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Aerospace Propulsion

Rocket Nozzles-ID Analysis III

Lecture 20

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In the last class, we had seen what specific impulse is, what vacuum specific impulse and density impulse is and how CF varies PC/PE and AE/AT. We also noted at the end of the last class that thrust varies with altitude for a rocket motor. Let us look at how it varies with altitude and what we can do to derive some benefit out of it.

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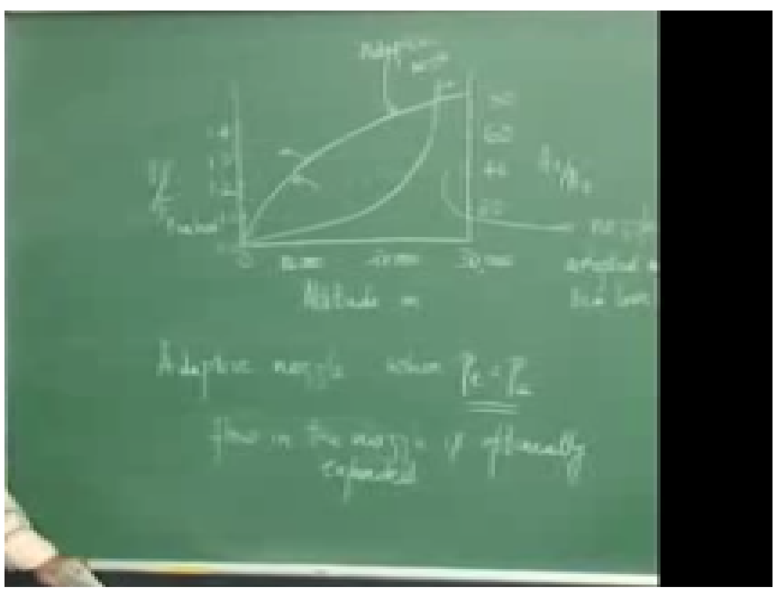

Suppose I were to plot the C level thrust at any altitude versus C level thrust, wherein if we plot the thrust at any altitude versus divided by the C level thrust for different altitudes and also find out how the AE/AT should vary if we had to have something known as adaptive nozzle. So if you look this plot what it tells you this dotted line here is for a nozzle that is adapted at C level if we use the same nozzle to fly to different altitudes.

Then this is the thrust that one can get. If we have a nozzle that is adapted at each and every altitude, what we mean by adapted nozzle is when PE=PA or when the flow is optimally expanded through the nozzle then we call it an adaptive nozzle. If it is adapted to every altitude then this is the thrust by its sea level thrust, so you can see that the thrust delivered by this is far superior to the thrust delivered by a nozzle that is adapted at C level.

Because it one accounts for the increase in area ration that needs to had because the ambien pressure is dropping. And if you look at what we need to pay or how do we do we account for it in terms of area ratio for an adaptive nozzle, this will go like this. Notice that beyond some altitude the increase is very, very steep. Obviously, we cannot have nozzle that is experiment that is also increasing its area ratio as the vehicle moves up that would be very, very difficult.

So let us look at there are other ways of getting to this or if improving this in some sense, so that we get as close to the adaptive nozzle as possible, but we always have said this that when PE=PA, we get the best performance that is when the nozzle is optimally expanded when the flow in the nozzle is, we get highest thrust is what we have said.

