

**Indian Institute of Technology Madras
Presents**

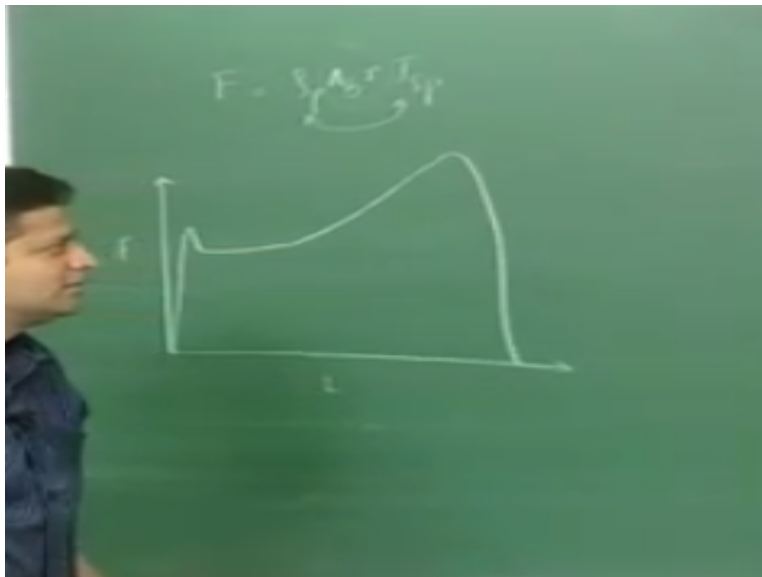
**NPTEL
NATIONAL PROGRAMME ON TECHNOLOGY ENHANCED LEARNING**

**Aerospace Propulsion
Solid Rockets – Grain**

**Lecture 27
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In the last class we have seen how the chamber pressure and thrust varies with burning surface area and throat area and initial temperature in this class let us look at how to design a grain okay typically if you are in an industry what is given to you is a thrust curve okay, project managers will hand down to you across time curve and you are supposed to meet it with whatever propellants that you have okay. Please remember whenever we are talking about thrust and curve of if you remember the thrust equation.

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F is equal to right now ρp and $I_s p$ are fixed depending on your choice of the propellant and nozzle right the only parameters that you can lay dear a B and \dot{r} let us say you are given a propellant with a particular I. how do you then design the grain to meet the thrust time

requirement the thrust time requirement can be varied depending on the applications launch vehicle might have a thrust time curve something like this right.

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Whereas a missile might have a very different thrust tanker yeah this might be the thrust time requirement for a launch vehicle where as this could be the trust time requirement for a mission typically a missile you would want to take off at very high acceleration. So as to overcome any wind loads and other things right, so you would have a very high boost thrust after which you will come down and try to maintain the sustained portion for a long time so as to be able to reach the target okay.

So how do we meet all these different requirements if we are given a propellant whose density and ISPs are fixed and r dot very little to play around with the margin for playing around with r is smaller, so how do we design such a propellant. Now if you remember our earlier discussion we said r is nothing but burn rate along the local normal right.

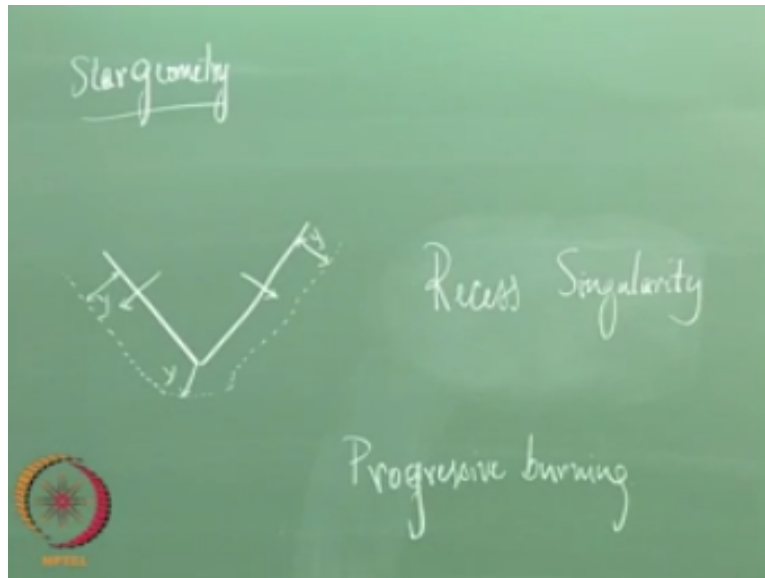
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We can now look at different kinds of geometries firstly let us look at a neutral burning grain that is if you have a cigarette burning configuration then as time progresses the burning surface will move in this fashion right. Now how does it look like if you have a tubular grain if you have a tubular grain and let us say it is burning from inside to outside you will have concentric circles right the same if it is burning from outside to inside this will give you a progressive grain, now as I said earlier this i. you have a small amount of flexibility here not a whole lot now we have to get everything that we want or a lot effect through the one burning surface area variation right.

