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Aerospace Propulsion

Rocket Nozzles – 1D Analysis II

Lecture 19

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Good morning, in the last class we had seen this that we derive the thrust equation and we had also derived equations for mass flow rate they A/At relationship in terms of Pe/Pc. And we derive the expression for the exit velocity Ue.

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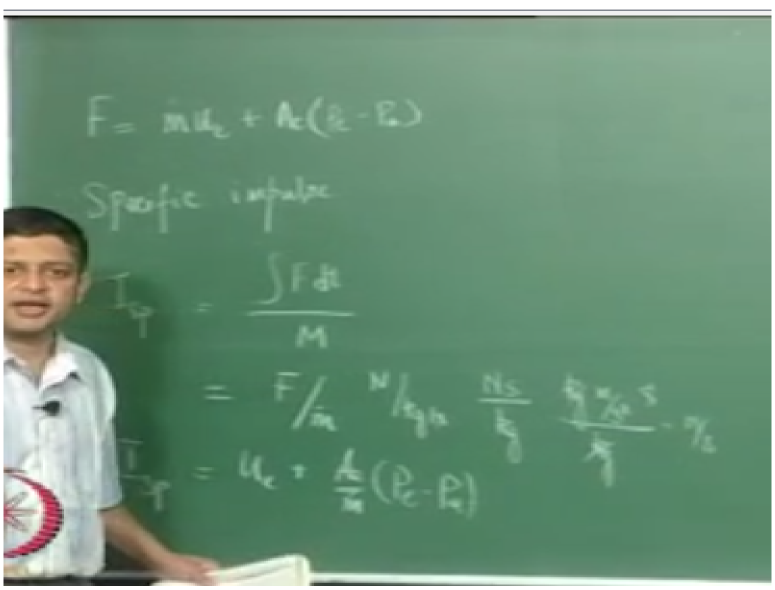

Now if you look at the thrust equation we had derived expression for m.Ue/AeAt in terms of Pe/Pc. We can know go head and define parameter that is of time importance and rocket propulsion. That is the specific impulse. Specific impulse as the name itself suggests is nothing but impulse per unit mass okay. If you look at the expression for impulse, impulse is nothing but ∫Fdt per unit mass of D propellant this is known as specific impulse or it is denoted by Isp.

And when you work the out you can get the expression for Isp cross per unit mass flow rate, m.s this, so you get expression as thrust per unit mass flow rate. Now we also know that if you take the m dot here you will get Isp=Ue+Ae/M dot. Now let us look at how to proceed on this further and get expression for Isp in terms of, if you look at the this equation we know they expression for Ue, we know M dot, and we know Pe/Pc in terms of Ae/At.

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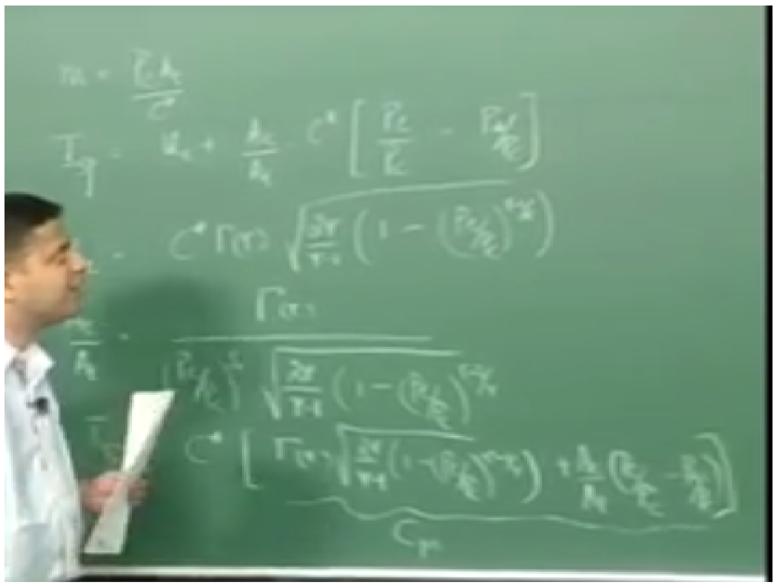

We will use that to get an expression for Isp now. M dot i we had got this last that is m dot = PcAt/C* okay. So I will use that and I will get. Now mUev also had derived an expression for in the last class that is Ue the exit velocity, and we had also derived an expression for derived Ae/At in terms Pe/Pc. Now if you substitute back all these terms we will get an expression for Isp.

