Acoustic Instabilities in Aerospace Propulsion
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Lecture - 09
Admittance and Standing Waves

Good morning everybody, today we will start with two problems, first we will work out a really simple problem.

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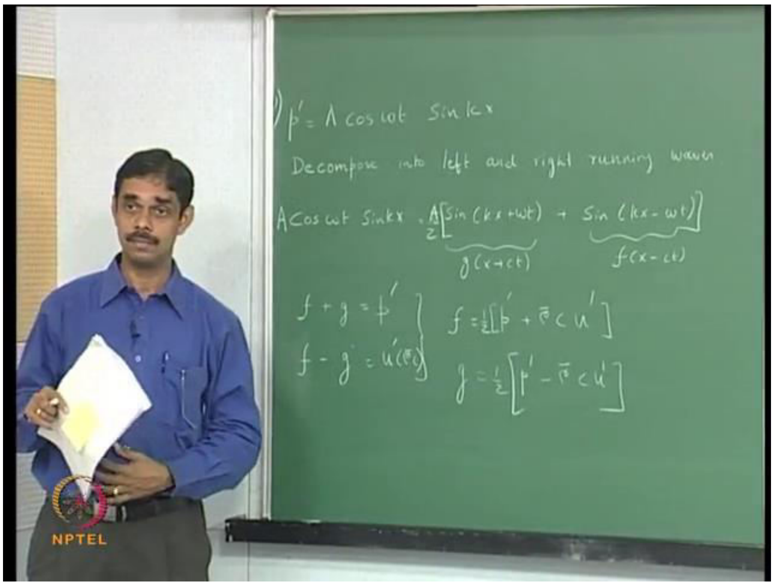

Let us say p prime is A cos omega t times sin k x, can we decompose it to left and right running waves, yes can we workout this exercise, it is very simple if you would be able to do it in over a half a minute and something. So, there are two ways you can do this problem, one is just to recognize that you can write cos omega t sin k x as sin k x plus omega t plus sin k x minus omega t over 2 so, this is a identity. So, we can clearly see that, when you talk about A cos omega t sin k x, this would be A over 2 sin k x plus omega t plus sin k x minus omega t.

So, this here is the f of x minus c t and this here is g of x plus c t so, it is kind of a trivial problem but, not always it may be so trivial I mean, this is the case where, you can very obviously visually see the identity. But, if it not so easy what we can do is, we can say we know that our f plus g equal to p and f minus g equal to u.

So, therefore, we can get f equal to, I made a slight mistake over rho c so, let me move this rho c here, this is right. So, f equal to p plus, put the primes rho bar c u prime over 2 and g equal to half into p prime minus rho bar c, please verify if this formula is right. So, how to do this problem, we are given only p prime, we do not know u prime so, how do we get u prime, momentum equation. So, let us write the linearised momentum equation.

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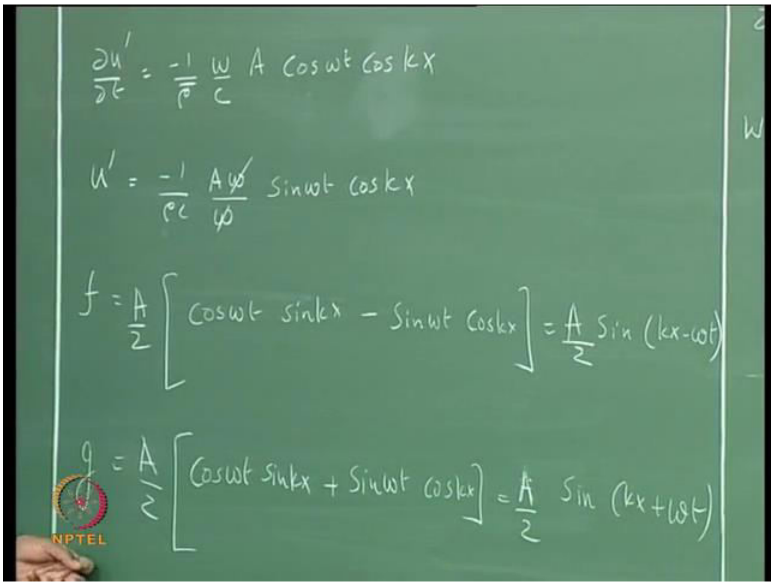

Dou u prime by dou t equal to minus dou p prime by dou x so, what would be u, minus 1 over rho bar, dou p by dou x would be, there will be a k coming out, which omega over c and I will get A cos omega t. Sin k x, when I differentiate becomes cos k x and now, if I have to integrate it, I get u prime equal to minus 1 over rho c A omega here and if I integrate this term, this omega will go, I will get sin omega t cos k x, I hope this is right so, I can say f equal to, this is formula of here half times p prime plus rho bar c u prime.

So, f would be equal to, half times p prime was so, let us take this A out, cos omega t sin k x minus rho c when I multiply this term rho c to u prime. This will go away so, I will get sin omega t cos k x, which is equal to A sin k x minus omega t and g would be equal to A over 2 times, p prime is the same term cos omega t sin k x. Now, there is a minus sign over here so, the minus and this minus becomes plus, sin omega t cos k x equal to A over 2 sin k x plus omega t.

So, this is like a proof first they are doing it, whatever function you are given, they can decompose into a left running wave and right running wave as this. But, sometimes as I mentioned, we can just without doing anything complicated, we can straight away write the answer as follows, is it clear, any questions on this. I will pass for a minute for you to digest this and those of you have finish this question, can think about the next question.

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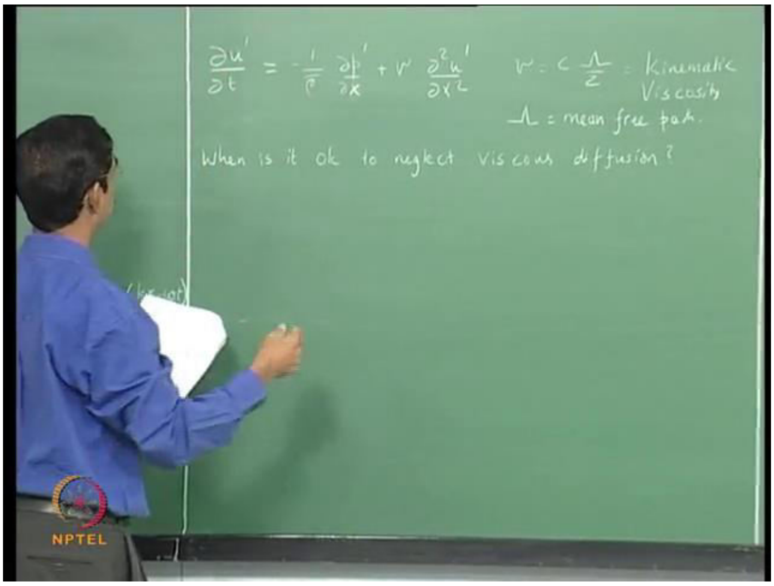

So, the next question is the linearised or momentum equation, which is of this form d u over d t equal to minus 1 over rho bar d p over d x plus mu times d square u over d x square. And this is the viscous term and mu is the kinalytic viscosity, which is written as c lambda over 2 with lambda is the mean free path. So, going back to this question, when is it ok to neglect viscose diffusion or viscose viscosity, we usually just say it is negligible or we will deal with earlier equation like that is what, I stated with and because, I did not want to discuss this.

