

**Indian Institute of Technology Madras
Presents**

**NPTEL
National Programme on Technology Enhanced Learning**

**Aerospace Propulsion
Solid Rockets – Burn rate**

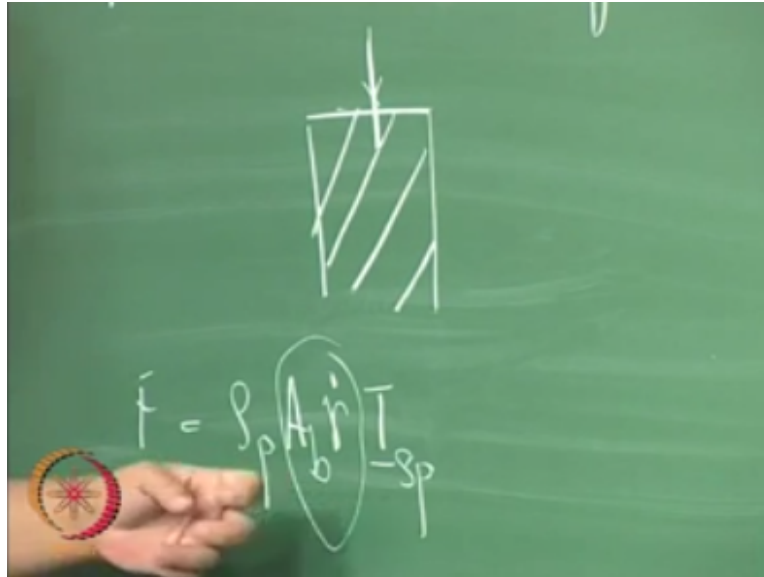
Lecture - 25

**Prof. Ramakrishna P A
Department of Aerospace Engineering
Indian Institute of Technology Madras**

In yesterday's class we had seen two kinds of solid propellants that I used namely homogeneous and heterogeneous propellants in this class let us look at how these burn and what is a function what does burn rate depend on and how can you vary the rest of a solid rocket motor, if you remember yesterday's class the figure that I would shown of a solid rocket motor it was very, very simple there are no moving parts and it is a self pressurized system right, but sometimes the simplest things are very difficult to design.

Because there is no control for you and if something doesn't work the way you want it to work there is nothing that you can do to bring it back to the way you want it to work so designing a solid rocket motor although it is very simple looking there is at times very, very difficult now if you remember our thrust equation.

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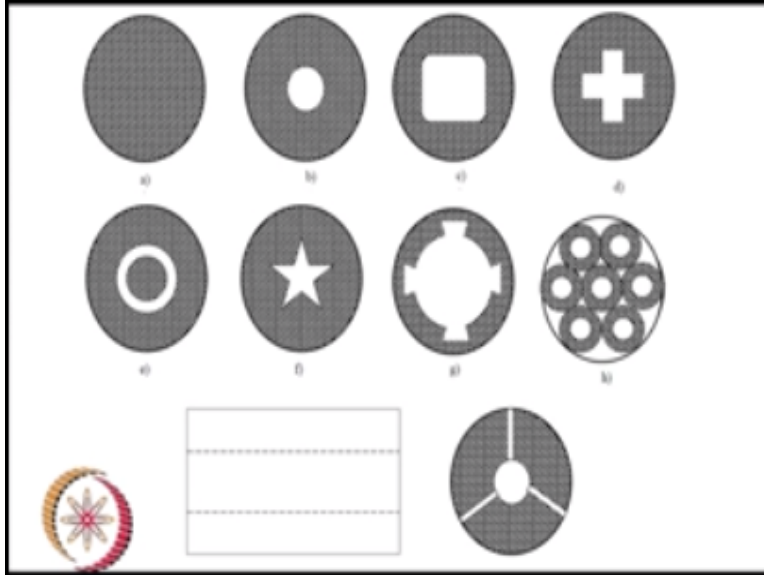


We had written $m \cdot I_{SP}$ right now let us say I pick a propellant combination then this is P is fixed right, so if I want to change the thrust in some sense the only way I can do it is by playing around with \dot{m} okay, so I can write this as F is equal to we also know that \dot{m} the mass flow rate is equal to you are aware of the expression for mass flow rate in a fluid it is ρAV very similar here it is $\rho_p r$ into A_b where ρ_p is the density of the propellant r is called as the burn rate and A_b is the burning surface area the burn rate is something that you measure along the local normal that is burn rate.

Along the let us say you have a propellant like this is the local normal to this surface and burn rate is measured along that okay, so this is r and A_b is the burning surface area, now I can rewrite my thrust equation as okay as I said earlier if I want to modulate the thrust, if I pick a propellant composition then I_{SP} is fixed and density is fixed the only parameters that are under the control of the designer are A_b and r okay, so in this class let us look at how these vary and how can we modulate the thrust.

