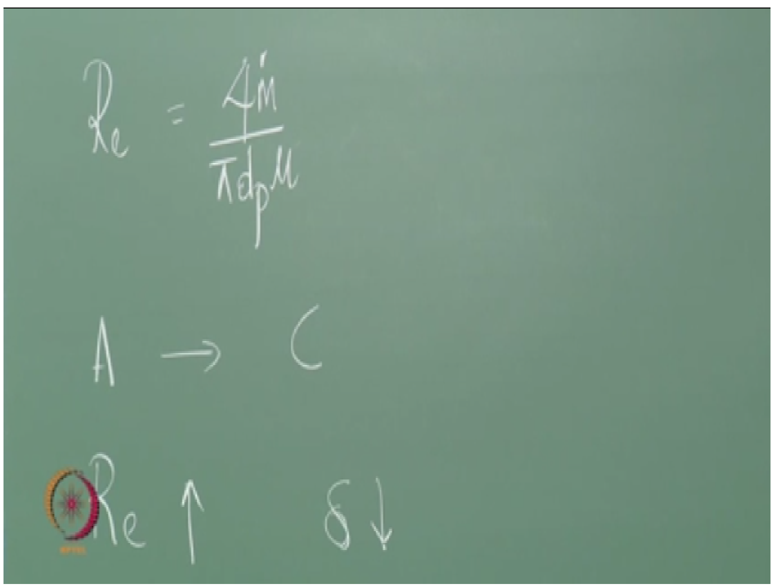

$4\dot{m}/\pi d_p\mu$ where $\dot{m}$ is the mass flow rate local mass flow rate, dp is the foot diameter and μ is the coefficient of viscosity, so if you have this as you see mass when it gets added more the Reynolds number will increase. So as you move from A to C Reynolds number keeps on increasing and the ∂ has a inverse relationship with Reynolds number that is as Reynolds number increases ∂ will decrease.

So because of it the boundary layer thickness decreases as you go from head into the nozzle end and because the boundary layer thickness decreases there is enhanced heat transfer at the nozzle and this is what we call as erosive burning. Now let us look at how does it affect the propellant combustion right.

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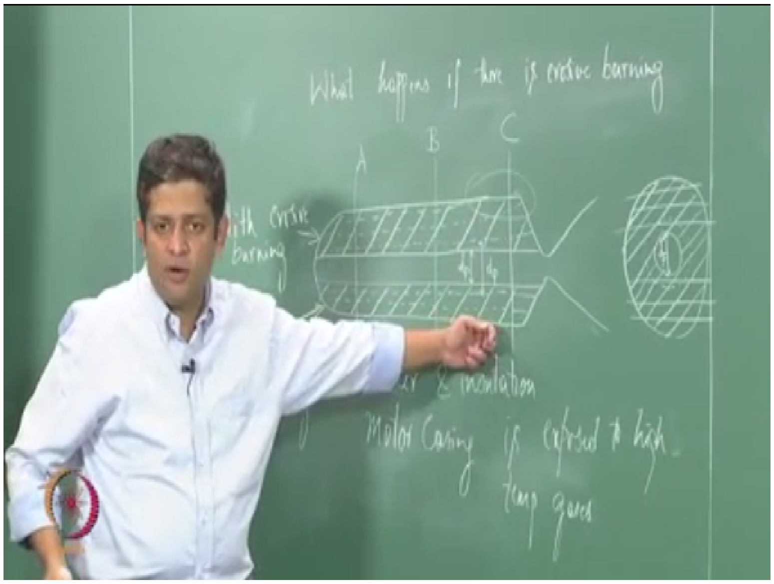

So let us look at what happens if there is a erosive burning, now when there is a erosive burning if we look back at the solid propellant that we had earlier, and if I look at the case with erosive burning on the top surface without erosive burning on the bottom surface we will get a picture like this, now I will take the top surface to be one where an erosive burning is predominant as the propellant burns upto some portion there will be uniform burn rate.

I am close to the nozzle end there is a higher burn rate so which means the burning will go in this fashion whereas if we look at uniform burning it will go in this fashion, here it will go in this fashion and maybe towards the end of the burning if you look at this portion the liner material, liner and insulation and also motor casing is exposed to high temperature gases.