So if you have if you have to design a large motor large thrust motor then if you cannot have an N burning grain and if you are still desiring something like a neutral burning up to some portion and then progressive how do we go about designing such screens, remember being in our earlier discussion we talked of something known as Star grain or star geometry.

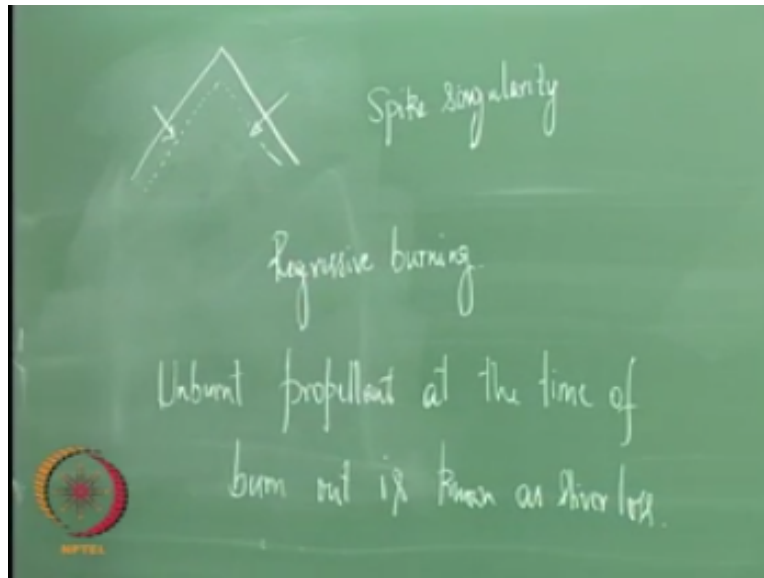
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And we said that it can give neutral regressive or progressive depending on how you design the ship right, so let us look at how that is possible right before we get into a star grain we have to look at how the propellant burns let us say we take a burning surface like this is known as spike singularity okay. Let us say it is burning along the local normal like this how would it look like after some time this would move parallel right and this would also move parallel this point here would get extended into an arc of a circle let us say this distance is y this should also be y and this arc could also be of Y radius okay, so a point here are sorry this is not a this is a research singularity okay.

So a point here the burning surface area is increasing right it was a point here and the burning surface area is increasing depending on this arc length right the longer the time you burn it for this arc length keeps on increasing. So you want to get if you have this kind of a singularity a progressive bunny right.

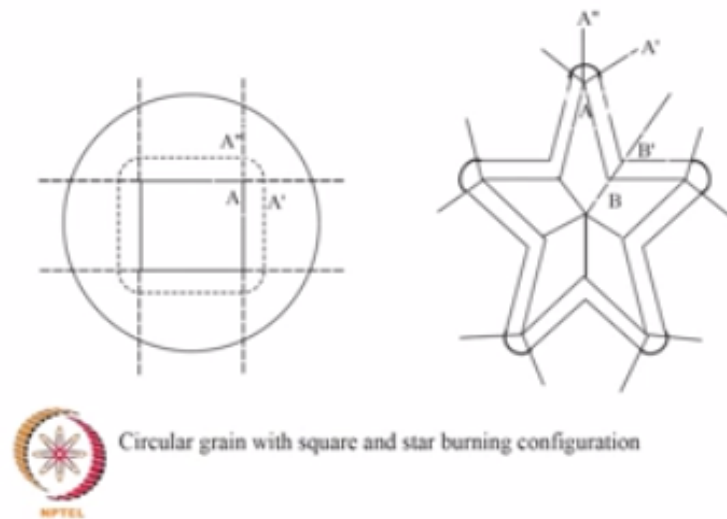
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Now let us look at how to get the regressive burning if you have something like a spike singularity okay. Let us say it is burning in this direction after some time what would happen sorry if it were burning in this direction after some time this surface moves in this parallel direction this moves in this parallel direction, so you would be ending up with losing certain surface area right the surface area keeps on reducing and over a period of time probably it will become horizontal right.

So this sir will lead to regressive burning okay so if we can have a judicious mix of the progressive and regressive burning or if we can have spiked and recess singularities then we have a chance of having a tubular grain and then even then having something like a neutral burning if you remember if you have a tubular grain like this you will always end up having progressive burning right, but if you remember we need a large surface area in order to get the required thrust so and we also are looking for something like neutral burning for a significant portion of the time.

