Acoustic Instabilities in Aerospace Propulsion
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Lecture - 6
Standing Waves – 2

Good morning everybody. Yesterday we were looking at standing waves and we derived expressions for waves in a closed type and open type so on. And the way we did this is we converted the partial differential equation into ordinary differential equation. And then we try to solve it by applying the corresponding boundary conditions, for the open end we assumed that the fustic pressure or the acoustic pressure 0. For the closed end we said the displacement is 0 and then we solve for the constants and then we got the wave fields; are they any questions?

Student: why do we assume p primary 0 for an open boundary?

So, the question is why do we assume p primary 0 for an open boundary?

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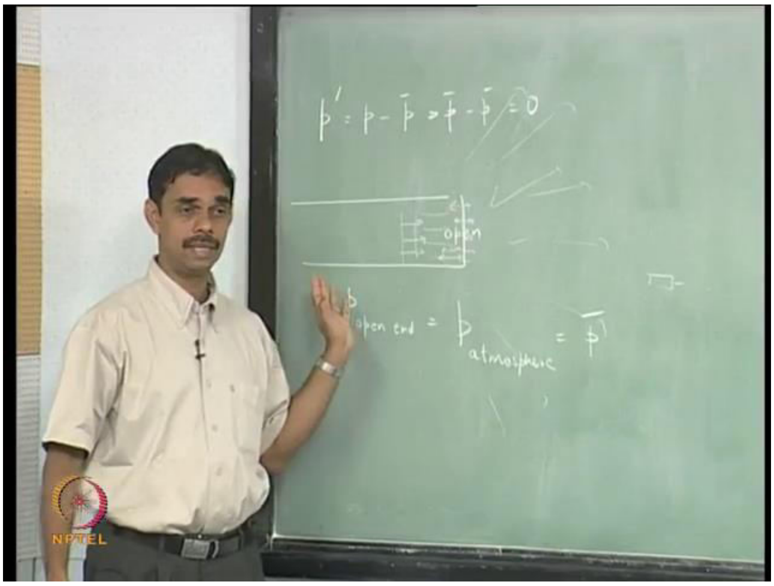

So, if you look at the definition of p prime, p prime is p minus p bar and when a tube is open; that means, it is open to the atmosphere of the universe. So, pressure here would be same as pressure here. So, pressure the open end equal to p atmosphere and since we are saying this is we are not have a base constant mean pressure and all that the pressure here would be p bar itself. So, if you say pressure is p bar and you have p bar minus p bar equal to 0. So, this is the reason why fluctuating pressure is 0 at the open end.

Student: In that case we cannot have wave's propagation in atmosphere.

We can have this is a very trick question or a very deep question. So, if you if you look at a low frequency wave in a standing wave coming, very low frequency wave. This entire wave will be reflected that is what we solve it is reflected such that pressure is going to 0. So, if you have a wave coming this way it is completely turned around and almost nothing is going out. So, if you really make a measurements with a microphone here and compare it with the pressure amplitude here, this would be significantly at least one order magnitude lower than this.

Now, will it be 0, no there will be some pressure. In fact, if you make if you make sound in a tube you attach a loudspeaker to a tube and if you listen outside you will hear a sound, but these inside there will be much more sound. Now, the question is how does the sound come out, everything is getting reflected as the theory says. So, here it is a 1 dimensional wave field and here there is a kind of wave suspending everywhere, so at the end what did we get we found that the pressure is 0, but the velocity will be maximum.

So, you can think of this the gas column at the end right at the end as like a diaphragm which is vibrating back and forth. And from there, so all the allowed speaker or crystal vibrating back and forth and these are open to the universe the atmosphere around. So, the sound field will radiate in a three dimensional fashion everywhere, but it will be radiating at a very poor efficiency at the low frequencies. Higher the frequencies the waves would radiate in a more efficient way, so that you will here more.

Now, typically the if you do experimental as standing wave tube, you can get of the order of mean maybe one-tenth or something only radiating or the power everything have should be here the inside. And if you look at the minima for example, if you look at the expressions 2 cos K x similar, that is what we get.

So, in principle the minimum should come to 0 the pressure should hit the maxima go to a minima and then come back to a maxima. To minimize 0 according to the formula, but never the less you never reach 0 you will get a ratio between the pressure maxima and the pressure minima not as infinity as it should be according to our theory, but it will be more like 10. In very good experiments tightly sealed and very nice experiments you could perhaps get 20 for low diameter tube, but so this is in practice there is a 3D radiation field outside and that is what is creating this.

So, the same thing with a close pipe, in a close pipe again you would not get this maxima to minima infinity ratio, but you will get something around 10 or 20. The reason being at the 10 is more realistic, 20 is if you have a set up really carefully, the reason is the end will I mean whatever you put it will have some level of vibration and then some feeling problems and so on; you have a question?

Student: The open tube, and the closed tube, frequency is coming identical.

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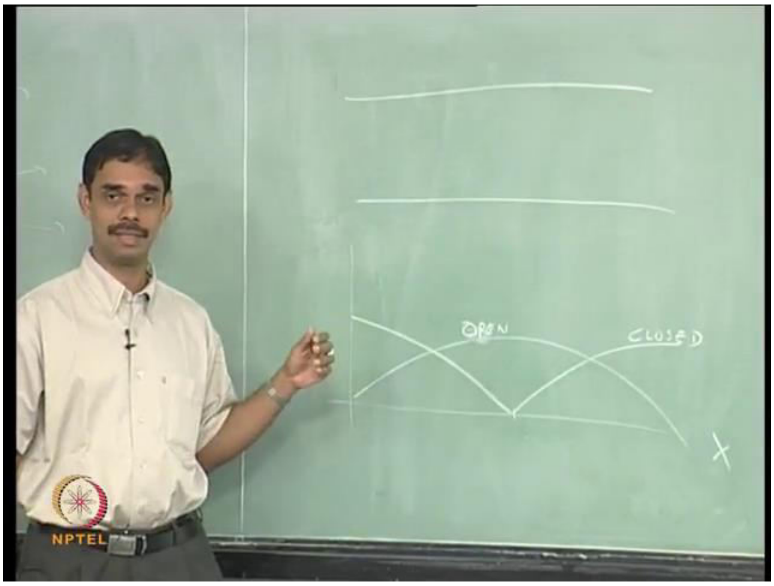

So, that is absolutely right, because if you look at the pressure distribution in a open tube. Let us take the first look the first mode, so will be this, so pressure versus x. And for the closed tube it would be this is open and this is closed. So, in here case you see that the fundamental, you have half a wavelength fitted into the ((Refer Time: 05:50)) only thing is offset by quarter wavelength. So, both case you should get the same answer, so it is perfectly fine that you are in the same answer, somebody answered the question.