And if it is exposed to high temperature gases it could have a detrimental effect you could have a whole being created here and the mission could not will not be sometimes accomplished and you might have a catastrophic failure which is not something that is desirable other than that even in case it does not happen this way reason to avoid it is what we will see next, if you look at the pressure versus time curve.

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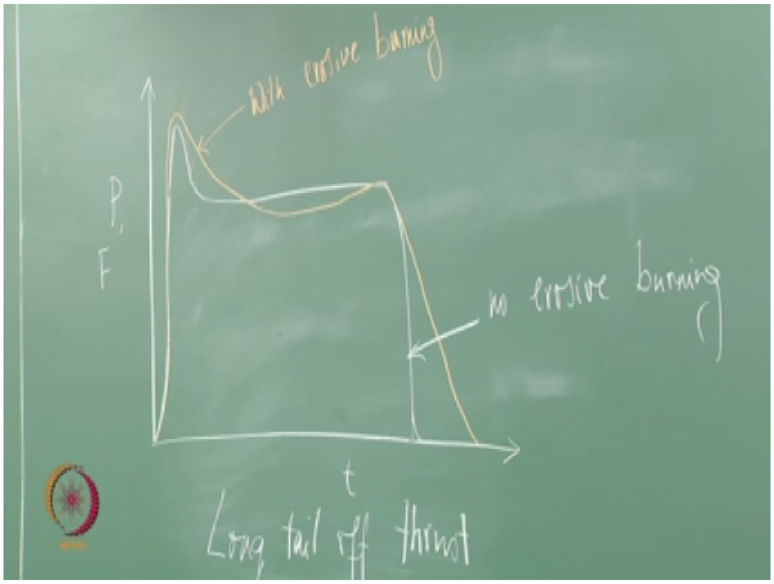

This could also be pressure comma thrust versus time, this is the line without a erosive burning and now if you super impose on the same graph the line with erosive burning, see if you look at the white line that is the line without any arrows you burning the pressure initially increases and drops to its equilibrium pressure and then remains there and the tail off is also very sharp, right.

So you have a very sharp tail off here whereas, whereas if you look at the case without a erosive burning you will find that the tail off is not sharp and the it takes in a much longer time for it to come back to the equilibrium pressure. So there is a much larger time that you have and there is a kind of hump that you see like a camel hump here.

This is a characteristic feature of erosive burning if you see a pressure tanker like this you can be very sure that there has been a erosive burn. So with a erosive burning you have long tail of thrust right, as seen here now why is it so detrimental although the area under the curve might be the same which means that the overall impulse is the same the thrust verses chi time curve is different which means that the mission might not be accomplished.

If the missile has to go to a certain distance might not go there because the thrust versus time curve is not actually followed, so that is something that is not a desirable feature and therefore we want to have very little arrows you burning as possible. If you come back to this figure here as I said earlier the top portion is a with arrows you burning, and the bottom portion is with no erosive burning, right. One of the good things about erosive burning is it is only predominant in the initial few seconds, right.

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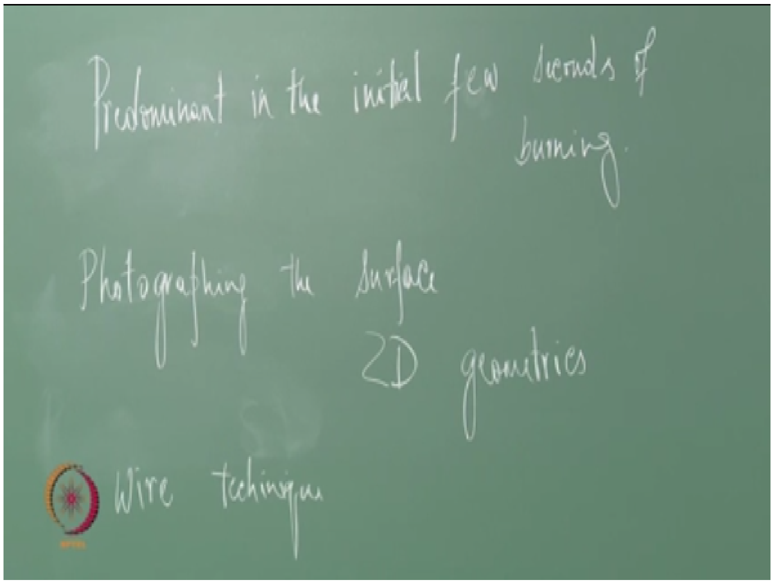

So it is its effect is predominant in the initial few seconds of burning, why is that if you look at what is happening here as the propellant burns the port area is from this to this there is an increase in put diameter so the port area increases the mass flux gets reduced and because the mass flux gets reduced the Reynolds numbers will become lower and that could lead to lesser convective heat transfer and therefore the erosive effects will come down within crease in burn time, okay.