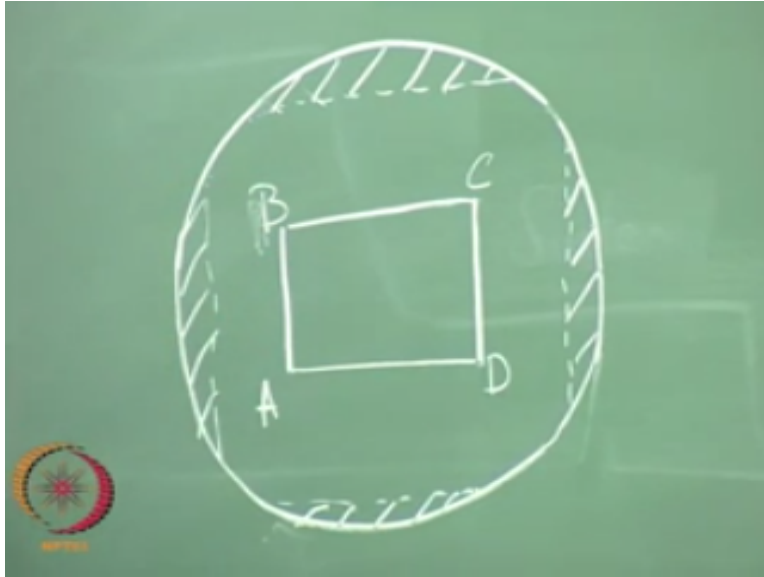
So having something combining spike and recess singularities will lead to what is known as neutral burning up to some time and that is possible with a star grain.
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If you look at this picture here on the right-hand side you have the star grain okay, and this is one two three four five six point star gray right, now what kind of singularity is this a s is a recess singularity okay it is burning in this direction. So a is a recess singularity and B is a spike singularity, so if you combine these two you will hopefully get neutral burning so that is the idea of having a hard rain on the left-hand side you have a picture here wherein you have a rectangular slot ok burning from inside to outside if you look at all the corner points ABCD right what kind of singularities are all of these this is a recess singularity.

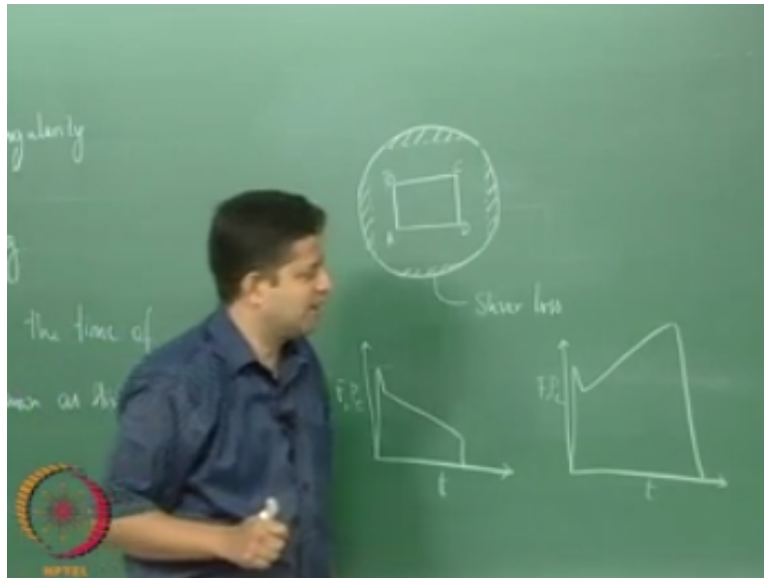
Because if you look at after some time the surface area is increasing okay and if it progresses like this in this direction in this direction there will come a stage wherein you will have some propellant un burnt okay that portion of the propellant that is un burnt is called as sliver loss okay at the time of burnout is known as slider loss okay. So if you look at the propellant that we had so towards the end if right this corner point reaches here much faster and therefore you will have some portion of it remaining unburned this is known as sliver loss.

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And as a designer you would want to minimize this there is no point in carrying propellant on board and not utilizing it is a waste that sense right so you would want to minimize sliver loss and if you look at the reason for wanting to have a neutral burn right why do we want to have a neutral burning I know it is not related to sliver loss yes if you look at the structural utilization okay let me first draw that figure.