If you notice in this equation Isp the unit of this is meters per second, and if you look at all the other terms here Pa/Pc, Pe/Pc and Ae/At one dimension, and also we have C* which is again meters per second. But if you look at this definition you get thrust per unit mass flow rate which means that this is Newton, or in other words Newton second per kg. If you do expand Newton then you will get Newton is nothing but kg meter per second square.

So what do you get, so you are now getting meter per second. And also if you look at most of the literature in rocket technology most of the books will give you unit for Isp in terms of seconds. Now if you divide this by the acceleration due to gravity that unit of that is meter per Second

Square then you will get the unit that usually rocket technology books will give you that is in terms of seconds.

So both of these are used Nsec/kg and seconds are the units that are commonly used. If they are using an Si system they will use this, otherwise they are going to use seconds. So now we have derived this expression let us substitute back I am now going to use this primarily, because if we remember the discussion last time we would be knowing the geometric parameters that is Ae/At, and we want to know what is the variation of Pe/Pc.

From this expression it is very difficult to extract out this. So therefore, what we said in the last class we either use graphs or tables to get this value. So I will not use this here, but without this I can write. This is the expression that we get, I have to have this brackets here. Now if you look at this look likes Isp is the function of two terms one is C* and there is another term in the brackets. This entire term we will denote it as Cf which is known as the thrusts coefficient. Why it is called as the thrust coefficient? I will be able to show it to you in a minute.

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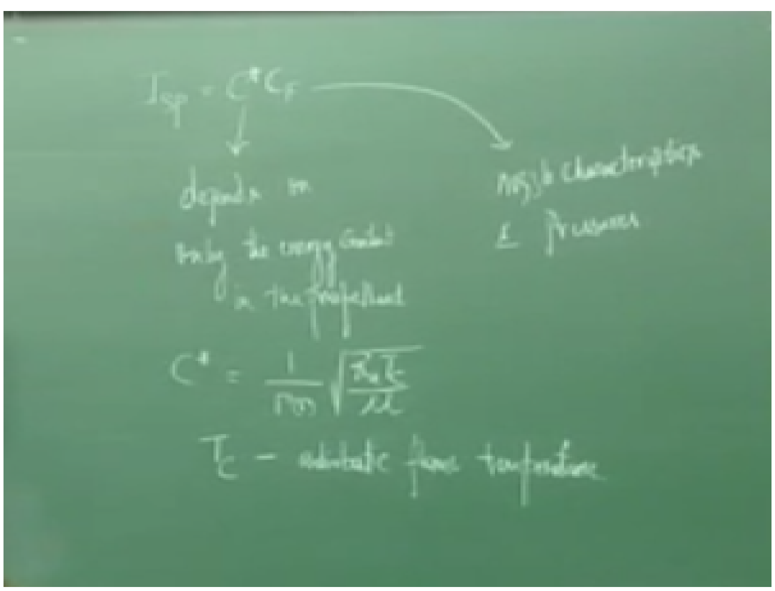

So Isp we can now write it is as a function. Now C* this depends on only the energy content and Cf, if you notice here it depends on the nozzle characteristic which is given by Pe/Pc and the nozzle geometry Ae/At. So if you know the nozzle geometry, and if you know the ambient pressure, and chamber pressure we can calculate the rest of the parameters. So it is only a function of nozzle characteristics.

So the reason for dividing this into these two components will be obvious in a short while from now. Now C* if you remember is given as 1/γ/γ(√) right. So C* depends on Tc and the molecular weight, Tc is nothing but the adiabatic flame temperature. So this is determined by what is the energy content on the propellant and M is the molecular weight of the gases. So if we divide it like this, we will be able to differentiate for a given nozzle which of the propellants gives us the best performance.

We will come to that in a little later; we will now derive what are known as rocket equation based on all these things.

