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NATIONAL PROGRAMME ON TECHNOLOGY ENHANCED LEARNING

Aerospace Propulsion

Rocket Nozzles – Real Effects I

Lecture 21

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In the last class we had talked about how to get the specific impulse for a rocket motor assuming simple one-dimensional steady flow nozzle theory now let us look at in this class what are the deviations from that one-dimensional steady flow theory that we had now if you look at what we had assumed.

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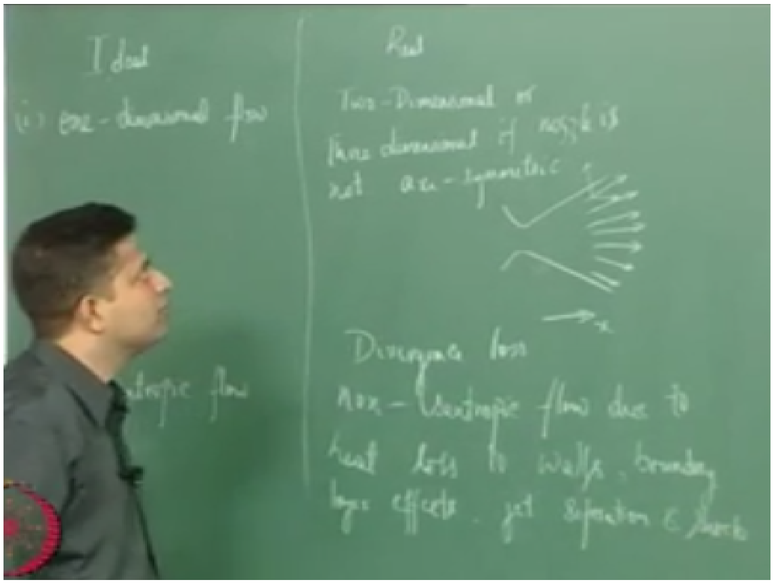

We assumed something and we will see what is happening in the real case firstly we had assumed that the flow was one dimensional flow right so but what happens in a real loss unless either the flow is two dimensional or three dimensional depending on the nozzle geometry if you have a rectangular cross-section then the flow could be three dimensional but if it is an axi symmetric geometry it could be two dimensional okay.

What does this lead to if the flow is two-dimensional or three-dimensional how is it going to attack how is it going to be different from what we had derived in class what is going to change if you look at typically what happens in a nozzles the nozzle is going to be something like this then if you look at the velocity vectors velocity vectors are going to be right so there is going to be a component in the X direction there is going to be a component that is going to go wasted if it is axi symmetric they will cancel each other out because of symmetry right.

So there is only one component of this in the X direction that we need to consider but we consider the whole thing to be the exit velocity so that will wait to something known as divergence loss now the second assumption that we had made was that the flow was isentropic right but in a real situation the flow is going to be non isentropic flow isentropic is reversible adiabatic it will not even be idea batted because there is going to be heat loss to the walls okay.

And because there is a boundary layer that is there the effects of the boundary layer will make the flow again non isentropic and get separation and shocks are something that are going to make it again non isentropic so this is going to be the second deviation now let us look at the other assumptions that we have made.

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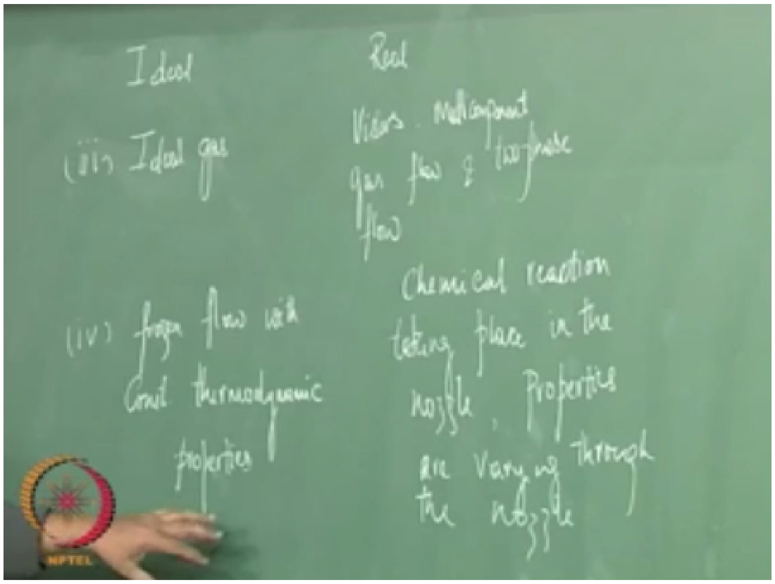

