Acoustic Instabilities in Aerospace Propulsion

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Lecture - 08
Impedance Tube Technique

Are there any questions from last class, anybody, so what we did last class was to define qualities such as impedance and admittance and then we look at the physical meaning of admittance. Particularly, we look at the real part of the admittance imaginary part we have not drives first. So, what does the real part of admittance means anybody, what is the significance, what does it show.

Student: ((Refer Time: 00:37))

So, if there is a power is coming in you will get 1 sin, if the power is going the other way you will get the other side and we will work this out as we go on. So, now I wanted to raise a question, we said that we have differential equations and we said that we can solve them as Eigen value problem depending on the boundary conditions. We can have boundary conditions need boundary conditions like open up and or closed open, open closed and so on.

But, then we said that the point was raised as to why a Rocket Nozzle would have a exactly close condition and why it should not be different and so on. And I agreed that a Nozzle will leak out some acoustic power, so you can have some non 0 value of admittance and so on. So, having said that and having a thought of the problem as Eigen value problem, it is squared likely that you would get a complex Eigen value.

And you will get a real Eigen value only if you have either velocities 0 or pressure is 0 at the end, as we saw in the simple problem that you worked out. So, let me before I proceed tried to figure out what is meant by complex frequency, any body know what is a complex frequency.

Student: ((Refer Time: 02:05))

What is the meaning of complex frequency I mean I can think of frequency as cycles per seconds.

Student: ((Refer Time: 02:15))

So, what was the and what will the real part integrate.

Student: ((Refer Time: 02:18))

So, that is absolutely right, let us look at this carefully before proceeding further.

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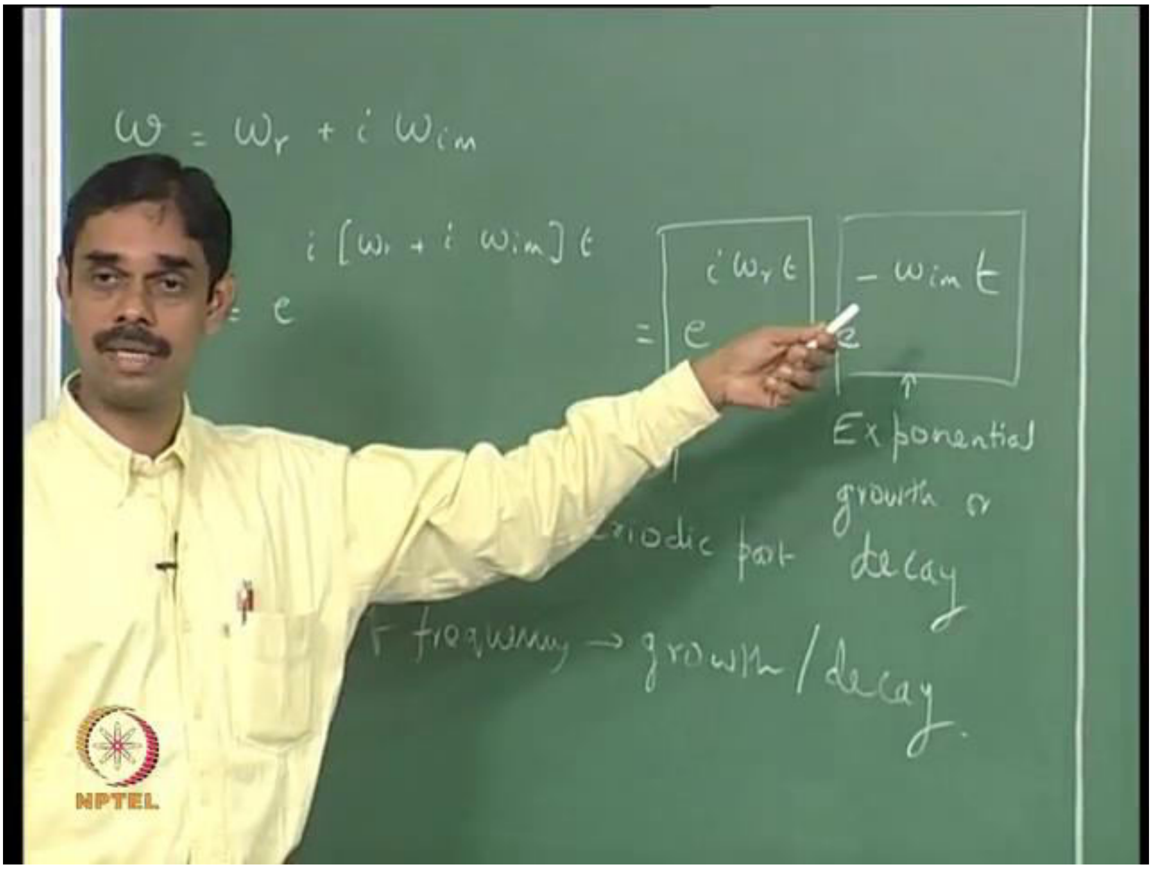

Let us think of omega as omega real plus i times omega is imaginary and let us look at e power i omega t which is e power i times omega real plus i times omega at imaginary times t which if you multiply out. We will get e power omega i omega real t as e power i times i is minus 1, so e power minus omega imaginary t. So, this is the periodic part and this is the exponential growth or decay. If omega imaginary positive you have exponential decay, if omega imaginary is negative you will have exponential growth or say this is exponential growth to decay.

So, if you have only a real frequency; that means, you have periodic component and it is neither growing nor decay, but if you actually under with a Eigen value which has complex part the imaginary part would indicate what is called a growth rate or decay rate. So, imaginary part frequency will be of growth of or decay, so I hope this part is clear. So, just fast for a minute for it make notes. So, let us think of situation when frequency is real, now we were speaking about impendence tube taker and we said that was nothing, but tube with loud speaker 1 and of some termination 1 side.

