Acoustic Instabilities in Aerospace Propulsion

Prof. R I Sujith

Department of Aerospace Engineering

Indian Institute of Technology, Madras

Lecture - 40

Response of a Diffusion Flame to Acoustic Oscillations-2

Good morning everybody, we are now studying the response of non prime explains to flow perturbation. We already studied prime explain response now we are studying this whatever non diffusion flame we are kind of looking at the classical diffusion frame and trying to follow the analytical approach, except the difference is that we are looking at the unsteady equation.

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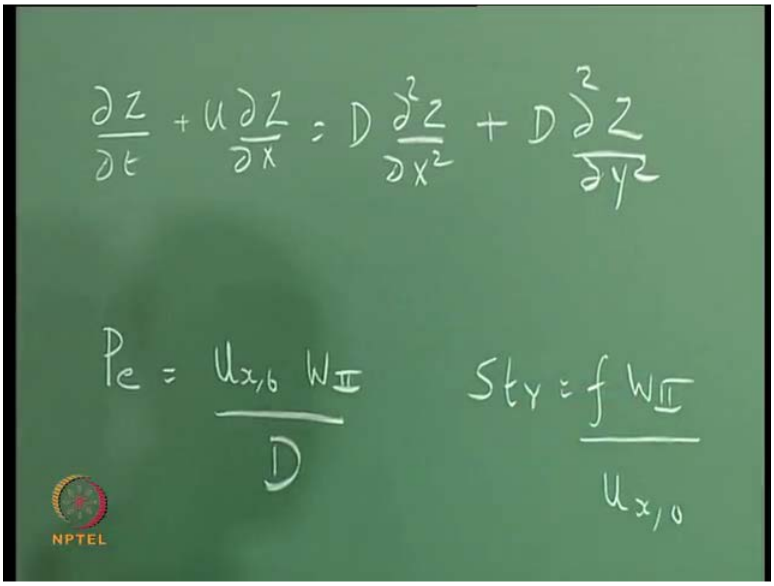

The equation that we are trying to solve is the advection diffusion equation which is and to obtain the steady part, we have to drop the first term and we have to get a steady solution or the stead base flow. Then on top of it we can linearise this equation and get a perturbation equation of course the equation is linear in Z. So, it is we do not have to drop anything, but the thing is if you separate in to steady and steady base flow and a perturbation equation, we can get neat analytical solution.

So, that is the big advantage and before you continue we emphasize that we identified 2 time scales or 2 non dimensional numbers, associated with this thing one is peclyn number which is U x, 0 W 2 over D. So, this is kind of like Reynolds number convection over diffusion, but here we are somebody deals with momentum transport here we are looking at mass transport similarly we have ((Refer Slide Time: 01:58)) number. So, W 2 over U x naught denotes one time scale, 1 over f denotes another time scale, so this is like a time scale of the flow, 1 over f is like the time scale associated with the unsteadiness and, so that is the other.

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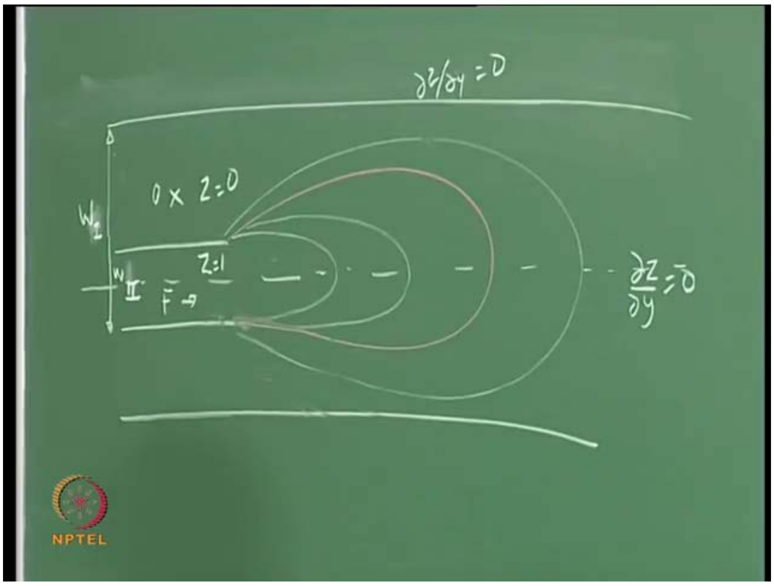

So, we say that, here in this inner block is that is one here is that 0 sulekha and then we have a center line, so we have to have symmetry. So, let us write this is Z equal to 1, here Z equal to 0, here and at the center line dou is said by dau it should be 0 also and at infinity, we should tend to some finite number and solution should not blow up and so on. So, this is what we need to solve I hope this much is clear to everybody, from last time are there any question have pass 1 minute to answer any questions.