Student: ((Refer Time: 06:00)) pressure and the ((Refer Time: 06:04)).

But you are connected to a infinite reservoir.

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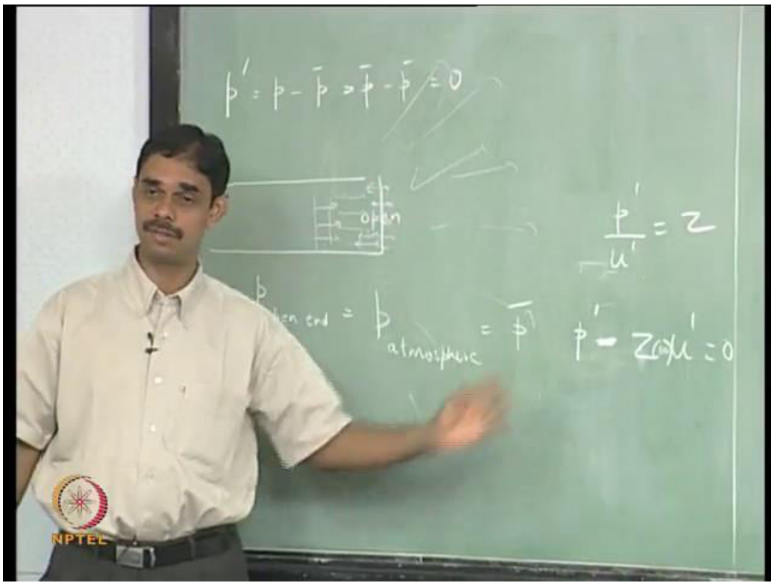

So, if the person is let us say higher here, so the expansion waves are keeping on coming inside and there are trying to make the pressure equalized. So, if this world had been only it would really be 1 atmosphere, but in reality it is complexes, so there is some delay and so on. And it cannot perfectly put 0, but within our theoretical frame, but this is probably the best condition that we can impose.

Well not quite you can some people measure the radiation efficiency and the impedance of the boundary. So, you can actually for every frequency you can measure the impedance and then you can impose a condition u prime minus no p prime over u prime is equal to z or p prime minus z u prime is equal to 0. So, you could impose a boundary condition of this form and z is z of omega plus as I mentioned what is being reflected or what is being lost is a function of frequency.

High frequencies radiate very efficiently which I think from a music instrument, you can clearly see that I mean the base sounds are hard to pickup and hard to how to get it out right. Whereas high frequency it is quite easy and so on. So, high frequency radiates out low frequency dose not radiates out very easily, so low frequency gets reflected by our. So, in any case z the impedance is a function of frequency, the input imposed a boundary condition of this form.

And also the reflection does not exactly happen at the end it maybe little bit inside because you know, when you start propagating as 3D you could think of it as being waves emanating from here. So, there is a l correction for the length of the ((Refer Time: 08:21)) that means the place very implosive boundary condition and also the boundary condition itself. So, you can get better results, but what I taught it is like this simplest form which is still reasonable useful; any other question? So, the you do not have any other questions, I have some questions, so we wrote pressure and velocity as f I mean, in term f and G. So, can you write sorry can you write f and G in terms of pressure and velocity.

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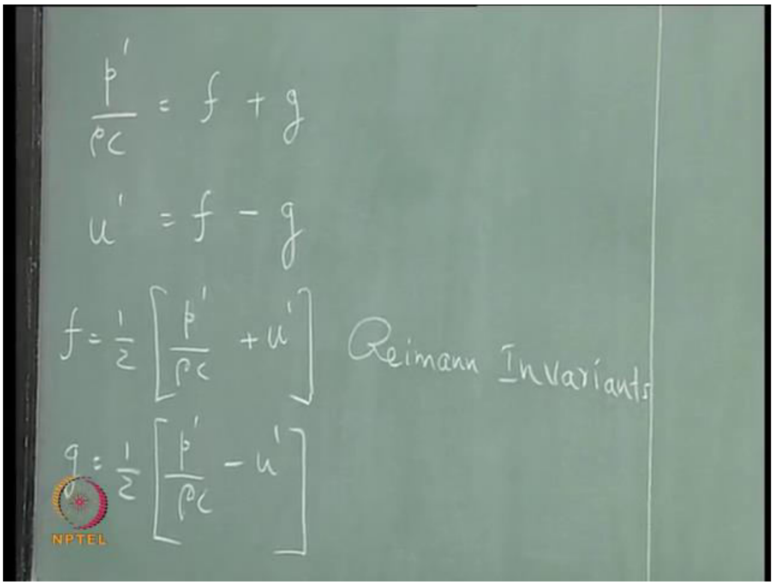

So, let us say, we write it slightly different way u. So, what would be the left and right running wave in terms of pressure and velocity, so f equal to half of p prime over rho c plus all right and G equal to half of p prime over rho c minus u prime. So, these are Reimann invariants, you must have studied Reimann invariants in gas elements. It is like u plus 2 a over gamma minus 1 equal to constant and u minus 2 a gamma over 1 equal to constant.

So, if you make the simplifications that we made for getting the acoustic equations, those Reimann invariant will boil down to this. So, these are Reimann invariants, so when we speak about Reimann invariants in acoustics we are speaking about f and g, but in ((Refer Time: 10:15)) which is ((Refer Time: 10:17)). Now, linear acoustics both are can be used interchangeably, your Reimann invariants are u plus 2 a or gamma minus 1 and u minus 1 u a over gamma minus 1.

So, what are the characteristics in gas elements, corresponding to this invariants. Here, we know the characteristics they are d x over d t equal to plus or minus c corresponding to each other corresponding to f it is plus e and g is minus c. What is the invariant ((Refer Time: 10:53)) in general high amplitude non-linear wave. It is d x over d t's u plus or minus c and there the u stands for the total velocity.