And you have lot of pressure transducer mount that or you have 1 micro phone which you are moving along. Now, imagine we turned the speaker on loud speaker on, actually in the beginning the amplitude will grow and the growth will happen over half a second, 1 second or quarter second of that times here. And then it will stay steady and we are stay steady, because as initially you are putting in power and the some losses, but as we increase the power you lose more.

And eventually you reach a kind of a situation where whatever you put in to is equal to whatever is lost. And then you maintain steady amplitude, so that would mean there is no further growth or decay. So, actually this reaching this final amplitude is a non-linear phenomena, but we will not worry about that and we will keep the frequency now as real, because we are not having a growth or decay and then we still try to apply linear theory.

So, just to code this over coded statements all theories are wrong, but some theories are useful. So, I think this is a we have to see if it is useful then it works. So, I must agree that I am doing this slight of hand because any time you reach in principle you are reaching a limit cycle because the oscillations are growing and saturating up. So, limit cycle is indeed a non-linear phenomena, there is nothing called a linear limit cycle, but then we will set up the experiment at low amplitudes.

And then we will say that the propagation and of the wave is linear and that is a slight of hand and hopefully you will get good results that is idea. So, like I said all theories are wrong, but some theories are useful, so we only after you do the theory, you will know whether it is useful or not. So, we will initially do stick with real omega and then you will try to see once you know the admittance of whatever is a termination. Whether, we can translate it in to had there be no loud speaker whether you get a growth rate or decay rate, so that is the ultimate objective.

That is to look at the stability of the system for example, suppose you found a stability of I mean you are impendence of profile and something by putting a loud speaker exiting it checking what the amplitude in the wave is coming and so on. And then that is not a n by it iself for us in the end what we want to do is well we want to solve for the Eigen value problem without any loud speaker, and see if the system will get self exited.

That would mean a self exited system would mean this would be a grow in term for alpha t and if it is a stable system it will be exponentially decaying in term. So, that is what we are interested in eventually finding out, so we will do it two steps first we deal with how to determine the admittance of a termination by you seeing this impendence tube technique. That is essentially a forced experiment; that means, you forced the sounds with a loud speaker, and then we will see if we can translate in to some kind of exited problems.

Where, you leave a system alone and see what happen? I hope this approach is clear sounds are little crocked, but that what it is, but a once you work out you will see it s not all that crocked. So, in this process we are doing the experiment with aloud speaker with a real frequency; that means, we are setting up the experiment and some kind of study states is that clear.

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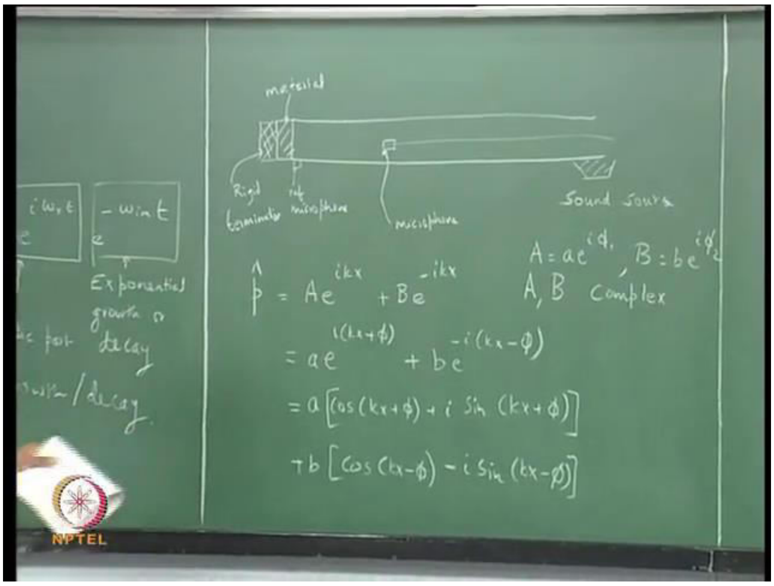

So, let me draw this picture once again, so we have, so this tube is the impendence tube and you have a sound source and you have a microphone. This is just symbolic of doing a throwers with the entire set up and as I mentioned you should have a reference microphone here. And you can either travels with a one microphone or you can put 20 microphones and get the acoustic field and you always need a reference to determine the phase.

Phase is bit by comparing signals from 2 different locations, so 2 different transducers. So, we need always a reference microphone either the end or some place and then we have this is the termination or the material you want to study. And then let us say we have a rigid biking to whole this implies and to make sure that nothing comes in from any other side and so on.

So, we will start with our equations usual equation p hat equal to A e power i k x plus B e power minus i k x. So, I will work out this problem in the harmonic domain or frequency domain; that means, it is implied that p prime equal to p hat e power omega t and then you have to take the real part and so on. But, we will just deal with the complex amplitude and then in the end if you want to find a instantaneous pressure we have to multiplied by e power I omega t and take the real part is that clear.

So, let us now try to do some algebraic man manipulations to get an expression for the amplitude which is the amplitude of this complex number which is p hat times p hat star. It is complex conjugate and take this square root and that would be the amplitude at each location in a standing wave. So, you have a complex amplitude that would mean it will have a both the amplitude information as well as the phase information. Phase means how the wave is going up and down in with respect to how the pressures going up and down and another location.

So, we want to try to get an expression for the amplitude separately and phase separately and then we will try to see what is the nature of the function. So, we can say that a in general A B are complex. So, you can say A equal to A e power i pi 1 B equal to B e power i pi 2, so if you put that in here. So, here you will get A e power i times k x plus pi plus B e power minus i times k x minus pi. Now, we will go to trigonometry to do some algebra. So, this can be written as A cos k x plus pi plus i sin k x plus pi plus B in to cos k x minus pi minus i sin k x minus pi. So, now what we can do is we can collect the real part together, yes some problem

Student: ((Refer Time: 12:20))

Thank you, yeah thanks a lot any thing else.

