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Aerospace Propulsion
Cycle Analysis – Turbojet I

Lecture 11

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Good afternoon in the last class we have seen how ISP and non-dimensional thrust depend on efficiency as well as θ B and θ0 okay in this class we will go further and find out what happens can we get a ramjet which can start below Mach number one is that possible or a subsonic in other words subsonic ramjets are they possible and then we will also look at whether ramjets are self-starting we know for sure ramjets are not self-starting whether our equations will bring it out is what we are going to see right and we will also look at what happens to thrust per unit mass flow rate and other things as we go along in this class.

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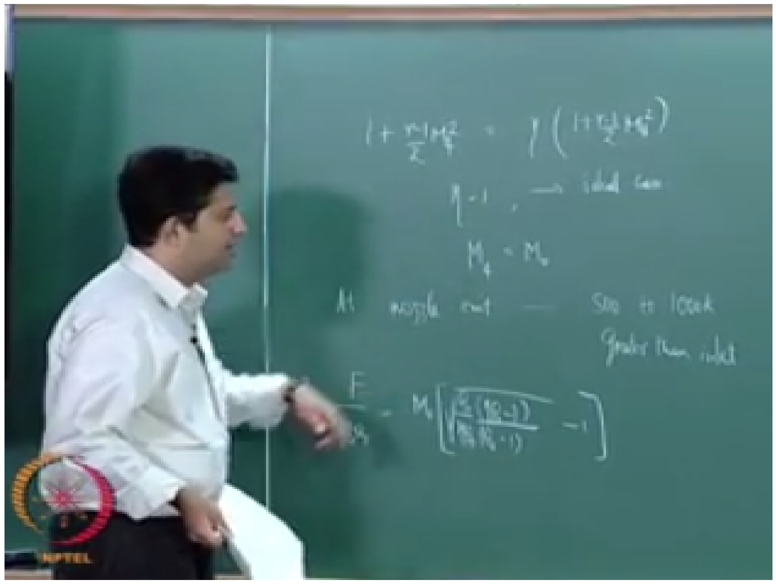

Now if you remember we had derived expression and said that $1 + \lambda - 1/2\ m\ 4^2 = \varepsilon$ x okay this was the expression that we derived in the last class now if you put $\beta = 1$ that is ideal case if we look at an ideal case what happens here we get this relationship that m4 =M0. So does it mean that at the exit Mach number is the same as the inlet Mach number then it would not produce any thrust is that the case what do you think will happen we are looking at an ideal case and this case exit and Inlet Mach numbers are the same, so do you think it will produce thrust or not produce thrust, yeah you are right if you look at Mach number Mach numbers might be the same what we are interested in this exit velocity and inlet velocity okay.

So Mach number being the same does not mean anything because if you know that at the exit of the nozzle temperatures can be 500 to 100Kelvin greater than Inlet, so if you have the flow through the nozzle where and the temperatures are higher and the speed of sound at that location will be higher and therefore you will be able to still produce first if efficiencies are one. Now we derived expression for non dimensional thrust as okay this was a non-dimensional thrust equation now in this if we put $\varepsilon = 1$ this simplifies to okay.

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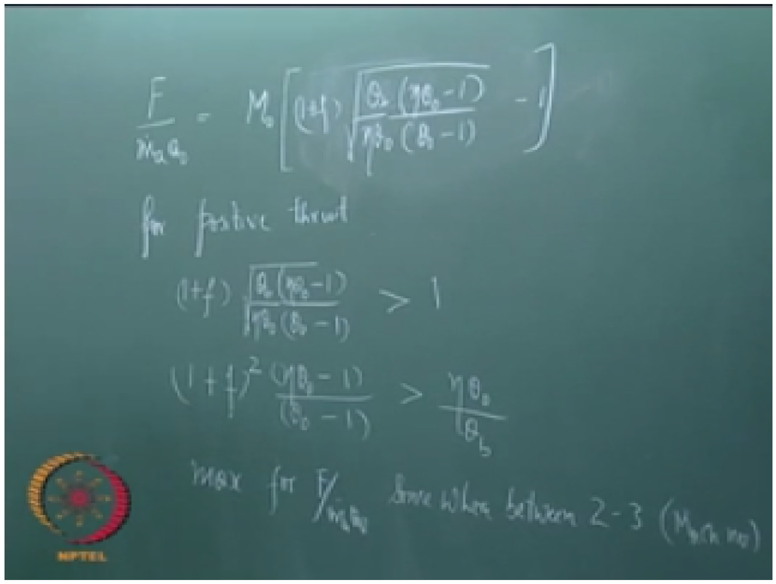