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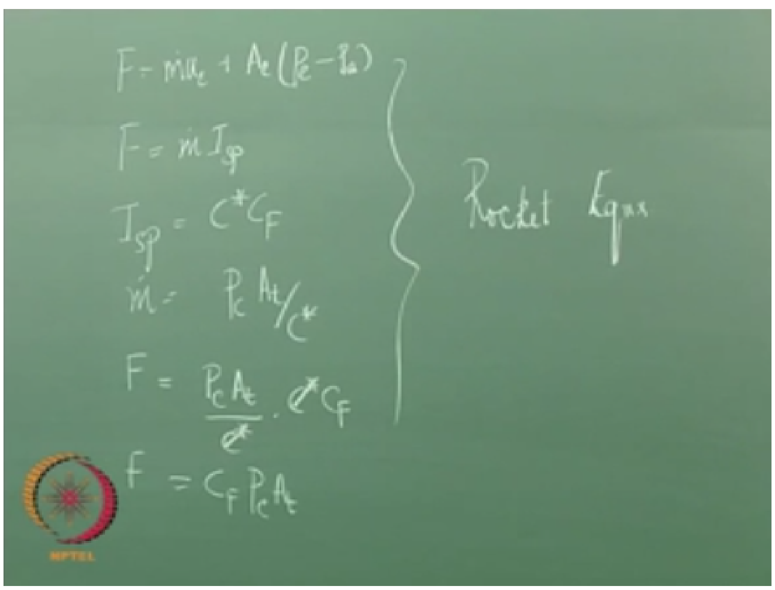

We now know that f=m dot Ue+Ae and we also know f= m dot Isp from the definition of specific impulse. And if I use this Isp is nothing but C* Cf, and m dot we know is nothing but Pc At/C*. So if we plug these two into that equation we will get f is equal to, I had call this as the thrust coefficient. Now if you look at this equation it is obvious why it is call the thrust coefficient.

Because these two terms multiple gets multiplied by a coefficient Cf to give as thrust which is why we named thrust coefficient. So these are known as rocket equations and we will be using them pretty frequently in this course. As I said if you look at Isp a define it is as C* Cf. Now C* I said depends only on the energy content of the propellant. Now if you are to determine which of the propellant is better?

Then we can doing a following exercise we can make it independent of Cf that is if we fix certain nozzle parameters, if we fix the chamber pressure of particular value. The area ratio is at a particular value, and then the ambient pressure is one atmosphere. Then we will be able to make the Isp independent of nozzle parameters right. So that is independent of Cf that is what is done.

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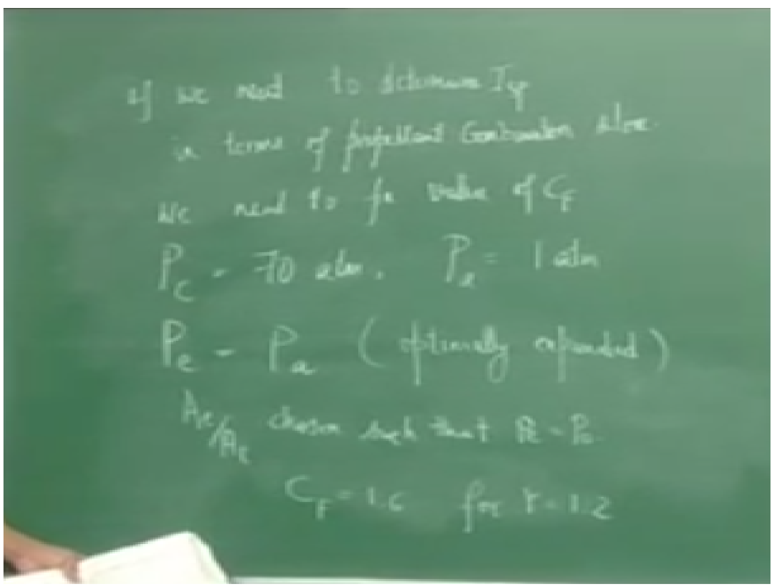

And typically, so we need to fix the value of Cf. And to do this we keep the chamber pressure that is Pc at 70 atmosphere, then the ambient pressure is the mean C level pressure that is one atmosphere. And we ensure that the nozzle is such that it is expands in way were Pe=Ta which is also known as the case where the flow is optimally expanded. And we choose the area ratio such that it gives this, that is Ae/At is chosen such that okay.