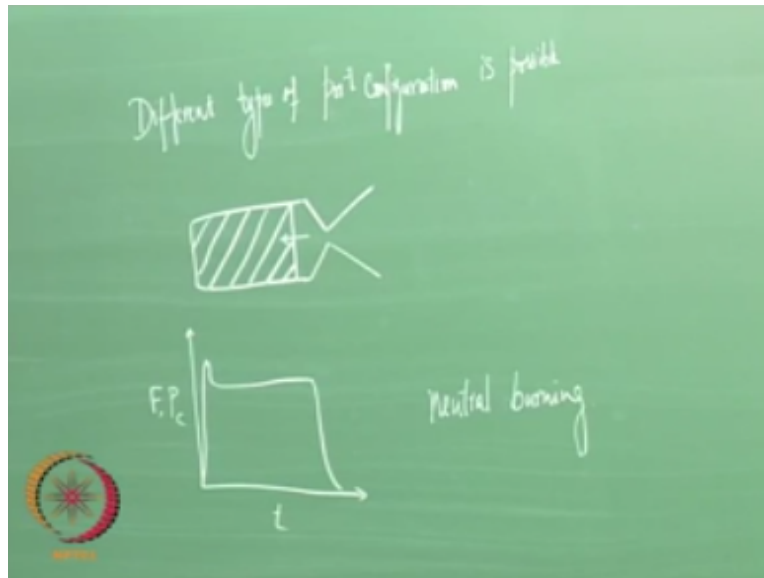
Now the burning surface area if you look at the various possibilities that you can have it is shown here in this figure wherein it gives you a lot of options for a burning surface area now the shaded portion is the one which is the propellant grain and it can burn for example in this.

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Along this surfaces okay this is known as a star grain wherein it burns from inside to out so there are various ways in which you can change the burning surface area right, now different types of.

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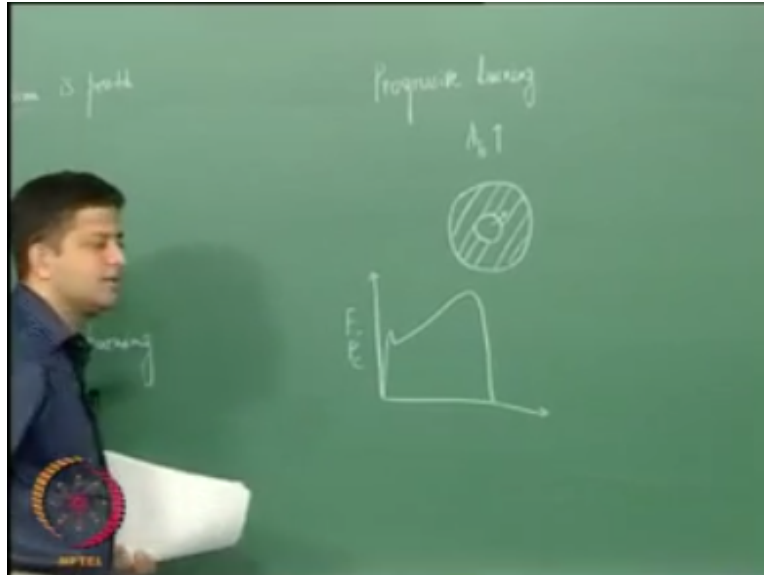
Port configuration is possible leading to changes in burning surface area now, if you pick something known as a cigarette or a end burning grain the burning surface area looks like this is the burning surface this is the burning surface area and it burns normal to that, so the burning surface does the area does not change with time what it will do is if you look at the pressure time or the thrust time for this we will remain a constant this kind of burning surface area is known as neutral burning surfacing.

That is as the propellant burns the burning surface area does not increase okay there are many other ways in which one can get a neutral burning surface, for example in this star configuration one can also get a neutral burning surface and also here it need not always be cigarette burning and if you look at this law rod and lever arrangement the propellant is burning from outside to inside in this portion and it is burning from inside outside in the other one, so the increase on one side is matched by the decrease on the other side.

And therefore you will again get a neutral burning yes that is should I hold on to this question I will answer it a little later in the probably in the next class very simply, if you go answer your question what you are replacing is if you look at this propellant has a very high density something of the order of 1500 mm 800 kg per meter cube you are replacing it with a gas of a very low density, so therefore if you look at it because of this the pressure does not drop too much.