So, our objective is, now 2 hold first is to get the solution for the steady base flow and once we do that, we will try to get the solutions for the unsteady part and once. So, here our objective is when we studied the G equation and the flame explains formulation we could directly track the flame. But, then the value of G itself was really of no major physical significance in that particular formulation here, whereas Z is a very physical quantity it is x minus y and x and y are very physical quantities dealing with the mass fraction of fuel and oxidizer. So, we are actually solving for the entire Z field and then we can track the flame which is happening, yet this Z stoichiometry. So, what wherever is the stoichiometry we call that the flame sheet and, so once we put that in and then we get a equation for the flame surface that match is clear.

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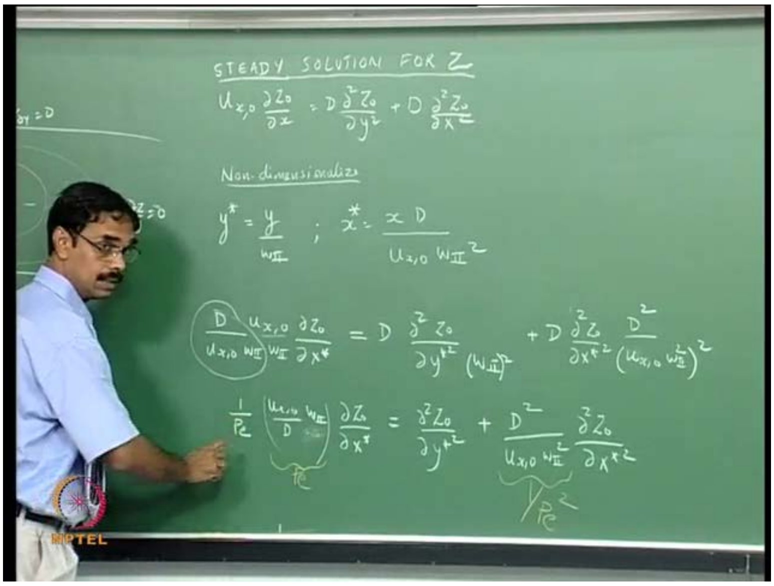

So, let us start first with the steady part governing equation was, so we are solving for steady solution, so following low evince convection, 0 means base flow or steady flow. So, let us do some kind of a non dieselization to keep things simple, so that the solution were come out nicely, so rather than copy the solution please work out the problem along with me and let me know than any mistakes.

So, we non dimensionalize y with this slot width W 2 and x with x, we have non dimensionalizing with this, now if you look we will get, 0, so x y, so you should get I hope this matches straight forward. So, we can club what was our definition of peclyn number it is U x naught W 2, so that was the W 2 squared give it, so not W 2 over D. So, this would be 1, 1 over peclyn number, peclyn number, so we are left with there is U and W 2, so I divide throughout by D, so how much you know over D there is a W 2.

Here, I multiplied throughout by W 2 squared, so this will come to talk, this is also this is like peclyn number, so I have divided by W, divided by D and multiplied by W 2. So, all I have, here is dau squared by dau by tau squared plus I took out 1 D, so I have t squared left and I remove W 2 squared, so I have, so this is it.

Student: Peclyn number.

Peclyn number and this is.

Student: 1 over P squared.

1 over?

Student: 1 over P squared.

P squared, so this should cancel with this, this and this are, so canceled.

Student: 1 by peclyn square into.

Where, here.

Student: Last one, last one, last one.

Sorry, I.

Student: That should.

Yes, now there is U x naught squared, here thank you, so this will cancel you have 1 by peclyn square, here this has a coefficient of 1, so we get a very neat equation.