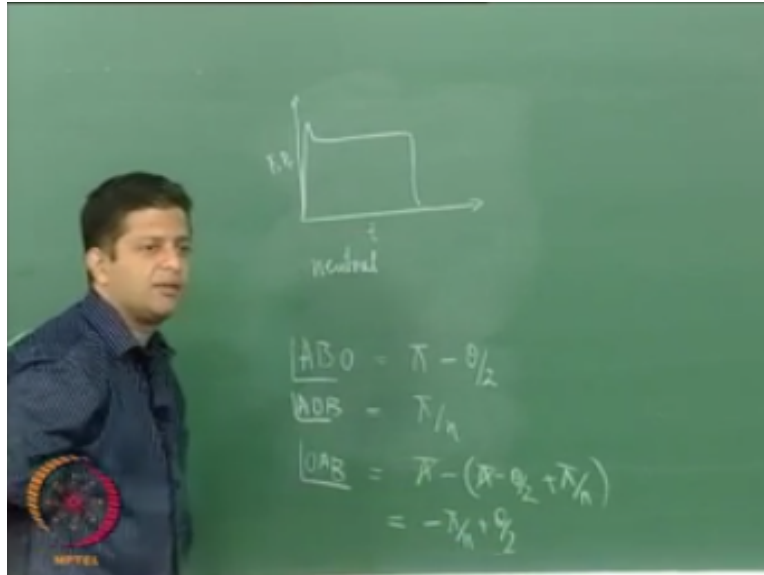
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If you look at a thrust time curve if you have let us say a regressive burning okay, and if you have progressive burning this is also the variation in some sense with the chamber pressure $F&V$ c go in a similar fashion okay, what pressure do you design your casing for it is for the highest pressure right in this case you design it for this pressure and in this case you design it for this pressure although you are designing the case for the highest pressure you are utilizing it for a very small time right.

If you were able to in some sense utilize the entire burn time at that pressure your thrust or the specific impulse would have been higher right you are not doing that and you are trying to kind of utilize that highest pressure only for a small time so from this perspective it is better to have a neutral grain.

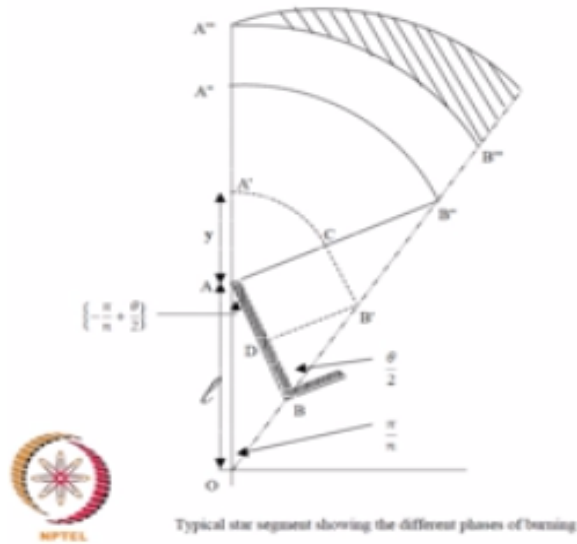
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As far as possible this part is the ignition peak will come to that a little later in the course, so if this is the mean pressure at which it is operating and if it is a constant then you can design the motor based on this pressure with a certain factor of safety and you will be utilizing it for a much larger pack and therefore you will have the benefit of higher ISP that is the reason why we would want to have a program you tool burning for as long as possible.

Now let us look at star gray if you look at this six point star grain that we have here you see that if you consider this segment right this small segment here because of symmetry if you if you consider not just that small segment if you consider the segment here if you draw a line from a to the center and long be if this is the origin AOB that segment is good enough because that is the segment that is repeating because of symmetry it is good enough if you consider only that segment okay.

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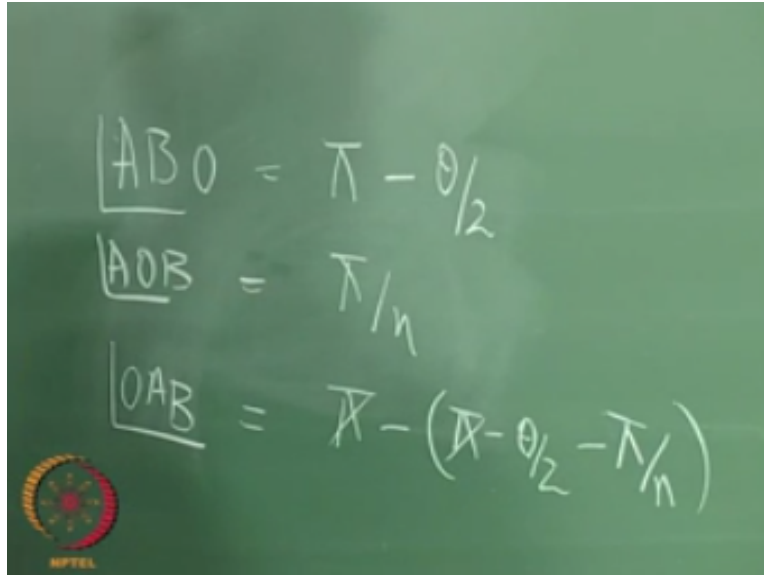


That is the segment that is shown here in this figure if you look at it this is going from 0 0 origin o a and B let this length 0 2 a b l now what kind of a singularity is this point B spike and a is a recess singularity, so there is a with burning let us say it is burnt up to some web thickness of why okay if it has burnt some web thickness of y the position of a will be shifted to a dash a dash to C is the arc that you are getting as an increase in the burning surface area right and there is a reduction in the burning surface area if you look at a to b is the surface area right.