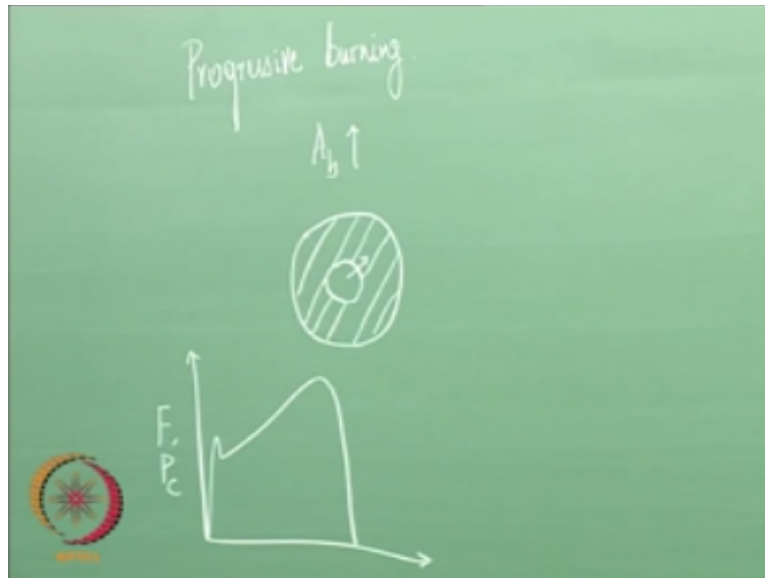
We will be able to show that in equations a little later now in addition to neutral burning we also have something known as progressive burning.

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Progressive burning is one in which the burning surface area increases with time, so a propellant configuration of the kind something like a tubular grain where in it is burning from inside to outside okay, when it is burning from inside to outside the burning surface area keeps on increasing with time and you will get something known as progressive burning and if you look at the thrust time curve for this it will look like this okay you can also have on the other and a regressive burning that is the burning surface area decreases with time.

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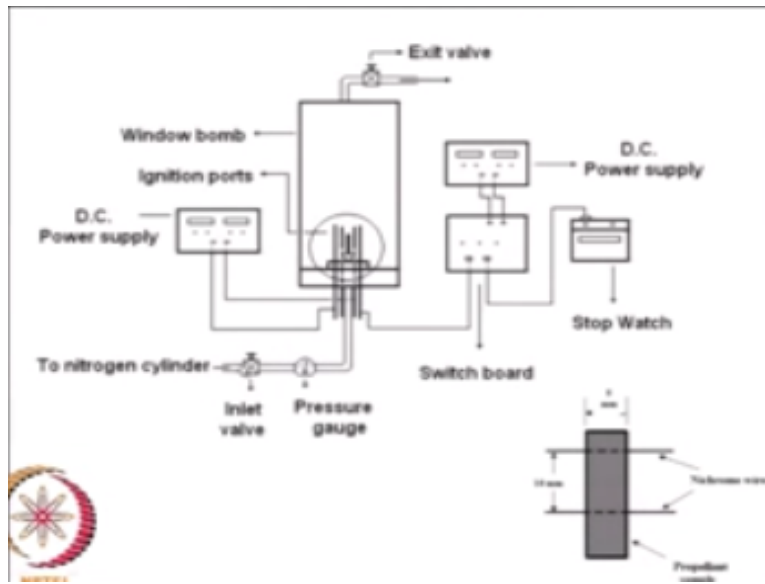


And a good example of that would be something where in a grain free standing and it is burning from the outside to the inside then the burning surface area shrinks with time and the corresponding thrust time curve will be something like this okay, if you look at this figure here the star grain you can configure it to be a progressive burning regressive burning and neutral burning surface area configurations also but some of them beyond a certain point will always be progressive.

Okay we will discuss that a little later in the course now if you look at all these configurations if you look at the propellant loading that is possible this will have the highest propellant loading right because you are utilizing the entire volume to fill the propellant, but in any other case if you look at it the burning if you want the burning surface area to be large then burn rate will be a little lower okay and the loading will also be lower, if you come back here to this equation you will find that if you want to get the required two thrust.

And A_b is limited as in the case of neutral burning then you have to have a very high $r \cdot \dot{r}$ in order to achieve the required thrust okay, although propellant loading is higher there the burn rate requirements are very, very high okay therefore these kind of configurations are seldom used now as I said earlier that the thrust depends on burning surface area and $r \cdot \dot{r}$. for a given composition right, now let us look at we looked at how burning surface area can be varied in order to get the required thrust let us look at the parameters on which the burn rate depends on.

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If you look at the burn rate burn rate depends on is a function of chamber pressure the initial temperature the cross flow velocity and the composition okay for a given composition it depends on only these three parameters okay, the change in burn rate that corresponds to this lateral velocity is something known as erosive burning we will discuss that a little later in the course, so for the time being we will look at how these two parameters affect the burn rate namely the chamber pressure and the initial temperature there is one difference between a composite propellant and a double base propellant.