Now a erosive burning has been known for a long time and people have done experiments to determine how to quantify erosive burning, there are two sets of experiments people have typically done to quantify this erosive burning, one of them is photographing the surface that is if you have a quartz window here and if you have a two dimensional grain instead of a three-dimensional green, right.

Then you will be able to see how the port evolves with time if you take high-speed photographs and you will be able to make out that burning here is normal whereas burning here is increased because of heat transfer that is what is done in photography technique, so they take photographs high-speed photographs and compare how these propellant surface evolves with time and they will be able to quantify erosive burning.

So this can be limited only to 2D geometries and there are x-ray techniques which allow us to even look at three dimensional geometries right that is also possible if you use x-ray photograph. Now the other technique is, wire technique that is let us say we were looking at this cross-section from this side, now this is the port right, this is the port dp.

Now if I have wires here right, and also on the other side and if these wires are electrically activated that is there is a current passing through them when the flame front reaches them as shown here as the propellant burns when the flame front reaches them these wires get cut because they are exposed to high temperature gases now and the connection is broken depending on the time at which the connection is broken you can figure out what is the burn rate.

And if you do that at different cross sections as I said earlier ABC you will be able to find out the burn rate across the length of the rocket motor. So how does the burn rate vary across the length is what we will get and you will also be able to quantify arrows you burning. Now let us spend the next few minutes trying to analyze if we can analytically predict the same that is what happens to the equations of burning and other things.

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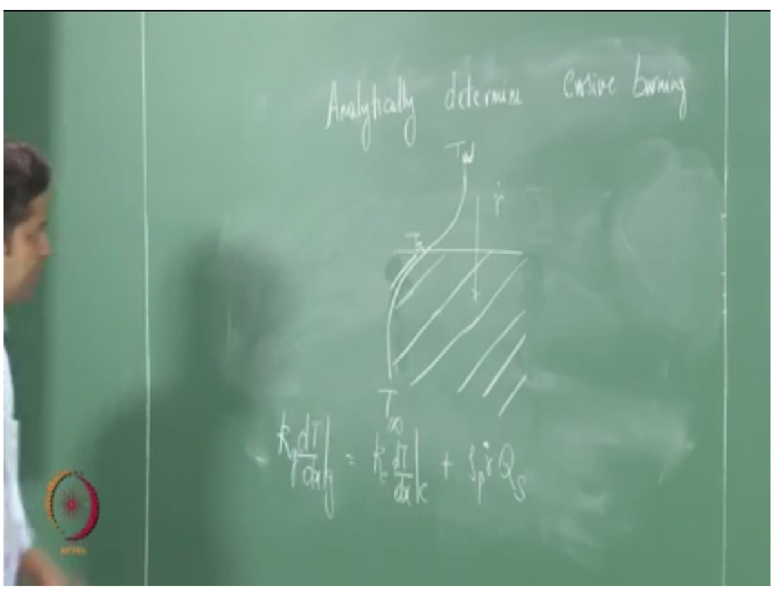

So analytically, so let us look at how to determine the burn rate analytically, now if you remember our burn rate equation for a solid propellant grain we are in this let us say this was the solid propellant and it is burning in this direction $\dot{r}$. Now if you look at the temperature profile temperature profile will be something like this right this is T_∞ T_s and T adiabatic right, so you have this kind of a situation we have discussed this earlier.

Now if we do the heat balance at the surface which we also discussed some time earlier you will get the heat flux coming from the gas phase must be equal to the heat flux going into the condensed phase right, $+\rho_p\dot{r}Q_s$ okay, this is the equation that we will get for the heat balance at the surface, so this is the gas phase heat flux not only must it maintain the temperature profile in the condensed phase.