So, when you are having non-linear acoustics, you cannot really write the equation in terms of you could write in terms of fluctuation, but so are the solutions do not add up. So, anyway here d x over d t is the total u plus c. Now, let us, so will at standing waves we will try to make a plot of the standing waves and look at the amplitude of phase. Let us just look at the simple case of the quarter wave tube.

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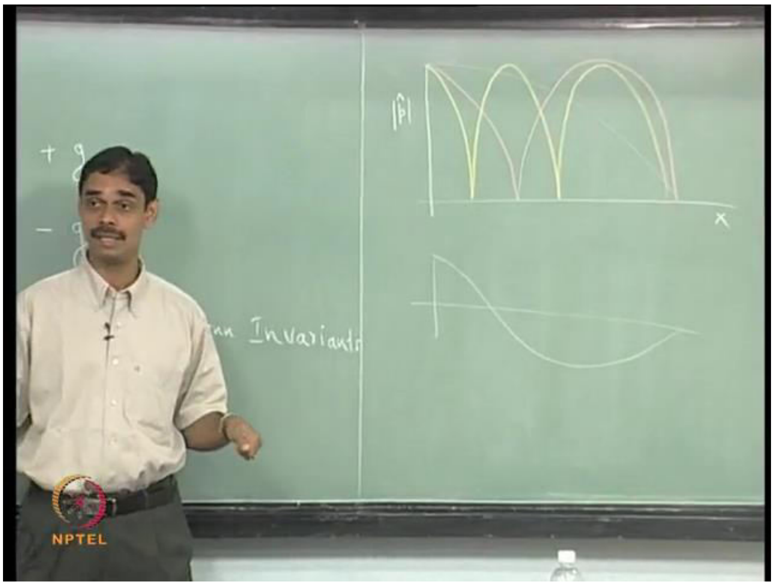

You can do the rest for other waves, so fundamental wave would be this way and maybe the next harmonic may look something like this and then another one would be and so on. And you know the natural frequency c over 4 l 3 c over 4 l or 5 c over 4 l. Now, how would the phase go? So, this is now in those of you studied vibration the way they plot the amplitudes different they actually draw this thing on the negative side also right. The mode shape when this plot the mode shape, they actually draw pictures of the poline form.

They would actually draw things like this, but acoustics we do not do that and there is a historic reason also when you look at vibrations you can see the beam go up and down like this; whereas, in acoustics we really do not see anything. So, what we see is if you plug in a pressure the microphone to measure the acoustic pressure, you see the pressure wave up and down. So, we have a in the computer or any data aquation system what you get is amplitude and phase. So, people plot the amplitude and plot the phase, so people get the feel in a certain different manner.

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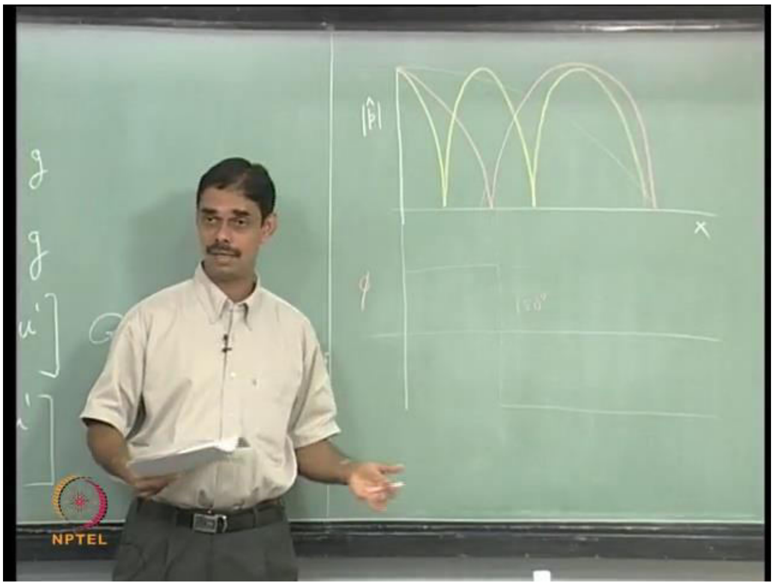

So, if you look at phase, how would the phase go for example, this one this pink colored one, what happens to this phase at a minima. So, I will give the answer for example, if you look at this the phase must stay constant here and then it will change by, so this is phase change by 180 degree across this minima and then again stay constant. And is it possible to imagine this or see it from your solution, what is it. So, phase actually means if you have if you look at this pressure here, now the amplitude at this way, but here the pressure maybe going up here it may be coming down.

So, that would mean a phase differs 180 degrees, if this is going up and that is also going up it is 0, so that means, if it is constant; that means, every part of the wave will be rising in pressure till this point. And then from this point every part of the wave will have pressure fall and in case of a perfect standing wave like in quarter wave tube. It is actually abrupt 180 degree. And can you try to look at the equation and see whether you can get it. So, what was our solution of standing wave for this closed and open pipe.

Student: ((Refer Time: 15:04))

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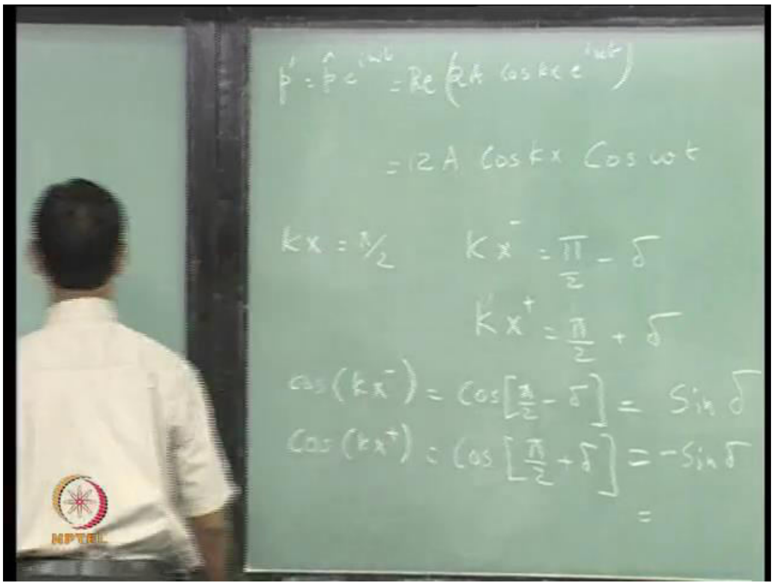

So, lets do it here, p prime equal to p hat e for i omega t which is 2 A cos K x into e for i omega t, but as I mentioned that this implicit or writing this for e i omega t is a fact that we have to take the real part. So, we got a real number for k, so cos K is a real number, so our actual, so let me do this. So, this is like 2 A cos K x cos w t. So, what will be the value of K x at the standing wave minima? So, K x equal to pi over 2, so K x, so we can think of K x minus equal to pi over 2 minus delta and a K x plus as pi over 2 plus delta. So, cos K x minus would be cos pi over 2 minus delta and this would be sorry. And cos K x plus equal to cos pi over 2 delta this is equal to minus sin delta which is nothing but no we will leave it there. Now, so we need to write a cos K x cos omega t, so this would be 1 second.