If you look at a double base propellant for a given composition you can change the burn rate by a small margin, if you add burn rate modifiers that is the only way it can be changed okay assuming that the burn rate modifiers are added in very small quantities but whereas, if you look at a composite propellant you can get with the same composition depending on the particle size you can get different kinds of burn rates okay which is why these propellants are also known as tailor-made propellants we will discuss.

That a little later so if you for a given composition depending on the particle size you can get different burn rates in a composite propellant, but whereas in a double base propellant you cannot okay, now let us look at how these two parameters affect the burn rate to do this about what people use is something known as a Crawford bomb which is shown in this figure here this

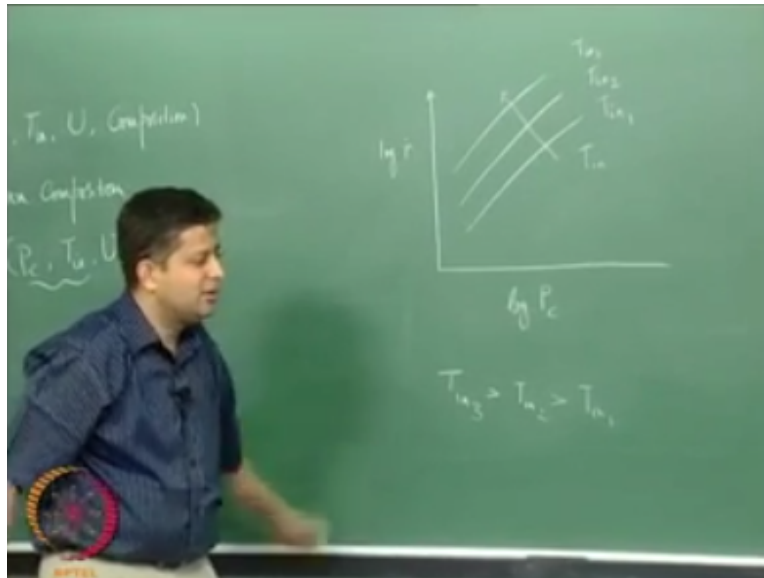
is nothing, but a pressure vessel wherein you can take it to very high pressures the pressure inside this chamber can be artificially changed by connecting it to a nitrogen cylinder.

And inlet one so you can set the pressure at which you want to do the experiment why nitrogen primarily because nitrogen is an inert gas and we would want to study the burn rate of the propellant in an inert atmosphere because, if you look at a rocket motor the combustion gasses are there is no oxidizer, if you do it in air then you will have to account for air also being a good oxidizer so which is why nitrogen is preferred, so using this nitrogen cylinder you can set the pressure in the combustion chamber.

Now if you have a small propellant strand as shown here okay and if you have nichrome wires running through them and, if you know the distance between these two wires you can have a simple electric circuitry wherein once the propellant starts burning from this side along this direction once it cuts the first wire the timer will or the stopwatch will start and once it cuts the second where the stopwatch will stop, so you know the time taken between the cutting of the two wires and you know the length at which or the distance between them then you can calculate the burn rate.

As an average value and that is what is done if you repeat this experiment at different pressures and different initial temperatures you will get the required variation.

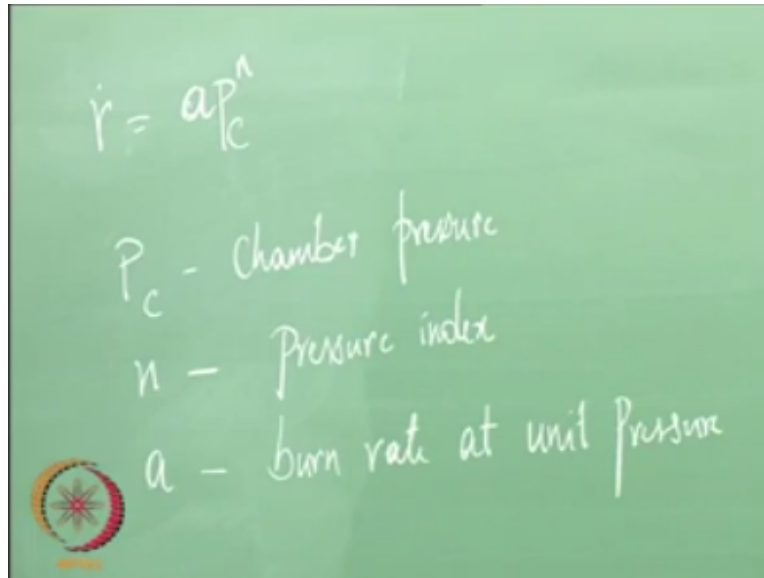
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And these can be plotted on a log-log plot and it has been seen that with increase in initial temperature the burn rate varies like this. For a given initial temperature that is T_{i1} , so the initial temperature 3 is greater than initial temperature 2 which is greater than initial temperature 1, so the burn rates are seen to increase with increase in initial temperature and they are also seen to increase with increase in pressure. Now why on a log-log plot simply because, if you have a variation that is following a power law.