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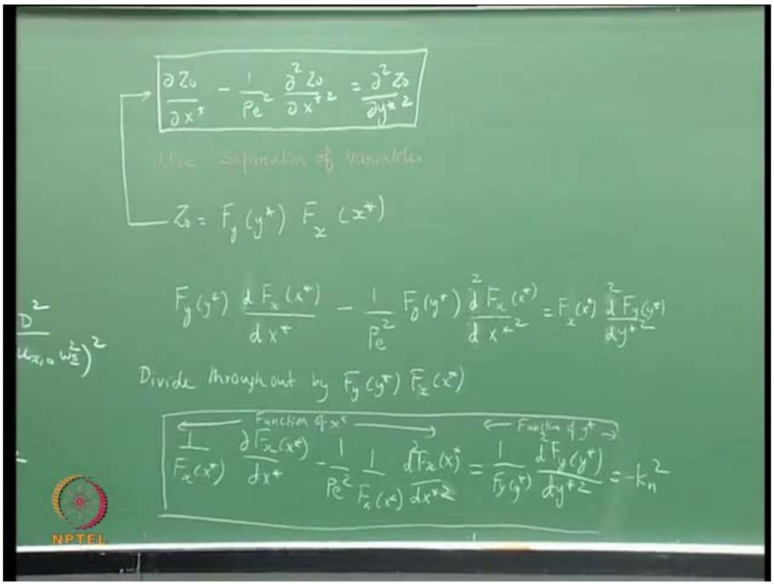

So, although for a moment the equation look like it was going to be a mess, but I think if you are doing right all things will combine together and cancel, now we need to solve for this. How would you solve for this use the meaning of separation variables is when you have a function of, here everything on the left hand side is a function of x, everything on the right hand side is a function of y. So, if we can manage to write it that way, then each of them should separately be equal to constant, so as a good chance that this would work and we should be able to write left, at the moment we do not know what is Z naught.

But, if we say that Z naught equal to F y of y star into F x, x star and substitute it, here what you would get is F y of y star dau squared F x of x star divided by, no dau x star minus 1 over peclyn squared F y of y star, dau squared Z naught by equal to F x of x star into dau squared F y of, I hope this is right please let me know. So, these are kind of the typical equations you would encounter and studying combusting, so it is good that we are going to x and separation of variables is a very simple solution technique.

But, you that many instances we can use it, so it is a good practice for you, so what we do next is to divide throughout by F y and F of x divide throughout by star. Then we should able to separate out or if we do, then we have success if we do not separation of variables is not working. So, if we divide this what you will get is one over F of x, x star actually I can write D instead of dau because each is a function of only that variables, so I do not need to write partial derivative it is simple derivative, so this particular path.

Student: D square affects.

Yes right, yes thank you, yes, so the left hand side yes, so the left hand side is a function of x this part x star actually and this is a function of y star. So, if a function of x star equal to function of y star then both function should be equal to a constant. So, we then say this is equal to minus K n squared, minus sign is simply for convenience there is no need to put it, but I mean just style I mean you can put the other one and you will get the same result.

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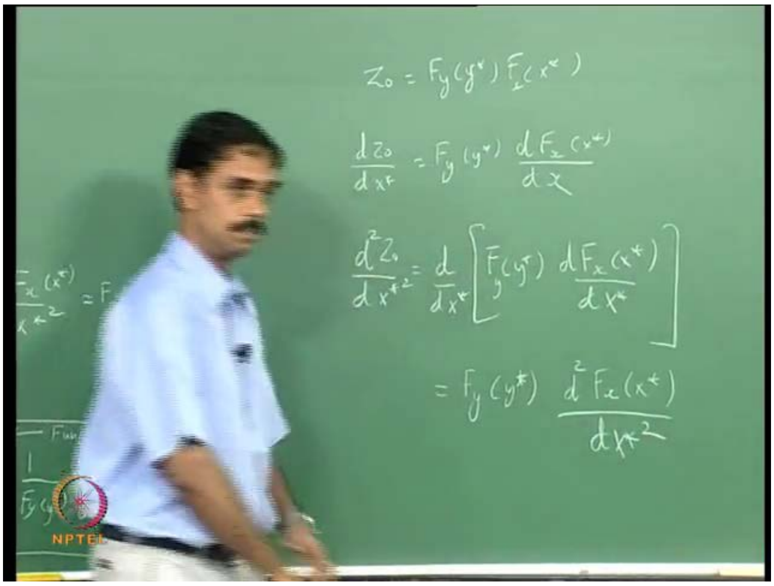

Here, Z naught equal to F y of y star F x of x star, so let us clarify this, so d z naught by d x star, x star equal to F y of y star d F x by d x d squared Z naught by d x star squared equal to d by d x star of F of y, y star d F x, x star over d x. So, as for as differentiation with x is concerned this is a constant, so it just comes out this is equal to F y of y star d square F x of x star over star square, so I hope it is clear.