As this point be moved from B to B dash the surface area is reduced from A to B to C B dash okay, so there is an increase in surface area there is a is in surface area if these two are equal then we will get neutral burning that is burning surface area is not changing with time okay. So let us try and derive the condition that will give us neutral burning if you look at this figure from the previous figure this is right you have this is a six-point star ok now the N here is six fine if you have a six point star this angle would be to $2\pi/n$.

Now this angle would be π/n okay because you are also considering the other portions, so this angle would be π/n and if you look at this angle here DB b dash this angle is nothing but $\theta/2$ okay, this will be given to you so this angle is $\theta/2$ and this angle you know so you can calculate the other angle that is a b o a B 0 will be nothing but $\pi - \theta/2$ and so therefore we can calculate the angle O a b right we know the other angle AOB is π/n so we need to calculate OAB we also know AOB is nothing but π/n then we can calculate angle OAB what will this be the entire A the sum of all the angles in a triangle is π . So you will have π -so this to cancel out.

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The image shows a green chalkboard with three handwritten equations in white chalk. The equations are:

$$\angle ABO = \pi - \theta/2$$
$$\angle AOB = \pi/n$$
$$\angle OAB = \pi - (\pi - \theta/2 - \pi/n)$$

In the bottom left corner of the chalkboard, there is a small circular logo with a red and yellow border and the word "NPTEL" written below it.

And you will get sorry this is Plus this Plus this so you will get $-\pi/n$ plus $\theta/2$'s the angle OAB okay now you know this angle the other angle that is b a c is a right angle right. So that is 90 and o a a' is a straight line, so this angle is 180 so 180 - 90 minus this angle will give you a dash a see okay.

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$$\angle A'AC = \pi - \left(\frac{\pi}{2} - \frac{\pi}{n} + \frac{\theta}{2} \right)$$

$$= \frac{\pi}{2} + \frac{\pi}{n} - \frac{\theta}{2}$$

For neutral burning we need

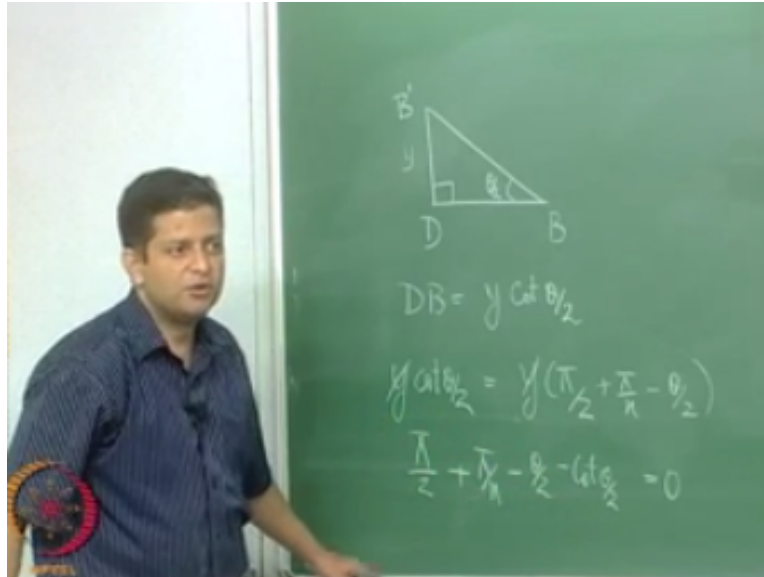
$$A'C = DB$$

$$A'C = y \left(\frac{\pi}{2} + \frac{\pi}{n} - \frac{\theta}{2} \right)$$

$$DB$$


So we have determined this angle this angle this what we had noted earlier was AC is the increase in the burning surface area and BD is the decrease in the burning surface now we have taken a two-dimensional grain and we need to look at this in depth okay into the plane is the depth right so this is a two-dimensional grain so that remains constant so we are essentially calculating a burning perimeter right, so for neutral burning a dash C should be equal to okay now we have determined the angle a dash AC so the length of the arc AC would be Y into the angle itself a dash C is nothing but Y into the angle that is $\pi / 2$ plus now we have to determine DB.