But now, I want to discuss this, do you have answer or can you work?

I do not know.

I do not know.

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No. yes, in some sense that is…

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Heat is there…

High temperature.

The dependency, you have a dependency on the mean path.

High temperature wherever the mean path is needed.

The high and low are, when I say that you are a tall person, may be you are tall compare to kids one over in school but, when I compare you to that men, you are short. So, it depends on, what I compare tall or short person. So, how is it and that is a number is what so, we have to compare mean free path with something, what is that something, that that is a length scale of what? molecular collision. So, and that is how, the transport takes place, whatever I mean the diffusion, and what is the appropriate lengths scale in acoustics?

Wavelength.

So, I think we have to compare the mean free path of the wave length and see, when they are of the same order what happens, when one is higher than the other what happens. So, how do we do this usually, I think you remember doing boundary layer theory, do you remember, like it is non dimensionalise the momentum equation and then, you recovered ((Refer Time: 08:58)) number out of it is not.

So, some like similar we have to do here, so we will do a dimensional analysis. So, what we do is, we can try to scale this with appropriate parameters so, what we can do is, I will scale velocity of a fluctuation with some kind of reference velocity and pressure with some kind of reference pressure.

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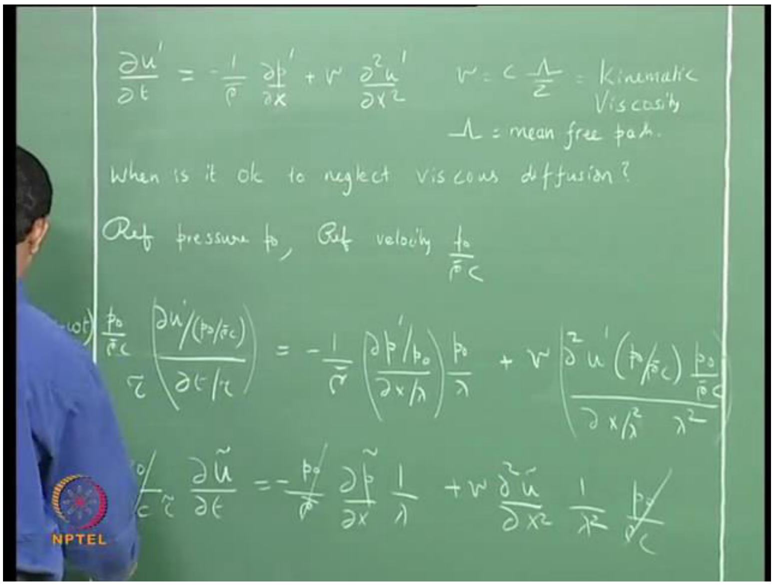

So, I will have a reference pressure p naught and this fluctuating for some value, reference velocity of the order of can be p naught by rho bar c, that is the order of magnitude, in one way exactly given a pressure, velocities of the order of p over rho c. So, if I non dimensionalise this I can get, Ii have a time square root tau so, all I am doing is the dividing and multiplying by the same number so, my equation is not changed. So, here for example, divided by p naught over rho c and I multiplied by p naught over rho c so, in the effect, I am simply multiplying by 1.

Now, if you choose the appropriate length scale and make these terms here of the order 1 and then, we can so, I will call the non dimensional terms as for example, non dimensional velocity is u delta, non dimensional pressure is p delta and so on. So, I will say, p naught over rho bar c term dou u delta over dou t equal to minus 1 p naught over rho bar dou p delta over dou x times 1 over lambda plus mu dou square u delta over by x square 1 over lambda square.

Now, I assuming that, I can use the appropriate scales and make all these derivatives of the order 1 then, we should be able to compare these terms and find out, when this term is dominant or when this term is dominant. So, the first term is of the order of let us cross of p naught over rho c, that is coming everywhere so, I will crosses p naught over rho, c is till there so, this c stay here.

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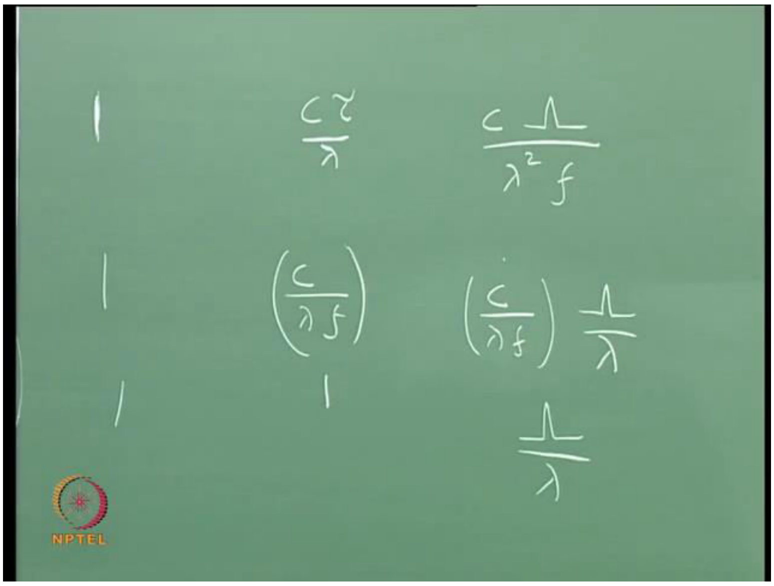

So, I will have the first term of the order of 1 over c tau, the second term is right now of the order of 1 over lambda, third term is and I have a new, which is of the order of c lambda mu is c lambda and thus, c here. So, these are the order of the 3 terms so, let me try to multiply all three terms by the same number. So, I will make this first one 1 so, this will be c tau over lambda and if I multiply this by c tau, I will get c cancels. So, c omega over lambda square f, which I can recast as so, this is c tau over lambda.

And if I replace tau by time period so, I can say c over lambda f and here, I will say c lambda f and c is equal to lambda f. So, this term is also of the order 1 and this is so, we will have 1 1 and (Refer Slide Time: 14:23). So, this term over here, mu times dou square u prime by dou x square will be important, if lambda over I mean, this capital lambda over small lambda is large and if it is the other way, will be negligible.

If the mean free path is much smaller than the wavelength, which is normally the case, the mean free path in the atmospheric conditions is a order of 10 power minus 6 meters something. And for typical ways, which you deal with wavelength is the order of meters 1 meter, 2 meter, half meter. So, 10 power minus 6 over a meter is 10 power minus 6 whereas, the other two terms we saw of r of the order 1. So, therefore, we can say that, this viscous term is several orders of magnitude lower than the other term that is, the unsteady term and the pressure flux.

