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Aerospace Propulsion

Cycle Analysis – Turbojet III

Lecture 13

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In the last class we had seen how to get the expression for non-dimensional thrust for a turbojet when we had the condition that the flow is optimally expanded through the muscle optimally expanded through the nozzle is a condition that is most times not satisfied the other condition that is the flow is choked as it leaves the nozzle is the most probable one in case of turbo jets okay turbo jets as I said in the previous classes other than what was used on Concorde all other turbojet engines are primarily a noisy nozzle.

They do not use a convergent divergent nozzle this is because the exit pressure after the turbine is very low so you do not have a scope for using a convergent divergent nozzle okay so let us look at the case where the flow is choked.

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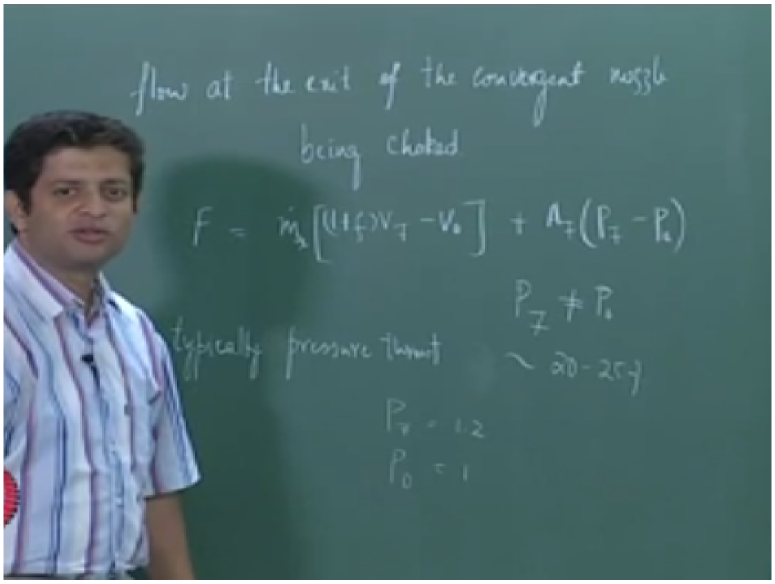

Now at the exit of the convergent nozzle what does this mean if we accept of the converging nozzle the flow is – what does it imply to our thrust equation if you remember our thrust equation was F = m. A into right this is our thrust equation if the flow at the exit of the nozzle is choked then what it means is this part is not going to zero in all the previous expressions that we derived we had assumed that the at the exit of the nozzle the exit pressure is equal to the ambient pressure that is the condition for optimally expanded flow.

Now we are saying that the flow is only choked and p7 need not be equal to P0 so here p7 ≠ P0 so therefore if we produce a pressure thrust also typically pressure thrust is around 20 to 25% of the overall thrust so there is not a small portion so in this class let us look at how to do this analysis when we have this condition before we go that what do you mean by flow in the nozzle is choked when do we say that when do we say that the flow through the nozzle is choked P7 =1.2 P0 = 1 this condition chopped and then what do we mean when we say that the flow is to go through the nozzle is going yes.

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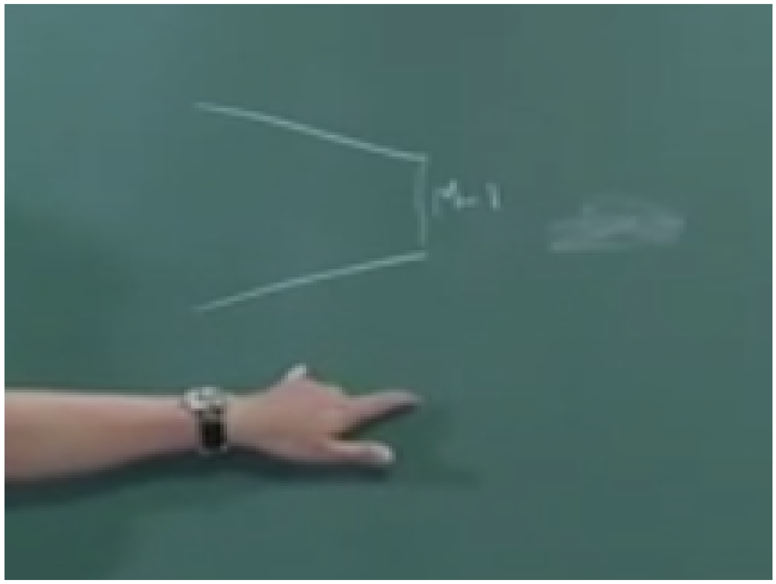

So if you have a nozzle a converging nozzle at the exit of it if the Mach number is one then we say the flow is choked the reason for that is the reason why we say the flow is choked is what happens when you have Mach number one here whatever happens in this side okay you cannot transmit any disturbance or any information upstream because the flow has already reached the speed of some disturbances propagate upstream at the speed of sound right, so if the flow is already reached speed of sound at the exit they can no longer the flow cannot feel anything in the downstream conditions.