Now having done this we will get value of Cf for this to be around 1.6 for γ=1.2. So having fix this Cf we can now evaluate various propellant and see how would each one of them is okay.

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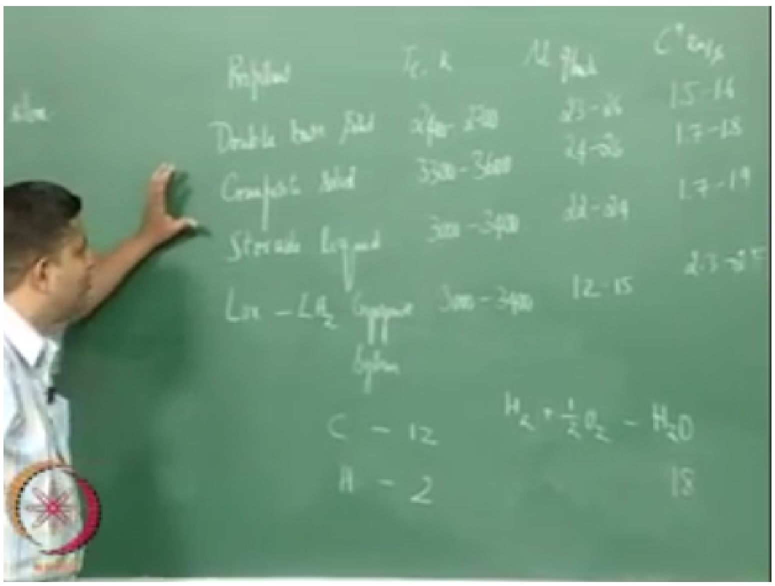

And if we do that, we will now try and see various combination of propellants and what is the Tc molecular weight, and C* that we get if we do this. There are two kinds of solid propellants, one is double base, composite solid, then there is storable liquid and lastly Lox-LH_2, that is gyagonic system. Now if you look at Tc for this these are the values that you will get. If you notice in this, there are several interesting things, the C* value for Lox-LH_2 system will be highest.

And therefore, the Isp for the system will also be highest and that will be around 400+ seconds. If we need to understand why this is happening? We also needs to look at this and this, if you look at these two the temperatures are similar in fact of in this it is little higher right. But yet the C* values of this is higher than this. The answer to this lies in the fact that C* is not only a function of chamber temperature, but also of molecular weight.

If the molecular weight is lower and the chamber temperature is high enough, then you will get a very good C*. And that is what is done in this LH_2-Lox gynogenic system okay, you not only get a lower molecular weight, but you also get a reasonably high temperature. And therefore, you will find that the Isp for this is the highest. It also true that any kind of propellant that we try to we always try to make it fuel rich.

The reason being, if you look at the molecular weight of fuels, the typical fuel elements are carbon and hydrogen. Carbon and hydrogen have molecular weights of 12 and 15 sorry H is 2 right. So carbon has a molecular weight of 12 and hydrogen 2. Now if you combine this with oxygen, if it is completely burns this will give to CO_2 and this will gives raise to H_2O. If it is completely burns in a liquid Lox LH_2 engine the reaction would be H_2+1/2 O_2 gives rise to H_2O.

If you look at the molecular weight of H_2O, H_2O molecular weight will be 18 right? But what we have got here is lower than that that is around 12 to 15. The reason being we want to make it fuel rich while not severely decreasing this component that is the chamber temperature okay. Then we will be able to get a higher C*. So in a sense all propellants we would want to make them fuel rich.

So that the C* value is that we get that higher okay. The other important thing that is present here is if you look at solid propellants to liquid propellants in a sense you will find that liquid propellants have a slightly better performance even in the case of storable liquid compared to solid propellants. The reason is like this, if you look at the solid propellants, solid propellants both the fuel end the oxidizer need to be present together and same chamber right.

So that imposes a restriction on what kind of fuel and oxidizer that you can choose, they need to compatible and they need to not start reacting as soon as they are mixed together. So that places is a severe restriction on the choice of fuel and oxidizer. But in a liquid propellant there is no such restriction, because stored separately, they are stored in different chambers, and they are only made to come and contact with each other in the composite chamber.