So, now, what we want to do is to we wish to club the real parts together imaginary parts together and then we can easily find the amplitude.

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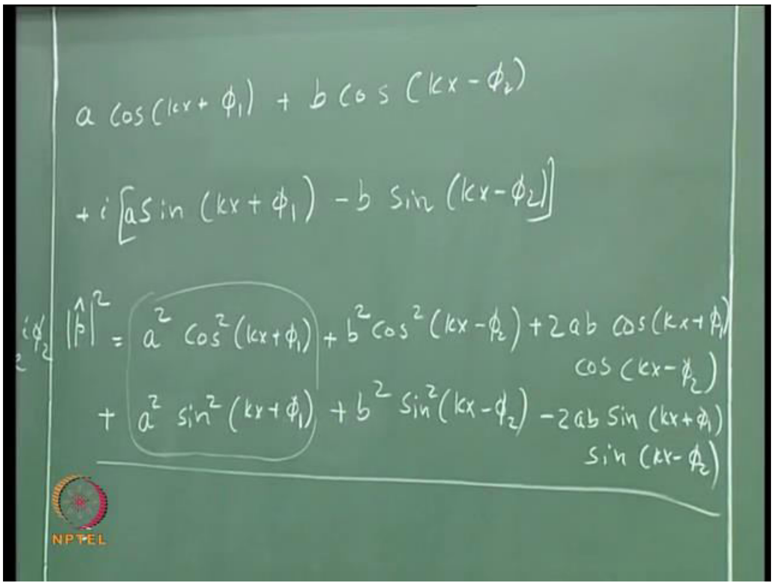

So, we can say this is a, so this should to be pi 2 actually sorry, if this any mistake please found out. So, if you want to take the find the amplitude of this. So, we can say p hat square equal to we just take square of that plus square of this. So, we will get a square cos square k x plus pi 1 plus b square cos square k x minus pi 2 plus 2 a b cos k x plus pi 1 multiplied by cos k x minus pi 2 plus the other set of terms a square sin square k x plus pi 1 plus b square sin square k x minus pi 2 minus 2 a b sin k x plus pi 1 sin k x minus pi 2. So, now if you try to add this terms it simplifies a lot here, so this term we know that sin square theta plus cos square theta is 1.

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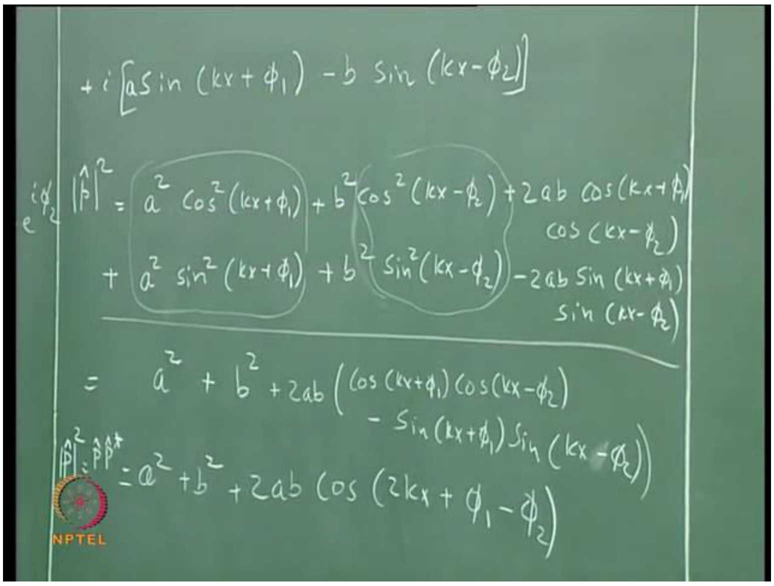

So, this whole term simplifies to a square and similarly here again we get sin square plus cos square. So, you will get that is 1 plus 2 a b into cos k x plus pi 1 cos k x minus pi 2 minus sin k x plus pi 1 and sin k x minus pi 2. So, this can be simplified you are having the form cos a cos b minus sin a sin b which is or the form cos a plus b. So, we can replace this term I hope I have the right result, so this is the expression for the amplitude of the standing wave.

So, just note that when we wrote p hat here there is a complex amplitude; that means, that a this location it is a complex number; that means, it is amplitude that is this directly square of the amplitude gives actually the amplitude of the stranding wave. But, the phase actually represents how the wave is moving with respect to other parts of the wave, so I hope this difference is clear

So, now if you put microphones at different locations, that each location if you measure the amplitude. This is the kind of the variation that you would get I mean, in fact you will get this square root because the square root amplitude. Now, the issue is that I mentioned that in a real experiment we actually know the pressure, but we do not know the admittance here. So, is there a way we can get this a b and pi 1 minus pi 2 from measurements, so that is the next question?

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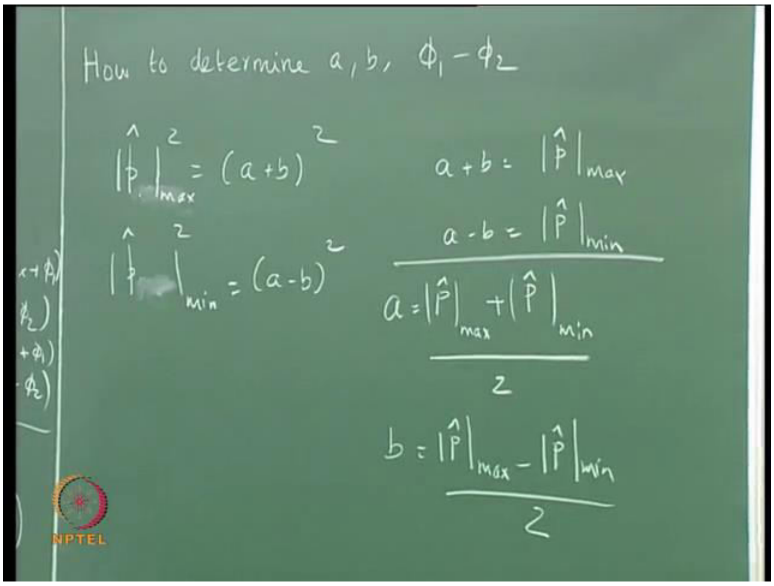