So for efficiencies being or one unity ideal system you get this expression for non-dimensional thrust and similarly one can get an expression for ISP now if you look at this expression for non-dimensional thrust what you see here is ifθB increases what happens to the non-dimensional thrust θ B remember is under the designers control that is you can choose what is the exit temperature at the exit of the combustor by burning suitable amounts of fuel there is an upper limit that is if you are using if you are using kerosene air there is an upper limit of two thousand three hundred Kelvin which is the stoichio metric combustion temperature but other than that you can vary the air fuel ratio and get different temperatures this number is under your control right.

And one it is very obvious that if you increase θ B you get a larger f / m. a0 now what this value F by m.a0 indicates is that you will see as this number is higher the size of the ramjet becomes smaller, so larger value of means smaller size of which is desirable because your drag will then be smaller okay. So you need a larger value of this and ifθB increases you will find that as a consequence f / m.a a 0 also increases okay, now when we were discussing ramjets earlier in the introductory classes I had made a point that transits are not self-starting that is they need something to take them to a slightly higher Mach number and then they will start from their own let us see if we can bring that out from this equation.

If we put M0 = 0 here what happens M0 is the Mach number at the inlet if we put that to zero that is if you are holding the vehicle stationary what happens to the thrust this goes to zero, so not self-starting or in other words static thrust is zero, so if you put the ramjet on a thread stand and you do not do you do not give the flow that is you do not induce the flow yourself then it would not be able to produce any thrust even if you switch on the fuel flow rate okay.

We know now that it does not produce any static thrust is there any value of Mach number at which it will start producing some thrust that is our next exercise we know that static thrust is zero here let us now find out what happens at what Mach number can we get some meaningful thrust okay, now we know that efficiencies if you remember efficiencies will always be less than one and when we multiply all of them the product range between product that is ε range between 0.84and 0.97 right so efficiency will always be less than one, so if you incorporate it and try and see what happens at what Mach number will we get some thrust okay.

To derive that we need to look at this expression here okay this is the expression and we know that if it has to produce positive thrust this number should be greater than zero right, so if it has to be greater than zero then obviously the product of the first two numbers must be greater than one or if you look at the terms in the bracket this should be greater than one. So we will put that dump I will also include the 1 + F term here they remember we had neglected this term for our convenience okay. So for positive thrusts 1 + f should be greater than one okay.

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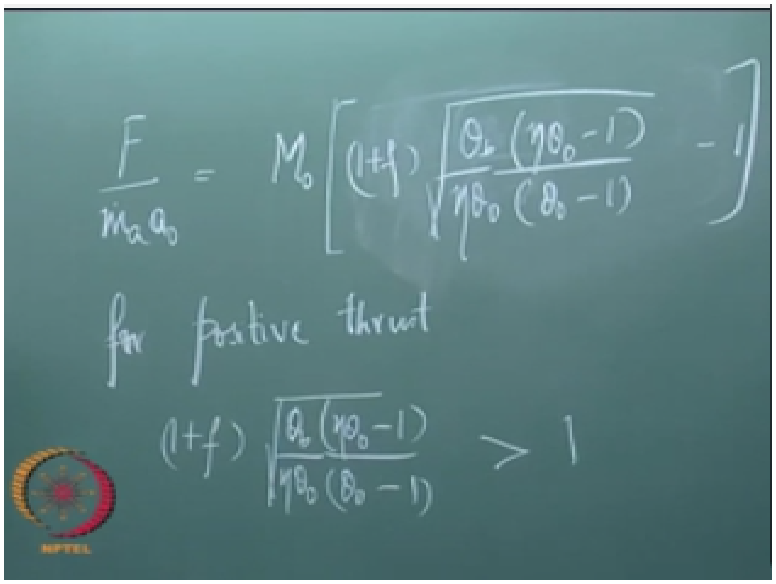