Ideal	Real
(iii) Ideal gas	Viscous. multicomponent gas flow & two-phase flow
(iv) frozen flow with Const thermodynamic properties	Chemical reaction taking place in the nozzle. Properties are varying through the nozzle

Now we had made this assumption saying it is an ideal gas okay it said that it is something that is a gas that will follow $Pv = Rt$ that is not going to be there and there is going to be viscosity so it the real flow will be viscous plus sometimes you will get multi-component gas flow and sometimes in solid Rockets especially with the composite propellant wherein it is aluminized you will get something like a two-phase flow also at times so and lastly we had assumed that in the ideal case it was assumed to be frozen flow with constant thermodynamic properties okay.

Whereas in the real case there will be chemical reactions taking place in the nozzles and therefore the properties are varying through the nozzle if you look at all the four of them the first three of them are going to reduce the specific impulse that we are calculated with our one-dimensional flow the last one is going to in a sense contribute positively all of them were a negative contribution this is going to contribute positively primarily because if you look at what is happening inside the nozzle the pressure and temperature are decreasing.

So that will favor exothermic reactions and therefore you will get a slightly improved performance then what we had calculated so we look at it in a little more detail in this class now let us firstly pick up divergence loss.

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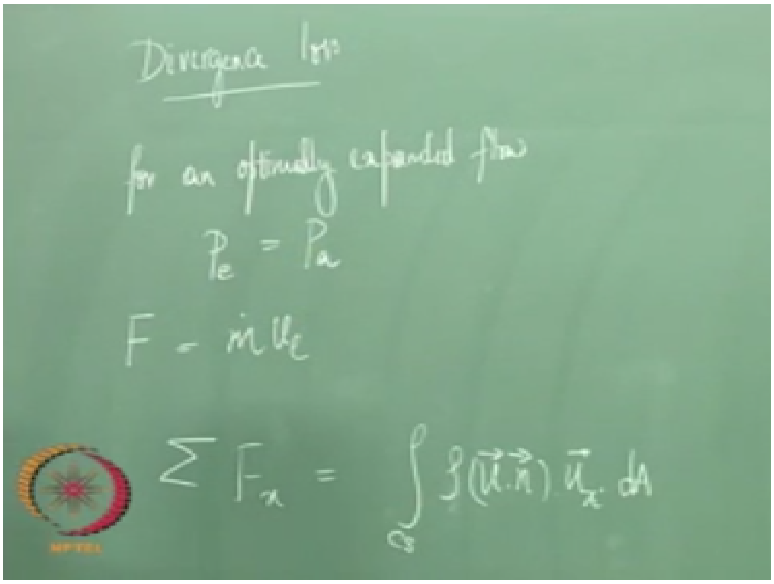

Divergence loss

for an optimally expanded flow

$P_e = P_a$

$F = \dot{m} U_e$

$$\sum F_x = \int_{cs} \rho (\vec{u} \cdot \vec{n})\, \vec{u}_x \, dA$$

Yeah now we are going to make an assumption saying that the nozzle flow is optimally expanded but that need not be the case for this derivation is still valid even if the nozzle flow is not ideally expanded or optimally expanded primarily because we can take out one term so if we assume optimally expanded flow then I can we know that Pexit = PA and therefore our thrust equation becomes f =m. Ue okay now what we are looking at is if you look at this figure we are trying to calculate what are the thrust in the X direction we do not worry about what is happening in the y direction because of axi symmetry they will cancel each other out.

So we are interested in calculating what is happening to thrust in the X direction so if we take the sum of all the forces in the X direction I can write this as ∫ over the cross sectional area ρ x u . x n where n is a unit vector x Ux which is the velocity in the x-direction x dA the surface area okay now we will get x a little bit of to trigonometric the and try and derive this.