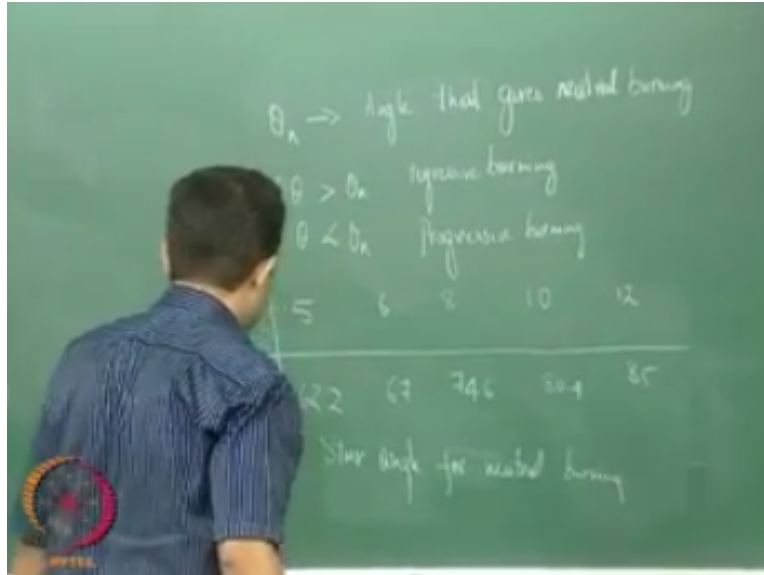
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So if you look at the triangle B dash d b right the D to B dash is nothing but y this length we know this angle is 90° that is B dash DB is 90° and we know that DB b dash is nothing but this angle is θ by 2 and we want to find out what is this distance d be okay this is nothing but $Y \cot \theta$ by two right DB is nothing but so if you look at the condition for neutrality then AC dash should be equal to DB, DB we have found out AC dash we have found out let us equate the two.

So $y \cot \theta / 2 = Y \times \pi / 2 + \pi / n - \theta / 2$ these two must be equal so you will get the condition for neutrality as $\pi / 2$ this must be equal to 0 right. Now as a designer you have the choice of choosing this n okay if you choose a particular n there is a corresponding θ that will give you neutral burning right for a particular value of n there is a particular value of θ that will give you neutral burning and you can get that value of θ using this equation.

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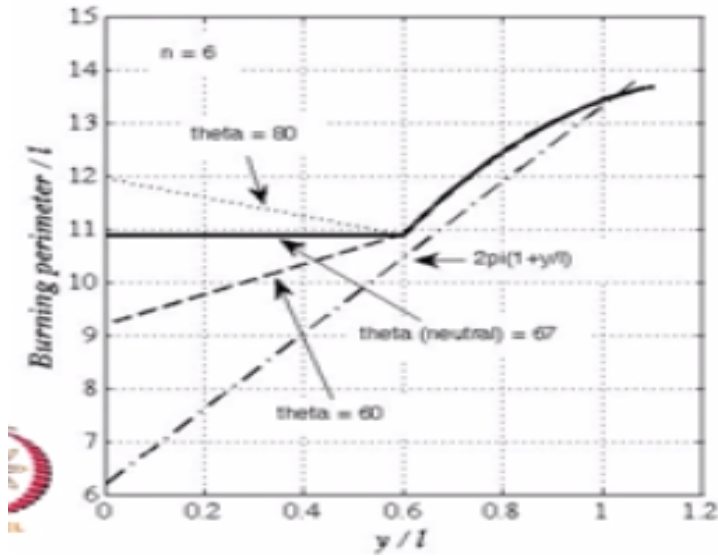


Let us call θ_n as the angle that gives neutral burning then if you have an angle θ greater than θ_n it will be regressive burning and θ less than θ_n it will be progressive burning if you look at this figure here you will see that beyond this point as far as it is going up to this point B Double Dash there is a decrease in burning surface area that is possible right after this point this entire curve a dash B dash moves parallel and therefore it will only be progressive burning after that okay.

So up to the point B dash B Double Dash you can have neutral burning beyond which it is not possible to have any neutral burning and it the rain will become a progressive burning grain okay, and this portion here is what is known as the flavor loss ok so you can solve this equation and get various values of θ_n four different point star grain which is tabulated here if you have θ_n for a five-point star grain 62.2 will give you neutral burning.

So you get various angles increasing from five it is 62 to 12 it will be 85okay, now we talked about how it can be progressive up to point B Double Dash here in this figure I said it can be neutral progressive or regressive after that it will only be progressive right.