So therefore, you can choose better liquids in this case, whereas the choice is most restricted on the case of solid propellants. And therefore, you will find specific impulse of liquids will be superior to those of solid propellants. We all know this that rockets also perform outside the sensible atmosphere, the kind of things that we discus here that is Pc Ae/At such that it is optimally expanded flow it is optimally expanded ambient pressure of 1 atmosphere.

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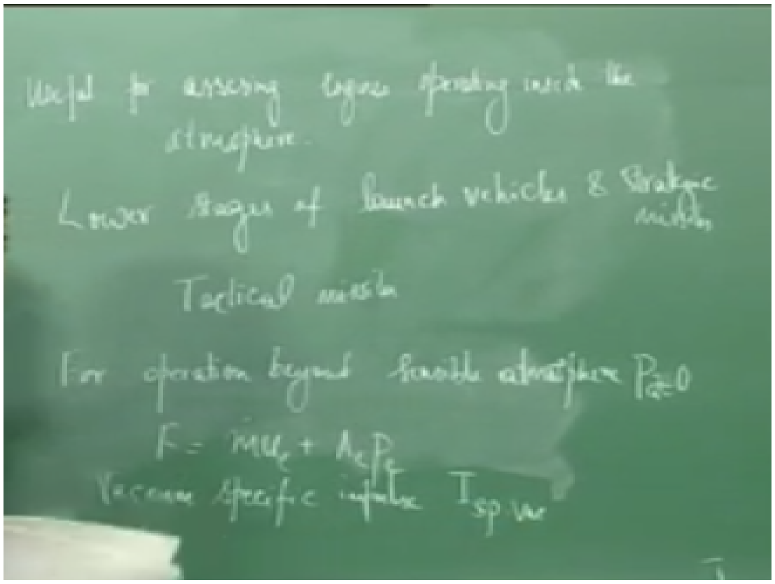

All these things are useful for assessing engines operating inside the atmosphere that is the lower stages of launch vehicles, and also certain tactical missiles. And what we do even when we have to look at things operating beyond the sensible atmosphere. Now what happens to the thrust equation that we have derived in this equation, if we are operating beyond the sensible atmosphere Pa goes to 0.

And therefore, we will be explain the expression Pa is approximately is equal to 0. And therefore, we will get F is equal to – notice that in the equation here the term Pa is with the negative sign. So therefore, if you remove that component your thrust will increase. So therefore, you will find that the thrust that is delivered by the rocket motor at vacuum conditions is higher.

And corresponding will be specific impulse which is known as vacuum specific impulse will be higher than the specific impulse that you will obtain when operating within the sensible atmosphere. Typically vacuum specific impulse will be 10-15 percent higher than the specific impulse obtain with in the sensible atmosphere or C level higher speed.

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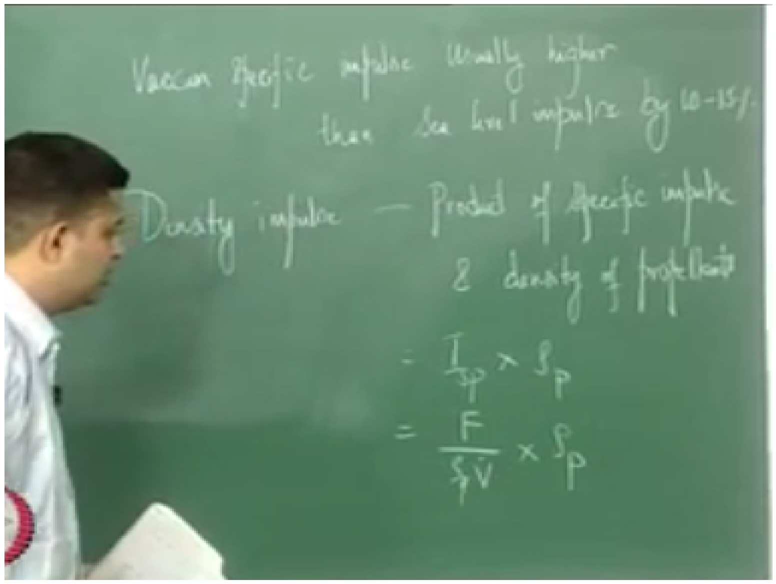

So we now learnt what is vacuum specific impulse and specific impulse. Those were another parameter called as density impulse. Density impulse is nothing but the specific impulse multiplied by the density of the propellants. That is product of Isp x Op, Isp we know is F/m dot that is f by m dot I can write it in terms of mass flow into volume flow rate, m = ρV, if I use that then I can write m.dot as ρP(V) dot.