Student: Sir is it same as sir K n, is it wave number or simply a constant.

It is a constant, we can it is some kind of wave number, but it is not the acoustic wave number, but it some other kind of wave number it is like a of course, here it just constant that is appearing. But, it is a Eigen value basically, so we will have to write this, so lot of K n can satisfied, so based on the boundary condition we have to sought out or find out what K n is and then we will.

So, if there are several K n's then this is a linear solution, so will have to write the function as summation of all this functions over all the K n's that are possible. So, if you write minus K n squared, so if you bring it over, here you will get these squared F over d y squared plus K n squared, F equal to 0. So, that will say that F is like sine, suppose I did not put minus then I would get exponential, but sine cosine, an exponential there interchangeable.

So, it depending on what actual boundary condition you have put that decides the sign of this, but here knowing since I have worked it out I know the answer, so I know that I will be a minus time some positive number. So, in some sense you work through on way even if it is a wrong thing at the end of it you know that you can swap it. But, then to keep things nicely you do this, so that is the advantages of doing things second time, so let us take a look at this part the right hand the right side of this equation.

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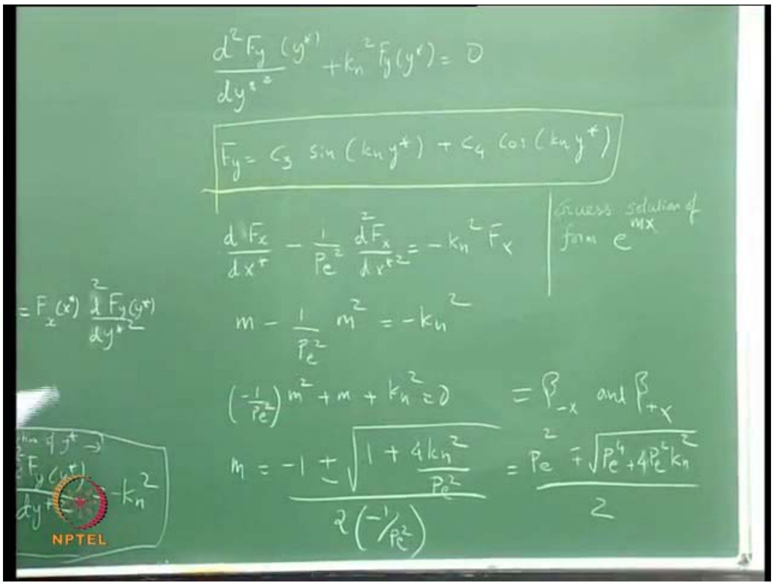

I have received C one and C 2 for x, but since y is simpler I am writing that first that is kind of box of the solution, so we do not know the constants we do not know the K n's. But, there may be many K n's that satisfy and then we will we will have to add all the solutions because in a linear theory all the solution can be superposed. But, what you superpose depends on the boundary condition that is someone substance what you asked, our next part would be d squared F of x, x star, sorry minus 1 over P e squared d F x.

So, it is all function of x star, but I think by now it is clear, so this is our function of x star part I would you solve this, I think we can guess that the solution is of the order of e power m x. So, let us guess solution of form e power m x, so then what would be get m minus 1 over P squared I made a mistake, here this is yes equal to minus K n square, so just writing it as nice quadratic form. So, m equal to minus 1 plus or minus root of P e squared minus 4 C, so 1 plus 4 K n square over P e square divided by I hope this is right.

So this we must be able to write this as P e squared minus or plus root of P e power 4 plus 4 P e square. So, let us call the, let us call this equal to beta, beta minus x and beta plus x, so P plus square root of P power 4 plus 4 P squared K n squared, we called that beta x plus. When we say P minus square root of P power 4 plus 4 P squared K n squared over 2 that we call it beta minus, so then we can write the solution as.