Now we can square both sides and cross multiply and we will get this condition that should be greater than θ B okay, even though F is smaller here this quantity can be slightly larger therefore I have taken that into account in this equation okay now this is a less cumbersome equation to solve and bring out what should be the value of Mach number at which it will give positive thrust so the easier way is you plot a graph and then try and find out where it leads you okay and that is what is done in this plot here.

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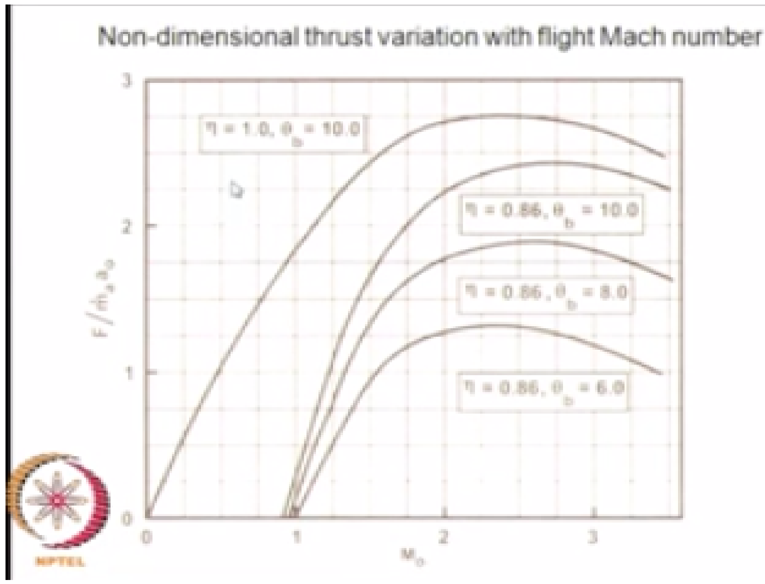

What is plotted as non-dimensional thrust variation with flight Mach number that is as you increase M 0what happens to f by m.a a0 you notice that there are four lines here each corresponding to a different value of efficiency this is for ε = 1, so you have its starting from zero itself okay whereas all the other lines are clubbed somewhere close to one where efficiencies are around 0.8 six okay.

So if you have a non unity efficiency that is for real systems you notice that it is very difficult to have a subsonic transit okay whatever be the θ B that you can go up to write θ B is the ratio of combustion exit temperature combustor exit temperature to the inlet temperature even if you add more heat even if you keep on adding more heat it does not matter we brain varied θ B from 6 to 10 almost by a factor of 2even if you add more heat it does not matter you still cannot have something like a subsonic ramjet this is purely determine by efficiencies for non-unity efficiencies you cannot have subsonic ramjets.

And what you will notice here is that all these systems have their thrust- or the non-dimensional thrust is maximum as somewhere between two to three okay, so it is Maxima for F by a Mach number range of two to three right but we need to take this result a little more carefully what we have looked at in this analysis is this is only the thrust produced by the ramjet we have not accounted for the net thrust that is thrust minus the drag right there is a drag component also that determines whether the vehicle will move forward or not and that will make it go even further right if you look at this graph in this graph at this Mach number f by m. a0 is this much there will be drag right.

So that drag will bring down this value and probably you will start producing net thrust somewhere at a slightly higher Mach number right, so typically you will not find any sub sonic ram dux that is the underlying message from this graph okay.