So if M =1 is reached then the flow becomes independent of downstream conditions and is only a function of upstream conditions right so which is why some people when they plot the mass flow rate with back pressure they say it reaches a maximum that is a little confusing you just need to say that it becomes independent of the back pressure okay typically if you take any nozzle when the flow is not choked the mass flow rate is determined by both the pressures only upon the nozzle being choked then it becomes independent of the downstream condition and becomes only a function of the upstream condition okay.

So now let us look at the case on our hand wherein you have to do the analysis for the flow when the reconversion nozzle is choked and this is our thrust equation and we cannot neglect the pressure thrust part okay.

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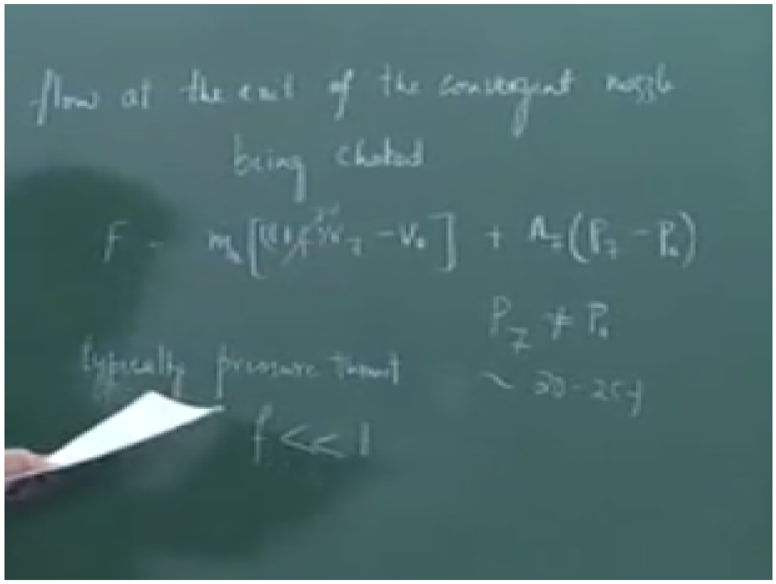

So we can again use or we can say that F is still very much less than one because most times the main gas turbine engine operates with somewhere around 0.02 or 0.04 okay so purely ratio of 0.04 so f is very much less than 1 so this goes to 0.

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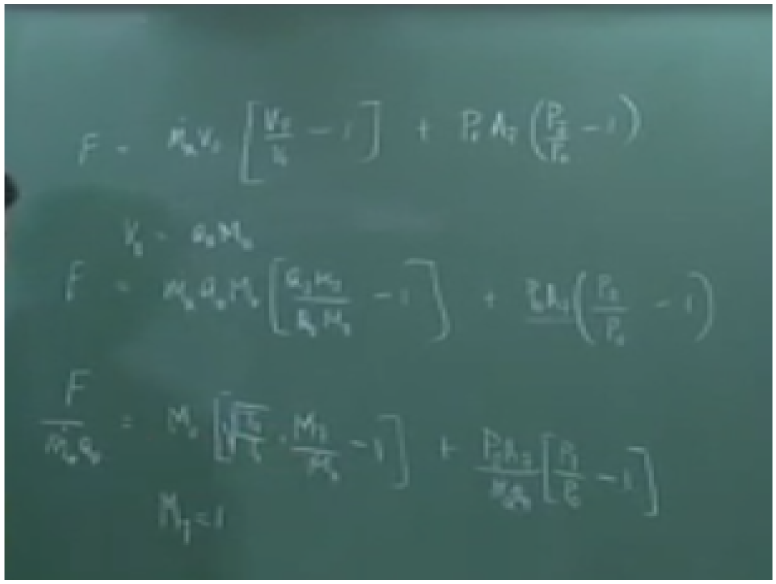

So you can rewrite our expression as f = ma.v0 into V7/ V_0 -1 + I will take out P_0 here so I will get P_0A_7okay now the this portion we can simplify as we had done earlier but in addition we have another component here so what is it that we do V0 = e_0 M_0 right so if you put that and M_0 is f is equal to m . a a_0 M_0 and V_7 / V_0 I can express it as a7 okay now using a familiar stuff that is we know that we assume that γ and r do not change across the gas turbine engine so you get a7 by a not as an expression in terms of temperatures so let us do that I can rewrite this as f/ m.a v0 = M_0 into under $\sqrt{}$ T_7 / T_0 into M7/ M_0 - 1 + B_0 A7/ m . a a_0 okay.

Now what do we do further we would do is the condition that we know we know that m7 = 1 okay at the exit of the nozzle the flow is took that as Mach number is 1 okay so we can substitute that here and rewrite our expression as.