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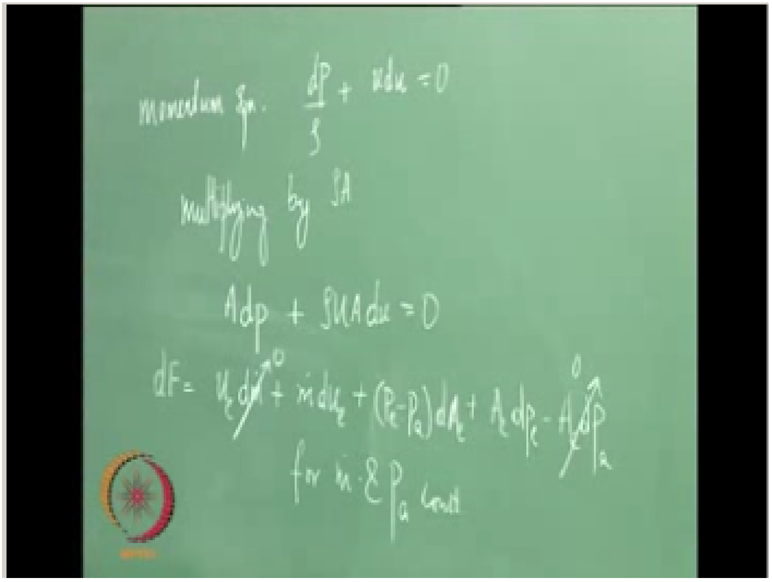

Let us look at how that is possible, if you look at the thrust equation, the thrust equation that we have is F=M.UE+AE, this is our thrust equation and if you look at it, we are saying when this term goes to zero, we get a best performance. From this equation, it looks like if we throw out a term that is causing a positive addition to it, it is still giving us a better performance this looks a little paradoxical to start with, let us look at how this happens.

How when PE=PA we get the best performance. Now, there are two approaches to do it, one is you know you have studied earlier classes that if you differentiate and show that the derivative is zero then it will be a maxima or minima and if it has to be a maxima then the derivative second derivative has to be negative that is one approach and we will also look at whether we can explain the same physically.

Now, firstly in addition to the thrust equation, we know that the momentum equation in one dimension is DP/row+udu=0. Now, let us multiply this equation by row A and you will get A DP+row UA. Now, we know that for this to be a maxima or minima, the derivative should go to zero, so let's take the derivative of the thrust, so we will get DF=UEDM.+M. PE-PAxDAE plus taking the derivative of the terms inside AE DPE-AE DPE fine.
Now, in this equation there are certain things that are constant that if you look at doing this derivative, we need to hold PA constant and the mass flow rate through the nozzle constant, so for M. and PA constant then what happens to these stems this goes to zero and this goes to zero, so we will be left with three terms here. Now, let us rearrange this a little differently and see what we can do.

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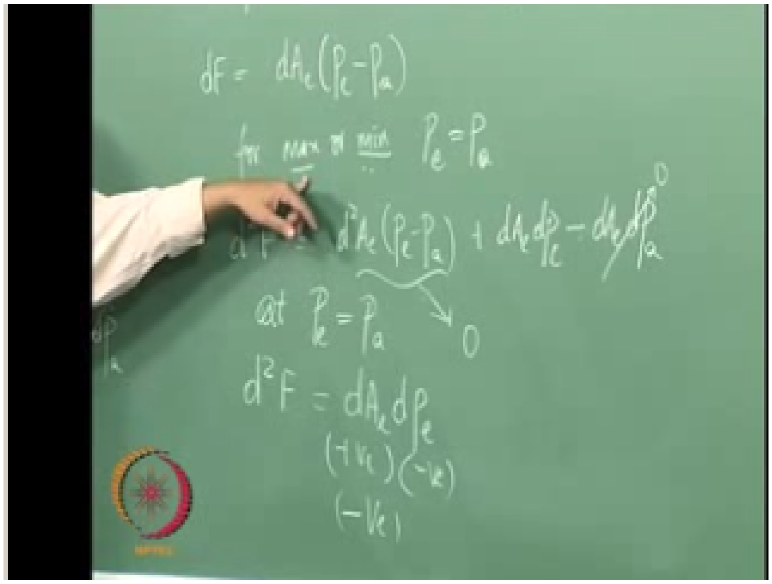

So I will get DF=+AE DPE now, if you notice we had multiplied the momentum equation with row x A and got this expression now we take this for the exit plane, I can rewrite this as AE DPE + ρ UA is nothing, but mass flow rate, so I will get. Now, if you notice the first two terms of the derivative of thrust as this, which means these two will go to zero, so we will be left with DF=for A to be maxima or minima, DF has to go to zero.