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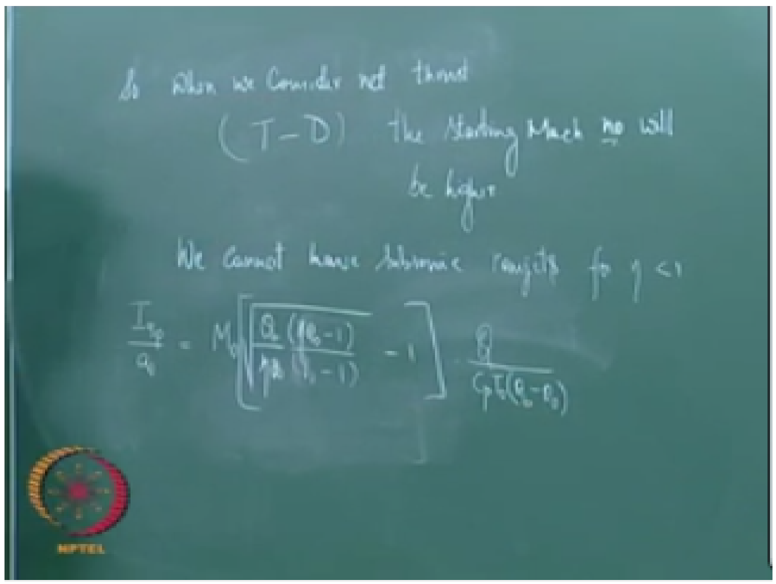

So then we consider net thrust that is thrust minus the drag portion then the starting Mach number will be higher and you cannot have subsonic ramjets or efficiency is less than what now in this graph we see that F / m.a a 0 varies in this fashion and if I were to look at ISP variation remember is P / a 0 is nothing but M 0 into okay now ISP what would be a preferable thing having a larger value of ISP or a smaller value of ISP you remember SFC we need a smaller value and ISP being one over SFC we need a larger value of ISP if you notice here you have θ b appearing here as well as here right in two places in this expression and here it is under root sign and here it is not under root sign okay.

So what do you expect to happen if θ B increases what happens to ISP is P should decrease okay and what are what should happen with efficiencies if efficiencies are less than one what should happen to ISPs it should also again decrease and the same thing is plotted here in this graph.

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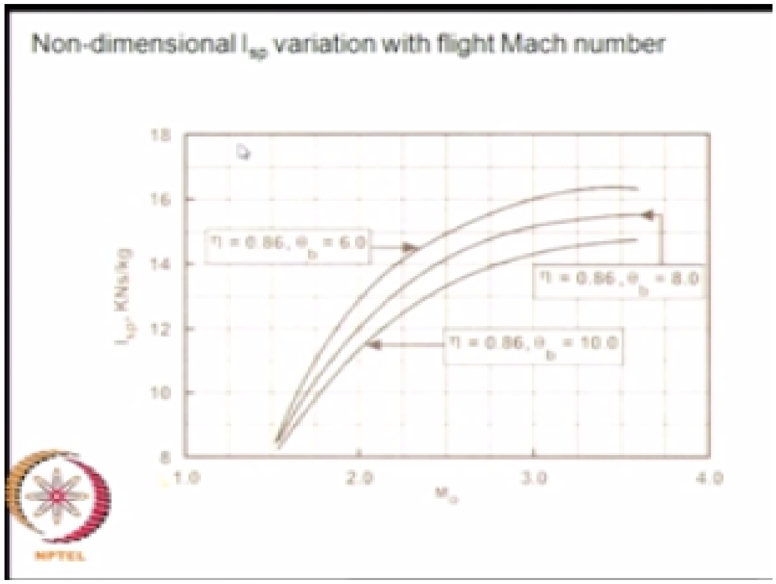

What you see here is ISP on the y-axis versus Mach number the flight Mach number for an efficiency of 0.86 with different θ B's as θ B is increased from 6 to 10 you see that the ISPs are lower okay, so which means SF C's will be higher or fuel consumption will be more which is not a desirable feature so you would like to operate such that θ B is lower so that your fuel consumption is less but then if you go back to the previous curve you will find that if θ B is less your f /m. a a 0 will also decrease or the thrust will also decrease.