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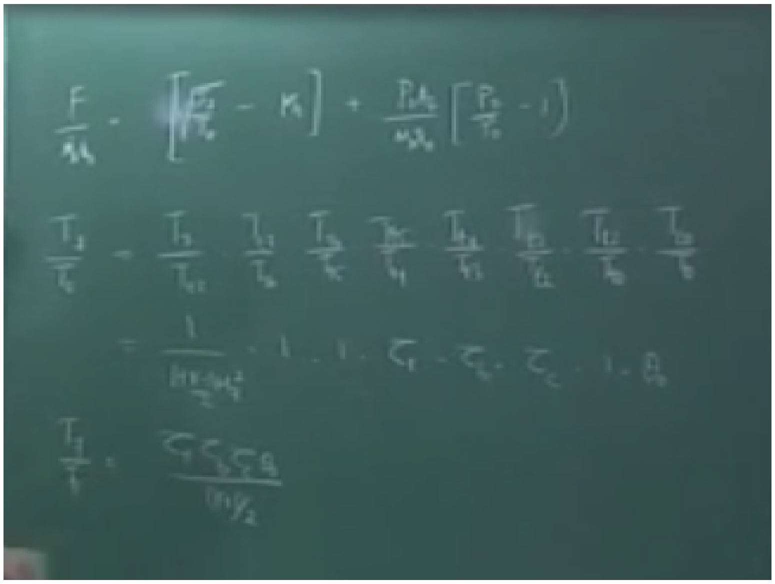

So n7 goes to one here I can bring in this M0 so I will get only under √ T7 / T0 - this will become M0 so this is the expression that we have now we need to find ratio of T 7 / T0 and also P 7 / P0 in the earlier case P7 / P0 we used to take it as equal to 1 and find using that find the ratio of Mach numbers now that is not equal to 1 so you need to find two ratios of temperatures and pressures now coming to T7 /T_0 we have done this exercise earlier.

Now that just that the nozzle exit is choked do you think this ratio is going to change so we need to do a first generation of this ratio you think it is going to change what will be conditions only thing let us see how it is going to change TT7 / PT7 into TT 7 / TT 6 into TT 6 / TT5 TT 5 / TT 4 okay now what is this is the only place probably it can change all the rest of the terms are similar to the previous conditions right only thing that will change is probably here right here we know this is the ratio of we can express this is ratio of static to stagnation.

So we can express it in terms of Mach numbers $m7^2$ then this is 1 this is again 1 after burner first one is nozzle this after burner this is τT flow through turbine then τ B flow through combustor or burner then τ C flow through compressor and this is flow through intake which is again 1 and this is θ_0 okay now here we know that m7 = 1 so we can get an expression for T7 / T $_0$ as τ T okay now we also know what is τ B if you look into your notes the previous class we derived this.

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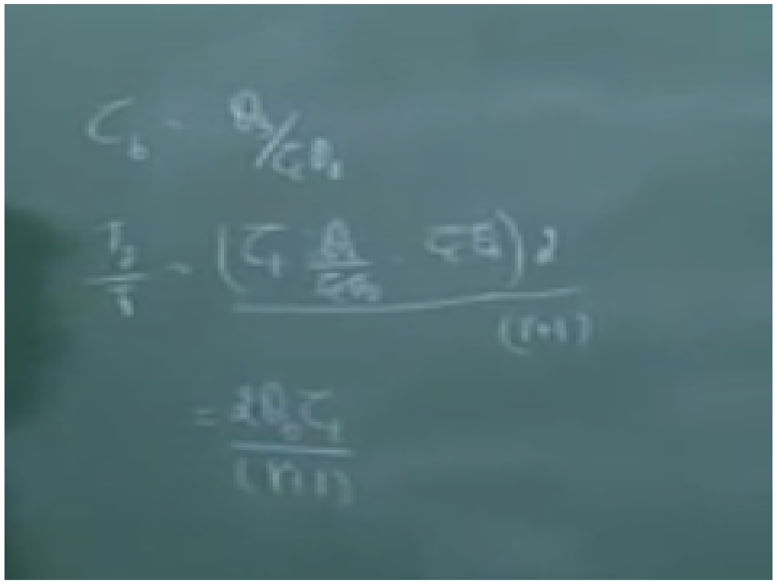

What is τT τ/ θ_0 okay and if you plug that in you will get T7 / T_0 = τT instead of τ B I have θ b by τ C we cannot into τCθ_0 okay so this cancels off and I am left with θB okay so this is the expression for T7/ T_0 for the condition when the nozzle is choked now what happens to the compressor turbine power balance does it change because of this or whatever were derived remains the same see what is happening is something downstream of the turbine right.