Now, the viscous term would be quite important, when you are having no lambda or high frequency and that happens in the case of ultrasonic. So, when you have very high frequencies and you have very small wavelengths then, this term will be very important. So, the question was asked yesterday, that you can have A as p max plus p min over 2 and B as p max minus p min over 2 or we can have the other solution A as p max minus p min over 2 and B as p max plus p min over 2 and which is a right one to do, I think you asked this question, Vishal.

So I think, this is very perform question actually and actually, what quite hard on this when I was a masters student because, I was measuring admittance, I could not recall what was the thing right away and this is very interesting. In fact, actually there is no way you can distinguish between A and B there, by looking at the standing wave structure from the pressure amplitude versus the space, the amplitude disturbance plot.

And this is no problem for someone, who is measuring the admittance of clothes or carpets or this holes in the walls, whatever or any damping because, there you know that, we have a damping martial. So, if the incident wave is left running wave and the reflected wave is a right running wave we will say that, the left running wave has more amplitude than the right running wave and you can fix the A and B appropriately, to make sure that incident wave has more amplitude than the reflected wave.

So, that is trivial obvious but, what I was doing during my masters was measuring admittance of flames and I found out, when I did the experiment that I could not make out between these two. Then, when I actually spend quite some time think about it yesterday, I just could not recall this actually, it turns out that there is a still a way to find out actually, which is more and perhaps somebody has the answer, I will just wait and ask someone else, if they know, how you can make out. And I will give you a hint actually this is how, I got the answer, think that was in 1989 when I was working in this, it is quite long time back.

So, if you have a left running wave and right running wave, those would be extreme cases for example, if I have a impedance tube, other way I do it and you have a left running wave coming in and everything got absorb so, there is no right running wave. Now, you consider the other situation where, no left running wave is coming and a lot of right running waves are coming, almost nothing is coming to left but, everything is coming from left to right so, that is like a right running wave.

What would distinguish these two waves, that are the first thing we started the class with then, left running wave is of the form…

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Yeah, when I take plus k x and the other one is of the form so, one is plus k x, the phase is going with plus k x, the other one is going phase going minus k x. So, it will be the same thing, but one will have a slope this way, one will have the slope other way. And otherwise, if you look at the amplitude of this travelling wave, it will be flat because, it is always A or always B, this will be flat . So, we have to look at the phase distribution that is the way, you can distinguish between left running wave and right running wave.

So, the same thing we can do and we can look at the phase distribution for a given admittance. So we say, if you take a value 0.1 plus so, 0.1 i or something and then, you take the minus of that. And then, you look at the amplitude plot, they you look exactly identical but, the phase will actually look different and from that, one would be falling and one would be raising. From that actually, we can make out which is dominated by left running wave, which is dominated by right running wave or alternately, which is directional energy propagation.

So, if you have mechanic engineer studying admittance of carpet or clothes and so on, we can take the answer for guaranteed, we do not have to measure the phase at all. We can simply use volt meter and measure the amplitudes and you peak with to I mean, A and B as p max plus p min over 2 and p max minus p min over 2. But, if you are a compassion person trying to measure admittance of flames, we have to look after phase.

So, I hope you you did the homework problem yesterday and these are working out, write down the answer and from there, we will conclude the answer to this question. But, this is indeed quite a important question, the question is how to distinguish between a impedance condition admittance condition with a sending energy into the pipe or out of the pipe. That is what, it boils down to, it is more deeper than, which is A which is B in some sense.

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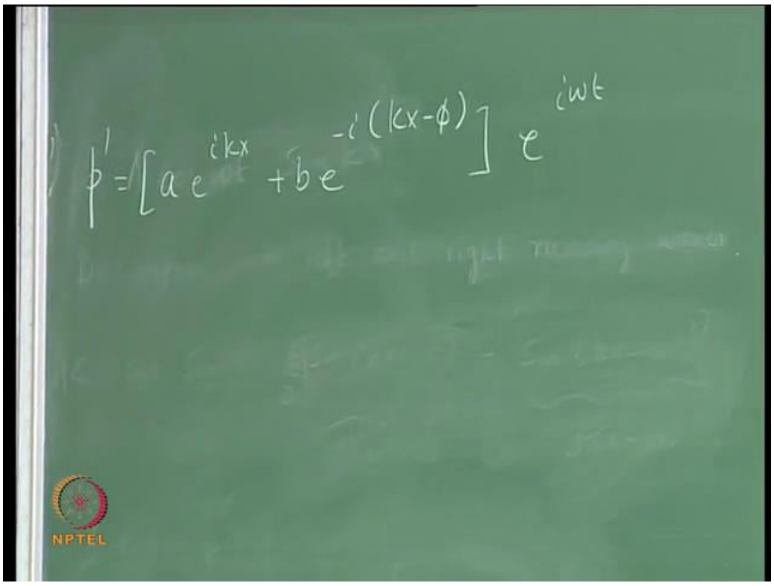

What is the question, Akshay you are, what is question something is bothering you, please ask.

Student: ((Refer Time: 21:18))

What is it?

Student: ((Refer Time: 21:22))

Which one?

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This one.

Student: ((Refer Time: 21:31))

Here?

Student: Left hand side

Here?

Student: ((Refer Time: 21:37))

Rho bar c?

Rho bar c tau.

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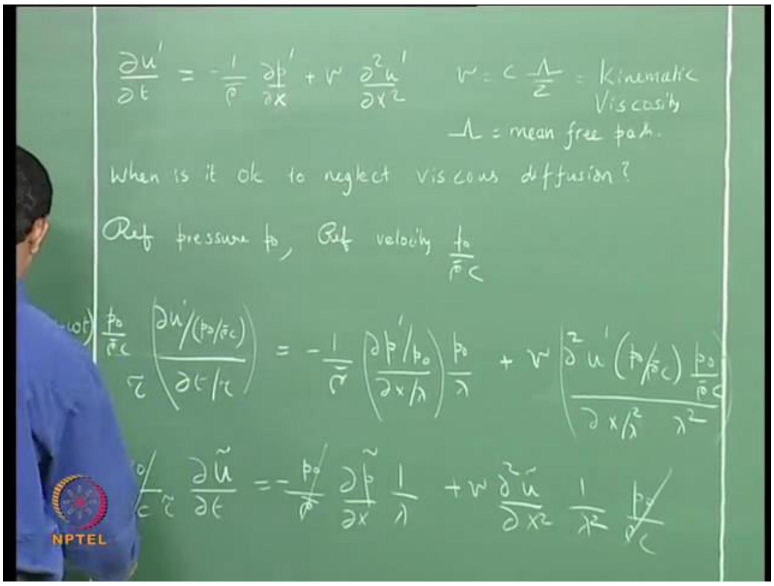

Tau is the times scale, t over tau, tau and period is a obvious time scale to choose for this of course, when you have flow and acoustics together then, you have one time scale corresponding to flow, one corresponding to acoustic but, here simple problem. So, I would not write 2 phis because, we saw that, phi 1 minus phi 2 it always appears together. So, because, I can always choose my clock to start my t equal to 0 such that, only one state, that is my right to fix the reference time.