Then when you plot it on a log-log scale what you will get is a straight line. Okay, so that is why these are plotted on a log plot and we can get the expression for burn rate as follows.

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\dot{r} is given by a P_c to the power of n where P_c is the chamber pressure and n is the pressure index and a is burn rate you need pressure that is, if you look at this graph when P_c is equal to 1 what is the burn rate that you will get as I said if you plot on a log plot anything that follows a power law it will be straight line, so if you take log of that you will get $\log \dot{r} = \log a + n \log P_c$ so n is the slope and a is the burn rate at unit pressure, so the slope of this will be n and its intercept on the y axis will be the unit burn rate okay.

Now this a has embedded in it the initial temperature variation part also if you look at this equation we have said that it varies both on chamber pressure and initial temperature chamber pressure is very obvious there the initial temperature part is embedded in the a we will see how it is a is given as.

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$$a = a' \exp\left[\sigma(T_{in} - T_{in}^0)\right]$$

σ - initial temperature sensitivity
 T_{in}^0 - reference initial temperature
 a' - value of a at T_{in}^0

$$n = \frac{d \ln \dot{m}}{d \ln p_c}$$

$$\sigma = \frac{d \ln \dot{m}}{dT_{in}}$$

V_0 into exponential of σ into okay σ is known as the initial temperature sensitivity and T_{in}^0 is some reference initial temperature and a_0 is the value of a at that reference initial temperature okay and if you work this out you can get the expression for σ and n will be very simple n is nothing but logarithmic derivative of burn rate with respect to pressure will give us n okay and σ is logarithmic derivative of burn rate with respect to initial temperature this we can simplify it further.

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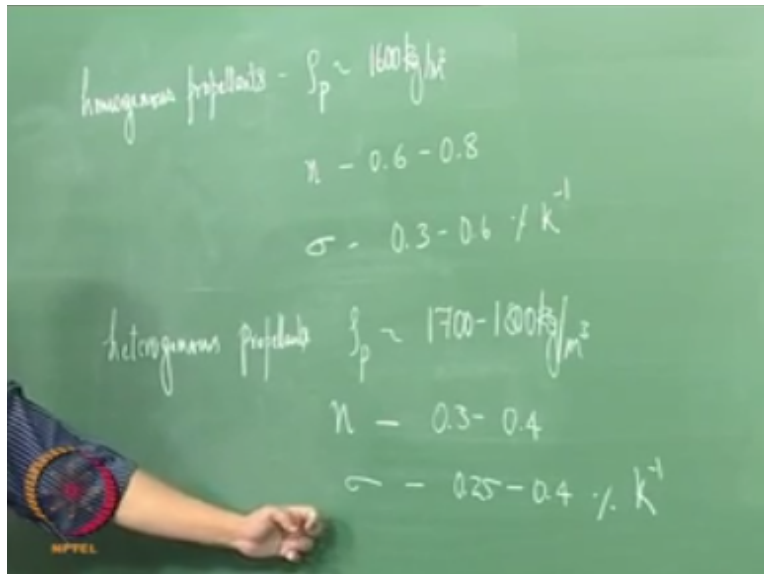
$$\sigma = \frac{1}{r} \frac{dr}{dT_{in}} \cdot \frac{1}{k}$$

$$= \frac{2}{r_2 + r_1} \frac{(r_2 - r_1)}{(T_{in2} - T_{in1})} \cdot \frac{1}{k}$$

And I can write it as the hog are too okay, so that will give you the slope so that is if you know the burn rate at two different pressures you can calculate the slope in this fashion it is nothing, but discretization of that equation that we have written there and similarly σ I can rewrite it as $\frac{1}{r} \frac{dr}{dT_{in}}$ that is we are taking the logarithmic derivative here and taking the r out okay and similarly if you discretize this you will get okay, this if you look at it is a non-dimensional quantity whereas this will have one by Kelvin unit okay.

The unit of this is $1/K$ and because it is usually very small it is usually expressed as a percentage okay, so you will have it as percentage $1/K$ let us look at the values of density typical burn rates that are possible and the pressure indices and temperature sensitivity for the two propellants homogeneous and heterogeneous.

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If you look at homogeneous propellant typically the density will be of the order of $1,600 \text{ kg/m}^3$ then n varies between 0.6 to 0.8 it is very high primarily because we have mixed the fuel and oxidizer at a molecular level and therefore its combustion will be something like a pre-mixed flame and which is the reason why we get a higher end we will see how to get to this a little later in the course and the σ values around 0.3 to 0.6K inverse and similarly for heterogeneous propellants the density is around $1700 \text{ } 2,800 \text{ kg/m}^3$.