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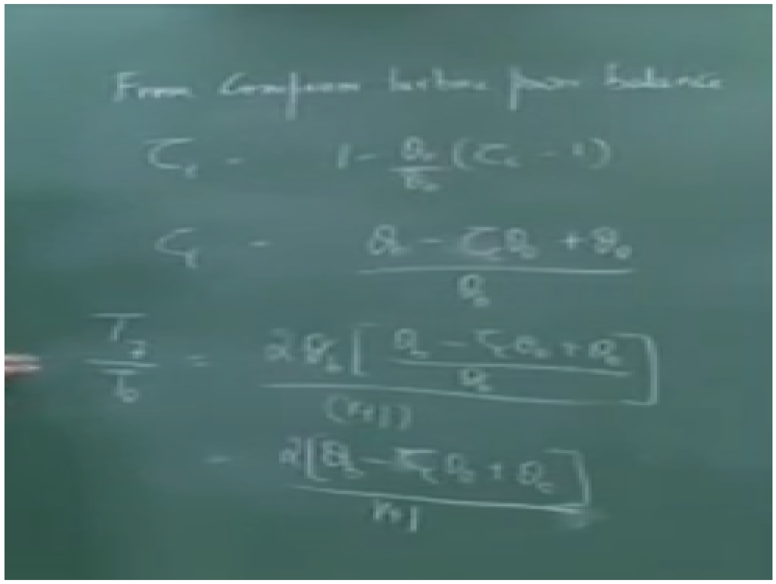

So whatever we are derived is upstream of this so this should not change so what we had derived for the compressor turbine power balance does not change so we derived that from compressor turbine power balance he had derived that $\tau T = 1 - \theta_0/\theta$ B into τC - 1 this remains the same okay now we have to find an expression for τT here so let us do that I can rewrite this as τT is equal to θB - and using this expression into this equation I will get $T7 / T_0 = 2$ into θB so this and this cancels off and finally left with 2 into $\theta B - \tau C \theta 0$ okay.

So we have been able to get the temperature ratio here that we were looking for right now what are the other things that we need to derive here we have got this part temperature ratio done we still have to find the pressure ratio and we have to derive an expression for this quantity here okay so let us do that now.

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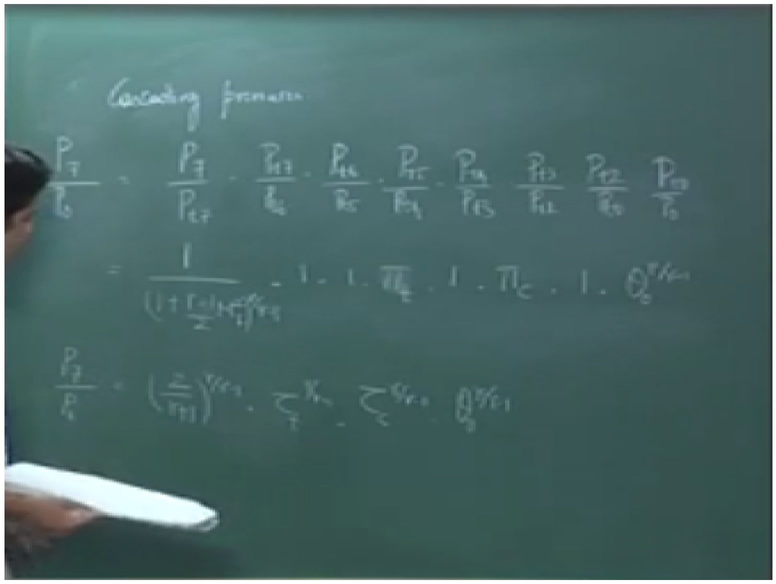

So again cascading pressures you are looking for an expression for P7 /P_0 we can write it as P7 / PT7 into PT7 /PT6 PT6 / PT5 okay now when we do this in the previous case we had assumed this I mean we had assumed the flow to be optimally expanded so this became 1 now it is a not equal to one so we will get the first term is ratio of static to stagnation conditions so I can express it in terms of Mach numbers okay the next one indicates flow through nozzle we have assumed all efficiencies we are doing this analysis assuming all efficiencies to be 1 so this is 1 again the next is flow through the after burner we have not switched it on so this is again 1 and what is PT5 / PT4 this is flow through turbine.

So this is this is pressure ratio so this is PI T into PT4 / PT3 is flow through combustor the pressure is the same because we are assuming an ideal cycle here so this is 1 and this is flow through compressor so this is π C and this ratio is for flow through diffuser or intake this is 1 because we are assuming efficiencies to be unity and the last term is $T2_0$ to the power of γ / γ - okay.

So I can write P7/ P_0 = this is 1 so I will get 2 / γ + 1 into $\tau T^{\gamma / \gamma - 1}$ into τC^{γ} by okay and we do know that τ T and τC are related through compressor turbine Power Balance and if we were to take that into account we can show that the ratio Ps7 / P_0 would be.