And that is possible when only PE=PA, so DF goes to zero now, let us look at what happens to the second derivative, so we will write equation. We had already said that PA is constant and therefore this term goes to zero, so we are left with two terms. Now, if you look at these terms for PE=PA the first will go to zero.

We are looking at what happens to the sign of D2Fwhen PE = PA, so as PE=PA this term goes to zero, so we will be left with only this term. Now, let us examine what is the sign of this term. How is in the supersonic portion of the nozzle, convergent divergent nozzle? What is DAE? Whether it is positive or negative? In the supersonic portion that is in the divergent portion, the area is always increasing.

So it is positive and if you look at what happens to pressure in nozzle it is decreasing, so the derivative will be negative, so you have one positive term and one negative term the product will always be negative and therefore this is a maxima, so we have a maxima at PE=PA. Now, let us try to understand this physically also, if you look at the convergent divergent nozzle, let us say you have rocket motor.

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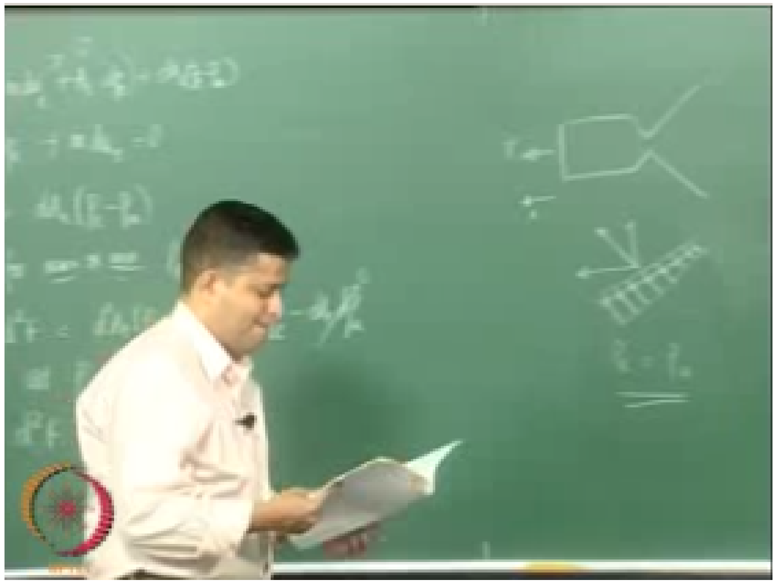

And then you have a convergent divergent nozzle. You all know in aerodynamics that you can get the lift of an air foil by integrating the pressure over the entire surface, similarly you can get the thrust by integrating the pressure over the entire surface, so if you do that if you look at this, all this portion to the left of this portion is immaterial to us because that will not change with change in ambien pressure or change in exit pressure. Up stream of this will not have any bearing and also.

If you look at these two surfaces they anyway cancel each other out. We are only looking for thrust in this direction, so it is good for us to only consider the divergent portion. Now, if you look at the divergent portion alone, I will take one section of the nozzle. Now on the inside what is happening to pressure as you move from the throat to the exit plane, pressure is decreasing. Now, let us take the case where the ambien pressure is equal to the exit pressure.

So then this is the exit pressure, so it is constant on the other side throughout, so for PE=PA. Now, if you look at this the net thrust if you look at what is happening here this will produce a force in this direction and you will have a component of the same in the direction that we want. The other component will get canceled each other out because of symmetry, so we are only interested in one component that is in the X direction, so if we are only interested in this component.

Now, let us say we add a little more nozzle what happens? Some extra portion of the nozzle such that if you add an extra portion what happens to the flow inside, the pressure will drop even further and you will get a situation where in the inside pressure is lower than the outside pressure, so in this very small portion, you will have a component of force in this direction, which will lead to something like this.

If you resolve it in these two directions, so this is adding to a negative thrust you do not want that so therefore, let us take this portion out and get back our earlier figure wherein we had PE=PA. Now, let us say the exit pressure is more than the ambien pressure then what will happen, we need to take out some portion of the nozzle. What we have essentially done is we have taken a portion, which was giving us net positive thrust, but we have taken that out.