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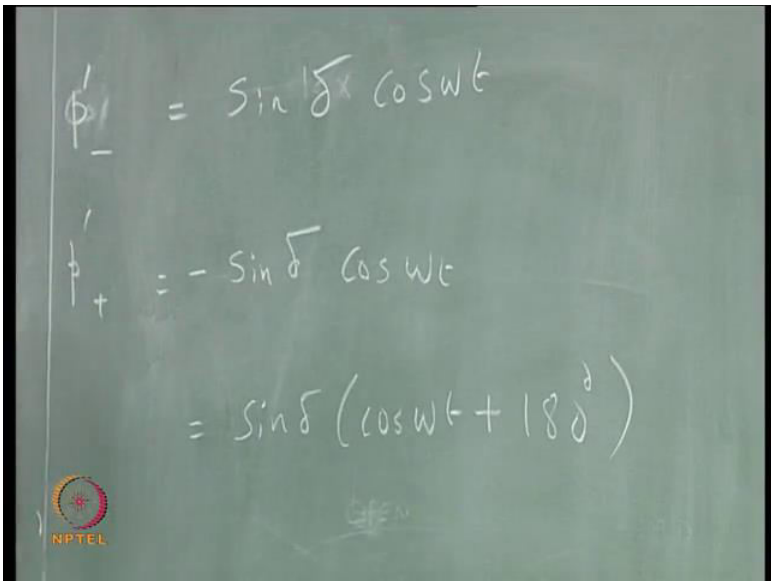

In one case it will be sin K x cos omega t, so this would be the p prime minus would be this p prime plus would be sin delta sorry minus sin delta cos omega t which is sin delta into cos omega t plus 180 degree. So, it is algebraic ((Refer Time: 18:04)) system, because if you think of this place pressure.

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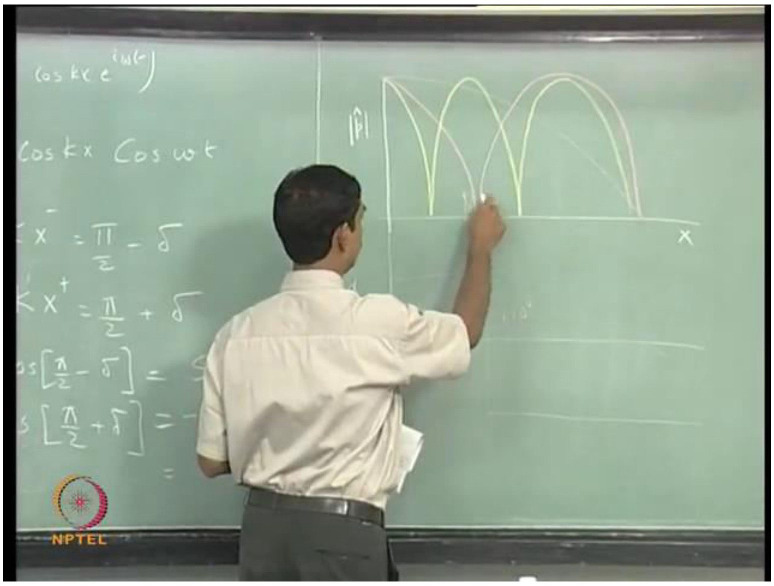

If pressure is going up here, it will be coming down if this is going up that will be coming down, so that is the meaning of phase. Now, maybe we should again harp on what is the meaning of complex pressure amplitude and so on. Are we have you taught about this seriously, why should p hat have a phase? And why should p hat be complex? or what is the implications? What does this complex number denote pressure is of closing real thing, where is it complex thing. Is there anything complex about complex number, is there anything imaginary about imaginary numbers, what is imaginary about imaginary numbers.

Student: ((Refer Time: 18:55))

So, there is nothing imaginary about imaginary number, it is just imaginary numbers are as real as real numbers except that we need 2 quantities.

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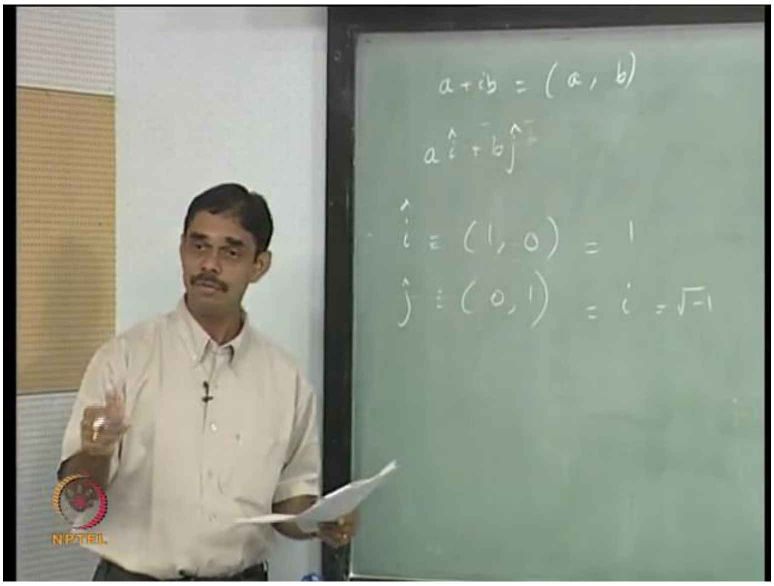

So, if you say a plus i b, it is just 2 numbers a and b, it is like a ordered pair and r 2 space. Just like if you want numbers in the r 1 space, so it is the same kind of notation as what you do when you have a vector notation. So, for example, if you write a i plus b j, we do not think as this is imaginary or complex or anything. We just think that to describe something in a plain, we need 2 coordinates 1 in this direction and 1 in this direction.