But it also has to provide heat for the heat of phase change it also has to provide for heat for phase change. Now how does this change when we introduce a erosive burning right, that is what we are looking at and we said if you remember that when you have a erosive burning you also will have in addition to conduction this is conduction heat transfer you will also have convective heat transfer, right. So if I rewrite this equal expression and if i assume an analytical solution for this term I will get.

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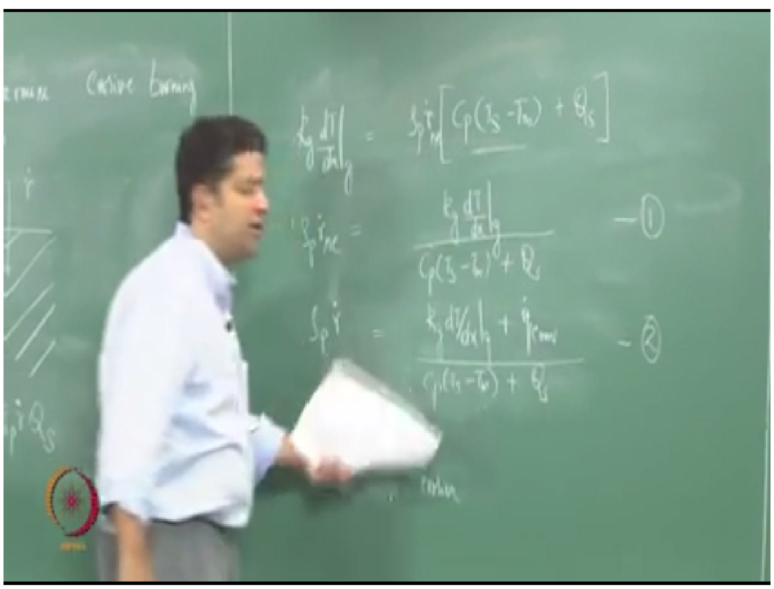

k_g must be equal to $\rho_p\dot{r}Q_s$ I can replace this term with the term that I have shown here this is from the analytical solution of this equation assuming that the flow is the heat transfer is in one dimension only and the heat transfer is steady. You can show that this term is equal to this term here. Now if I look at this from this I can get my burn rate expression as $\rho_p\dot{r}$ is equal to let me call this as the non erosive burn rate right.

Because we need to make a distinction between a erosive burning and non erosive burning so I get for the non erosive burning this kind of an expression. Now I also said that when you have a erosive burning you will have in addition to conduction heat transfer from in the gas phase you will also have convective heat transfer, so we do that and if we call the burning because of erosive burning effects also included as a $\dot{r}$ right, you will get where $\dot{r}$is nothing but $\dot{r}$ non erosive indicated by ne +$\dot{r}$ erosion okay.

So this $\dot{r}$ includes both the erosive component and the non erosive component so we have these two equations if we divide let me call this as 1, let me call this as 2 if we divide 2/1 what do we get.

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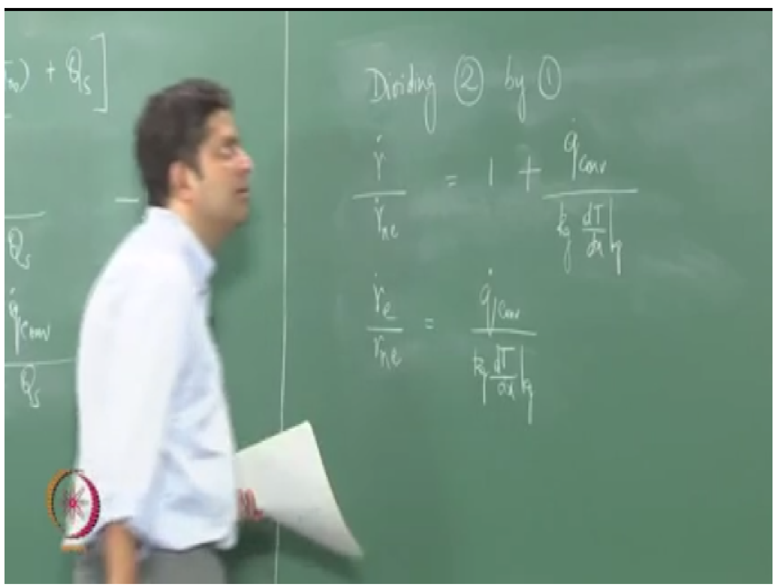

So we get firstly you get $\dot{r}$/$\dot{r}$ non erosive must be equal to these two terms will cancel each other out these two terms will cancel each other out, so we will get this is the kind of expression that we will have for the ratio of erosive overall burn rate to the non erosive burn rate and we also know that $\dot{r}$ is given by this so if we were to incorporate it we get this ratio for $\dot{r}$e which is the erosive burning component to the non erosive burning is, right.