So therefore, we will get a reduced thrust, so it works out that physically PE=PA only, we will get the best thrust. If you look at the thrust equation here, this term is increasing when as you reduce pressure. The velocity keeps on increasing as you reduce the pressure and when PE=PA you will have this portion as the maxima and that is why you are able to get the highest thrust when PE=PA okay.

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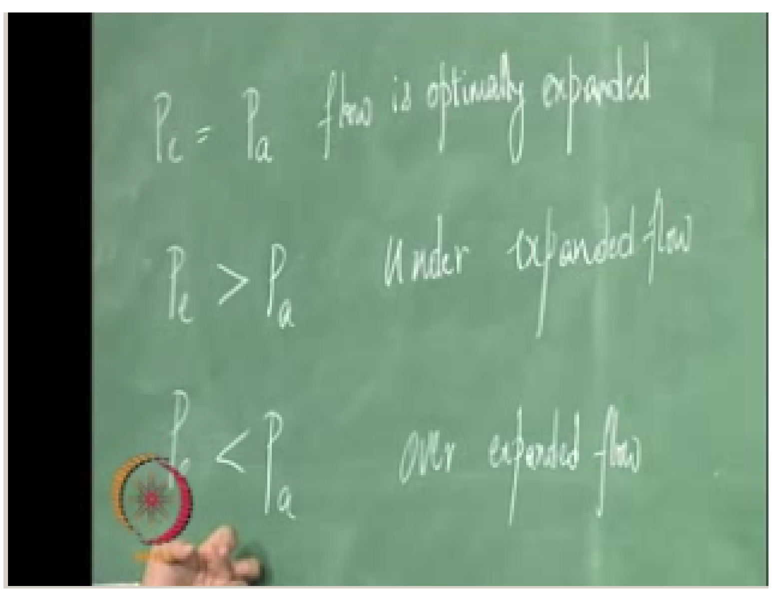

Now, let us look back at some gas dynamics and try and understand what happens in the supersonic nozzle, so now we know that in our thrust equation, F=M.UE. If PE=PA then the flow is optimally expanded and gives us the maximum thrust when PE is greater than PA, what is this case, this is case when it is under expanded that is there was scope for this to expand up to PA.

But we have left it some portion unutilized and when PE <PA, it is over expanded flow. Here, we have taken it more than or lower than PA and therefore, it is over expanded, so now let's look what happens inside the CD nozzle when all these three conditions take place, I mean whether what will happen to the flow as we look at it when these things happen, so to do that let us look at a convergent divergent nozzle, I will only take one half of it up to this is the reservoir, this is the throat.

And this is the supersonic portion. We know from gas dynamics that if we have a reservoir pressure PC then depending on what is the ambien pressure outside, we have derived that in the class that for γ=1.2 this ratio should be around 1.7 for the nozzle throat to be choked. Let us say if the ambien pressure is somewhere here, ambient pressure.

And the pressure inside is the same then what will happen, there is no flow, so there will no flow and pressure is constant throughout. Now, let us keep reducing this pressure on the outside, so then there will be flow and if the fluid is viscous then we will not recover the actual pressure, but otherwise we will recover the pressure and if you still keep reducing it further once it reaches a critical pressure.

The ratio here then it will expand and it will reach MOC number one, M=1 at the throat and then if the pressure on the outside is still lower it will expand further and let us say this is the point where now, there could be two solutions possible when PA is there if you after you have reduced the PA if you increase it further, let us see what happens to the flow in the nozzle. Now, we have gone to a case where in PE=PA and the flow is optimally expanded.

Now, let us say if we increase the pressure in this direction what happens? As we increase all that flow now says as soon you are confining it within the nozzle here, so till it is in the nozzle, it cannot experience what is the outside pressure and also this flow remember is supersonic, so it does not have anything, it does not know what is going to come ahead, so as long as you find it, it will not feel what is the outside pressure.

But the moment you release it outside, it knows that is has to equilibrate with the surrounding fluid, so it try to process itself either through shocks or expansion fans and then equilibrate with the surroundings. If the pressure here is slightly greater than PA, let's say at this point, it will go through this and it will process itself through a series of oblique shocks and expansion fans and then it will equilibrate over some length with the external pressure.