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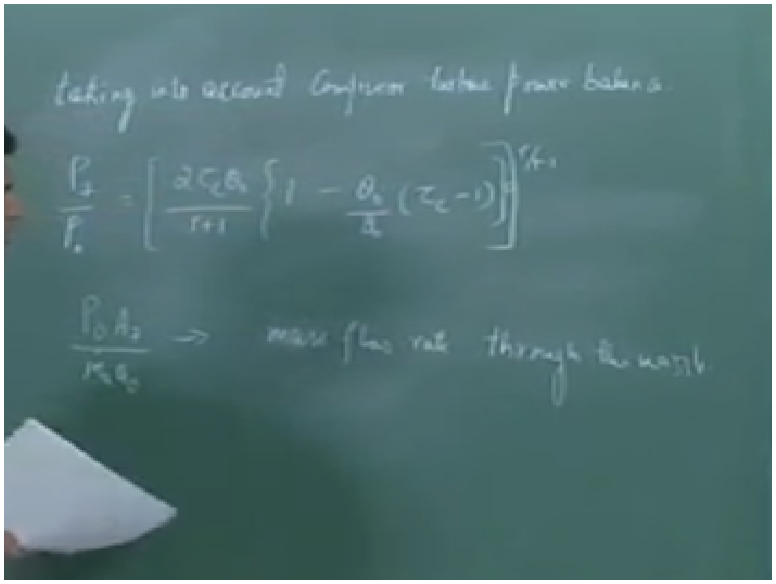

Taking into account compressor turbine power balance we saw that the compressor turbine power balance was unaffected and we could get this expression for τT now using this in that expression there we can get okay right okay I have only substituted for τT there $\tau C\theta_0 / Y$ and so this is the expression that we have now we have been able to get expression for two quantities the last quantity that we need an expression for is we need an expression for okay so how do we go about doing this how do we get this expression yes they just like to get an expression for F you said he said compressor turbine power balance.

And then to get an expression for τT in terms of τC sorry in the last case energy balance across the combustor what was what we used to get an expression for F to get an expression for τT we said compressor turbine power balance and we got the expression similarly is there something that we can do here to get this expression yeah what we need to do is we need to look at mass flow rate through the nozzle we will see how we can use that to get this expression. Now mass flow rate through the nozzle what is the expression that we know.

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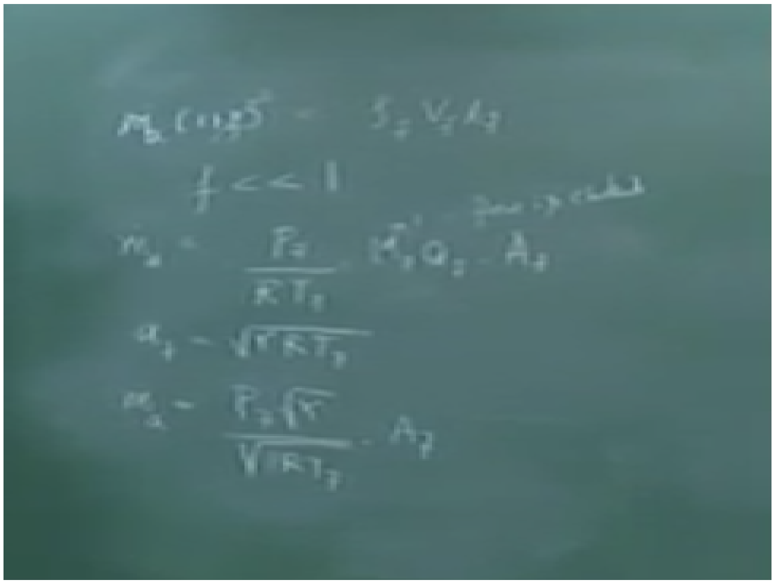

We know that m.a into 1 + F that is mass of F + mass of fuel burnt this must be equal to 07 v7 into a7 right now again we can make this approximation that F is very much less than 1 so therefore this goes to 0 so I get m. a = δ7 v7 a self now what is δ7 in terms of pressure. e is equal to using the equation of state I can write P7 / RT7 for δ 7 into V7 I can rewrite it as what do not number into a7 right.

So into A7 do I get what we are looking for is P_0 into a7 / m . a a0 okay so what do we need to do here this we know is this quantity is what is this because this is flow is so m7 = 1 so I am left with what is a7, a7 is nothing but γ RT7okay so if I substitute for a7 here rewrite m .a I will get P 7 under $\sqrt{\gamma}$ divided by this would not be there RT7 into a 7 okay now what do I need to do I need to multiply by if I multiply by a0 on both sides I get one part that is m . e a 0 okay.