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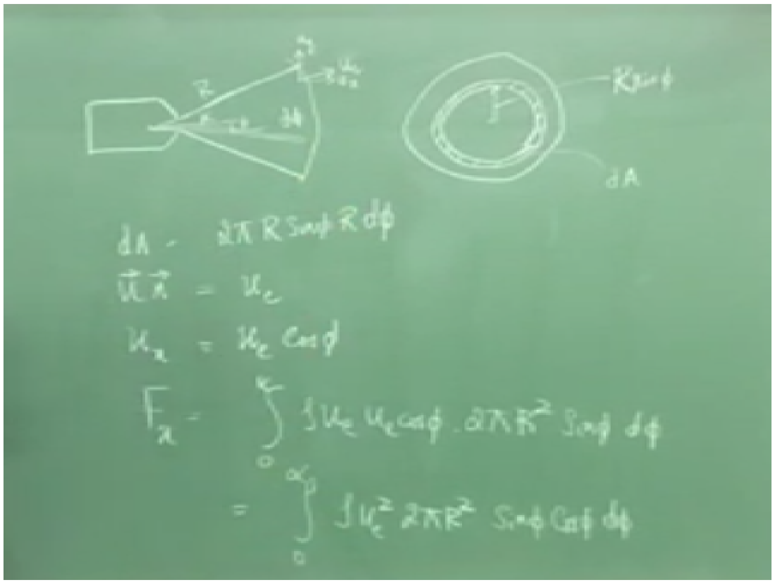

But before we go there if we look at this nozzle okay we could look at the flow that is coming out through this nozzle as something that is coming out from a point source somewhere here right coming out from a point source at some and we are only interested in that code that is seen here okay now let me call this radius from the point source as R okay then this half angle that we are looking at let me call it as α right then we need to integrate this equation so we need to determine what is this da okay to do that this is Ue which will have a component in the Y direction and in the X direction right.

Now if you look at what is the area that we need to consider let this angle be ϕ and we are interested in the small elemental area Dϕ now if you look at it from the other direction you will get something like this okay this is the area that we are interested in and if this entire radius is R and this is at angle ϕ what does this component are sin ϕ right, so this would be R sin ϕ and this elemental area would be da fine.

So now dA would be this is 2π R sin ϕ right x what 2π R sin ϕ is the circumference x Dϕ right Rdϕ R x this small angle let me call this as Dϕ okay so it will be 2πR sin ϕ that is the circumference into this area that is nothing but Rdϕ right and we know that the normal SU. n not a vector ue u . n as what we are looking for this is nothing but Ue right and the component in the X direction what is that going to be this is the component in the X Direction is nothing but U Ue cos ϕ right.

So sorry u. n is ue ux is ue cos ϕ right and what we need to do is integrate this so let us do that Fx is equal to ∫ 0 to α u this u. n becomes ue + ux is nothing but ue cos ϕ and x the elemental area Da which is given by this 2π R^2 sinϕ Dϕ so we will get okay yeah yes normal you are looking at if you take this is a you have to integrate it towards this area so normal to this will keep changing right so now let us look at simplifying this would need a little bit of your trigonometry you know that sinϕ x cos ϕ is what.

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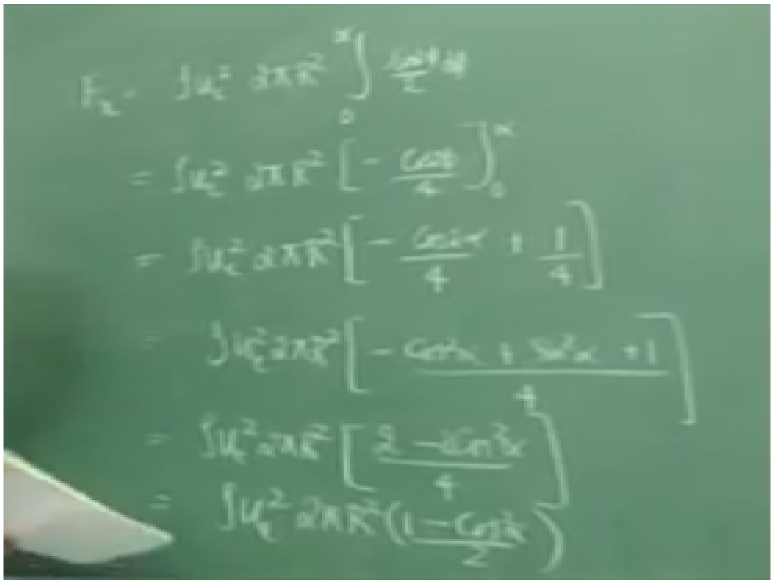