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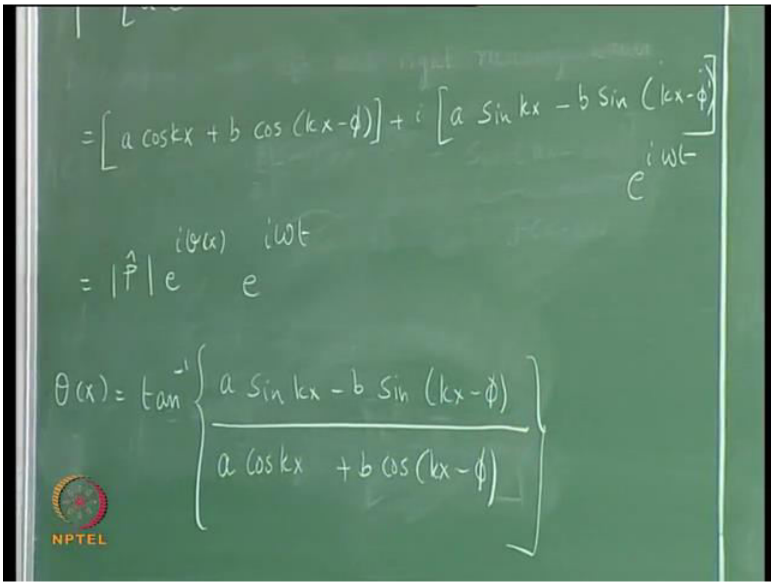

So, if I rewrite this so, we can write this complex amplitude as some real amplitude times e power i theta, which is the complex number to times e power i omega t. And we have to note that, this phase is actually a function of x, there are cases where, in may not be but in general, it is a function of x and that is determine from here. So, theta x equal to tan inverse, this over this.

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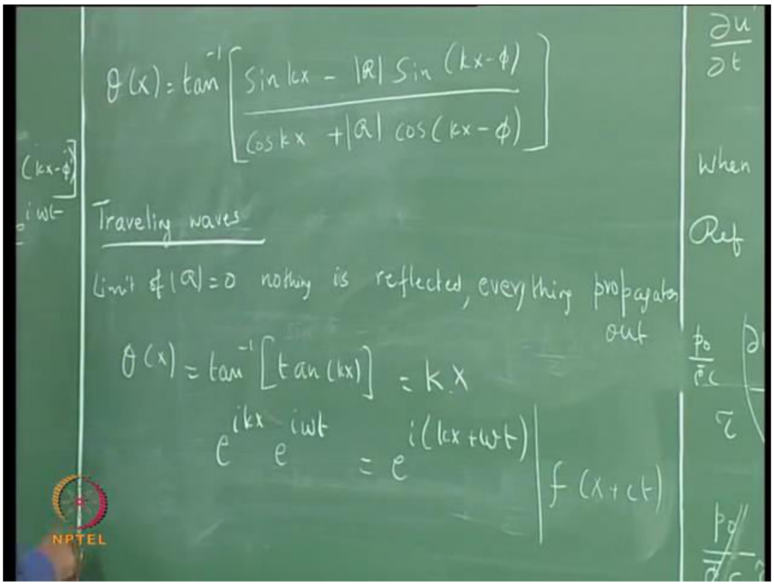

This we can recast perhaps as so, this is just tried for algebra now, hope there is no algebraic mistake, if there is something please point out because, the result will dramatically alter, if there is a minus sign. Actually, the books do not speak anything about phase because, none of the acoustician are ever concerned about measuring a substance, which is driving. Because, all things they encounter with the damp in general, everything damp but, only in blasting or aerospace we are having flames, which actually impart energy into the wave and then, your reflected wave is more than incident wave.

So really, I have to work this all out by myself and I will go by the exact thinking process. So, we look at the travelling waves so, will have a limit of R equal to 0 or so, nothing was reflected, everything kept propagating as left running wave. So, what will be theta, theta of x will be tan inverse tan of k x, this would be k x and this is pretty obvious that, this is a left running wave, we are recovering a nice result because, this is. So, you have e power i k x e power i omega t, which is e power i times k x plus omega t so, this is of the form f of x plus c t or something like that.

So, this is indeed a left running wave and how do you recover right running wave (Refer Slide Time: 27:11). So here, we put a equal to 0 then, you will get a right running wave so, you will get theta equal to so, you be say a equal to 0 so, you will get theta equal to tan inverse tan of minus k x. This is nothing but, we should not write at the bottom.

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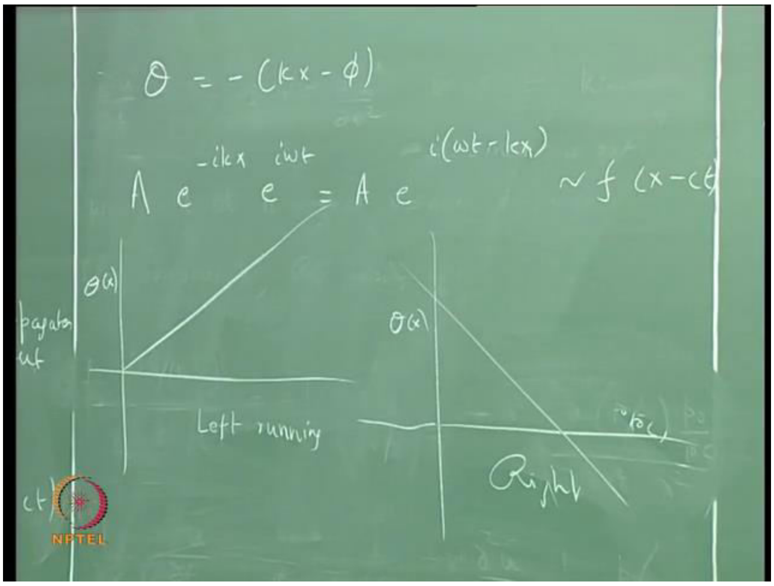

so, if you have the pi, can be observed with the constants e power minus i k x e power i omega t and this constants contains the e power pi, this can be plus A e power minus i into omega t minus k x, this is of the form f of x minus c t, which is a right running wave. So, I mean this is trivially obvious but, we are recovering from our general results. So, a left running wave would have a phase, that goes this way and for a right running wave so, you will get some such curve.

Now, if you take a look at the for a general termination but, we will consider 2 cases when something is driving and something is damping. So, next some plots of this standing wave distribution, both the amplitude and phase. So, what I would very strongly suggest is, you can write your own computer program, we can pick a b and phi and then, play with how the amplitude looks like, phase looks like and then, you can get a feel for it. I actually did this things, when I was a student I wrote a proton program and plotted now a days, things are very easy. You can make this formula and excel with one click and drag down and it is within a seconds, you will get all these things but, I will draw some hand draw so, how the curve looks like.