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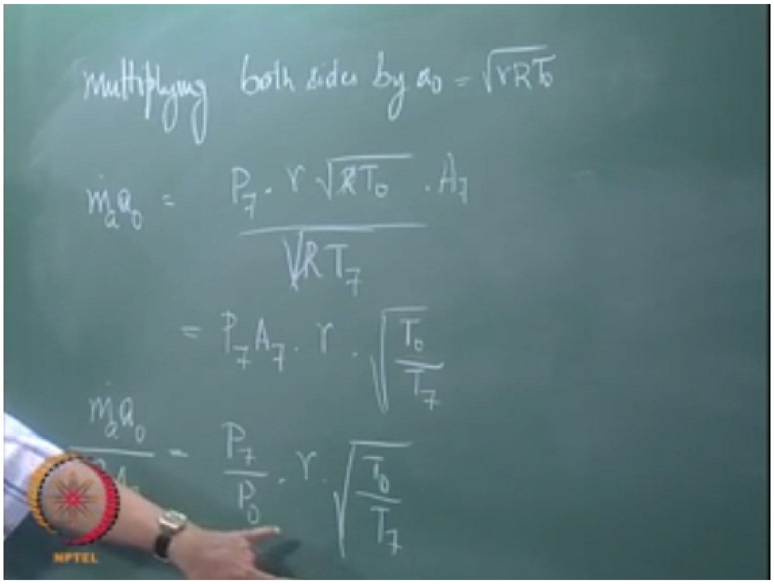

So let us do that I get m .a a0 is equal to again a $_0$ is nothing but right so I will use that on the right hand side so I get P7 into γ under √ RT0 okay so R and R cancels off here so I am left with P7 a7 into γ into okay right now what do we need to do again so if I want m . a / a_0 / P7 P0 by a7 right so I can do this I can write this as m . a_0 / P0 a7 = P7 / P_0 into γ into under √ T7 T_0 / T7 okay.

So we will been able to reduce m.aa $_0$ into two known quantities right we already know the expression for P_0 / P7 / P_0 and we also know expression for 7 /T $_0$ so we have been able to reduce it to this form now let us go back and substitute it and see what we can get if you remember our expression for our expression for.

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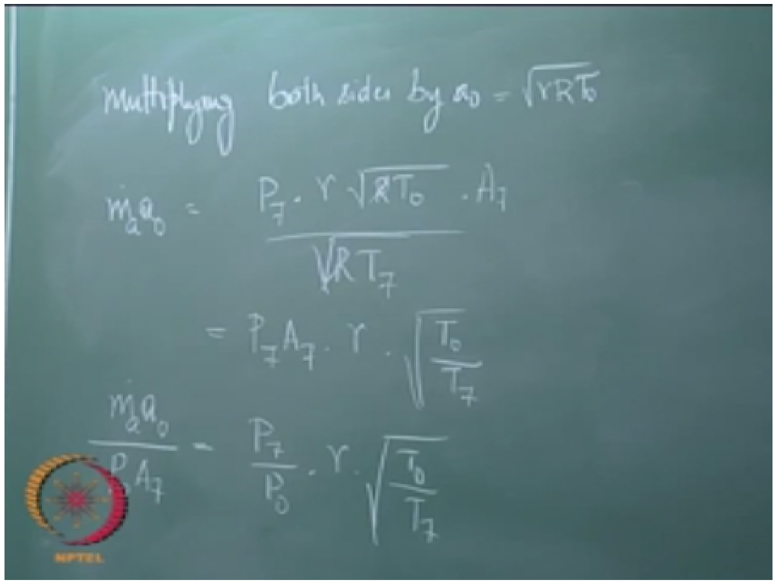

F/m . a a not great watt was under √ T7 / T0 - M 0 this part was taken care of and sorry P0 a7 by m . A a0 into what was the P7 / P0 -1so we have got expression for this as well as this so when we substitute we get this part remains as is P7 / E_0 - M_0 + P 7 / P7 P0 a7 / m . a a0 is 1 /γ into okay.

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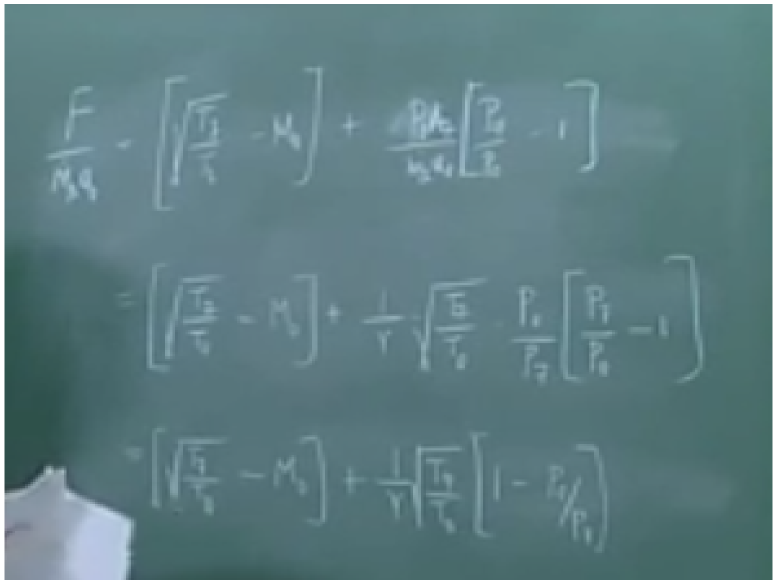

Now we can simplify here and rewrite this expression as okay now we know expression for T $_0$T 7 /T $_0$ and we also have no expression for P7 / P_0 so we will substitute that and see what is the final form that we can get it.