$$F_x = \rho u_e^2 2\pi R^2 \int_0^\alpha \frac{\sin 2\phi}{2} d\phi$$
$$= \rho u_e^2 2\pi R^2 \left[-\frac{\cos 2\phi}{4} \right]_0^\alpha$$
$$= \rho u_e^2 2\pi R^2 \left[-\frac{\cos 2\alpha}{4} + \frac{1}{4} \right]$$
$$= \rho u_e^2 2\pi R^2 \left[\frac{-\cos^2\alpha + \sin^2\alpha + 1}{4} \right]$$
$$= \rho u_e^2 2\pi R^2 \left[\frac{2 - 2\cos^2\alpha}{4} \right]$$
$$= \rho u_e^2 2\pi R^2 \left(\frac{1 - \cos^2\alpha}{2} \right)$$

So sine ϕ x cos ϕ we will get the Fx = ∫ρ Ue^2 2 π^2 x ∫ 0 to α sin ϕ x cos ϕ is sin 2 ϕ so sin2π/2 right so now if we integrate this what do you get – cos 2 ϕ/ 4 okay 0 to α so what is this I can rewrite this as ρ right first term will be - cos 2 this I like to know this right now cos 2 ϕ we know that it is nothing but- $\cos^2$ ϕ + $\sin^2$ ϕ + 1/4 right we would like to retain it in terms of cos ϕ I will come to why we are doing that in a little moment so $\sin^2$ ϕ I can write it 1 - $\cos^2$ ϕ so I will get.

Okay this is nothing but I can take or two as a common factor so I will get okay now if you see here we have a term in terms of art which we will not know we would want to eliminate this one way to eliminate it is to look at what is the mass flow rate through this and then we will hopefully get a function in are in which we can use to eliminate this so if you want to look at what is the mass flow rate.

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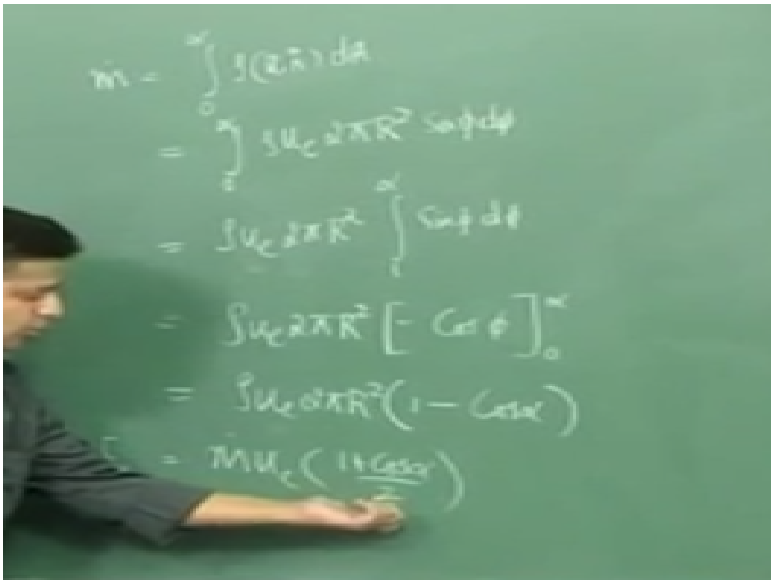

$$\dot{m} = \int_0^\alpha \rho(\vec{u}\cdot\hat{n})\,dA$$

$$= \int_0^\alpha \rho u_e 2\pi R^2 \sin\phi\, d\phi$$

$$= \rho u_e 2\pi R^2 \int_0^\alpha \sin\phi\, d\phi$$

$$= \rho u_e 2\pi R^2 \left[-\cos\phi\right]_0^\alpha$$

$$= \rho u_e 2\pi R^2 (1-\cos\alpha)$$

$$= \dot{m} u_e \left(\frac{1+\cos\alpha}{2}\right)$$

m. = again ∫ 0 to α dA right now da we know is nothing but 2 π ^{2} sin ϕ sin ϕD ϕ so we will use that and u . n is nothing but Ue so we will get if we integrate this to ρe 2 π R^2 ∫ of sin ϕ D ϕ 0 to α so this would be - cos ϕ right and so you will get 0 is 1 1 - cos α is what you will get here so we have what two terms here one is the thrust and one is the mass flow-rate mass flow rate if you can see I have put it as a function of cos which is why I wanted to retain this in terms of cos now if you look at this I can Club these two and write my thrust in the x-direction s Fx = you will have a Ue term and 2 π R ^{2} will go away.