So you should design such that this is a large quantity so that your size becomes okay and then the other thing will automatically get fixed okay now we looked at how the non-dimensional thrust varies with θ B let us look at what happens with M 0 you have already seen that in graph let's try and bring out if we can do some analysis and find out what happens with Mach number Mach number if you notice here it appears this is the expression that we are looking for it expressed it you find Mach number in M 0 and θ 0 it appears in both places.

So it is not obvious as θ B is we noted that with the increase in θ B it is this becomes smaller which is what we found NIC curves but Mach number is not such a straightforward relationship let us try and find out what happens there, that is what we are trying to find out us is there an optimal Mach number at which if we fly it seem that it should lie somewhere between two and three can we bring that out through some analysis here.

So what do we do we take a derivative with respect to M 0, so firstly we will assume ε to be 1 so that we do not end up with a large or a cumbersome equation so if you look at ε = 1 the expression for F / M 0 a 0 would be M0 x √θ B by θ 0 -1 right so now if we take a derivative of this with respect to M0 et me also include 1 + s now if we take a derivative we will get 1 x the rest of the things + F √θ B / θ 0 – 1 – ½ m0 x 1 + F θ B θ B does not vary with M 0 so θ B does not get change here so θ 0 to the power of 3 / 2 x λ – 1 m0 okay.

And for a Maxima this must be equal to 0 so if we equate this to 0 then both these two must be equal we will get 1 + f = 0 √ θ B /θ 0 – ½ x λ -1 m02 okay, what is this quantity here what is the definition of θ 0 θ 0 is so what you have here is θ 0 - 1 so if you put that you will get 1 +F θ V – θ 0 x and do the simplification that you get this θ 0 -1 is divided by this must be equal to 1 or θ B by θ 0 this becomes θ 0 - θ 0 + 1 so θ 0 θ 0 cancels out you get 1/ θ 0is equal to 1 θ 0 to the power of 3 / 2 must be equal to okay this is the expression that we get for maximum value of f / m. aa0 occurs when θ 0 is in this is the value of θ 0 again.

Now you plug in some typical values we will find out what we had discovered earlier that it ranges somewhere between 2.5 to 3.

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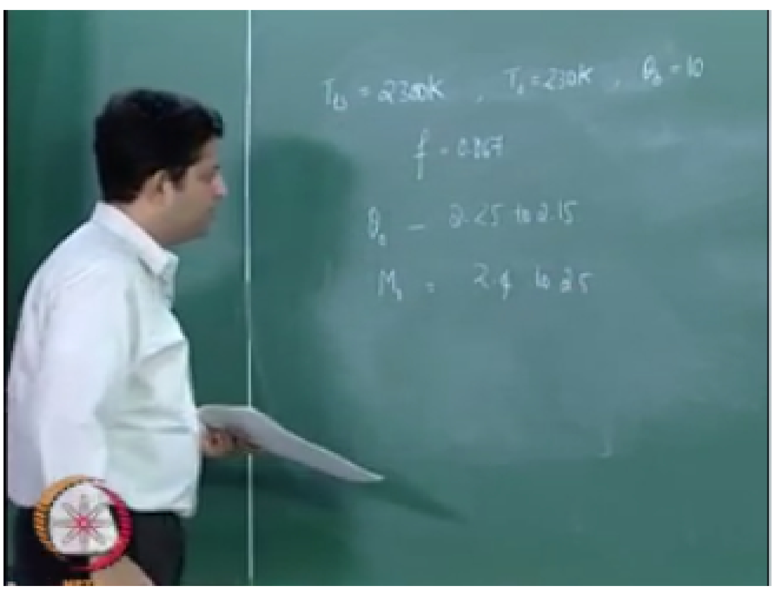

So if we take et3 to be 2300Kelvin which is what you will get as the adiabatic flame temperature if you use kerosene air system at special metric condition and I will take T not to be 230 Kelvin that is as you go higher in altitude temperature drops, so at some particular ask you will get this temperature and therefore my θ B will be N and if I use seven okay you will find that θ 0 you will get as somewhere between 2.252 2.15 this depends on what value of F you consider and correspondingly this will also change a little okay and therefore M 0 to be somewhere between 2.42 0.5 okay.