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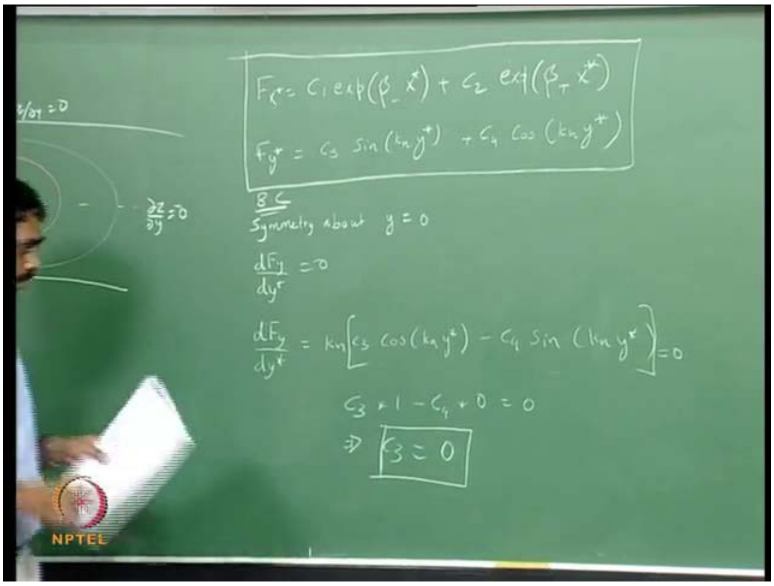

So, this is our solution and our Z is if we take product of F x and F y, so if you take the product of these 2 we should be able to get the solution. Now, we have to apply the boundary conditions and we will apply symmetry about y equal to 0, so that will say that d F y over d x should be 0 because the slope should be 0. Only then you will have a symmetry part, if you have a profile which comes like this which is symmetric then the slope should be 0 here. So, what would this give, so let us differentiate this will give K n C 3 cos K n y star minus C 4 sine K n y star, so if you put y equal this should be equal to 0, so if you put y equal to 0, cos 0 will be 1. So, in this K, so C 3 into 1 minus C 4 into 0 equal to 0, which will give C 3 equal to 0 fantastic, so that simplified our solution quite a lot.

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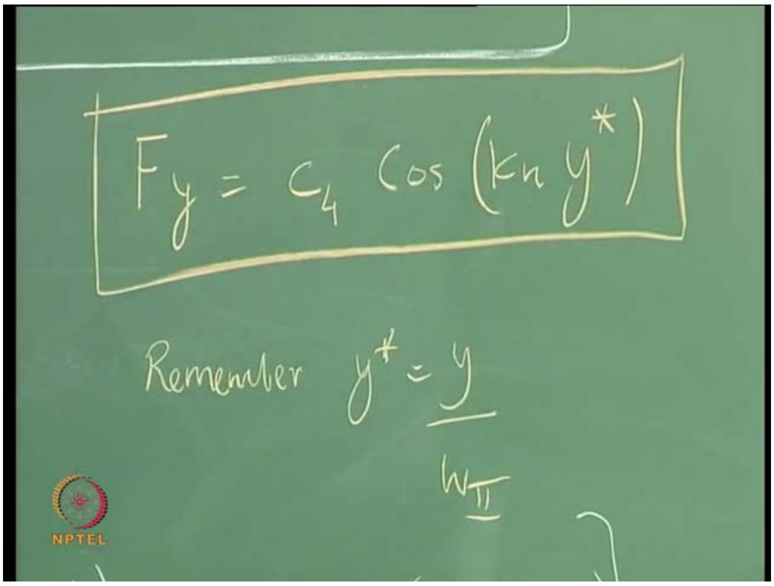

So, we can write here, now F y equal to C 4 cos K n y star our solution is in terms of y star that means what was y star.

Student: y y W 2

y y W 2, so we have to remember, so next boundary condition is we have this is wall they should not be any diffusion through the wall so D is over d y should be 0 here.