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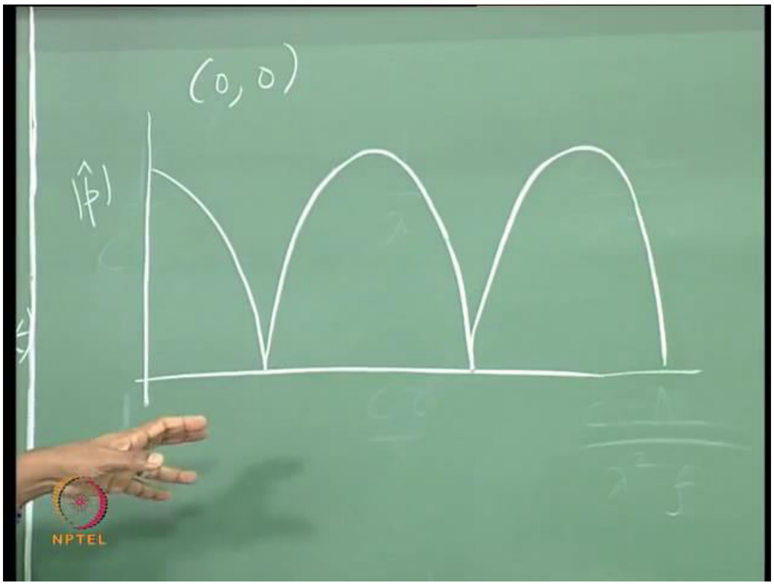

So, let us look at the admittance at 0 comma 0 so, I will locate the pressure amplitude, this way would look like this, the dominating feature was that your minimum will touch 0. So, you can make this plots and logs scales or linear scales, the acoustician sometimes would plot in logs scale in decibels but, I have plotted in a linear scale. So now, why does it touch 0 that means, the incident wave and admittance of wave plot admittance is 0 that means, or a equal to b.

So, you have a perfect reflection, you can have a equal to minus b also, you have perfect reflection and therefore at the minimas, the wave is able to cancel precisely. Of course, it is that only in theory in reality, we never have perfect reflection. If you have a closed end as I mentioned, they were still vibrate and if a open end, some radiation will happen. So, in a practical context, I think you can a typical maxima to minima ratio of 10 would be what you expect for a perfect termination that is what, I would expect.

Now, I have seen values as I has 25 and so on for smaller dots and for bigger dots, it can be instead of 10, the ratio can be 8 or something but, that is a number. And if you have maxima divided by minima of 2 over 3 then, definitely you are far from a perfect termination. So, these are just thumb rules and it depends on the frequency, it depends on how you tighten the nuts and bolts and so on.

And if you do very well, I think you can get I mean, 10 is considered very good, I think maxima to minima ratio. But, I have seen one of my student Librica, she can get make it nice such that, you get that ratio of 25, that is a best I ever seen in my life, maxima to minima ratio for a standing wave now, if I raise the real part first let us look at that, that is easy.

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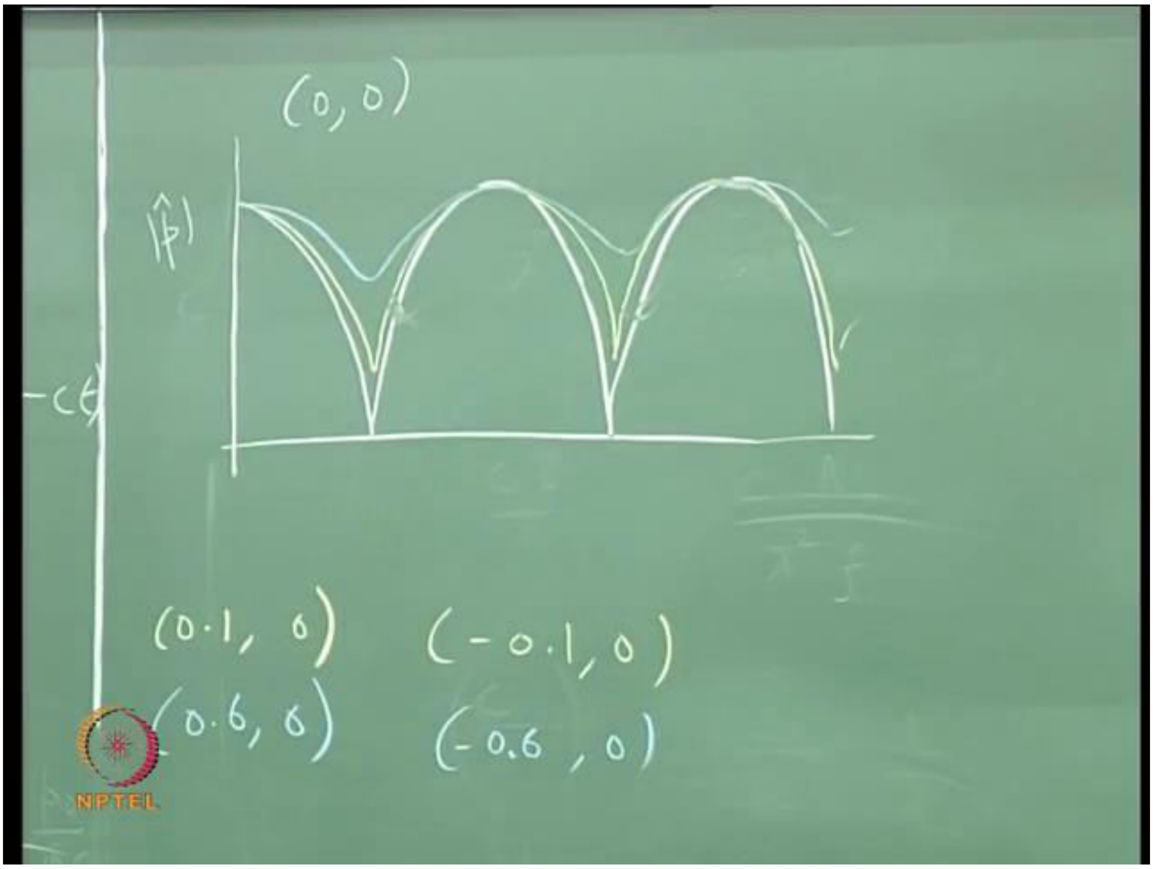

What will happen is let me take a color and draw 0, 0.1 comma 0 so, it would look like this, the actual amplitude it is of does not have any meaning because, we are having a linear theory. So, you can multiply this by any number and you will still have a solution so, actual value does not mean much. But, what happens is, this as you increase the real part of admittance, the ratio of maxima to minima will keep coming down that is what, you would observe so, let us try another one.

I urge you to plot this graph and see but, you will get something like this now, what I wish to emphasis, as what Shabari just pointed out the other day. I will get exactly the same result, even if I had minus 1 over 0 or minus 0.6 I will get exactly same result and I would not be able to distinguish it. For doing that, we will have to draw the phase now, if you look at the phase, let me 0 comma 0, put this over here.