So, pi 1 minus pi 2 phases always on with reference to another, so it is not necessary to get pi 1 and pi 2 independently, but it is enough to find pi 1 minis pi 2. So, the simplest way is we can see when this expressions are maxima and when this expression is minima. When would this be a maxima? What is the maximum value of this expression? And what is the minimum value of the expression? a minus b whole square. So, we can say a plus b equal to p hat max and a minus b equal to p hat min which should give a equal to p hat amplitude max plus p hat amplitude min over 2 b equal to p hat max .

So, now, we have determined a and b now we are asked to determine the pi 1 minus pi 2, how would you determine pi 1 minus pi 2 which is a simple way to determine can we think about it. One the best way to look for is to find, if you can locate the minima that is a standing wave minima. So, then what will be the value of this term at a minima will it this term should be minus 1 at a minima plus 1 at a maxima. So, when will this be minus 1 this is equal to phi..

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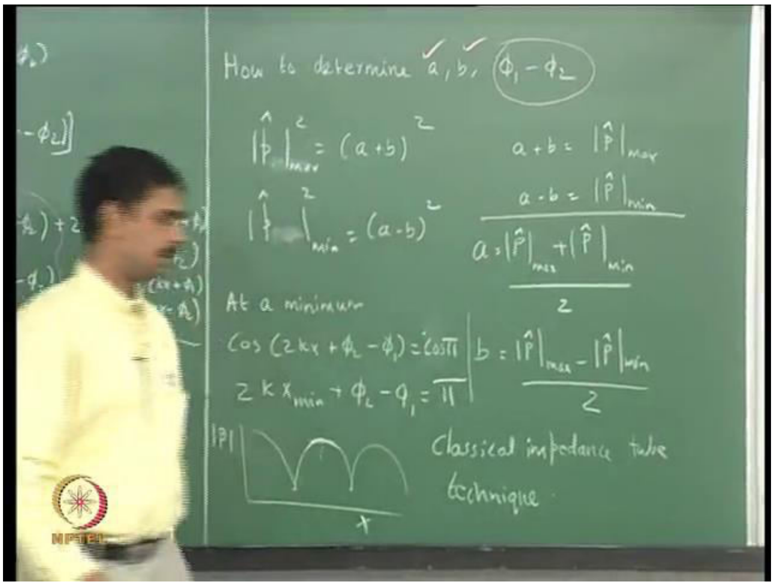

So, we can say at a minima cos of 2 k x plus pi 2 minus pi 1 equal to phi or 2 k x min plus pi 2 minus pi 1. Now, you could use the maxima also, but in general in a wave it is easy much easier to determine the minimum than a maximum, because if you look at the structure of a width. At the maxima the values stays quite for some time, so we would have difficult in determining which is the process s location, where is you can pin point this minima quite easily and also if you look at the phase at the minima the phase changes dramatically.

So, that is a reason why we go for determining the minimum value and using that to determine the phase pi 2 minus pi 1 rather than using a maxima. So, in summary the minima standing wave distribution is quite sharp at maxima it is not quite sharp therefore, we choose the minima. I just plotted standing wave distribution. So, this is a minima I can surely pin point it because I can I mean is taken the pattern, but where as the flat portion is I mean it is not that hard to I mean it is quite hard to pin point.

Where exactly it is a maxima because for example, you can imagine change ((Refer Time: 22:18)) a microphone with the values do not change you do not know whether here or there. Because, over a big region the changes would be of the order of the uncertainty intimation that is from practice concentration in principle you can use the maxima also

Student: ((Refer Time: 22:38))

So, you are saying that this is not differential this is amplitude, so we are taking a square root that is why of this found. So, you should look at this square there is no problem. I mean this is a continuous function it can differentiate more than the twice, and so this is the classical impendence tube technique where whatever I just mentioned let me from that the classical.

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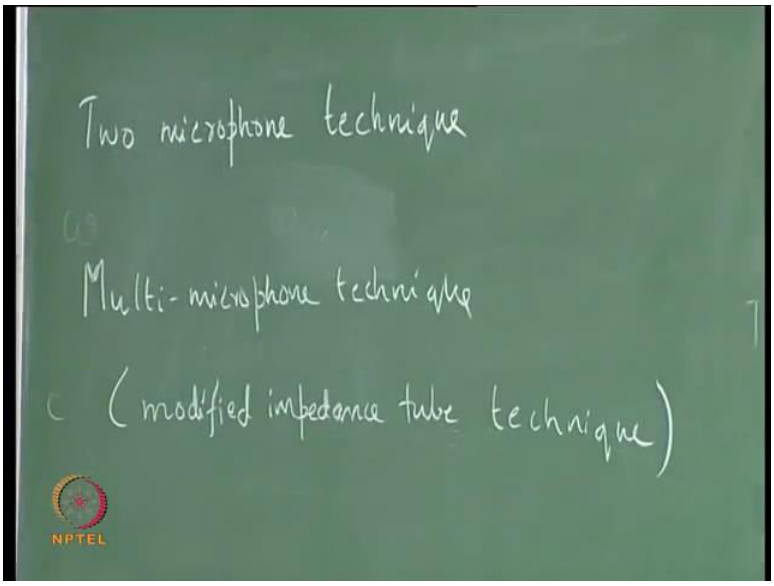

So, there are 2 variants to this technique 1 would be called 2 microphone technique and another 1 is called multi microphone technique or alternate can be called modified impedance tube technique. I personally think well I am not assuming any thing if you get the other way.