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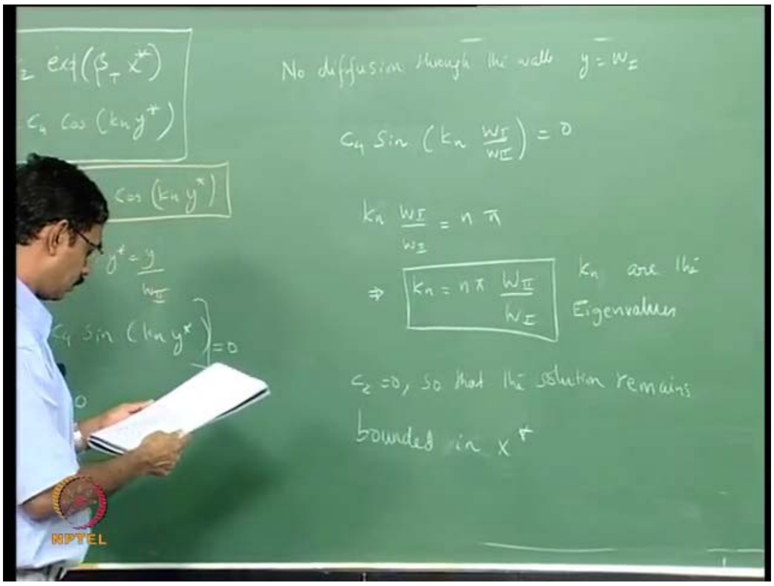

So, wall is, so once we enforce W 1 y equal to W 1 y equal to minus W 1 will automatically be enforced because we already enforce this symmetry. So, this is the solution C 4 cos K n into y star, y star was y by W 2, this would be W over W 2 equal to 0. So, this should give K n, sorry it is a sine because in the condition is on D is said by d y, so when this is y F y, d F y by d y will be like sine n by n by, so this is the answer to what you are asked what are those K ns. Now, they are being determine these are the only K ns that is n pi times W 2 or W 1, these are the only K ns that can satisfy the boundary condition.

So, the solution will consist of several K n 1 will be pi times W 2 by W 1, second would be 2 pi times W 2 by W 1 and so on so forth. Now, they will all have a cohesion in front that will have to be determining from the next boundary condition, yes I think natural frequency is a very specific term used for wave equations solution to that. So, this would be Eigen value that is the more general terms, so K ns are the Eigen values Eigen comes from the germen word from own, once own or unique something like that. So, it is very similar to natural frequencies, but we are not solving a string vibration problem, so it is not exactly natural frequency.

But, something like that actually, now we have to look at let us take a look at F x star, so what happens to this term as we go down straight this C 2 e x p beta plus x star beta plus means that is a positive number. That is all it means it will keep on increasing unbounded we cannot let that happen or we cannot this templar, so C 2 is 0. So, let us write this C 2 equals to 0, so that in the solution remains bounded in x star, so we just have this term multiplying this term C 1 e power beta minus x times C for cos K n y star.

So, since K n takes infinite values that is there is no upper bound on what is n and all that, so the solution to Z naught is an infinite sum over all integer values of n with cohesions some kind of cohesions which are like this C 1 time C 4. So, these cohesions and we are adding a n number of them, so they have be solved using the boundary condition at the inlet plane, so it is clear, so how do you do this.

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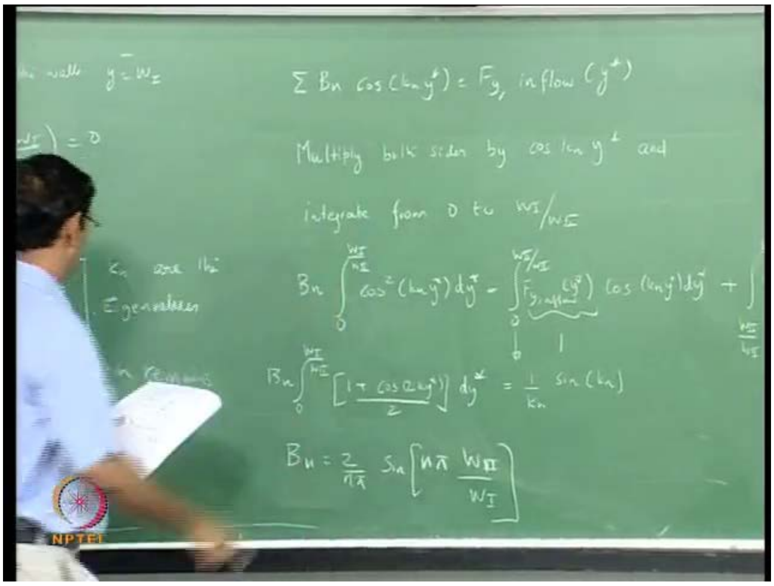

So, e power 0 would be 1, so we put our x equal to 0 here, so our solution is equal to F y comma in flow y star, so let us multiply both sides by cos K n y star and integrate. So, this would be 1, this would be 0, so we can drop that, so you should get we look at the right hand side. So, this would be 1 over K n, sine K n is that let me know if this is not working it, so on the left hand side we will do this integration. So, you will get B n times integral 0 to W 1 over W 2, this to integrate we will say it is 1 plus cos 2 K by star over 2 d y star.