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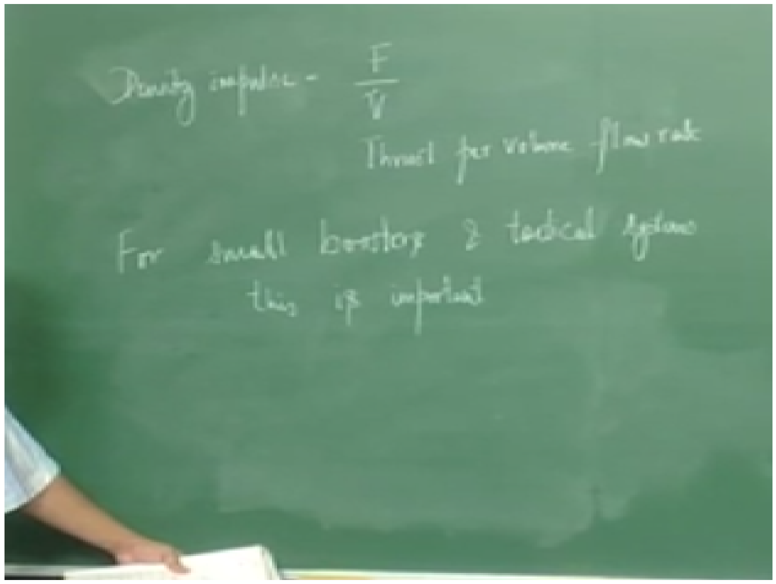

So using this that is thrust per volume flow rate, please remember the ρP that we are using there is not the density of the product gases, but it is the density of propellants. So where do you think this would be useful, if you look at systems that operate within the sensible atmosphere you would be want them to be as compact as possible primarily because then you will be able to reduce the track or your net thrust will increase.

Your net thrust is nothing but thrust of the system minus the track. So therefore, if you want to net thrust to be higher you need to also have drake load. So if you have a system that is operating within the sensible atmosphere that is very bulky or a large volume, then that will not produce the lowest drake. So therefore, you need to have this optimize for systems that operate within the sensible atmosphere.

That is for a small boosters and tactical system, the reason for this if you look at the Lox-LH_2 system what is the density of liquid oxygen, any idea around 1100. And if you look at the liquid hydrogen the density of liquid hydrogen is around 70 kg per meter cube, it is a very low density liquid. If you look at the space settle, the large time that is on the back side of the orbiter most of it is the liquid hydrogen tank.

Because it is very low density fear, if you have such a system for a tactical system then the volume will be very large although the higher speed is better, the volume is very large, and therefore, the drake will be also the very large. So in such cases is it is better to go in for a solid system where in the density will be higher if you compare the density of this to density of solids, solids will be of the density will be of the order of 1600-1800 k/m^3 which is very large.

So the system will be much more compact. So for tactical system and small boosters, it is better to go in for solids and for large rocket rotors that operate we on this sensible atmosphere it is better to go for storable liquids or gynogenic engines, because then your higher speed will be higher. It would be a combination of these two, typically as I said earlier we do not use the stoichiometric ratios.

We use it fuel rich that is typically oxygen will be the order of 5 to 5.5, the oxidizer to fuel ratio. So you need to take that and then calculate the density. So we now learnt the what is it that we need to use if you looking for a large system that is we need to use storable liquid or gynogenic system when the operation is beyond the sensible atmosphere. But when we are looking for a tactical systems that operate within the sensible atmosphere and for small boosters, it is better to go for solids.

Now there is also if you look at thrust equation there are two things that are varying, the Cf will vary with respect to the area ratio that we use.