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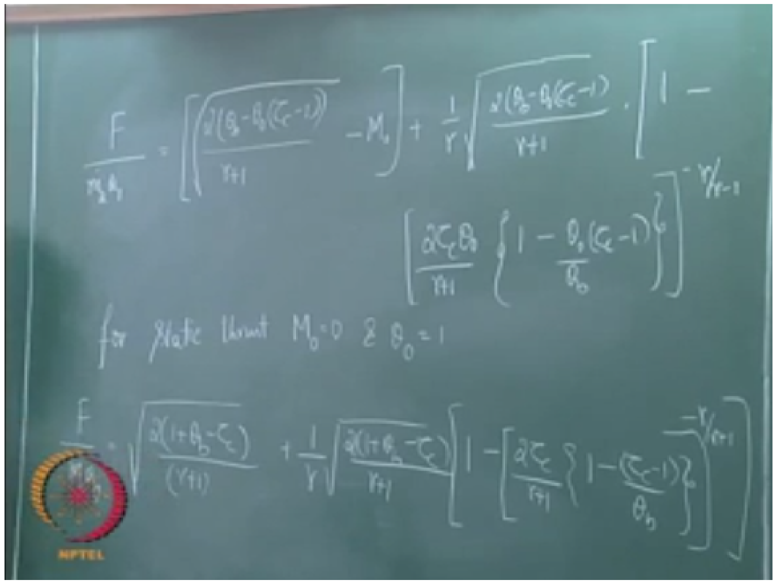

Into 1 - is a really big expression okay this is the final expression that we get now if we were to substitute here what we did earlier that is does this produce static thrust or not. So let us do that for static thrust M_0 = 0 and θ_0 =1 so if you substitute that we will get yeah there is the expression that one can get and further simplification is possible on this I will leave it as an exercise to you okay you can take out $\gamma + 1$ and further simplified and this part also you can take out so we have been able to derive this expression for non-dimensional thrust.

Now this is a fairly complicated expression compared to that we derived for optimally expanded flow through the nozzle and much more complicated than the expression for tan jet okay. Now the next part that we need to address is what is ISP okay so ISP part what we will do is ISP we know the expression for ISP b/ a0 as.

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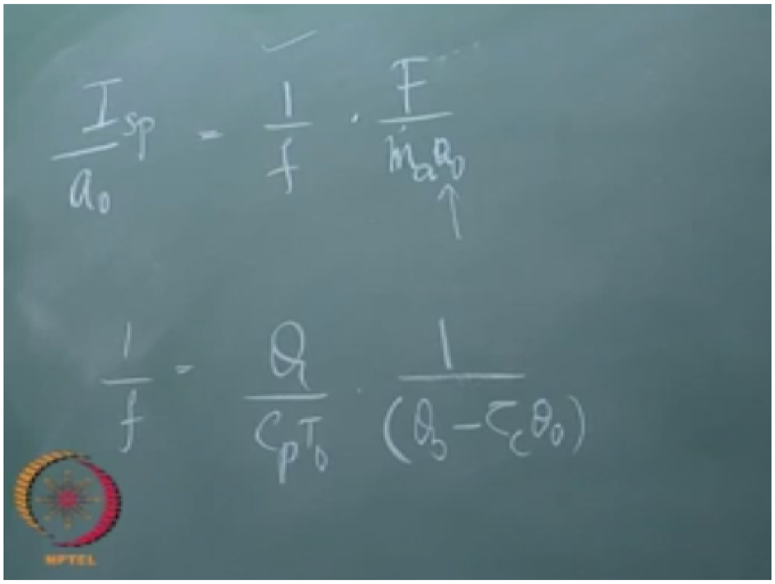

Right and we have spent a considerable amount of effort trying to derive this expression we already got this expression now what we need is 1 / 8 now in the previous case when the flow was optimally expanded through the nozzle we had derived this expression for 1 / F now what we need to look at is what is the difference between the expression that we derived there and will there be a difference here now again if you look at the flow process what is happening in the nozzle is much more downstream than what is happening through the combustor.

So the expression for 1 / F that we derived earlier is the same so I can use 1 / F that we had derived earlier as Q/ C_PT_0 into okay this expression remains the same so I know that 1/ F is known F / M_0 a is known so if you substitute this expression there sorry this expression you will get is P okay right now having derived these expressions let us look at what is it that we can understand from these expressions remember we did the same exercise we found out what is the Mach number at which the non-dimensional thrust would be a maxima depending on what values of θB and other things for the turbojet let us do the similar exercise for turbojet okay.

Now in a turbojet we will again look at the condition where the flow through the nozzle is optimally expanded simply because it is something that is easier to do in the classroom doing something on this is a little more complicated so I will look at what we can find out using the expression that we derived for optimally expanded flow.