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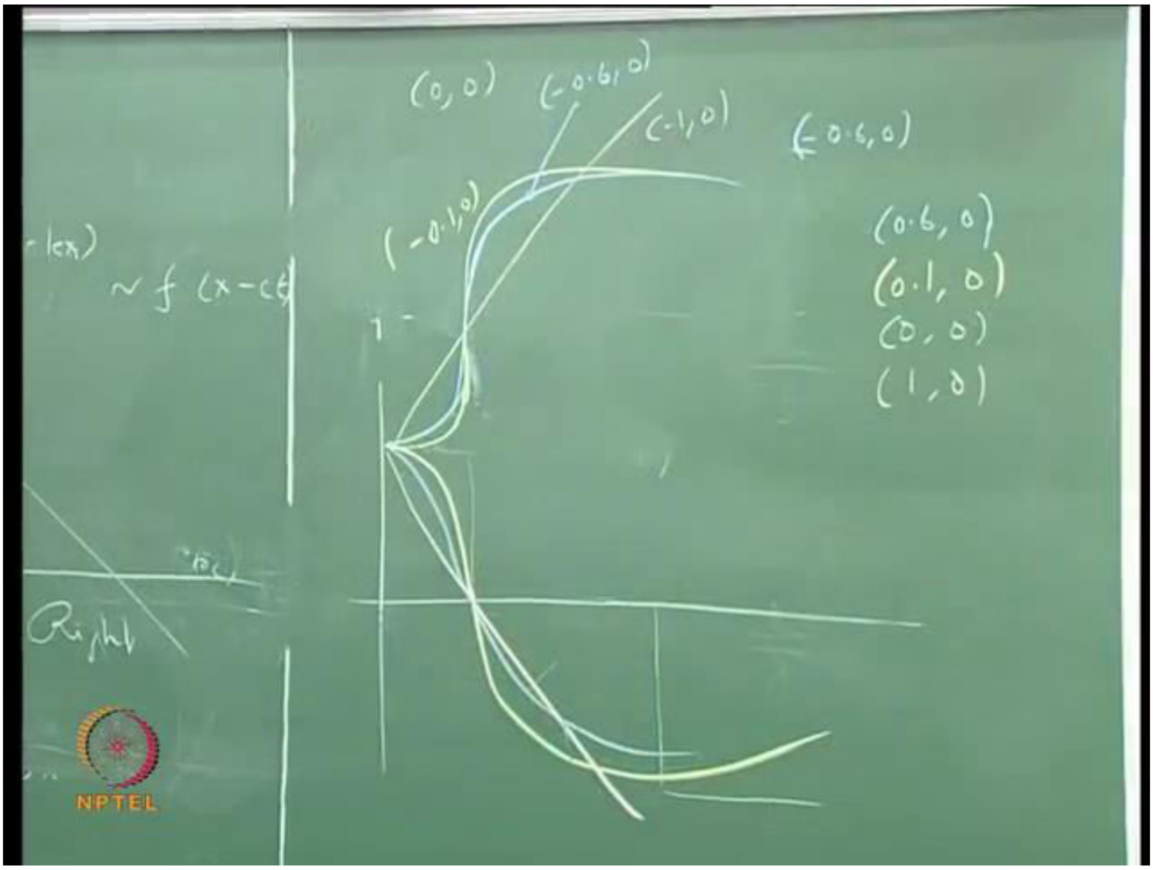

Now, if I were to plot this phase and I said that, for a perfect termination at the minima, you had a phase shift of 180 degree of similar. So, but really for a perfect termination, no energy is going this way or that way, it is just staying there. So, whether you draw it this way down or going up, it does not make any difference but, if you now speak about the 0.1 comma 0, if you have a driving, it would go this way and for the next one, it would be give one more this way. In the end, if you get a left running wave and that would be, what would be that, will be the admittance of left running wave.

Let us look at right running wave, if you have right running wave rho c, may 1 over rho c that I mean, that is a non dimensionalising with that so, you will get 1 I mean, these are the non dimensional values. So, I should get a straight line like that because, we saw so here, your energy is coming from left to right. So, this reflected wave is dominating b is greater than a so, you now can pick your b as a plus I mean, p max plus p min over 2. Now, if you had minus 0.6 for example, I will draw it whole top of this, the curve would look like this. So, this would be minus 0.6 comma 0 and the limit would be a travelling wave, which would go like this, I think I should erase the curve, it is confusing. So, this blue is minus 0.6 comma 0 this would be so, you have to determine the phase and that is the only way you can be able to distinguish with it, a left running wave and a right running wave, is this clear.

I would a suggest that, you make a plot of this things and get a feel for it and in the examination I will give some admittance values and give you a few plots and I ask you, which one would roughly corresponds to that the quantities. So, I ask this every year and every year most people get a problem, I just want to distinguish between the imaginary parts I will give 2 values.

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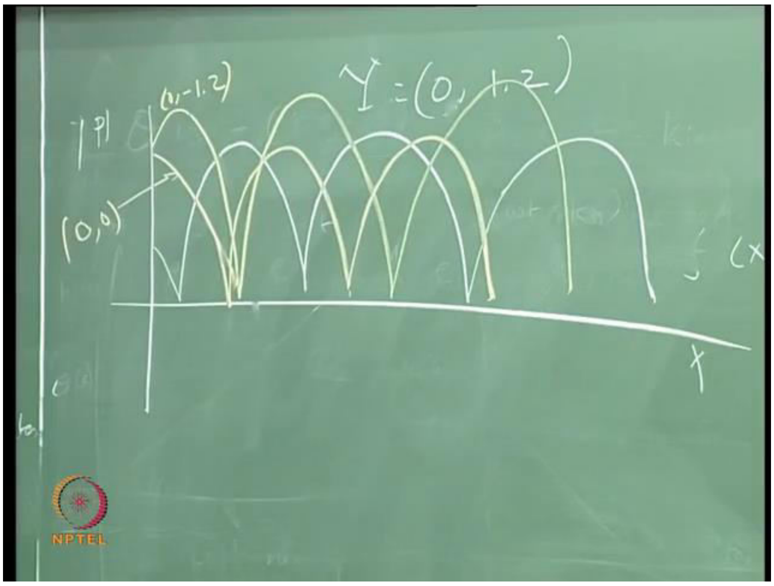

So, let us say, I have 0 comma 0.2 of 0 comma 0 would perfectly line up 0 comma 1.2 would look like this, this is x. So, the imaginary part actually decides, it is an indication of where the reflection is taking place. If you have 0 comma 0, you would so, this would be 0 comma 0 so, the reflection can be thought of, it just taking place right at the surface. But, you can think of it, the reflection to be happened little bit ahead or little bit behind so, the imaginary parts actually indicates that, I will work out a problem to give you more feel of this.

But, this is how, you would obtain in the pattern and like I said, we really need to see the pattern we able to tell what kind of admittance you have. But, to be able to do a good job was that, you have to be able to play with numbers and see, what numbers give what pattern. So, as I mentioned to you, I did this experiment when the conditions on the data type systems were very bad.

We did not have lab view, we did not have proper computers, we do data accurate system on a paper tape and so on, it is really difficult. So, it is really important that you knew what you are looking for, otherwise you would spend lots of effort and not get anything. I think, had I been doing the experiment since the modern times as it now, I would not probably think about these things so much.