So, just to write the final result you will get, so this term becomes half this will become sine, so and then you can do the further simplification get this just write down and use this value of K n. So, this term will drop and you will get this, so now we need to get B naught, so B naught we just we do not multiply anything we just multiply we just integrate that equation.

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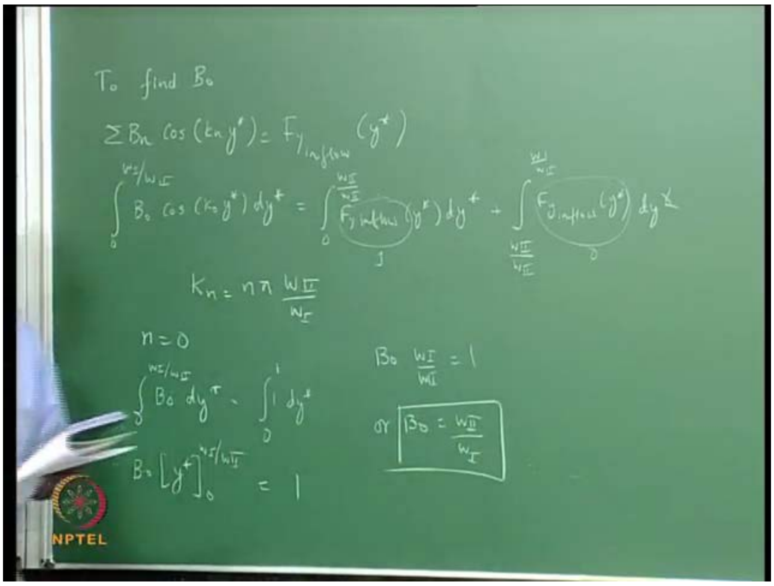

So, we have to find B naught, so this is our equation, so we just integrate this W by 2 by W 2 is nothing but 1 and, so we know that K n equal to n pi W 2 by W 1, so we use n equal to 0 that is what is of interest. So, 0 2 W one over W 2 b naught d y star equal 2 integrate 0 to 1 because this is W 2 by W 2 and this would be 1, here this thing has a value 1 into fuel slot, this has a value 0 for Z. So, you will get 1 into d y star, so B naught y star 0 to W 1 over W 2 equal to 1, so y star is the upper limit is W 1 over W 2 lower limit is 0, so you will get B naught W 1 over W 2 equal to 1 or b naught equal to W 2 over W 1.

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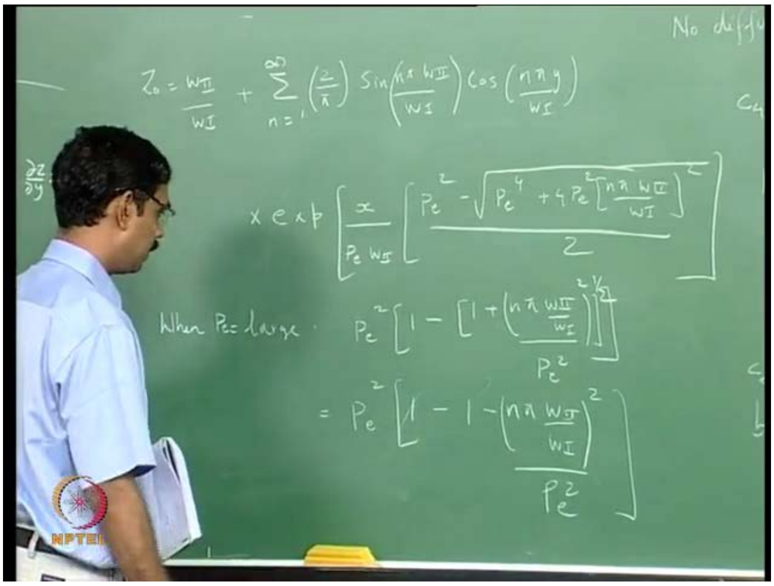