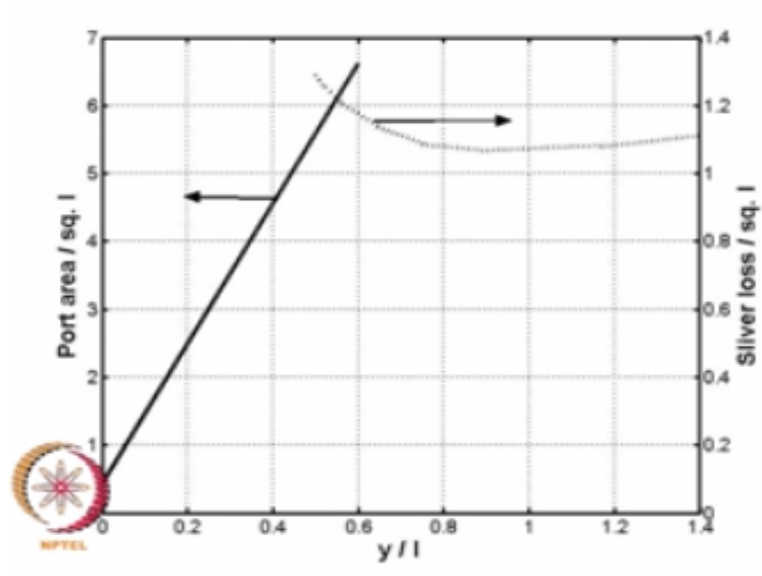
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In this figure here what I have plotted as burning perimeter versus Y / L burning perimeter by L and on the x-axis it is y / L so this is a no dimensional plot irrespective what size of the grain you have you can use this plot, so if you see here the point B Double Dash prime that I was talking about in the previous slide okay corresponds to this web burning of around point six up to this you can have a regressive neutral and progressive beyond which it will only be progressive this dashed line that you have here corresponds to a cylindrical grain if you had a cylindrical grain you would have had it being progressive all through.

But if you have a star grain with a n of six then you can have neutral burning up to some point and after which you can have a progressive burning okay, now this tells us that if you depending on the Y / L you can have up to a certain portion only neutral burning right and what we also need to keep in mind is what is the slider loss that accompanies it okay. If you look at the previous figure here you would be tempted to say if I have a grain stopping somewhere here if I have burning only up to y by l of around point eight it will be very advantageous because you have Mew two burning through most of the portion.

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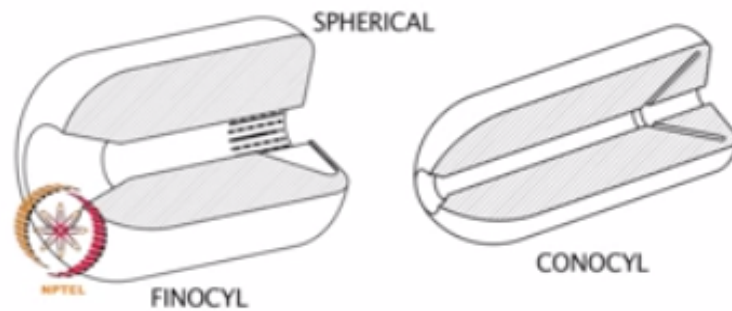


But the other factor is if you stop it at a y / L of around point eight you see here the sliver loss is slightly higher and if you burn it further this sliver loss tends to reduce on the axis here you have sliver loss by L square here you have boat area by l square and the x-axis is again y / L then again for a six point star if you see the port area is increasing as you go from zero to a y / LF point six but this sliver loss fraction is reducing, so if you stop it early then you will have a higher sliver loss which is again not an advantageous thing you want to minimize this sliver loss okay.

So as such in a sense the design is constrained you want to get the thrust and curve that you want you also want to have neutral burning for as far as possible because then your structural utilization will be better and you need the sliver loss to be as minimal as possible always in a sense are conflicting at some point. So when you need to find out for each particular grain which is the best possible design right.

Now what we had discussed here is or something like two dimensional geometry right we said that the along the length the same geometry slip it right they taken across-section and we said along the length the same cross-section repeats itself so this is something like a two-dimensional grade we could get all the parameters that we wanted namely how long will it burn neutral how long will it be progressive all that by doing a simple analysis but if it were a three-dimensional geometry as shown here.