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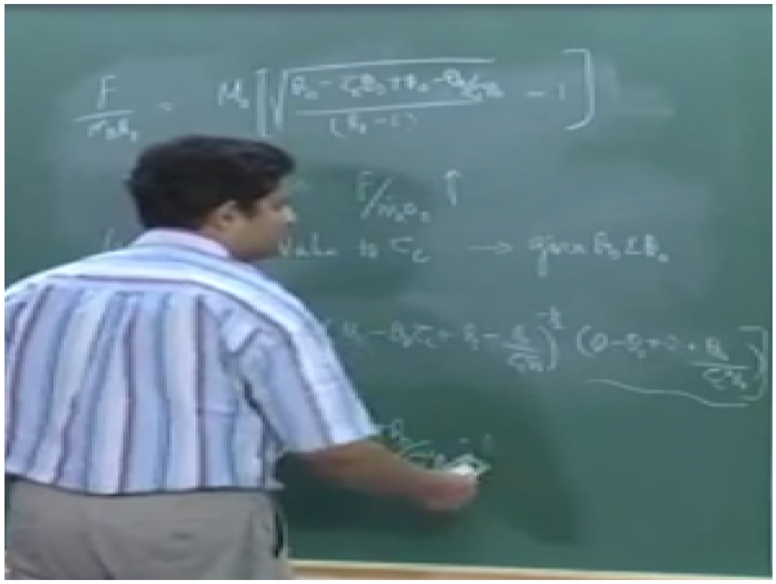

So if you look at the expression for optimally expanded flow through nozzle f / m . a a0 was M_0 into θ B - τ $C\theta_0$ +θ_0 – okay. Right this was the expression that we had if you see here as θ ba increases what should happen yes θB increases what should happen to F/ m. a a0 this also increases because you can pull out the θ B terms and you see that you get 1- 1 / τ C θ_0 so as θB increases it is fairly obvious from this expression that as a consequence f / m . e a_0 also increases right which means what that if you have a large enough θB then the size of the engine is going to be smaller and smaller for the same thrust.

Okay fine or if you keep the dimension same right then your thrust is going to increase if you increase θB okay this is preferred because you will have lesser drag on the engine okay then this is fairly clear then what we need to look at is what happens to what is there an optimal value for the compression ratio if I fix θ B and θ_0 is there an optimal value for compression ratio okay if I am flying at a cruise Mach number let us say if I am flying at a cruise Mach number then my θ_0 gets fixed remember when in this previous class one of the previous class we had derived an expression wherein we looked at what is the range and how the overall efficiency gets affected by it right.

In that we had said most of the flight takes place in the cruise range right so if you substitute θ_0 for cruise and you know the θ B value then you can find an optimal expression for τ C let us do that in the next few minutes so we are looking for optimal value of τ C given θ B and θ_0 so how do we go about it again take a derivative with respect to τ C so you get- θ B τCθ not this goes to the power of – ½ into the derivative of the terms containing τC that is the first term is 0.

Then you have a - θ_0 then again you have + 0 + because this becomes - so you have θb/ $\tau C^2\theta$ -not okay right derivative with one numb goes to 0 so you get this expression now this goes to the denominator because you have multiplying I mean you have a power of- ½ so what you need to look at is only this part of the expression because for this to be for f by m. a0 to be maxima this should go to 0 this can only go to 0 if the numerator goes to 0 okay so this is the numerator in this so for Maxima θ0 must be equal to 0.

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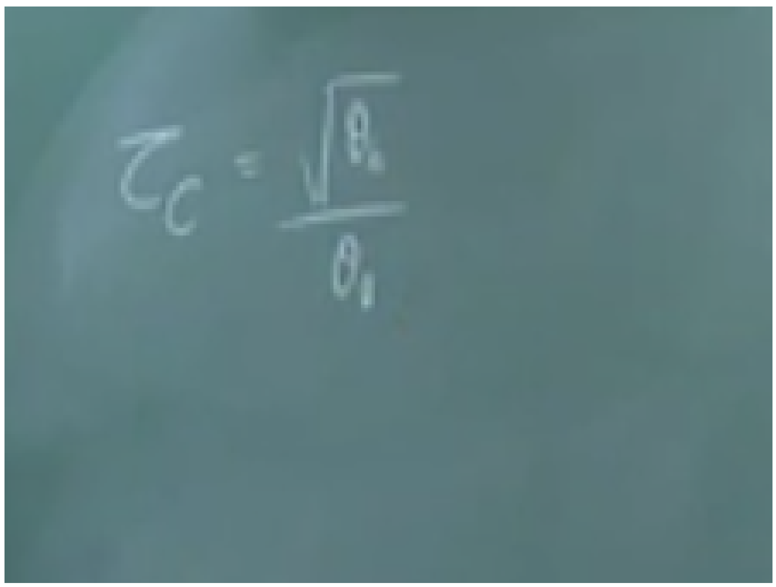

Which means that τ C must be equal to okay right sorry yeah so you get this condition so we will stop here and continue in the next class thank you.

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