So, will write the solution, this is what I should get we are taking the beta minus solution, so this is the full solution will fix that that part next class. Now, we want to look at what happens when peclyn number is large, I will just wait for you took look at this I hope this is alright then we can simplify this expression. So, this term inside the bracket it will become P e square into 1 minus 1 plus n pi W 2 over W 1 whole square divided by P e square over half. So, this would be P e squared into 1 minus 1 minus, so when peclyn number is large we can simplify it as this particular term therefore our full solution.

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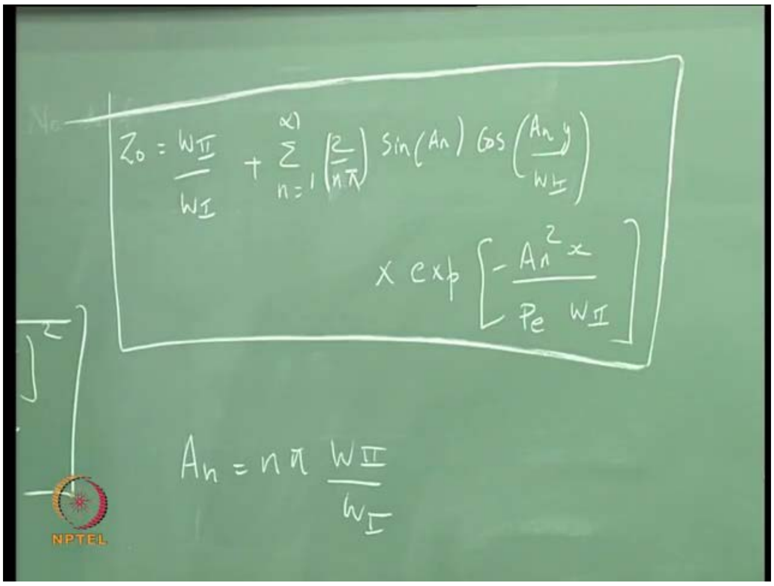

So, this is the solution for the steady part of the Z function.

Student: Second one, 2 by n pi.

Yes, thank you and remember A n is equal to n pi, so wherever this term comes we replace that by A n just force being simple. Thank you.

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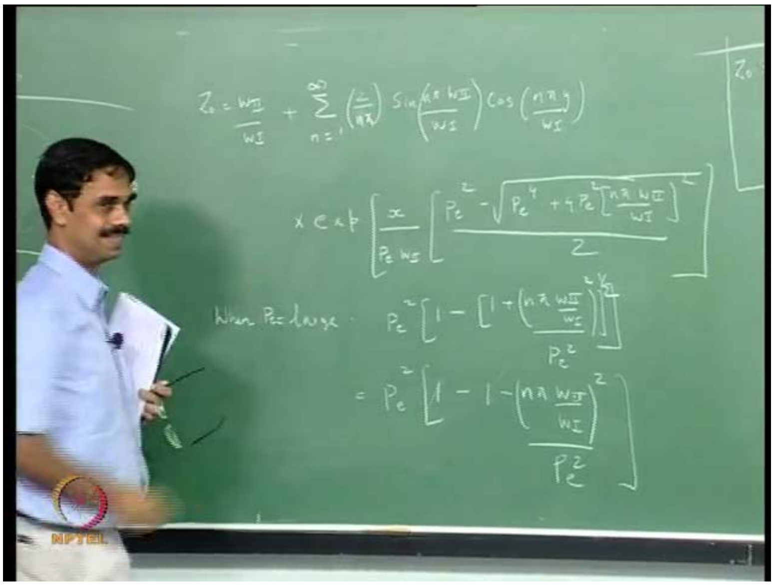

So, in the next part what we have to do is we take the perturbation equation, then we force that equation with a velocity fluctuation which close like cos omega t or sine omega d. Then we have to write the solution for that for a given U prime equal to some epsilon cos omega t, so what we have to do is to split the solution into a homogenous solution and particular solutions find each of them. Then add them and find the cohesion using the boundary condition, so that is we will do on next class, we will stop here for time being.

Thank you.