Student: ((Refer Time: 24:25)) b e is p max by p e min by 2 we also have the same solution.

So, it will switch.

Student: So, we have 2 solutions sir.

I mean, but that is alternately pick 1 you are fine what you call a you will end of calling b.

Student: ((Refer Time: 24:43)) we basically have 2 amplitudes for the plus k x term and minus k x term.

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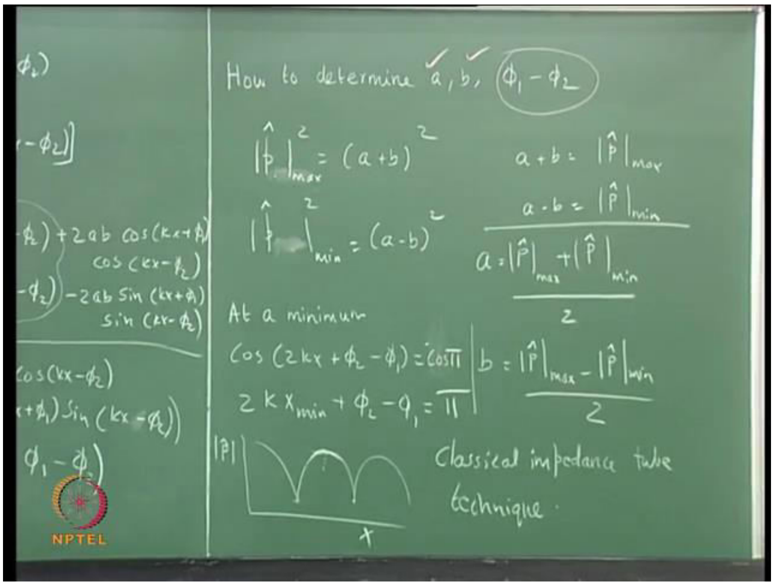

You are saying that if a can be if I take the minus sign here sir.

Student: there sir p min square is a minus b whole square, it can be or p min can be b minus a which was a minus b in that case b will be p max minus p max minus p min by 2 and a will be p max minus p min by 2 So, that would mean the amplitude of the e power i k x term will be a lesser and the amplitude of e power minus i k x will be higher and pi 1 and pi 2 are they do not really depend on the choice of the here

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Student: So, I mean.

What you saying this a here a be can reverse yeah it can reverse, but I thought that till would not.

Student: ((refer Time: 26:04))

Let me think about it and get back to you, but you are saying that I assumed that a is greater than b.

Student: No.

Implicit in that this a is greater than b. So, let me think about it I do not have a clear answer I think both are, but let me get back to you sorry about this I do not know.

Student:((Refer Time: 26:37)) If you take the problem then the ((Refer Time: 26:55))

What can come in that way also I mean if there is some flying sending and the power

Student: in this particular problem b can be grater or less.

B can be grater or less. So, I think you will have to 2 states such that depending the way the power is going in or out.

Student: In that case it is like you said the a final objective is to know the system grows or decays.

Yeah. So, let me get back to you I i am at the moment being stumped on this one I give answer tomorrow the moment when it is working sorry about this. So, we will get back to this same thing tomorrow. At the moment I am not recalling the answer

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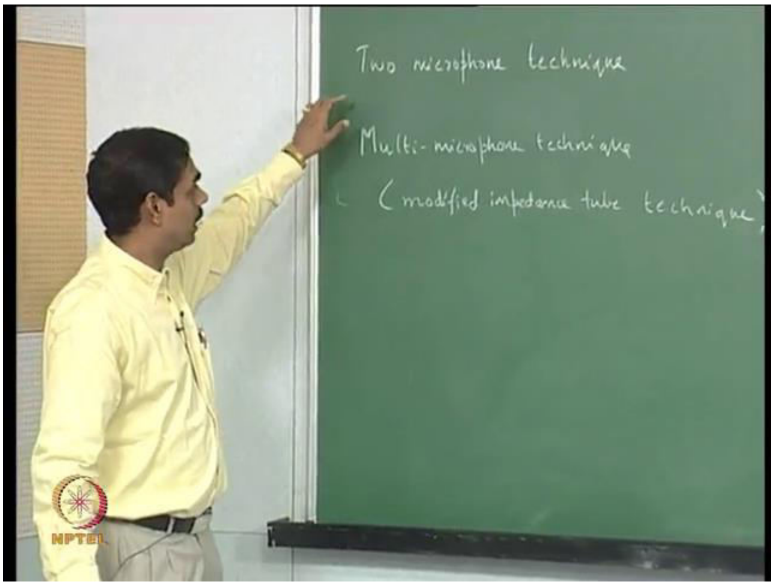

So, we can have a suppose we are unable to travel or something in experiment that is possible, if you somebody gives a engine. And you can not your microphone and say that a version you are not allowed to drill 20 holes then 1 possibility is to keep the microphone at two specific locations. And then you have to know x the locations you have fixed x and then you to make the pressure measurements both the amplitude and phase at each of the locations. So, and then we can also determine the pressure and velocity phase that would be one thing I would not go in to it, but if you can read up only this.