So, the last question is Anwikasha ask this morning so, what is the procedure to determine admittance. So, it is very simple, you determine the pressure maxima and the pressure minima and the location of the minima and then, we can determine a as p max plus p min over 2 and b as p max minus p min over 2 or vice versa. Now, we do not know which is which so for that, we have the phase plot.

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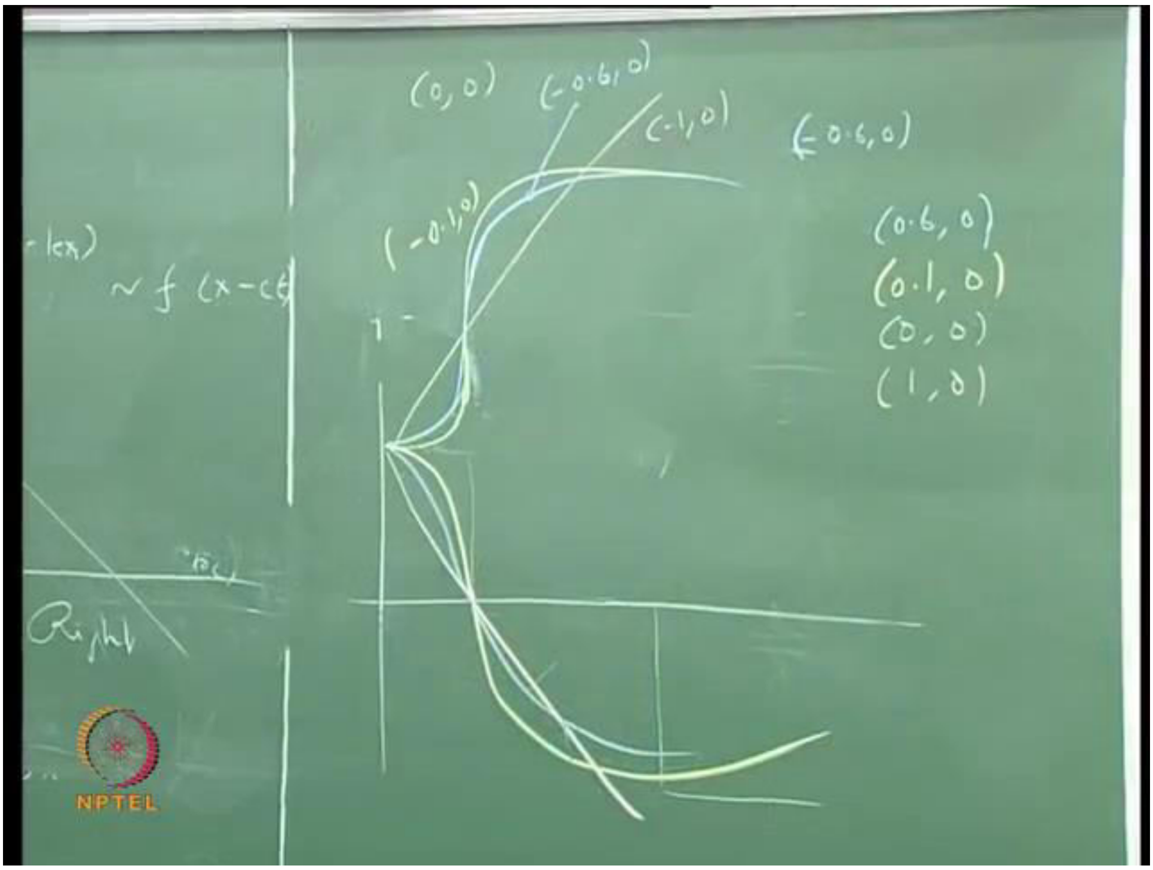

And if the phase plots are coming this way if they are falling that means, your reflected wave is stronger than the incident wave and of the phase plots are going that way that means, the incident wave is stronger than the reflected wave. So, you pick your a and b appropriately, we also know your phi because, there is a relation for the x location of the minima and phi, we are derived in last class in terms of k x and so on.

So, now, we know a, b and c, and now a, b and c we can get the general expression for pressure and velocity, which we wrote a e power i k x plus b e power minus i k x, that is a pressure and similarly, there is relation for velocity. So, knowing that, we can evaluate that the expression at the any location and you can get the admittance at any location, I hope this is clear, I will just summarize it, list it.

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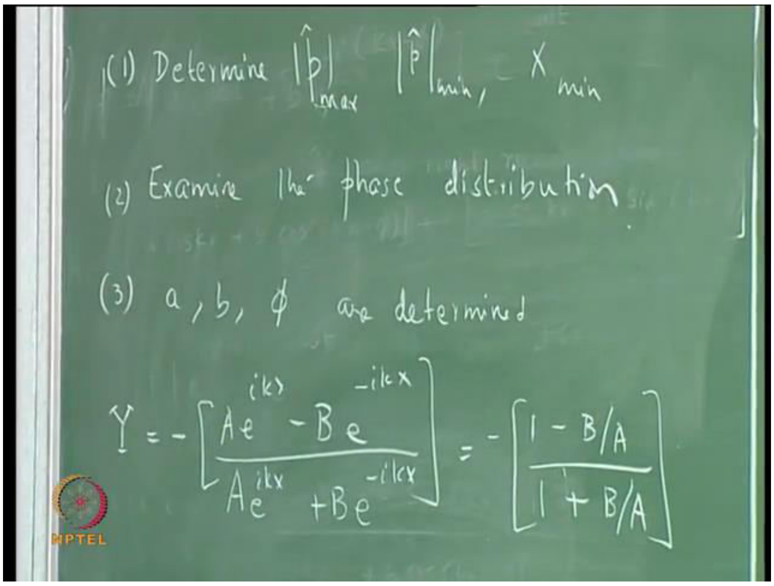

One so now, we know a, b and phi, and capital A and capital B can be determined from small a and small b and phi. So, if you want at x equal to 0, if you fix the reference flame as that then, that would be equal to so, this is the simple procedure to determine admittance. Now, you can do this, if you can precisely determine the location at maxima and minima. But, if you are doing a experiment like if you are studying the admittance of solid propel and or something burning propel in a impendence tube with flame then, we may not have time to locate the maxima and locate the minima.

Because, you have to move a prop somewhere and find it, and the experiment is over let us say, 0.2 seconds and there is no question of moving anything to find that and so on.

So, therefore, what you do is, you have to mount a lot of transducers and even if you had 15 transducers, you still cannott pick them minima exactly. So, we will have to use data from everything and do like a curve fitting kind of situation and you can curve fit for the amplitude and the phase, that would be the best thing.

And then, you can get the a, b and phi and then, you can actually do the same procedure, that will be how you would obtain the admittance, if you are unable to pin point a maxima and the minima. And if you have only 2 microphones or there is of course, the variant of 2 microphone technique, which you have you derive a expression in terms of some specific locations, you have the microphone p 1 and p 2 and then, you can do that so, I would not deal with these things but, you can look it up on your own.

So, I will stop here, the next thing to look at is that so now, we studied a problem where, we are driving with a loud speaker and you are having a standing wave, which is not changing in amplitude. But now, often we are interested in stability so now, let us say, we have a termination of certain admittance, it can be positive or negative, you can send energy in send energy out, how would you relate that to the growth rate or decay rate. So, that is the next question so, what we would do is to, we did the problem of closed open end.