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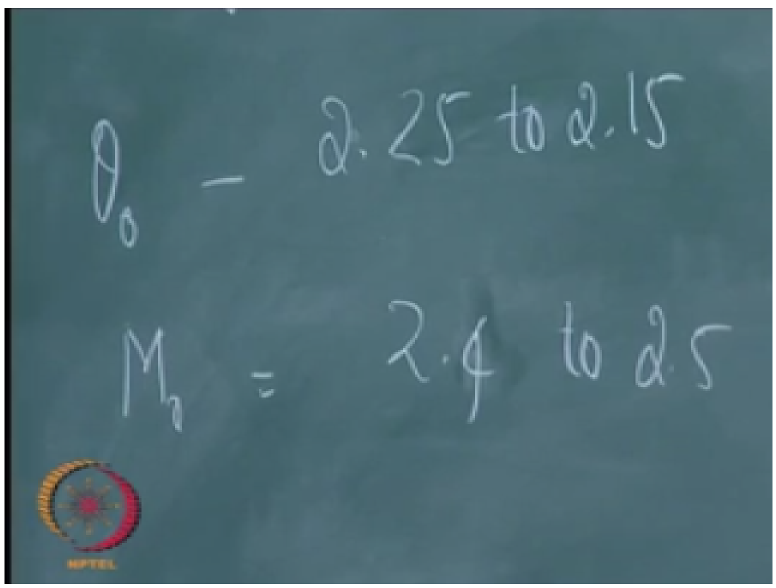

So you will notice that most systems most real systems although all these analysis we have carried out with ε to be one even if you put in the efficiencies we have seen in the graphs that Mach number range for high value of F / m. a a 0 ranges between two to three which is where you will find most operating systems okay most systems would want to go to that condition and then fly in that Mach number range okay.

So that you have a larger value of F/ m. a a 0 okay this finishes our discussions on ramjets now let us move on to the next engine that is the turbojet okay to do rate has a slightly involved system it has a compressor and a turbine, so we need to account for that here okay now when we look at turbojets there are two things that we can look at one is when the nozzle is choked and the other is when the nozzle or when the flow through the nozzle is optimally expanded.

Firstly we will consider the case wherein it is the flow through the nozzle is optimally expanded and we will try to find out what derive what equations will get.

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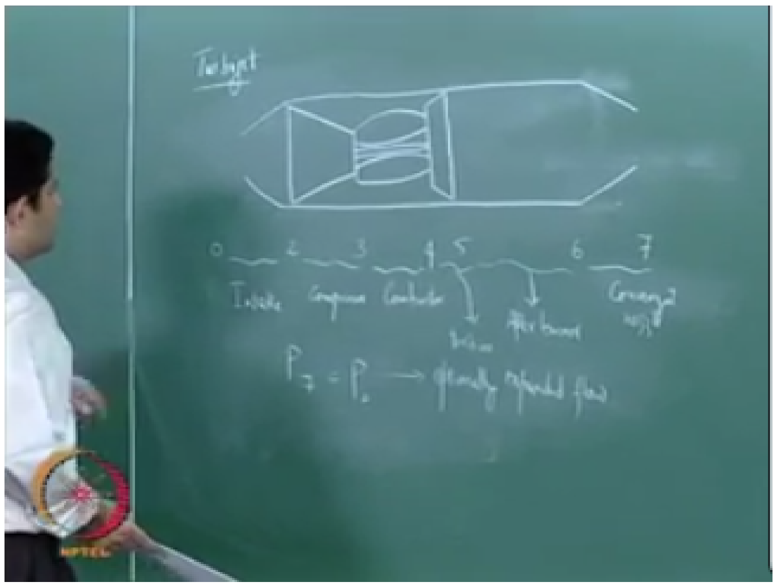

The turbojets at about that similar to a ramjet we will have an intake and then we will have a compressor which is connected to a wine and you have a combustion chamber in between and then you have a jet pipe or an afterburner and then convergent nozzle. So let me again call these stations as one two three four five six and seven this is the zero to two is the intake two to three is compressor three to four is combustor this is the turbine and this is the afterburner and then the converging nozzle as I said earlier we do not use typically CD nozzle the only aircraft or a rail system that has a turbojet which uses a convergent divergent nozzle is the what does the engine Olympus engine write on what is that a Concorde aircraft okay.