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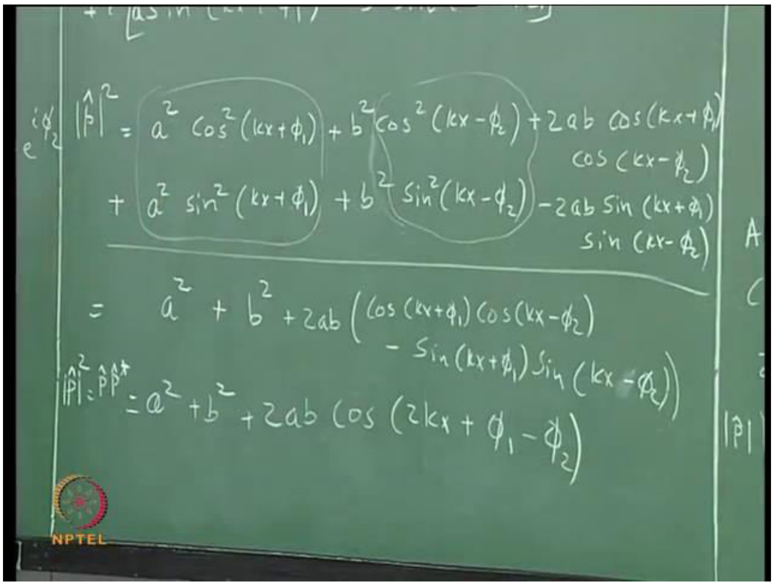

Another one is what is called multi microphone technique. So, you can see that if you make many measurements, you can actually do a curve feet, like a lee square feet for a b and pi 1 minus pi 2. So, from that you can actually get a better estimate of a b and pi 1 minus pi 2, but that would mean that you have to make many measurements rather than this relay on measurement at a maxima and measurement at a minima.

So, that would be called a multi microphone technique or they modified impedance tube technique I think the multi microphone technique sounds as a better name, but both essentially both the terms are using the literature and they mean the same thing. Now, we wish to recover some of the classical results from this problem. So, let us look at the case of open end and closed end and see if you can recover these results.

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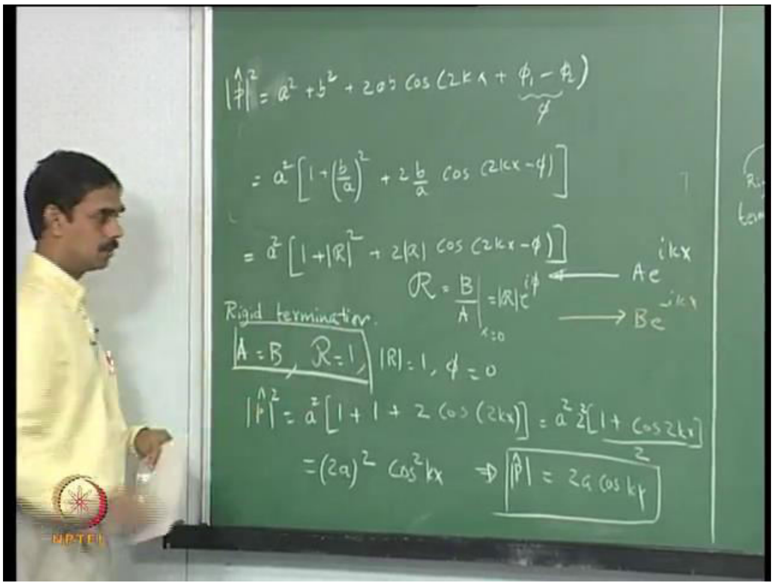

So, I will for convenience denote this pi 1 minus pi 2 as pi and then we can rewrite this as a square into 1 plus b over a square plus 2 b over a cos 2 k x minus pi. So, if you think of a wave A e power i k x using this way and you can think of another wave B e power minus i k x. We can actually think of a reflection coefficient R which is B over A, let us for convenience we say that this is at x equal to 0 that is we are determining impendence or the admittance. This can be written as, so this would be a square into 1 plus.

Now, I have rewritten the expression for the standing wave in terms of a reflection coefficient. So, that is another way of writing the admittance or the impendence in terms of saying that I send an a e power i k x b e power minus i k x comes out. And let us put our coordinate system such that the termination is at x equal to 0. And then a reflection coefficient can be defined as this complex number B divided by the complex number A, so this is as equivalent definition of defining admittance.

So, if you say A equal to B, then R equal to 1 that would mean R equal to 1 and pi equal to 0. So, this would be p square equal to a square into 1 plus 1 plus 2 cos 2 k x, this would be a squared into 1 plus this can be recast as. So, what is this termination actually we worked out this problem earlier, when this reflection coefficient equal to 1. And when do we get this kind of pressure distribution closed in rigid termination.

So, this would be for a, A equal to B and R equal to 1 this for A. So, what we have this is like a general result this distribution which we get and we are able to derive this previous expression that we got for the simple standing wave with 1 end closed and 1 end open which is 2 cos k from that.

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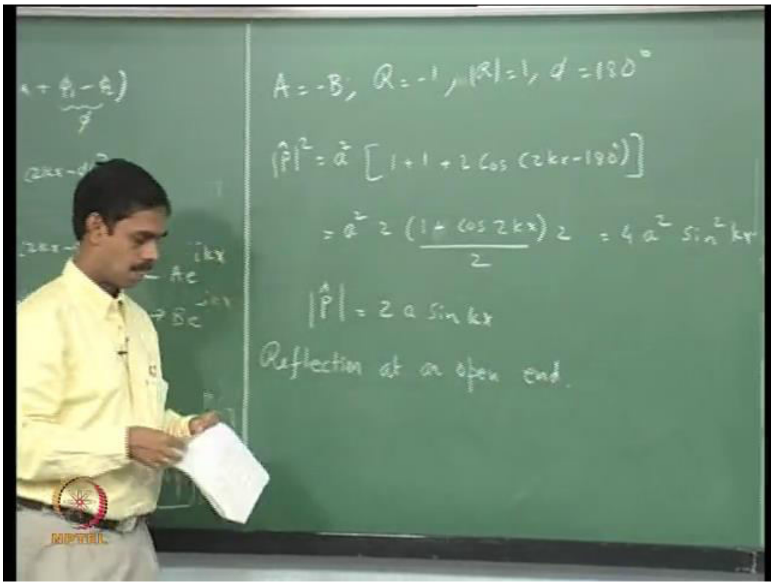

So, now we can look at what happen for the reflection coefficients minus 1, so this minus all will have 180 degree, where 180can be removed. And you can put a minus sign here and then you can, so this is 4 a square you will get. So, this is the other classical this is which you have for reflection at a open end. What happens when B is 0, it is a progressive wave. So, you do not have wave patterns standing wave pattern, but it is just the constant amplitude that is a travelling wave when B equal to 0 will give you wave.