We will do another problem let us say, one end is closed and the other is having admittance boundary conditions and then, we will derive the eigen value of that problem. And look at the real and imaginary part and see, how the periodic part is affected that is how, the real frequency is effected and in terms of admittance. And then, we will look at, what is the growth rate or decay rate in terms of the admittance. So, you can actually get the expression for change in frequency in terms of imaginary part of admittance.

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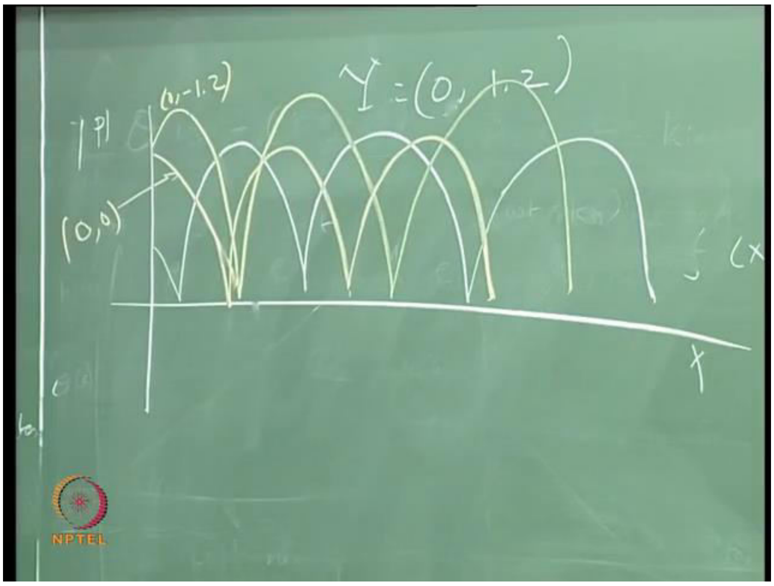

Because, as I mentioned, it decides where the reflection happens or it indicates and you can also get the growth rate in terms of the admittance. Because, the growth rate depends on how much energy is coming or going out so, this we will do in the next class. So, I stop here, is there any questions?

How to get a continuous graph for phase.

How to get a continuous graph so, the question is, how to get the continues graphs for phase.

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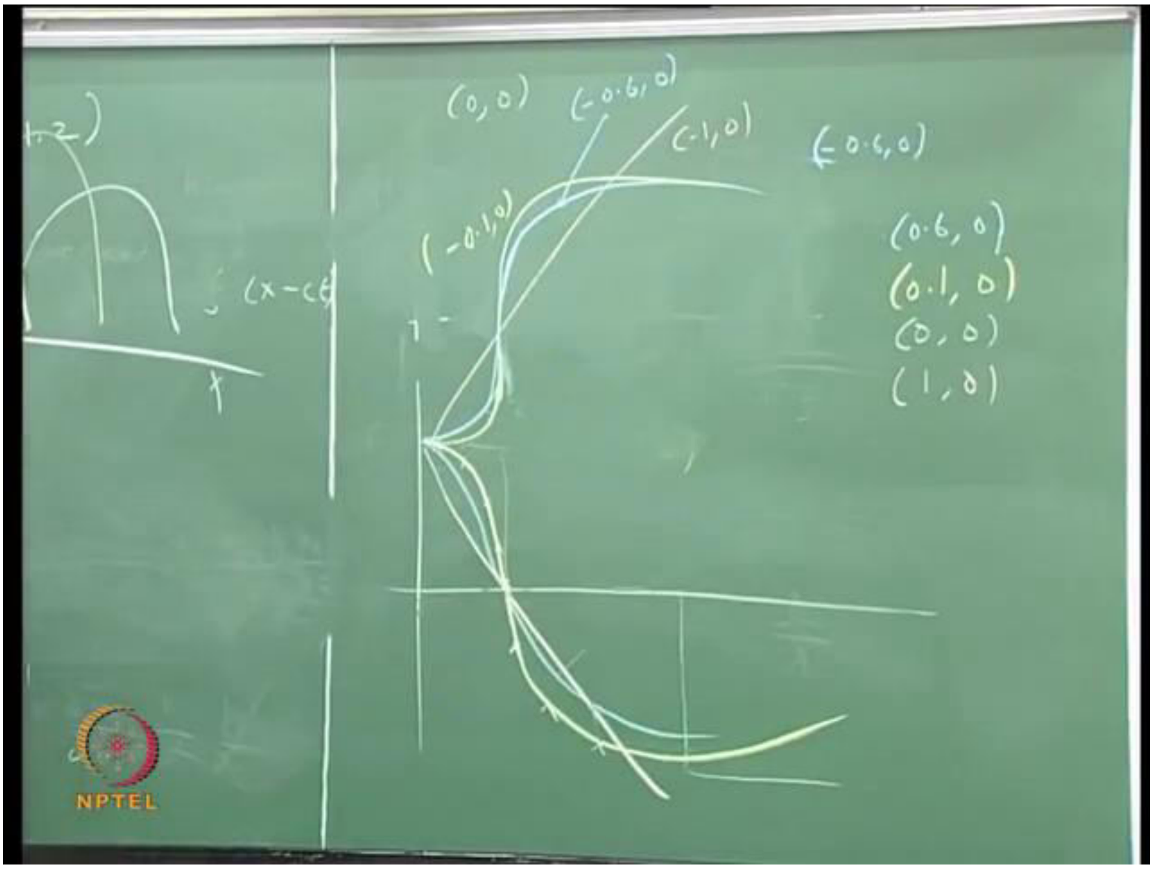

So, what you really get is a lot of discreet points actually so, I mean, same with the amplitude, amplitude you can not get, you may we are making discreet measurements. And then, we are plotting it and then, we can curve fit so, you have measurement is several locations and you can curve fit or something.

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Yeah, that is true so, we have a reference microphone and with respect to that we measure and if it is 0 degree, it can 0 or 360 or 720 and so on, we have that much of ambiguity. But, we can do this, what is called phase and rapped and get it continuous because, it does not matter. So, we can actually get it to a continuous curve, you can actually take off this point or add 360 degree to that but then, solving is I mean, your tan inverse formula also has a problem.

So, I would make it smooth and work out and the other thing I want to say is, I will never use tan inverse command and computer, I will use a tan 2, which gives both the numerator and denominator. So that, you can distinguish between the various quadrant and in general, I would not curve fit for the phase, because of this problem. But, you can always curve fit for, you can write things as real part and imaginary part and then, curve fit those things, that will be easier to do rather than write it as complex number and curve fitting for that.

So, that way you are sight step changes but in general, while plotting we can unwrap it that means, you can keep on plotting it as a continuous thing. But then, if you are having several wavelengths, you will run out of the page and then therefore, you can actually add 2 pi or subtract 2 pi and make it fit with a new page so, that is the purpose, anything else. Then, these are practical issues, very good you ask this question, any thing else so, that is all today.

Thank you.