So you will get m. Ue 1 + okay now if you remember what we started with was f = m . Ue so we have now got a multiplication of 1 + cosα / 2 this term is also denoted as λ.

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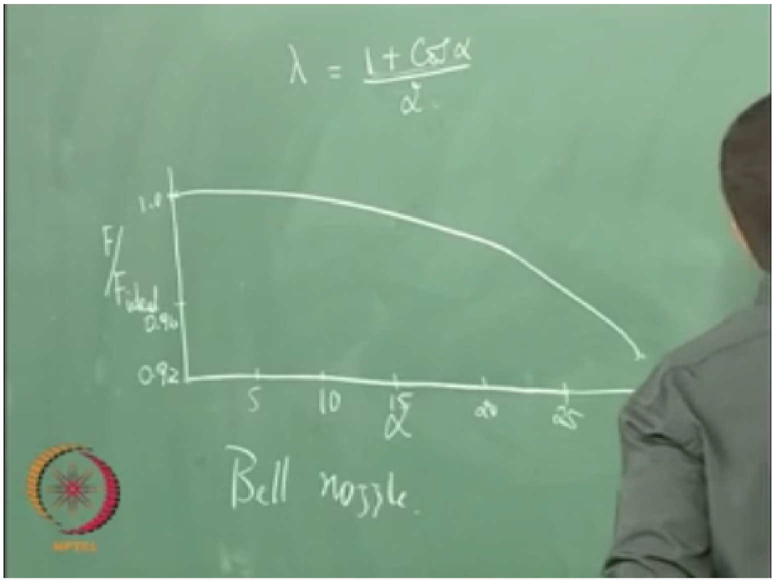

So in a sense ideal thrust is multiplied by a factor λ where λ is equal to so 1+ so what is the maximum value of λ we can have α cos α cannot best be 1 right so the maximum value of a λ can only be 1 and in real cases it will be less than 1 depending on what is this angle α let us look at how it affects by plotting this if we plot F / F ideal versus α if α is 0 then the ideal thrust will be equal to the or the actual thrust will be equal to the ideal thrust.

So this factor will be one but I have α keeps on increasing cos α reduces and therefore this factor λ will be reducing so this is how the graph looks like and you see that typically we tend to use something like a 15 degree α conical nozzle in the subsonic portion so that will mean there is already some loss to the thrust that we get now how do we minimize this thrust loss one way to look at it is make this angle exit angle very small right if we make this exit angle α small or 0 is impossible right 0 would mean that you cannot have a supersonic portion at all.

So you need to have this angle as very small as possible now if this angle is very small then you need a very large length to get to the exit area that you want right which is also not going to work out because you if you have a very long length then your vehicle length will increase and therefore that is again a problem so how do we do this is something that was thought about long back and people came up with a concept known as bell nozzle.

Now if you look at this figure what we are saying is you will get a component only in the X direction right if somehow we were to make the flow as parallel to the x axis as possible at the exit then we will be able to recover a larger fraction of the ideal thrust right then we will be able to restrict our loss to a smaller value right that is what is the idea of a bell nozzle.