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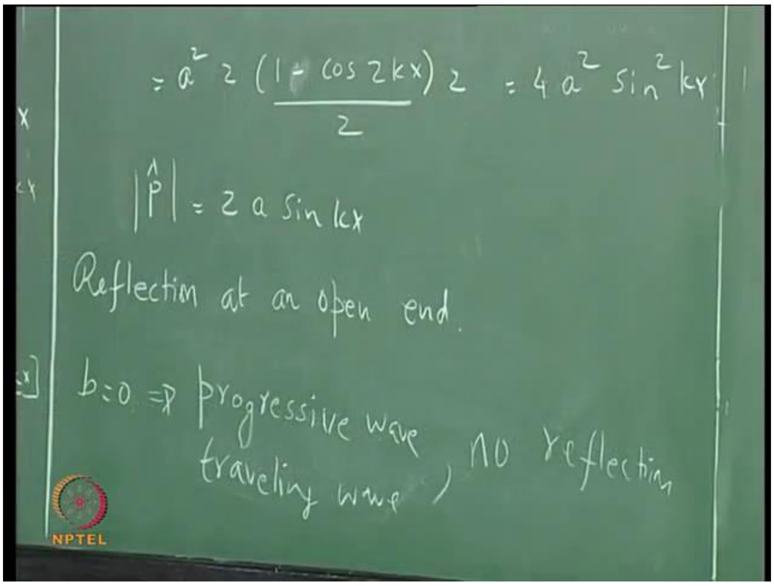

So, the actually have the pressure distribution, now what we need is to determine the acoustic velocity. And then once you know have a expression for acoustic velocity we can determine the acoustic velocity at your x equal to 0. And then the admittance is just acoustic velocity amplitude divided by complex amplitude divided by acoustic pressure amplitude, both are complex amplitude. How do you determine acoustic velocity? Rajesh how do you determine acoustic velocity?

This is the fatal mistake we all make, in fact I must want to know that acoustic pressure divided by rho c will be the amplitude only. If it is a right running wave, if it is a left running wave what will be the velocity and we have both left and right running wave. So, you have to take the amplitude of the right running wave divided by rho c minus amplitude of the left running wave and divided by a rho c. So, in general pressure divided by rho c is not a solution because pressure, it is of the form F plus G and velocity of the form F minus G over rho c. So, you take your F plus G and divided by rho c that is not equal to acoustic velocity is that clear.

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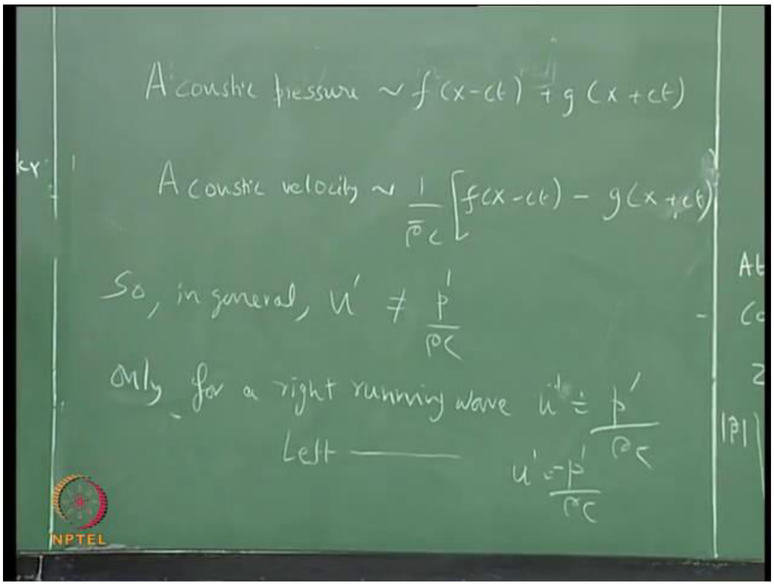

So, let me just restate this acoustic pressure is of the form f of x minus c t plus g of x plus c t and acoustic velocity of the form 1 over rho bar c f of x minus c t minus g of x plus c t. So, in general u prime is not equal to p prime over rho c only for a right running wave, u prime equal to p prime over rho c and for a left running wave u prime equal to minus p prime over rho c.

Because, the wave which is going to a right if you think of a compulsion wave going to the right, the gas will move to the right the same compulsion wave of the same amplitude if it going to the left gas will move to the left. So, that is why velocity has a direction associated with that is by the sign changes when that pressure is just it does not have any direction is that clear.

Student: If I take the both together how do we do.

It that was an original question, but we do not know beyond a 8^{th}, so that is it again. So, in general how do we get. We can use the what equation liberalized momentum equation. Absolutely, the liberalized momentum equation if you given pressure you can determine the velocity and the liberalized momentum equation I think we should write in big letters.