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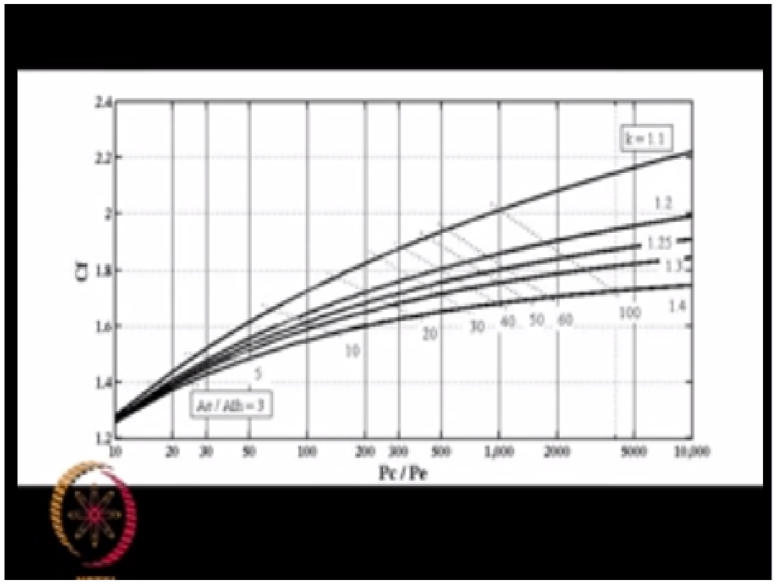

So if you look at this graph here, on the Y axis you have Cf variation, and on the X axis you have Pc/Pe variation, and the graph also shows for different area ratio of the nozzle, and for different values of γ right. Notice that for a low Pc/Pe and for a low area ratio things do not change too much all the values different values of γ give raise to a almost a single value of Cf around 1.2 okay.

As we go to a higher and hig her Pc/Pe the value of Cf changes with values of γ right, and for large area ration, you see here that Cf will change with the value of γ, and that change is not very small if you notice somewhere around 1000 it varies between 1.7 to 2. So that variation is not small because Cf and C* is work uses Isp, so the Isp would severally change depending on the value of γ.

So also we need to keep in mind that the value of γ that we made use of in all are calculation is like this that we have taken γ to be constant and not varying beyond on the nozzle entry point. As I said earlier also the value of γ will change because these are reacting compounds an temperature and pressure are changing. And therefore, you will find that γ will change during the expansion process.

And it is important to note that during the expansion process if the γ changes, then there would be significant change in Cf depending on the change of γ. We look at all this things little later in the course. Now we know that thrust varies if you have a rocket system that is operating through the atmosphere axie level that is a ambient pressure, and the ambient pressure keeps all following as you go higher in altitude.

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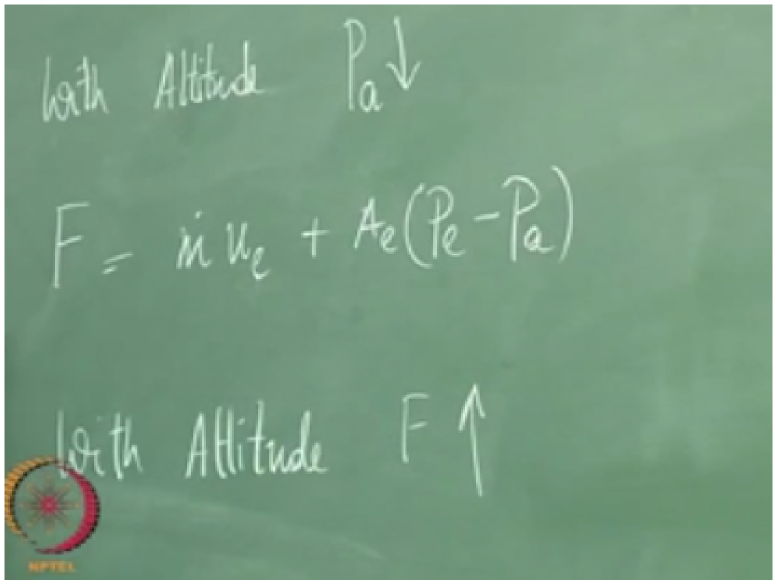

So if you go with altitude Pa will decrease. So therefore, if you look at the thrust, thrust is nothing but m dot Ue(Ae Pe – Pa). Let us say we have a fixed area ratio nozzle which is typically the case. Then the Pe is also fixed only the Pa keeps on decreasing as you go higher in altitude. And therefore, you will see with altitude the threats increases, because of this increasing thrust, people have been looking at what is known as adoptive nozzles which we will discuss in the next class. Thank you.

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