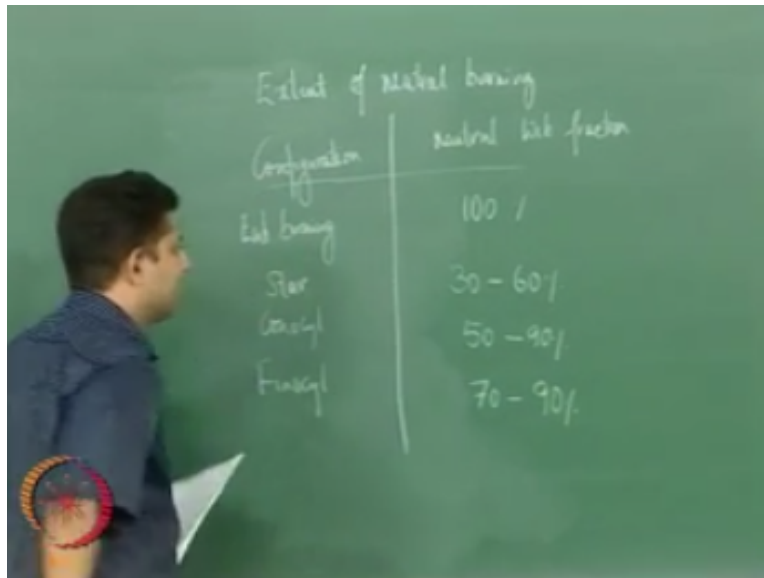
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This is a FINOCYL this is a CONOCYL okay these are three-dimensional geometries that is as you go from the nozzle end to the head end the same geometry is not repeating itself there is a slightly difficult to do this based on simple analysis that we have done you need some computational strategies to find out how the burning surface area is evolving with time okay then you will be able to calculate how whether you can have neutral burning up to some lengths or not right.

And people have done this before so for a particular configuration you can look at how long you can have neutral burning.

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In the case we had considered neutral burning was possible up to 60% of the web thickness for a six point star gray right, now let us look at other possible grain configurations and how long you can have a neutral burning if you have an N burning grain what would be the extent it 100% right. So you can have 100% of the web as neutral burning then if you have a star grain depending on the number of star points that you have it can vary from 30 to 60% percent and if you have CONOCYL that we just saw it can range from 50 to 90% in FINOCYL these are two-dimensional geometries this is in fact a one dimensional geometry there is a two dimensional and these two are three-dimensional geometries.

So if you go for a three-dimensional geometry you could have a slightly larger compared to a two-dimensional one but the end burning is the best that is you can have 100% of the web as neutral burning what it will do if you can design such a grain is not only are you going to utilize your structure very effectively right but you are also going to operate it at a same pressure for a the entire burn time.

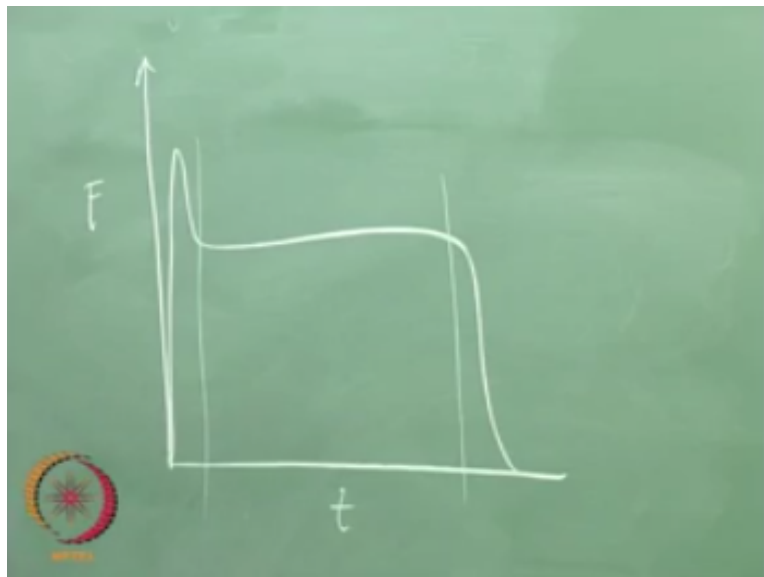
So your ISPs will be optimal also and in addition you can load a lot more propellant right the propellant loading will increase if you have any other kind of geometry namely port burning configuration there will be a certain fraction of the port that will be empty so your propellant loading will be lower and therefore you will lose out on in some sense the structural factor right, that is what is the fraction of the structural weight that is required to hold certain amount of

propellant if that is as small as possible then your what the payload do you want to carry will keep on increasing right.

So it would be nice to have an N burning grain from all these considerations but if you look at the burn rate requirement for these the burn rate requirement will be phenomenal if you work it out it will be very large, so therefore in a sense we are not able to get those kind of burn rates and which is forcing us to go towards a port burning configuration when that then we are looking at all these kind of geometries to give us neutral burning up to a certain extent of burn web or a certain extent of web fraction okay.

In this class we looked at how to look at what is happening when the propellant is burning in a steady fashion if you notice while the propellant has to be switched on and also when this propellant has to be at the end of burning there is a non steady burning that is if you look at the thrust time curve.

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We have considered how it operates in this zone we still need to find out what happens in these two portions okay. We will do that in the subsequent class and there was a question that was posed sometime back wherein you had asked me what happens as the propellant burns there is more free volume right. So soon the pressure drop and how do we account for it right because the free volume is increasing and therefore the concern was the pressure would drop and how do we account for it will also discuss that in the subsequent class okay, thank you.

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