So this is the expression that one gets connecting the erosive you and the non erosive burning rights, and you see that if Q convertor is small or Q convertor is nearly 0 then there is no erosive burn right. Now people have done a lot of experiments on erosive burning and people have found out what are the effects of erosive burning with a high burn rate propellant with a low burn rate propellant what happens.

Which do you expect will have higher what should I say higher erosive affects the one with higher burn rate propellant or lower boundary program, which would you expect will have higher erosive affects the high burn rate rope lines or the low burn rate propellants, we will see that just now.

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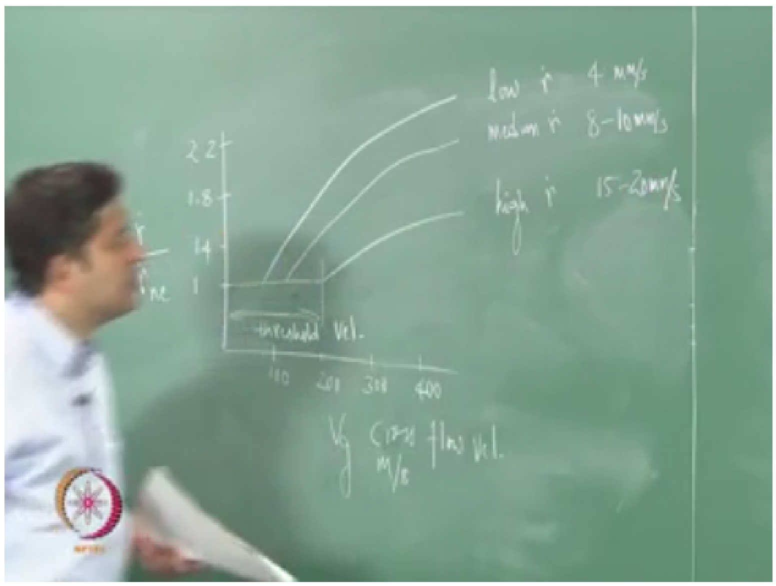

So if I were to plot on the y axis $\dot{r}/\dot{r}$ non erosive okay and, on they-axis plot vg or cross flow velocity it is very easy to varying the velocities so people have done experiments by having different velocities flowing over a propellant cross-section then you get the increase or decrease in burnet because of this so that is what you have on the x axis this is in m/s, high order there are three cases here high burn rate propellant which is indicated by this line lower burn rate indicated by this line and medium bond rate.

As you can see the high burn rate propellant is least affected by erosive burning and there is a velocity up to which this is known as threshold velocity, the velocity up to which erosive burning effects are not felt at all it shows one as the number of this ratio and beyond which the erosive burning effects kicking and the high burn propellants are the least affected by erosive burning and the low burn rate propellants are the most affected by erosive burning.

So if you were to ask ourselves this question why is it that this is happening right, why is it that low burn rate propellants are more affected by erosive burning whereas high burn rate propellants are least affected by propylene burning I am sure most of you might have studied some things like boundary layer flow control and other things in your in your fluid mechanics wherein they do section of the boundary layer.

In order to reduce the boundary layer thickness what we have when we have a propellant burning here is when you have the propellant burning.