So other than that typically only a converging nozzle is used because the pressure at the exit of the turbine is very low and it would not be useful if you have a convergent divergent nozzle and therefore only a convergent nozzle is used, now here we will take up the case where P 7 = P 0 okay, that is Emily expanded flow notice here that you have compressor extra and the turbine extra. So we need to define certain other parameter that is we will define compressor pressure ratio.

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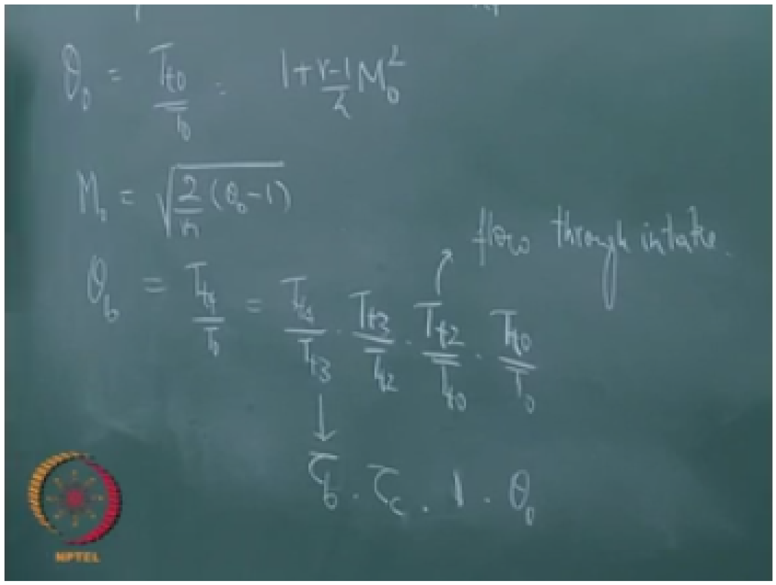

I will call it π C, π C is nothing but PT 3 / PT 2 and E again here indicates stagnation conditions from this we can define τ C that is nothing but t t3 / TT 2 which is π C $^{\lambda - 1/\lambda}$ okay, and similarly I can define a turbine pressure ratio, so from this I can define τ T which is nothing but TT_5 / DT_4 okay. So now with this and we know that from a previous calculation θ0 is nothing but TT 0 / T 0 that is 1 + λ – 1 $n0^2$ right.

And based on this I can get M0 = √ 2/ λ – 1 θ 0 – 1 the other parameter that we need to define is θ B in this case I will define θ B as t t4 /T 0 okay so θ B is nothing but t t4 / T 0 which I can rewrite as tt 4 /tt 3 x dt3 / TT 2 x T T 2 / T T 0 T t 0 / t0 what is tt 4 / TT 3, this I will call it as τ b similar to what we had done earlier I will call this as τ b and tt3 / TT 2 again we have hid here it is π C and no τ C sorry and what is TT 0 T T 2/ T T 0 if we assume isentropic processes this will be 1 this is flow through this is flew through intake and just like in the previous case if the process is isentropic we can get this to be one and lastly little this is nothing but θ 0 okay.