So, anything in a plain we can describe it 2 qualities, so instead of calling j as j they call it i is equivalent to 1 comma 0 and your j is equivalent to 0 comma 1. This we normally call 1 this is called the i the imaginary number which is square root of minus 1. So, there is really nothing imaginary about imaginary numbers at all. So, now, the question comes as to if you have measured with a microphone, let us say you bring microphone there are having microphone in this cameras. And what is it measuring it is measuring the real part of p hat or absolute value of p hat or what is it.

Student: variations in ((Refer Time: 20:38))

The other answer

Student: Real part

Anything else

Absolute ((Refer Time: 20:48))

It is none on the above very interesting. So, let us expand out the complex numbers and follow the notation in the book, so last sheet what I wrote.

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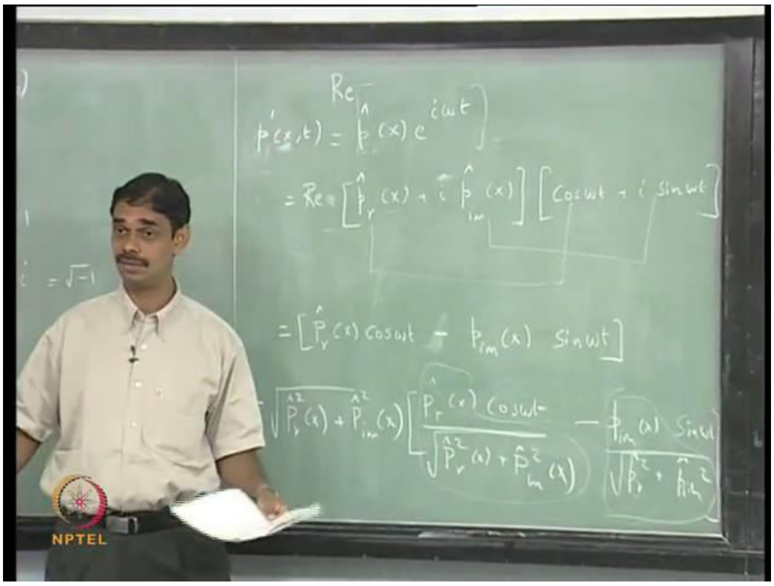

So, p prime that is what we are interested in it, that is what the microphone measures. Every instant the microphone is listening, so what we listening is really not an average value that every instant in time, it is listening only thing it is fixed in space is it that clear. So, p prime is actually p prime of x and t. So, we are measuring it on particular x, if you want to measure several locations you need several microphones that is quite clear. So, x is fixed for a given microphone and t is continuously a variant.

So, I was just put a vigal under it to denote that microphone is fixed in space ((Refer Time: 21:54)). But, in principle you can get the field in field by adding by keeping various microphone. So, this is nothing but by definition we said p hat of x times e power i omega t, so this lets write in terms of the familiar real and imaginary parts and then see what we can get. If this is p hat real of x plus i times p hat imaginary of x times if i omega t is cos omega t i sin omega t, do we need to evaluate this entire expression.

No, we just need to look at the real part of this, so because it is always implicit that we are taking real part of this thing. But, if you are too smart and if you take real part of p hat of then there is a problem, because we see you get a real part by multiplying this and this, but also get the real part by multiplying this and this. Now, is that some mathematical thing which is not physical or is it something physical to it lets take a look.

So, we will write out this expansion, so this is the real part, so you can also write the imaginary part as i times p hat of imaginary x cos omega t plus p real times sin omega t. Now, there is nothing glorious about taking the real part you could have rigged up the subject such that the imaginary part denotes the actual pressure also anything is possible, but the what is followed in convention. So, now we have an imaginary part and what is the imaginary about imaginary thing or can we get some reality out of this imaginary thing, so that is the next step.

So, if you do this high school algebra, we can write this as p real square x plus p imaginary squared x and when I mean by p real squared of x. It actually means p real times, so it just means you take the real part of p r and then square it that is all. So, times p real at of x of omega t divided by the same thing p r squared x plus, if I forget my hat somewhere I am, sorry minus p m x sin omega t divided by the same factor p r y plus p in square plus p hat.

Now, you see these factors over here and this factor over here, what is the unique about that, so if you square this factor and square this factor what do they add up to 1. So, therefore, you can think of this as cos and that as sin or vice versa when you are perfectly legitimate in thinking of one thing as sin and one thing as cos. If there are squares add up to 1 because you have the identity sin square theta plus cos square theta equal to 1.

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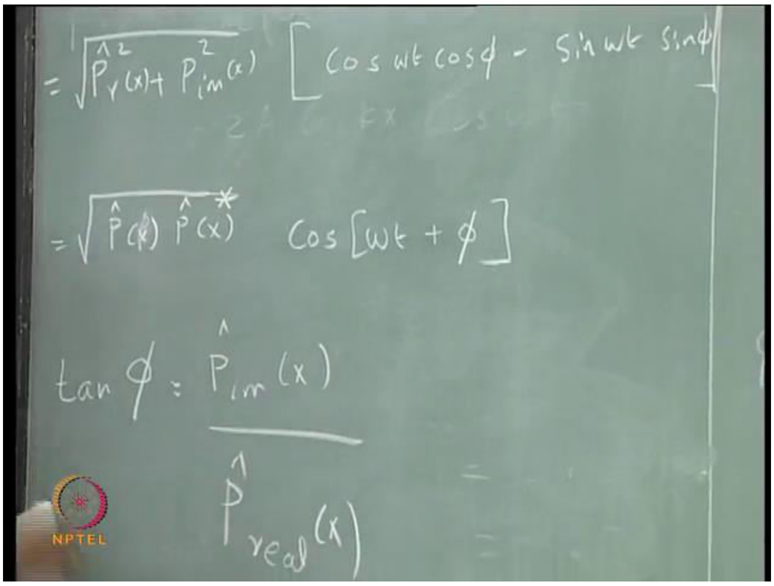

So, this could be root of ((Refer Time: 26:07)) equal to cos omega t cos pi minus sin omega t sin pi, this can be written as root of p hat star denote the complex conjugate. So, this p a complex number to a magnitude is multiplied by it is complex conjugate and you get the square of the magnitude, multiplied by cos omega t plus pi. And pi can be written as tan pi equal to, so we saw that this wave can be written as for, any travelling wave we need a phase.