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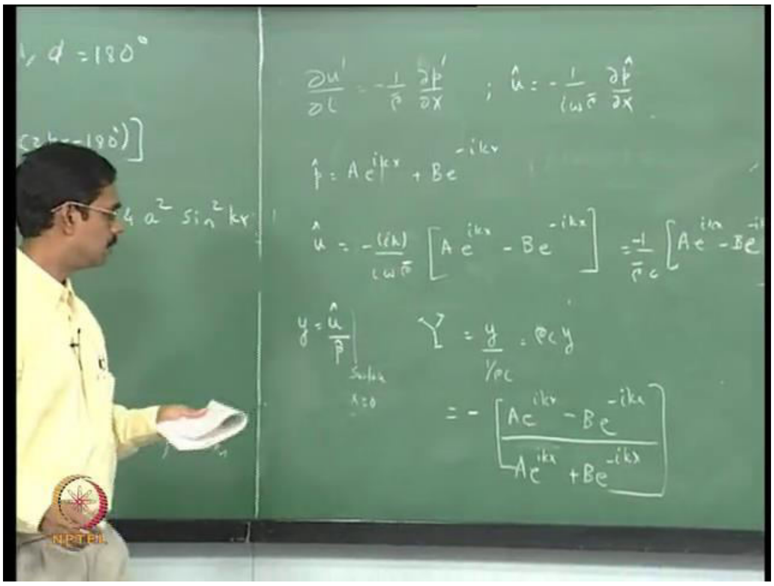

And the harmonic domain, you have u hat equal to minus 2 over omega rho bar rho p hat by rho x. And again it is quite important remember whether you are using e power i omega t or e power minus i omega t both are completely, but we have to use 2 from start to end. Because, if you u power minus i omega t you would not have your sign here would be different.

Your wave equation and harmonic equation would be same, but the sign would be different, but in the end the answer would still be the same because when you in the end take the real part you will get the same expression, but we have to keep track of what you are using and used consistently throughout your exercise. So, we know that p hat was A e power i k x plus B e power minus i k x. So, u hat equal to minus 1 over i omega rho bar into dou p by dou x that would be A e power i k x you take i k out. If I differentiate e power i k x I get i k, if I differentiate e power minus i k x I get minus i k.

So, once we have A and B and pi; that means, we essentially have capital A and capital B. So, you can actually get the velocity and then you just have to divide this velocity by the pressure and you will get the admittance right is that clear. So, just to go back to remind you have the definition admittance y was defined as u hat over p hat at which ever surface you want, and here it is x equal to 0 I have chosen my coordinate system that way.

And I also have the non dimensional admittance script Y or capital Y as Y divided by 1 over rho c or rho c times Y. So, this would be equal to I think I have missed a sign yeah I should have minus here, yeah thank you. So, we know A capital A we know B capital B, so therefore, we know the value of Y anywhere, so we can know at x equal to 0. So, that is basically it does not matter, once you have that small a and small b, we have actually solve the problem.

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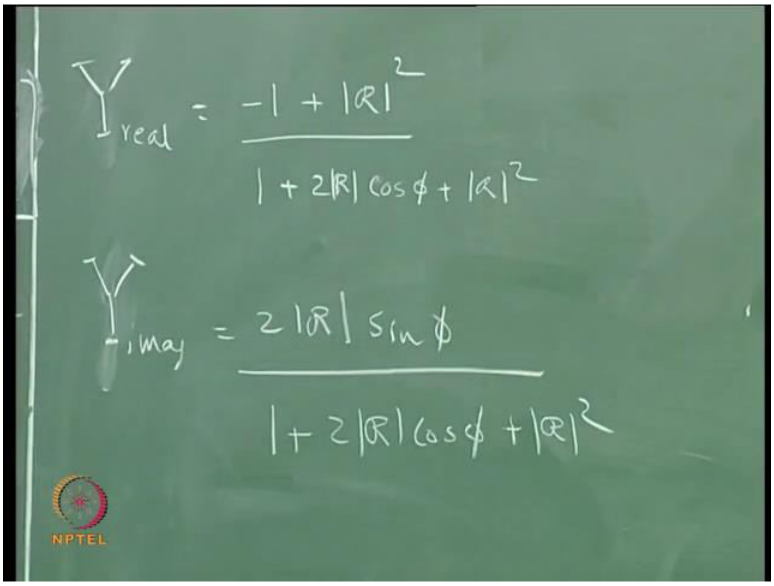

So, if you look at Y at x equal to 0 this will be minus, if I divide by A I will get 1 minus B over A divide by 1 plus B over A, which is equal to minus 1 minus R divided by 1 plus R which is same as minus 1 minus. So, we can do some algebra and separate it into real and imaginary parts I will just write down the final expression. We can check at home by real equal to minus 1 plus R square divided by or this is capital Y actually sorry thank you. So, I want to give you a home work you please do it.

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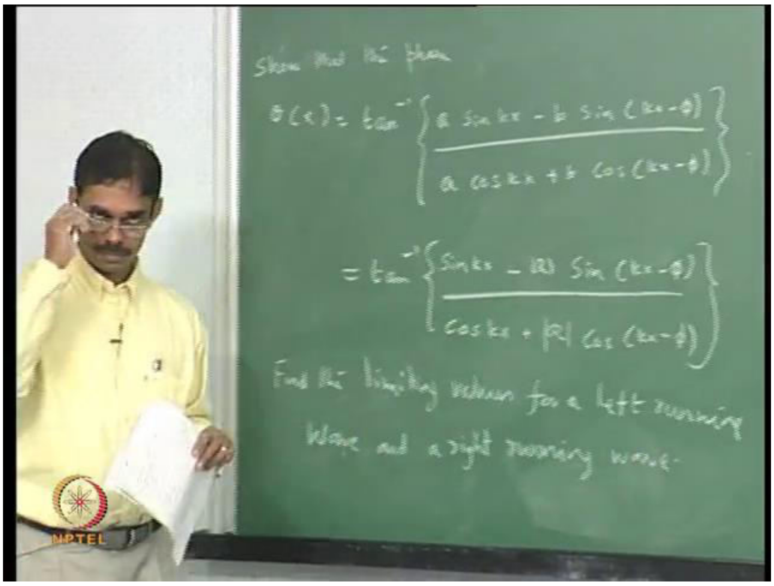

This shows that the phase, so if theta x equal to tan inverse a sin t x minus b sin k x minus pi divided by a cos k x plus b cos k x minus c which it is same as tan inverse sin k x minus r. So, derive this expression for phase and then show the I mean find the limit is when you have a left running wave and a right running wave. So, please do this at home and make it look at the results next tomorrow.

Thank you.