Depending on the composition if you have a highly aluminized propellant it goes towards the same then the index here if you remember the ingredients are mixed in a mechanical fashion and therefore you will not go into this, if the combustion of this will be more diffusion dominated and therefore you will find the burn rate pressure index lower in this case something 0.3 0.2 0.4 and the initial temperature sensitivity will be of the order of please remember it is a percentage which means the actual value will be 0.02K.

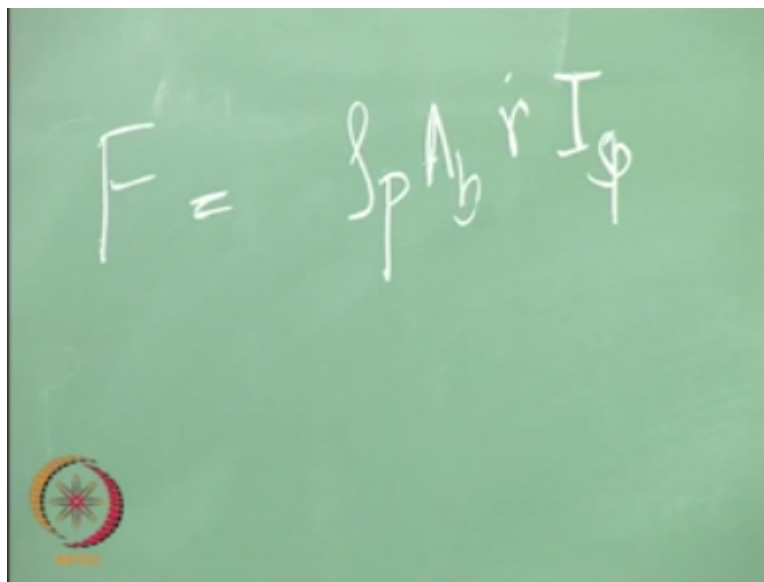
Inverse now what do these quantities mean to us in terms of how do they affect the thrust of the rocket motor we will see that next, firstly let us take the initial temperature sensitivity right, why is initial temperature sensitivity very important, now it will not be so important for a launch vehicle industry primarily because you can choose the time and place of the launch that is the place will be usually fixed okay in India's case we launched it from Sri Hari Kota.

So the place is fixed and you can choose the time such that the temperature is within certain bounds but whereas for any military application depending on the temperature variation a

temperature will vary both day and night and also from place to place, so usually any missile will have to take into account - 52 + 70 okay temperature variation of - 52 + 70, now what does this how does this affect what we are talking about if you recall the burn rates were higher at a higher initial temperature and lower at a lower initial temperature.

So if burn rates the motor is going to be the same irrespective of which place you launch it from right and if you are launching from different places which are at different temperatures then the thrust of the rocket motor will also be very different, if you look at the thrust equation.

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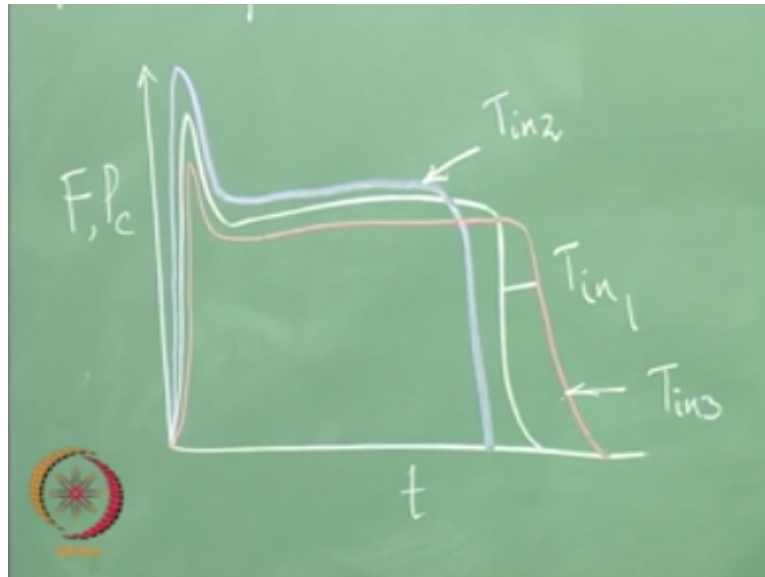

$$F = p A_b r I_\phi$$

F we had written as right now this our dot will vary depending on the initial temperature and if this varies the thrust will also vary, but because we have loaded the same amount of propellant the overall impulse is the same so therefore what can one say will the mission be severely hampered or will it not be hampered because we are saying the overall impulse is the same right, only the thrust time curve changes for the overall impulse will be the same how does this affect the mission.