Because, what does the phase denote we always start our work at sometime t is equal to 0. For example, today's class we started at 8 o'clock and then we put the clock at 0 there. So, when is your reference time when is your reference in 0, so that is what it is. So, here we our daily reference is at mid night 12 o'clock or something like that, so like that we can keep a reference.

And so, without that we cannot write when you describe a wave the function we always need a reference as to when the time starts. So, that is what the pi denotes and the complex notation is just another way of expressing this. So, they are completely equivalent writing something as some A cos omega plus omega t plus pi that is all that is. Is same as writing something as p hat of x e power i omega t of course, implicit in this is, the fact that we have to take real part of this. So, now, is it clear what is phase.

So, this pi is same as the phase of this complex amplitude, so complex amplitude is a complex number and it is phase is same as phase of this wave cos omega t plus pi. Now, one more thing to denote is if you write a computer program it is important to use a command A tan 2 because tan can 2 values and therefore, you can and get into mistake. So, please remember to use a tan 2 or whatever is that command a tan 2, these 2 quantities you do not give the ratio because you can have in 2 quarters the same value, any questions in this, is it clear what is phase now.

So, when the phase changes by 180 degrees; that means, an on place it is some values times cos omega t, the other place it is some value times minus cos omega t it is go up or down by the ((Refer Time: 29:34)), any questions. We talked about aquatics waves and we said that we have particles moving as well as the wave itself moving and there is phase speed and the particle motion. So, there is particle displacement also right, what is how does this relate particle displacement to particle velocity.

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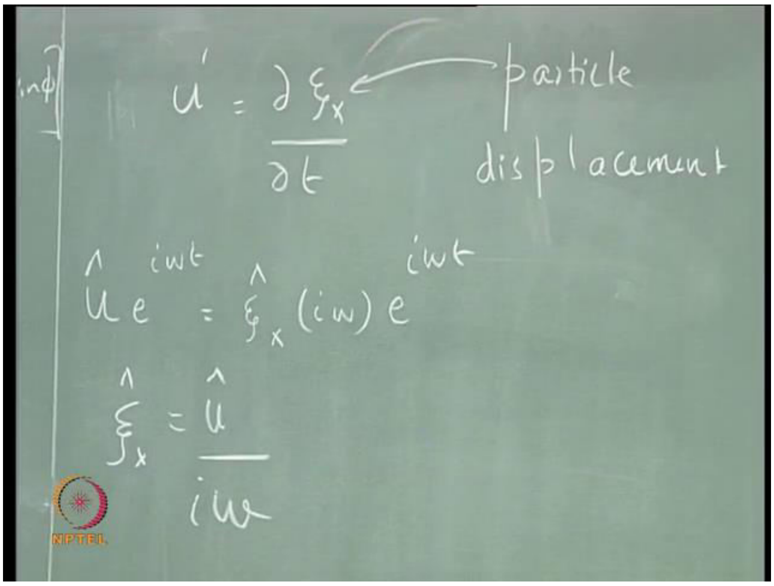

When for example, for harmonic, so lets see what is particle velocity throughout this is particles, so if you now think of this u hat e power i omega t. Let us say harmonic function is, so the particle displacement is goes like particle velocity divided by i omega, so for the higher frequencies for the same aquastic velocity you get a lower particle displacement, any questions.

So, in the last class we said that we need some kind of measure for the energy and if no energy is coming in or no energy is going out this measure or the energy will stay constant. If energy is going out if there is flux going out of the aquatics power. So, aquatics power is radiated out then energy will come down if the aquatics power is coming in energy will move up.

So, we need to write some kind of expression between aquatics energy and aquatic power that is coming in or out through the boundaries. And reality that can be other ways in which aquatics power can be generated. You can have aquatics power being a generated by chemical reaction that is the combustion stability or you can have aerodynamic ways of generating it like a what extending and so on.

So, there is aerodynamic sound, we will come to those things later, but we will look at just the aquatics power flowing in or flowing out through the boundaries. How do we do this? So, we derive a new equation or how do you start.

Student: to derive ((Refer Time: 33:07))

Homo ((Refer Time: 33:09)) equation

Student: You can get energy ((Refer Time: 33:11))

So, we need some kind of the expression for the energy and I told yesterday that if you think of it as a dynamical system. Then it is norm of system it should go like energy 2 norm, but if you do not want to think of that that way also it is fine. So, what we can do is we can we need to come up with a expression for energy. And when we it should be how to come up with is it is quite easy for aquatics this equation, but anything more the more complex in this you bring in even a simple mean flow bringing in. This will complicate the equation means quite a long and we will see that later.

But, we need 2 expressions one part should correspond to what are the energies in it. Energy is like a measure of whatever is there in the system and if you do not take away or put in energy it should stay constant that is the idea. So, what are the 2 components of aquatics energy?

Student: Kinetic Energy

Kinetic energy and

Student: potential energies

Potential energies; so, we should have those 2 things kinetic energy should go like u squared and potential energy should go like p squared. So, we have to keep these things in mind and try to derive it, so let us do that.

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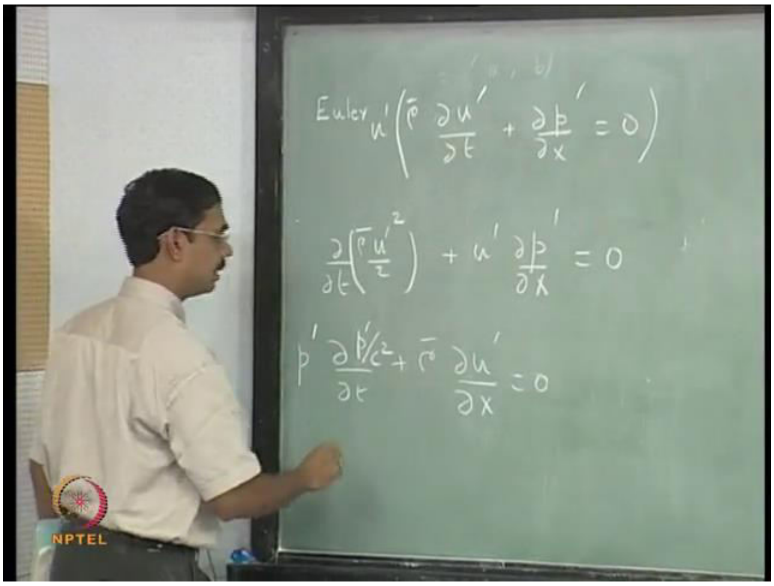

As Rajesh pointed out, Euler equation or the momentum of equation here we are always writing the linear rights version. So, this would be rho bar do u by d t plus dou p by dou x will be 0 and I must remember to put my primes, but in case I forget I think I should ok. So, we multiply this with u prime, so if it was like a vector equation as Rajesh mentioned to take a dot product with you. If not I mean here it is a means we are just writing on components like a scalar equations, so I just multiply by u prime.