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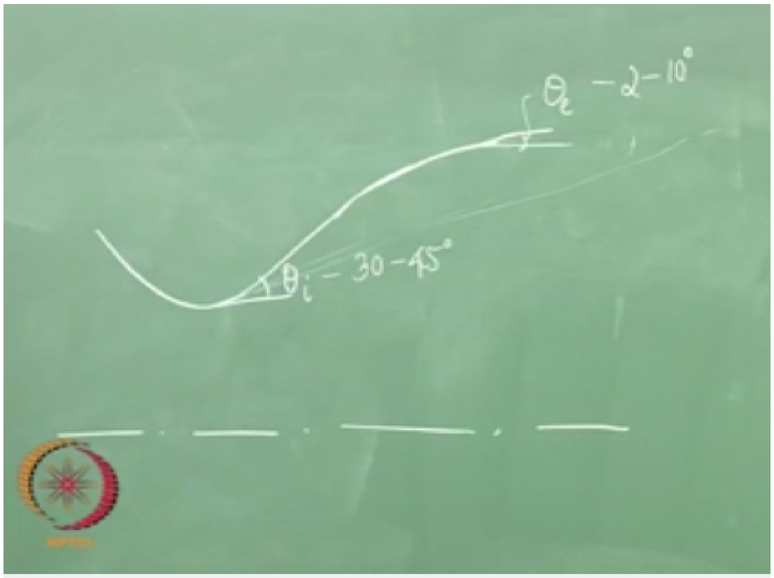

Suppose this were the axis then you could have a nozzle like this instead of the conical nozzle which would have been something like this so you see that if you want to get to the same exit area your length of the conical nozzle will be much longer and if you have a bell nozzle you will shorten it essentially what is done in the bell nozzle is we increase the angle after the throat right we make the flow go through a turn through a very large angle after the throat and then make it go as parallel to the x-axis as possible okay.

So there are two angles here that we would be interested in that is θ I and θe θi and θe in a conical nozzle will be identical right will be the same whereas here you have the option of having a different θi and θ, θi will be typically very large around 30 to 45^0 and θe will be somewhere between 2 to 10^0 and in between you have a parabolic profile connecting these two okay now this would mean that you would have a shorter nozzle therefore the nozzle weight will be smaller and also your cooling remember nozzle is exposed to high temperature gases your cooling load will also come down.

Because you are now looking at a smaller area to cool the and also you will be having a smaller area wherein you need to cool the nozzle now if you were to calculate what is the length saving that we will get we can look at this table.

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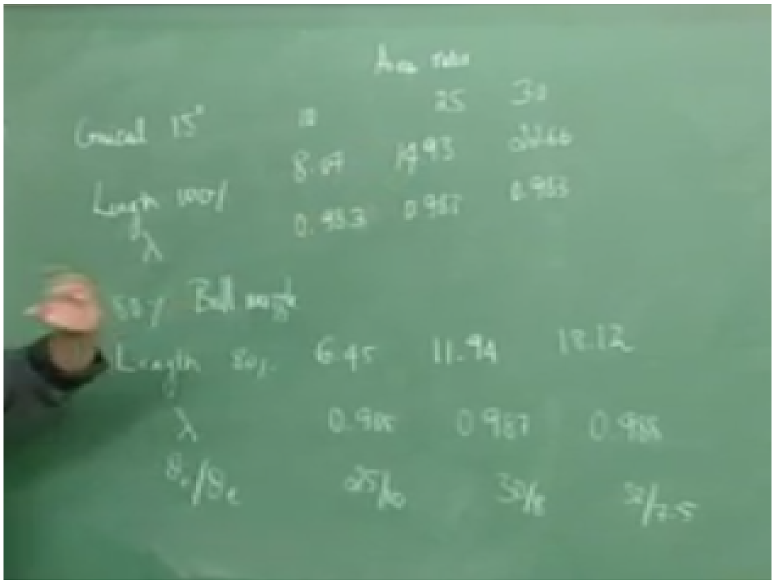

	Area ratio		
	10	25	30
Conical 15°			
Length 100%	8.07	14.93	22.66
λ	0.983	0.983	0.983
80% Bell nozzle			
Length 80%	6.45	11.94	18.12
λ	0.985	0.987	0.988
θ_i/θ_e	25/10	30/8	32/7.5

Let us say we have a conical 15⁰ then for different area ratios what is the length that we are going to get this would be something like 8.07 this is normalized with respect to the throat radius okay so you will have a non-dimensional number here it does not depend on what is a throat radius you can multiply it by the throat radius and get this value and the λ would be hey this λ here would be constant now if we take well nozzles are usually denoted as 80 % or 60 % of the conical nozzle okay.

So if you take an 80 % bell nozzle then the length would be 80 % of the length of the other nozzle so it will be something like 6. 54 and the λ would be depending on what is the θi to θe okay now if you look at this the λ values here and here they are comparable and in some cases you will get a higher λ here but whereas the length is smaller than the other one so in a sense you have saved quite a bit on the length which means you will have a reduced nozzle weight okay.