Let us say we increase the pressure beyond this, there is a point at which the fluid cannot process itself through these oblique shocks outside and you will get oblique shock inside and the flow will separate and therefore you will get something like this. Further still, you will get a normal shock and after that the flow will be subsonic and therefore, you will get a pressure recovery. Now, let us look at what happens if we go below this. If we go below this, the flow is going to be the same up to this point.

And then it knows that the outside pressure is much lower than the exit pressure, so it will expand further and then equilibrate with the surrounding fluid, so it will process itself through a series of expansion fans and these will get reflected and we will see that here.

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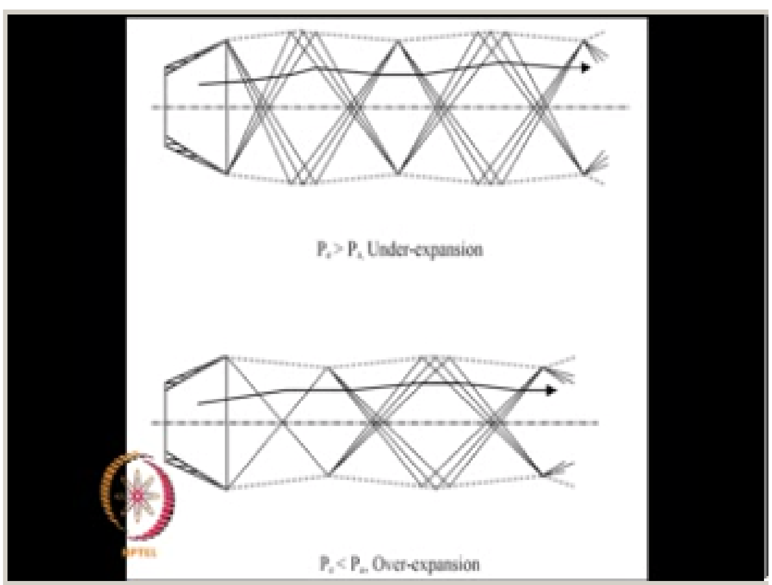

If you see this case here, the first figure, this is when PE > PA so therefore you will have expansion fans and the flow will process itself through these expansion fans. These expansion fans when it hits the get boundary will get reflected as compression waves. These are MOC waves and they will get reflected as compression waves, which will again get reflected from the free jet boundary as expansion fan.

So it will go through series of expansion fans and compression waves and process itself and finally it will equilibrate with the surrounding pressure now, if you have a case wherein PE is less than PA then it knows that it has process itself through an oblique shock, so you will have an oblique shock first and then this oblique when it hits, the free jet boundary gets reflected as a expansion fan.

And the same here on the other side, oblique shock gets reflected as an expansion fan and then these expansion fans will get reflected as MOC waves, so it will process itself through a series of this till it equilibrates with the ambien pressure. Now, how does this effect or thrust equation in water jet's rule is what we have to see.

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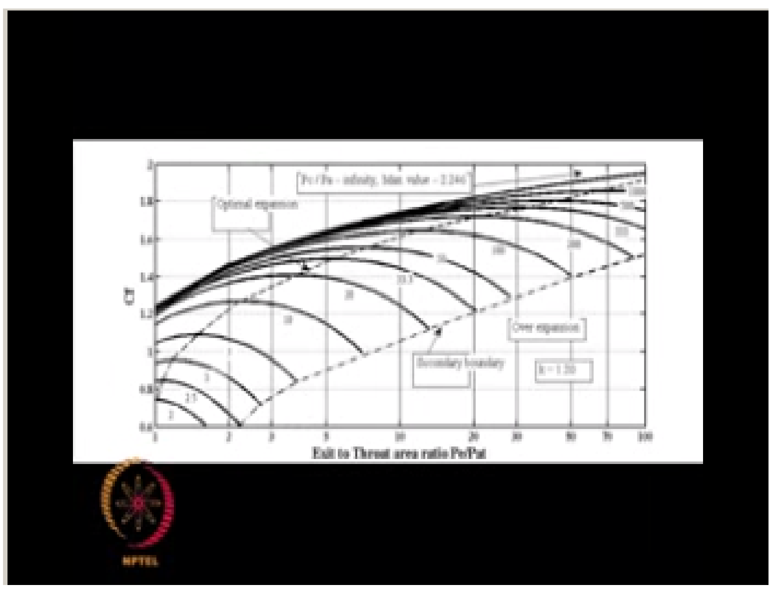