So, this would be and let me, now let me write the equation for continuity equation which is dou rho prime by dou t plus rho bar dou u prime by dou x is equal to 0. So, what we do is to we had our relationship between, the density fluctuations and the pressure fluctuations. What was it?

Student: that replied this by p prime or c square.

So, that replied this by p prime over c square right is this. So, now, let us multiply this by p pi.

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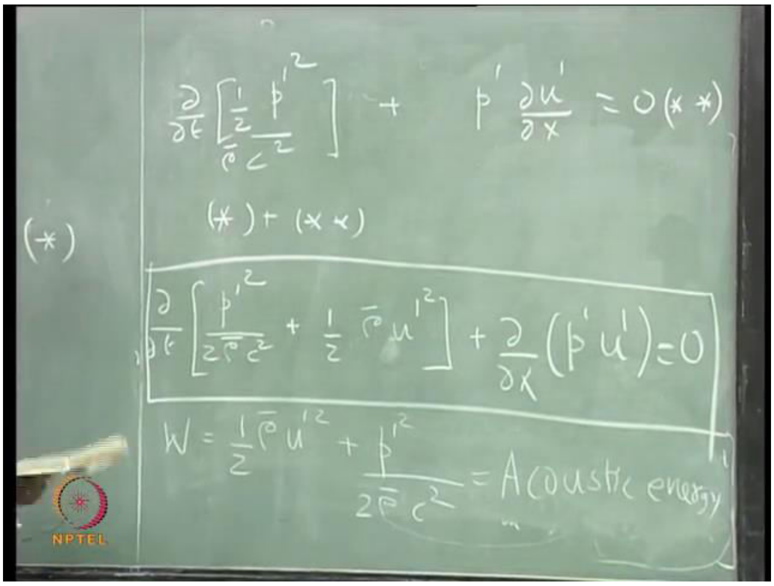

So, what you would get is dou dou t of half p prime square over c square plus rho bar p prime dou x is equal to 0. So, let me just bring this rho bar to the bottom here, so now, let me add these 2 terms. So, let me call this equation as star and call this as star star. So, I can add star plus add these 2 equations, we will get dou by dou t of p prime squared over 2 rho bar c square plus half rho bar u prime square plus I will get p prime d u prime by d x plus u prime dou p prime dou x equal to 0.

Now, this term can be recast as d by d x of p prime u prime. So, that p prime u prime equal to 0, but this is now looking quite physical. So, what would this already we this is like a measure or energy. In fact, a half rho bar u prime square, if you multiply it by like a d v delta volume that would be like kinetic energy of I mean a kinetic energy of that certain mass of gas right. Like, half m v square you just have to multiply a rho bar and some volume then.

So, let us call w as half rho bar u prime squared plus p prime square over 2 rho bar p square. So, this would be aquatics energy it is a result of by definition. Now, this would mean that what is the physical meaning of this, so there is some quantity here and it is time rate of change depends on p prime u prime it is derivative. Now, we can get a more secured meaning for this if you actually integrate this equation and use what is Gauss theorem and write it. Now, so let us first write this vector form let me call this as that as w the aquatics energy.

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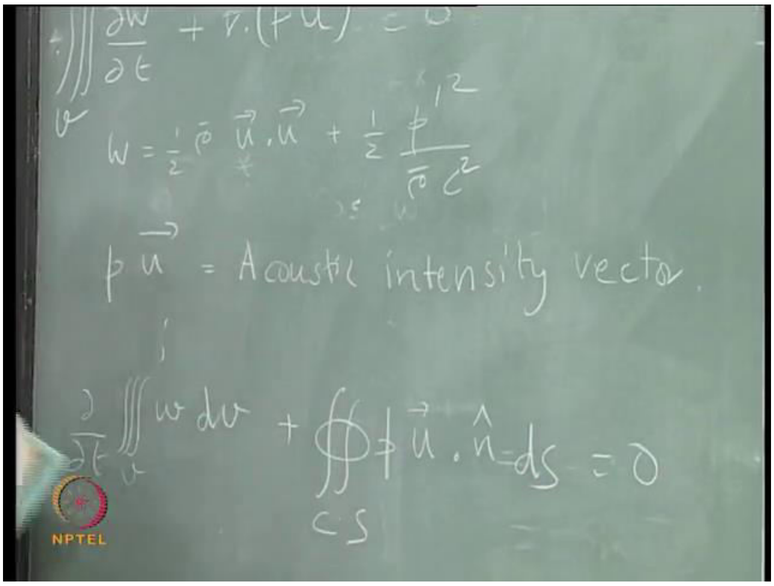

So, then in the vector form this will be dou w by dou t plus del dot p u can use the vector equal to 0, where w would then be half rho bar u dot u which is the kinetic energy plus half p prime square over rho bar c square. And p u is called acoustic intensity vector. Now, if I were to integrate this over the control volume, so let us integrate this over a control volume. And let us assume that the control volume is really not distorting or anything it is just a fixed control volume. Then it easy to do integration even otherwise you get the same results, but I will keep it simple.

So, you can also swap the derivatives and say, and what happens to this integral this can be converted from a volume integral to surface integral by graph. C S means control surface equal to 0. Now, there is some physical thing to it p u is actually called acoustic intensity, an acoustic intensity as over a surface would be acoustic power.