So what will happen discipline meditation yes if you look at it the burn rate will change if the thrust if the time for which the thrusting is there is altered then the drag characteristics will be very different as long as the rocket motor is thrusting the drag will be a lot less, but as soon as you switch it off the drag will increase, so if your burn time changes your drag will change and

the missile will not hit the entire net target which is why the variation with initial temperature is a very great significance to military applications and not so much for the launch vehicle industry.

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The typical thrust time curves or the pressure time curves with different initial temperature will be something like this, so you notice that the burn times are very different for the three cases and therefore as I said the drag profile will also change which means that the mission may not be fulfilled we need to keep that in mind because the range of the missile is determined by this okay, so what one would want in terms of initial temperature sensitivity is a slow a parameter as possible if it is a slow then these dispersions will be smaller okay these changes in burn time will.

Be smaller and therefore you are within a small variation right now let us look at how the other parameter namely N or the pressure index affects performance the first thing that we are going to look at is what is the value of n that should be there sure, if a value greater than one is good or if a value smaller than one is good and then how small do we want this value or how large do we want this value.

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$$\dot{m}_{in} = \int_p A_b a P_c^n$$

$$\dot{m}_{out} = \frac{P_c A_t}{c^*}$$

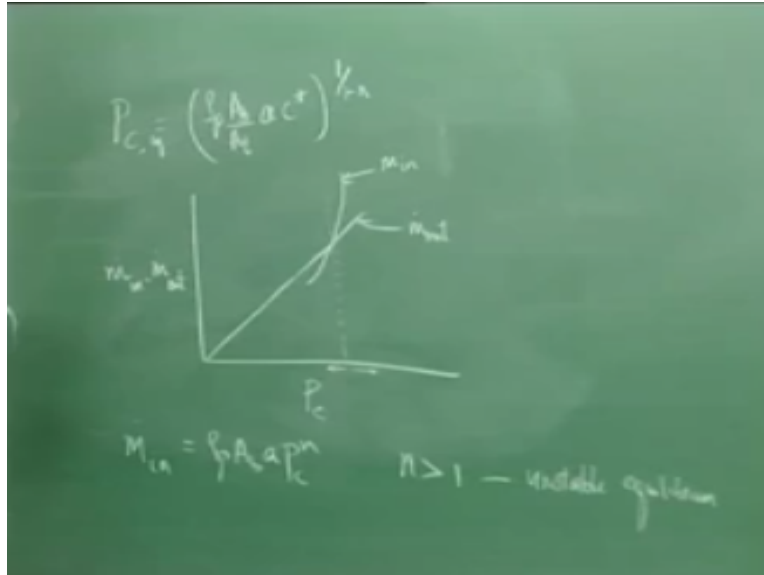
$$\dot{m}_{in} = \dot{m}_{out}$$

$$\int_p A_b a P_c^n = \frac{P_c A_t}{c^*}$$

Now if you draw the pressure versus mass flow rate, but before we go there if you look at a rocket motor okay, if I take this control volume up to the throat okay now the mass is being added because of the burning of the propellant, okay and the mass is thrown out through the throat right, so I know that for steady operation the mass that is coming in must be equal to the mass that is being thrown out so I will call \dot{m}_{in} is the mass that is coming in \dot{m}_{out} we know is dependent on the burning of the propellant.

So it is given by \dot{m}_{in} and \dot{m}_{out} is it if it is a choke flow then this can be written as $\dot{m} \propto P_c^{0.5}$ alright now here in this equation I can rewrite the burn rate as a P_c to the power of n there is one assumption that we are making, if you look at this term pressure what we had got here was based on the experiments done in a Crawford bomb for a window bomb okay so this is a static pressure right and if you look at this is a stagnation quantity the assumption that we are making is that the static and stagnation pressures are nearly the same.