Now, if you look at this figure here, what you see plotted on the X axis is A/80and on the Y axis it is CF and this is plotted for PC/PA different values of PC/PA that is seen to increase in this direction and finally it reaches ∞ and the dotted line here that you see is the locus of all this maxima that you have here for each AE/AT, so if you connect them you will get this line. This is actually the line if you say that an adapted nozzle will have the CF variation.

If you have an adaptive nozzle that is if you continuously keep on increasing the AE/AT, an adaptive nozzle will give this kind of CF profile. If you have an adaptive nozzle that is allowing the flow to be expanded optimally at each and every altitude then the CF of that will be something like this, but let us say you have a nozzle that has a particular area ratio, let us say you have a nozzle that has an area ratio of 5 then what happens to the CF, we look at 5 is somewhere here.

So if you look at having only area ratio of 5 then depending on PC/PA, PA keeps on decreasing as you increase in altitude. PC let us say we hold it constant, so what happens to this ratio, this increases, so you will go from smaller values to larger values and then there is a point at which it will be optimally expanded and then beyond which it will also drop.

Now, there is if you look at this dotted line here beyond this the flow will separate and you will have recirculation zone in the supersonic portion of the nozzle and the flow will separate here. So if you want to have a single stage to orbit vehicle, it is extremely difficult to design this, it will not function optimally

Because what will happen is, if you look at having a single nozzle, let us say you have a nozzle where in which gives optimally expanded flow at C level then if you continue to use it at higher altitudes, it will perform badly as we go in altitude compared to an adaptive nozzle, so we are losing out on some thrust that we could have probably got.

But let us say we do the other thing. We take the nozzle that is expanded at some altitude and try to use it from ground level to higher altitudes. What happens is beyond a point, the flow will separate and you will get to this kind of situation, so either ways, it is very difficult to have this, which is why having a single stage two orbit is very difficult and also if you look at rockets as soon as you expel out some propellant the structure is a waste.

The structure is not useful enough, so therefore if you multistage it then at each stages you can have the optimally expanded flow for some altitude and therefore you will get a much better benefit. We will not probably be able to do if we look back at the adaptive nozzle figure.

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That is altitude, we had seen that we use a nozzle that is adapted at C level and if we use it for higher altitudes also this would be some performance and if we were to use continuously adapted nozzle then the performance was superior, if we do multistage, we will probably get firstly, it will be adapted at C level let us say and you go to some altitude then you make sure that it is adapted at that point.

So it will probably go like this and again, if you separate that stage, it will go in this fashion. In a sense, we are trying to do the adaptive nozzle in stages that is what we do in multistage. Also, if you look at PSLV, PSLV has six traps on motors they do not switch on all the six of them at ground level they will switch on four of them in the ground level and two of them at higher altitude.

The reason being if you switch it on a at a higher altitude then you can have the nozzle adapted to a higher altitude and therefore, it will perform better, so that is in some sense, some kind of multistage or that is done in PSLV, so such things are possible in order to get a slightly better performance. Now, there are also other issues that sometimes if you use the nozzle wherein the flow is under expanded, you will get into some kind of other difficult situations, I will explain that with a small figure here.