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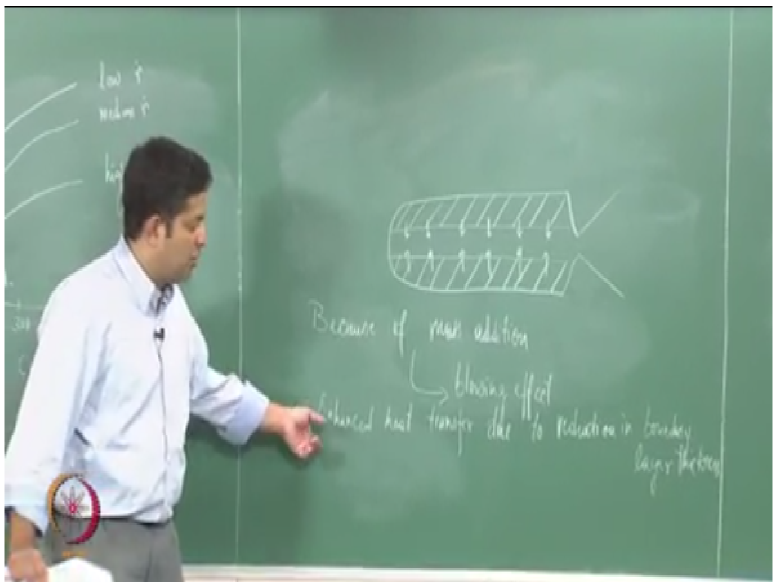

There is mass addition in this direction or there is something known as blowing effect that is this tends to blow away the boundary layer because of mass addition there is something known as blowing effect that is the up to a certain threshold value. Now if we see there is one blowing effect then there is enhanced heat transfer due to reduction in boundary layer thickness.

So there are two effects one is the blowing effect one is the enhanced heat transfer because of reduction in boundary layer effects upto this threshold value there the this effect is not as pronounced beyond this point the enhanced heat transfer because of boundary layer thickness kicks in and you will have a erosive burning effects. And if you look at it the one prop the propellants that have high burn rates right, are the ones that have a very large mass addition because of burning here.

And due to which the boundary layer tends to be blown off, so therefore you will find that high born bred propellants are least affected by a erosive burning, whereas low burn rate propellants are most affected by erosive burning. Yes, these are really something like low burn rate is somewhere around 4mm per second then this is like 8 to 10mm/s then this would be something like 15 to 20mm/s right.

So if you have very low burn rate the more effective boundary layer additional heat transfer because of boundary layer thickness reduction, and if you have high burn rate least effect of that right. Now if you see this analysis there was a lot of analysis like this done by a lot of people working across countries and lots of groups worked on this and they came up with such correlations.

All of them are dimensional in nature and therefore you will find that it is very difficult if you are designing a new rocket motor to understand whether it will have a erosive burning effects or not, what would be very useful for the industry would be if you can have a non dimensional analysis and then using that non dimensional analysis people can a priori predict whether it is going to have any erosive burning effect or not. And Paul and Mukunda have done these kind of studies and they are shown here.

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Now what they did was they collected all the data that was available in the literature of experiments compiled by various groups but just one group were various groups and put them all together on a non-dimensional axis and I will show you what the non-dimensional parameters where, they had a plot of $\dot{r}/\dot{r}$ non erosive verses g and what they found was I will explain in a minute what g is, g is some non dimensional parameter.

And if they plotted $\dot{r}$ versus $\dot{r}$ non erosive burning verses g they found that all the data of all the experiments that were conducted prior to this work fell in this range that is you all the experimental points falling within this range right. So the curves here essentially captured all the experimental data and they derived an expression for this dotted line here. Now let us look at what g and V represents.

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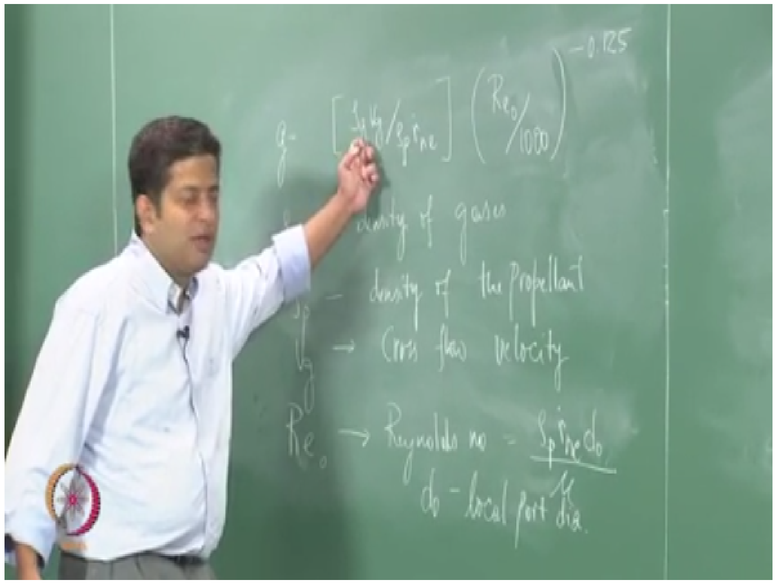