So you need to bear that in mind that this is stagnation and this is static and can be different okay so now for equilibrium or for steady-state conditions the mass coming in must be equal to the mass that is going out, so we will get \dot{m}_{in} must be equal to \dot{m}_{out} which means you can rewrite this expression and get an expression for P_c alone, so if we equate the two and try to obtain an expression for P_c .
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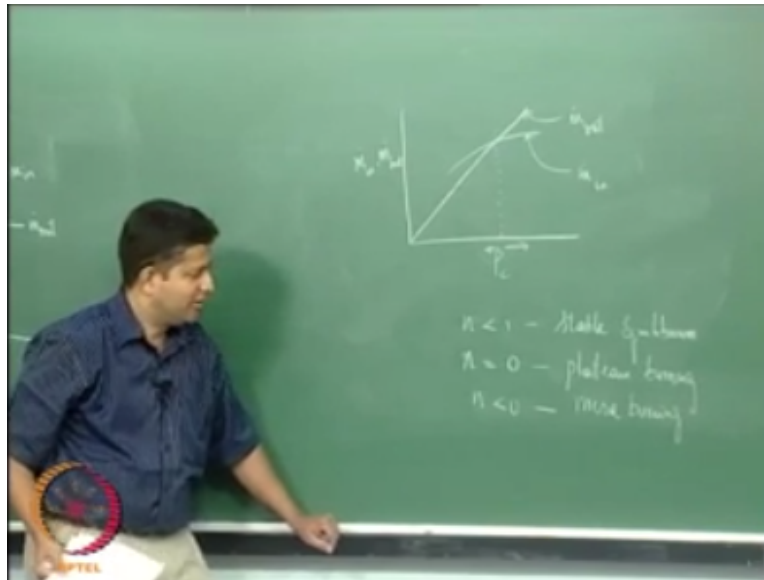


We will get P_c is equal to okay this P_c is also known as P_c equilibrium okay. that is once it reaches this then there will be steady flow okay now let us look at what value of P_c what value of n will lead to stable operation of the motor and what values do not lead to stable mode motor operations, if you plot the mass flow rate m . and P_c here I am plotting both m dot in and m . out okay m . out is a linear function remember it is nothing but P_c a T by C star, so it is something like this now there are two possibilities that we can look at for m . in.

m . n is nothing but when n is greater than one and when n is less than 1 for n greater than one if you say something like this then at this point the mass flow rate that is coming in is equal to the mass flow rate that is going out okay and this would be the equilibrium pressure, now let us say for some reason or due to some disturbance, the pressure changes if it goes in this direction right what happens then this is m . in and this is m . out okay, if you look at this the m . in as much greater than m .out in this region.

And therefore the pressure will tend to keep on increasing and this could lead to an explosion okay whereas if you crumb on this side, if you if there is a disturbance in this direction then the mass flow rate that is coming in is lower than the mass flow rate that is going out and it could finally lead to quenching, so for N greater than one the motor operation is unstable and is not preferred let us look at the case where n is less than one.

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Okay this is the n for n lower than 1 if you look at this is the stable or the equilibrium pressure for n less than 1 if let us say the pressure is change from this point in this direction $m \cdot out$ is greater than $m \cdot in$, so which means the pressure will tend to fall and come back to the same value okay and if it goes in the other direction $m \cdot in$ is greater than $m \cdot out$, so it will tend to increase the pressure and take it back to the same value, so this equilibrium point is a stable equilibrium and n less than 1.

Is preferred for rocket motor operations so we would want the value to be lower than one but how much lower than 1 or how much are we going to have is what we need to look at, if you look at this equation here you see that n appears in the denominator okay, let us say we have a low value of 0.2 or something right what will this be $1 / 0.8$ right $1 / 0.8$ is 1.25, so any changes to either the burning surface area or the throughout during the operation of the motor the pressure is only going to go up by a $1^{1.25}$.

So it would be good if you can have a lower value let us look at the other extreme that is let it be less than 1 let us say we take a n of 0.8 then, if you have $1 - 0.8$ that will be 5, so the pressure for any changes here will increase by the power of 5 which is not a desirable thing, so it is important to have less than 1 for stable operations and as low as possible, so that changes in burning surface area or throat will not affect the chamber pressure much okay.

There are also propellants that are known to have a burn rate pressure index of 0 okay these are known as Latu burning propellants and there are propellants with n being negative these are

known as Meza bond, if you have n equal to 0 then any change in pressure or any change to any of these parameters will not affect the pressure at all okay, if n goes to 0 then changes here will not affect P_c okay and that is preferred we will stop here we will continue in the next class thank you.

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K.R. Mahendra Babu
Soju Francis
S. Pradeepa
S. Subash

Camera

Selvam
Robert Joseph
Karthikeyan
Ram Kumar
Ramganesha
Sathiaraj

Studio Assistance

Krishnakumar
Linuselvan
Saranraj

Animations

Anushree Santhosh
Pradeep Valan .S.L

NPTEL Web Facilities, Faculty Assistance &

Allen Jacob Dinesh
Bharathi Balaji
Deepa Venkatraman
Dianis Bertin
Gayathri
Gurumoorthi
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Lakshmi Priya
Malarvizhi
Manikandasivam
Mohana Sundari
Muthu Kumaran
Naveen Kumar
Palani

Salomi
Sridharan
Suriyakumari

Administrative Assistant

Janakiraman. K.S

Video Producers

K.R. Ravindranath
Kannan Krishnamurthy

IIT Madras Production

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