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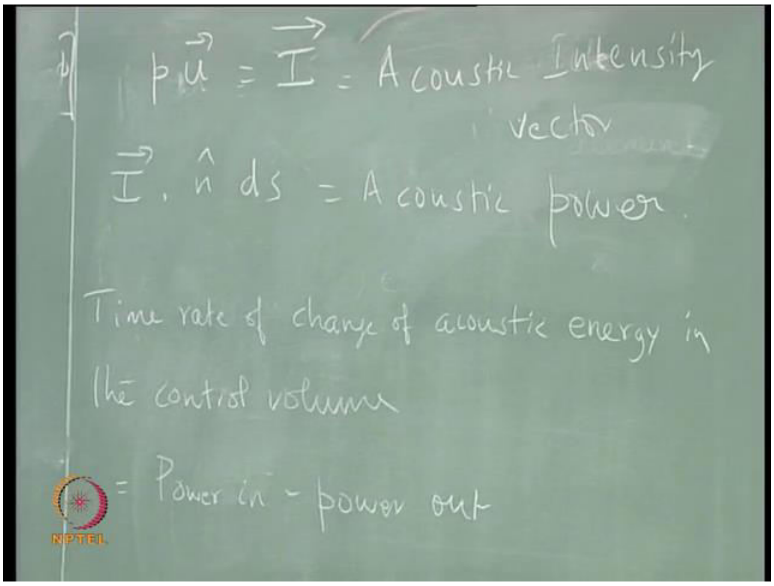

I will pass for a minute to rho. So, the this equation means that the time rate of change of acoustic energy is inside the control volume is equal to the intensity flux in minus intensity flux of across the control surface or the power in minus power out power flow in minus power flow out. That is what it is physically mixed, so the time rate of change of acoustic energy that is here, this plus the power in minus power out be 0 or should be conserved.

This is the conservation of acoustic energy is the physical meaning quite clear. I think you can note this down, I will write it the net power in. And I also want to draw your attention to note that this acoustic intensity is the vector, but power is like a scalar, because it is intensity you are say I dot n d s that power. So, one key thing to notice is that we did not evoke any new conservational loss, this was consistent to the conservational loss which we already use, which are conservation of mass and momentum and the isotropic equation that we used p prime over c square equal to rho prime, we did not use anything here.

We manipulated them and the way it was caved it was such that see we could have multiplied by any other constant also, but we wanted this p prime d u prime by d x and u prime d u by d x to come into a flux kind of form. So, we wanted like a del dot p prime u prime term, then you can re-write in terms of the surface integral that was the recent which the scale is done and then we integrated it to get the equation in this form. So, this if you remember the equations the way I wrote it in the state vector of form this would like the norm of the state vector that available.

And what else, so if this was kind of not, so difficult exercise of trivial exercise for acoustic energy in the absence of mean flow in the absence of any homo-genetic and so on. But, the moment there is flow then I mean the homo-genetic it is exercise gets really complicated and there if there is no. The 3 waves which are coupled together acoustic entropy and would see and they are completely coupled if there is no flow and everything is constant, but the moment there is flow of things to the couple.

And even so there is quite a bit of debate on what is the expression we have produced for the acoustic energy. And it is a still ongoing research topic towards end of the class we will have some look get it, but the moment I wish to say that it is really complex and deep equation. And one more warning I should say when we write pressure and velocity we write in depth of complex quantities, and there first order quantities, but acoustic energies is a second order quantity.

So, we should not write that in terms of quantity like quantities you will be much better of taking the u prime and p prime as it is and writing expression for that rather than dealing with the complex numbers. Because, you know you have square qualities and then everything is messed up. So, I think I will stop here for today there any questions you may ask any questions?

Student: Actually the ((Refer Time: 47:23)) about the harmonic ((Refer Time: 47:29)).

We found that along the characteristics d x over d t equal to plus c, the pressure would be constant. So, something that is constant in variant there is another word for it. Now, why is it, so that is a characteristics of wave if you think the 2 waves you can think of, if you think of it mathematically we have a hyperbolic differential equation and it will have characteristic lines along which certain invariants exists. That the property of the hyperbolic equation or you can hyperbolic.

If it is gives you a direction along which something will stay invariant that is the correct full matter of everywhere p d I, I think you can look mathematics book, but physically what does this invariant mean. If you have wave a vigor and if there is a wave going to the right I mean any vigor which will transport and. So, it will go right or left or whatever. So, when it is moving a left running wave or right running wave it will just preserve the wave ship and keep on going in a 1 dimensional sets. If it is 3 dimensional then of course, you will have this 1 over r d K and so on.

So, but a wave it if it is propagating this side it will just keep something like this it would not change at all. Same wave is moving to this side it will just have whatever is the vigor and it will move to this side. So, what is invariant itself is a invariant, so whatever is moving to the left will continue to move to left it is only just translated. It does not get expanded or change in amplitude or anything under coaxial conditions things may be different when temperature is non-uniform as if pointed out.

So, the this the way the whole structure is invariant, if for the left running and right running. And that is the I mean there is nothing which can make it spread for example, if you had non-uniform temperature. The front part of the wave if you are going into a, which the higher temperature. The front part of the wave can travel faster than the back part and that will run away.

So, it is relaxed expand and if you are going to core medium the back part of the wave will catch up with the front part. So, the wave will ((Refer Time: 49:43)). In fact, if you have a finite amplitude for example, not the acoustic wave, but like a like a higher amplitude. Then even for a uniform medium uniform temperature as the front part goes it is like a shock it is heats up compression will heat up. And the back part will tend to catch up because it is seen a higher propagation velocity and eventually the format shock.

So, then once you have non-linear acoustics you do not have invariants property at all. The wave can change the shape, but when you are corrosion medium and there is no in homogeneity and so on. Then there is nothing in it which can actually disturb the shape of the wave any disturbance or pressure or velocity will propagate as expansion wave compression wave isotropic waves left and right.

So, that is what it is physically mathematically you are guaranteed a solution and if you think about non-linear acoustics with higher amplitude and so on. You still have a hyperbolic definition you still have a characteristic d x o d x u plus or minus c. And there also you have invariant 2 a or dou minus 1 plus u a is constant, but it is you can not write in terms in simple pressure waves and velocity waves, because the pressure as I mentioned a pressure wave can consists of en-velocity you can step and or relax or, but this full thing is invariant. Any other questions did I answer it we will stop here.