G is nothing but this is what g is where ρg is density of gases in the combustion chamber ρp is density of the propellant vg is cross flow velocity and Re0 is Reynolds number which is given as ρpṙne d0 which is the initial diameter or the local pore diameter this is the d0 is the okay, so if you are able to calculate Reynolds number Re0 and also ρg and vg you can get g, now using this g they came up with an equation for this dotted line here.

That is not because it depends on which cross section you are at right, there is across flow and depending on ABC if you remember earlier figure ABC depending on that cross section ρg vg will be smaller at the initial cross sections a and will be larger towards c right, so this varies with along the axis along the length of the rocket motor whereas this is a constant number, right.

Any location that you take ρp vg this is again a local diameter at that location okay, so now you have to write an expression for that expression for that is given as follows.

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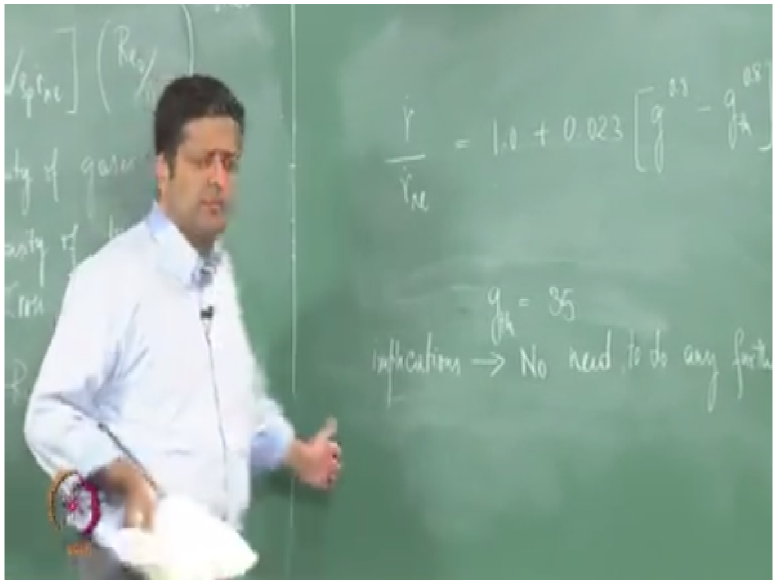

$\dot{r}/\dot{r}$ine is nothing but 1.0+ this H is a heavy sided function right what does the heavy side function you if g is less than gth it will be 0 which means that the equation will recover 1 right, and which is which represents this part and if g is greater than the threshold value then it will have a value of unity, so this will have two values 0 when z is less than gth which is nothing but g threshold right.

And one when g greater than or equal to gth so yeah, g threshold value is 35 gth=35 that corresponds to this location here okay. So if you have 35 as the threshold value up to 35 this will be 0 this entire term will be 0 so you will get one which represents this portion and after that it represents this dotted line. Now what are the implications of this the implications of this are irrespective this is a non dimensional analysis hence it is very powerful because for any motor that you design you can get these numbers.

And once you get these numbers you can find out whether there is a erosive burning or not you just have to look at this either this graph or use this equation and find out whether there will be erosive burning or not. So the implications are very strong no need to do any further experiments okay, yes. No, wherever diameters are large let us say you have a complicated green grain geometry right, it might be large at some cross-section wherever that is this is d0 so you can take the local pore diameter at that cross section you can calculate it for each and every point across the cross section of the across the length of the motor it need not be at one point you can do it at various points and you can look at the erosive term as being added to the non erosive term and you can come up with a software for how the grain evolves with time, right.

Because you are now able to predict the burn rate so this is away powerful tool because it gives the designer the freedom to use this and to come up with burn rates prediction of burn rates even without doing any further experiments, okay. So I will stop here this finishes our discussions on solid propellant burn rates, solid propellants and solid propellant burn rates grain evolution and all that that we looked at erosive burning then grain geometry in all these discussions over in the next class we will look at liquid rockets, thank you.

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