So that is the advantage with a bell nozzle very nozzles are typically used only in the upper stages and not in the lower stages we will come to why that is so analysis in fact if you look at the ideal thrust to the actual thrust.

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And if you please remember that if you have a rocket motor this is going through the atmosphere and therefore the altitude will change right so we are depending on what is the altitude let us say you have a bell nozzle and a conical nozzle designed for some high altitude around 14 kilometers or so then it will give its best performance there and what happens to it at other altitudes is what is of interest to us right this is how a conical nozzle will be and a bell nozzle now if you see this the off design condition this is the design condition right off design condition is poor for the bell nozzle at lower altitudes.

If it is designed for a higher altitude review if you use it at a lower altitude the off design condition for a bell nozzle is poor compared to a conical nozzle so which is why we would typically not want to use bell nozzle at lower altitudes also if you look at the rocket motors that are typically used in the lower altitudes these are usually solid motors and the pressure in the combustion chamber not very large this is around 70 or something and you can expand it only to around 1 atmosphere or so what you would do is you would be have a conical nozzle whereas if you look at high-altitude motors they can expand to vacuum near vacuum conditions.

So there you would need a very large area ratio nozzle okay so there you will try to use a bell nozzle and at lower altitudes you will use a conical nozzle we will look at a little more in detail about why this happens in the next class okay but this is about the first component that is the difference between real and ideal conditions that we assume now let us look at the non isentropic factor show do they contribute to the reduction in specific impulse.

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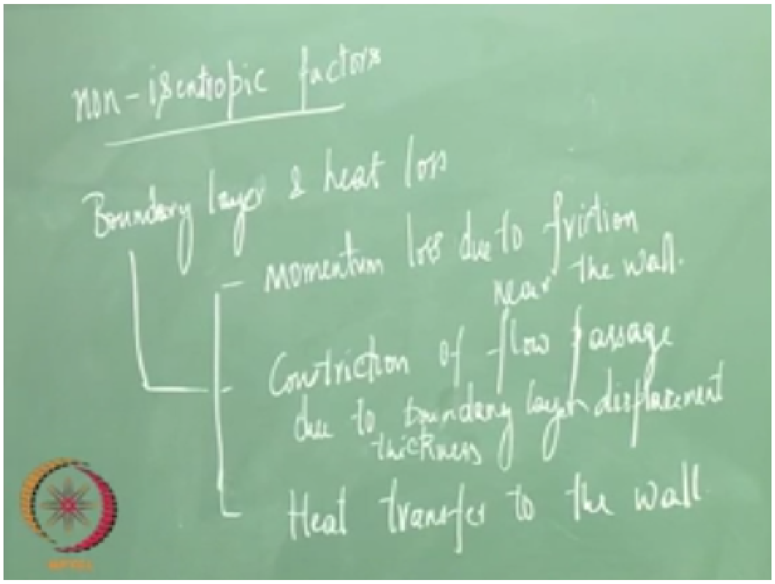

non-isentropic factors

Boundary layer & heat loss

- momentum loss due to friction near the wall.
- Constriction of flow passage due to boundary layer displacement thickness
- Heat transfer to the wall

Typically this comes in because ofboundary layer effects okay so because of the boundary layer and heat loss so you will have typically some three components momentum loss due to friction near the wall the flow needs to go to rest at the wall and therefore you will have a boundary layer in which you will lose some momentum because of the friction okay then you will have if you look at a nozzle if there is a boundary layer then you could also assume it as something wherein the flow is displaced by a certain margin right so that will reduce the mass flow.

So construction of the due to boundary layer displacement thickness and lastly heat transfer to be what if you look at these three this will be very small right this is not a significant component and this will be the highest component now the constriction of the flow passage if you look at a very large motor with the throat radius of 0.5 meters or something like that then the boundary layer thickness is very small compared to that you will not lose out too much but in a very small Motors.

This can be of significance remember the equation for mass flow rate that we have derived in class we have not taken into effect this while deriving the equation that is PCt / C * it needs to be multiplied by a suitable coefficient of discharge when you are looking at a smaller motor for a larger motor it does not affect too much but for a smaller motor it does affect quite a bit okay we will stop here we will continue in the next class thank you.

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