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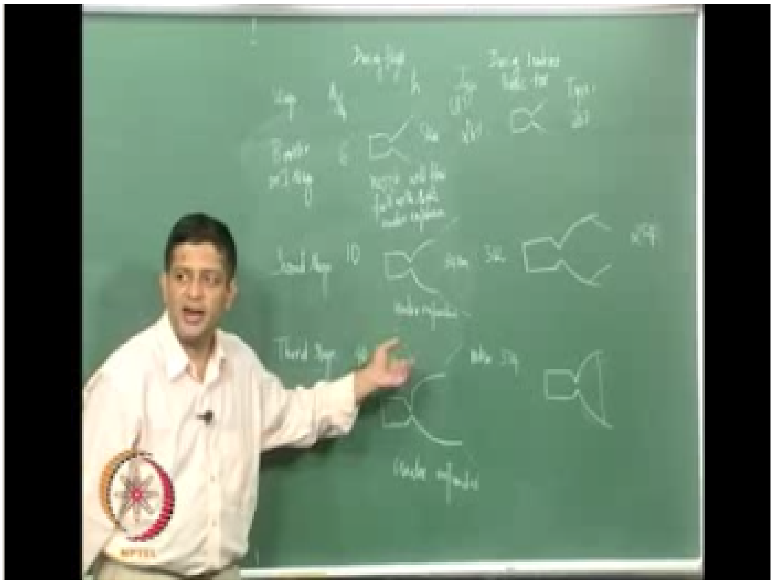

Let us firstly consider a booster or first stage then a second stage motor and then a third stage motor. What would you use for a booster, what kind of area ratios would you want to use for a booster? Is it going to very large or is it going to be smaller number, so you will typically use something like six to eight, let us take the case where it is six and for the second stage, you will use a larger area ratio.

And then also for the third stage, you will use a still larger area ratio. If you take the nozzle for this, during flight the pressure if it does go to a very altitude that is the maximum altitude is something like 5 km, the nozzle will flow full with slight under expansion and ISP will be the order of 2670, this is in or I will put as 267 this is in seconds okay.

Now, if you test it at sea level conditions remember whenever we want to send a rocket up, we need to also test it in a static test facility, so if we test in a sea level static test facility, this nozzle would probably be flowing full if you test in sea level conditions, there is not going to be too much of variation here and you will probably get similar values for ISP. Now, if you look at the second stage it has a fairly large area ratio.

And the flow will be under expanded here. Here again, the altitude is something like 24 km and the ISP that you get will be something like 312 seconds, so the flow is here under expanded. What happens if you use these nozzle at sea level conditions, it is in such a way that it is optimally expanded at some higher altitude.

Now, if you use the same nozzle like sea level conditions in a static thrust, what kind of ISP will you get lesser because if you look from the graph, the CF will reduce because there will be probably flow separation that takes place and the nozzle might not flow full, so therefore you are probably going to get something like ISP of 254 seconds.

And if you use a third stage motor wherein it is expanded for something like 100 km altitude and it gives something like 334 seconds, then the flow beyond this will expand further, so it is under expanded again and if you use this at a sea level condition probably you are going to have a normal shock setting somewhere in the supersonic portion of the nozzle and you might get a very low ISP because the flow beyond the supersonic after the shock.

The flow will be subsonic and therefore, probably you might end up getting a very low ISP here. So one of the problem is if you looking at this scenario, wherein the flow expands even further you should be careful to avoid having any instrumentation in this region because at a higher altitude the flow might turn back and you might have hot gases impinging on this equipment, which it might not able to withstand.

So therefore it is either better to have some insulation there so as to prevent the heating of these equipment. Till now, we have looked at an analysis wherein it only looked at was one dimensional flow we had assumed the flow to be isentropic then we had also assumed the flow to be an ideal gas and then all the parameters that all the parameters that is the CP thermal conductivity viscosity whatever in viscid flow, so there is no viscosity. The CP does not change with flow in the nozzle.

We had assumed that CP is constant and thermodynamic properties are also constant. In the next class, let us look at how changes in the real world will affect whatever we had derived. In most cases in Engineering, it would be nice if we can get a closed form solution that is if you have a real situation.

And if we can get the entire solution for that particular situation, otherwise the next best thing that we can do is let us say we can get the bounce for it. if you have an upper bound and a lower bound and if you say the solution might be somewhere in between these two that is also good for us because then we know it is going to be within these two values, so that is what we are going to do in the case of nozzle flow, which we will discuss in the next class thank you.

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