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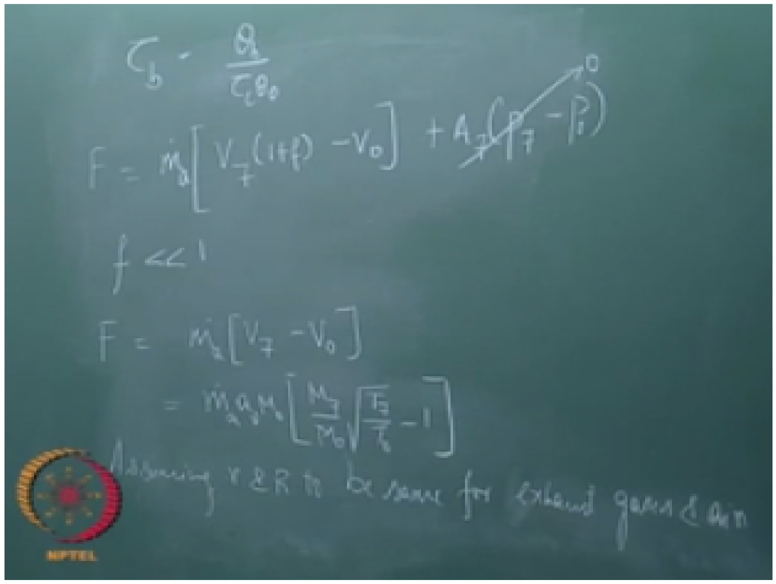

So I get my expression for θ B or tau B I can write it as okay, now we want to derive expressions for thrust so the thrust equation is f / f = m . a x in this case the exit velocity is at v7 okay and intake is at 0 so you get v7 into okay we are assume P 7 = P 0 and therefore this goes to 0 and will also make the other assumption that F is very much less than 1 this is more true in a run in a turbojet compared to a ramjet because in the main combustor it will be much lower than 0.067 because you are worried about the turbine Inlet temperature so you want it to be much lower than the stoichio metric condition.

So f is less than 1 so I can neglect this part and I get F = m. a v7 - V 0 and we will do the similar procedure as we did last time so I can write this as m. aa0 m0 x m7 / m0 x √ t7 / t0 – 1 again we have used the relationship for speed of sound here okay and we have again made the assumption that λ and r are the same for exhaust gases as well as incoming air, so assuming λ and r to be same for gases and air okay. So we get this relationship now again we need to look at these two ratios or we can write the non-dimensional thrusters.

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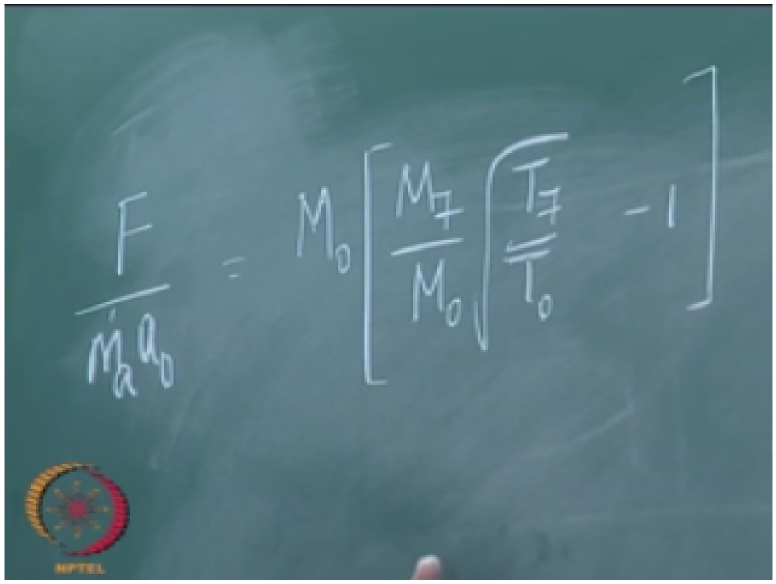

We can write the expression for non-dimensional thrusters m0 x m7 / m0 okay so we are again left with the task of trying to find these two ratios again we will follow a similar procedure of cascading temperatures and cascading pressures right, now what I want you to think about is all the while I have been telling you that you are looking at flow through intake and I said we are looking at isentropic conditions in the previous case that we took up ramjet we looked at all the temperature ratios and efficiency is never figured in that place right why does the efficiency only figure when we are looking at pressures and why does not it figure here.

Now in the previous case also we looked at a realistic system right, so we cannot look at realistic systems and say the flow through the intake is isentropic. So what really is the crux there why are we taking this is one and why do we take the other thing wherein when we look at pressures we are looking at efficiencies here okay we will discuss this in the next class okay, thank you very much.

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