Acoustic Instabilities in Aerospace Propulsion

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Lecture - 39
Response of a Diffusion Flame to Acoustic Oscillation-1

Good morning everybody, we will study today.

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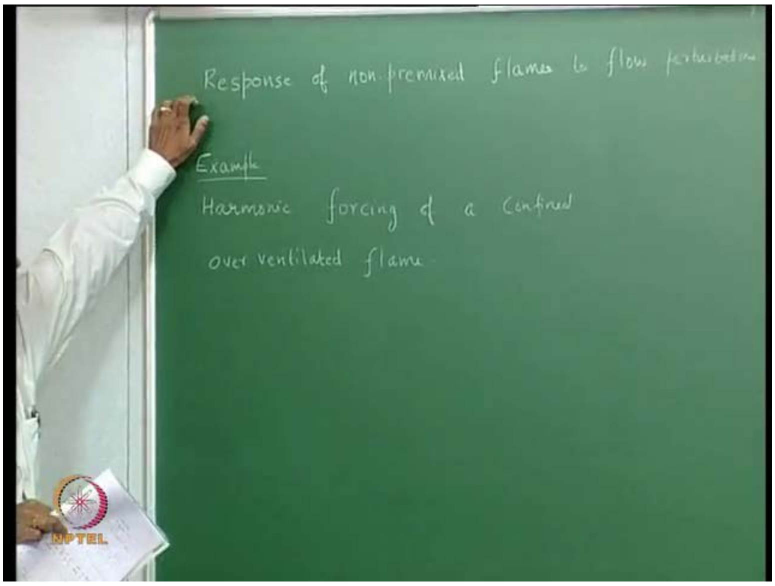

We will study today the response of non premixed flame to flow perturbations. So, we are done, some study of how we can calculate the response of premixed flame to flow of perturbation. Do you remember how we how we did this?

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G equations so what is G equation do ((Refer Time: 00:43)) then we saw for a flame flow, G equation what is G actually it is some kind of variable, but it really has meaning only at flame surface. And now we want to see whether a similar approach can be done for diffusion flames. So, we will specifically follow the approach of a Balasubramanyan and Sujit and these references are given here.

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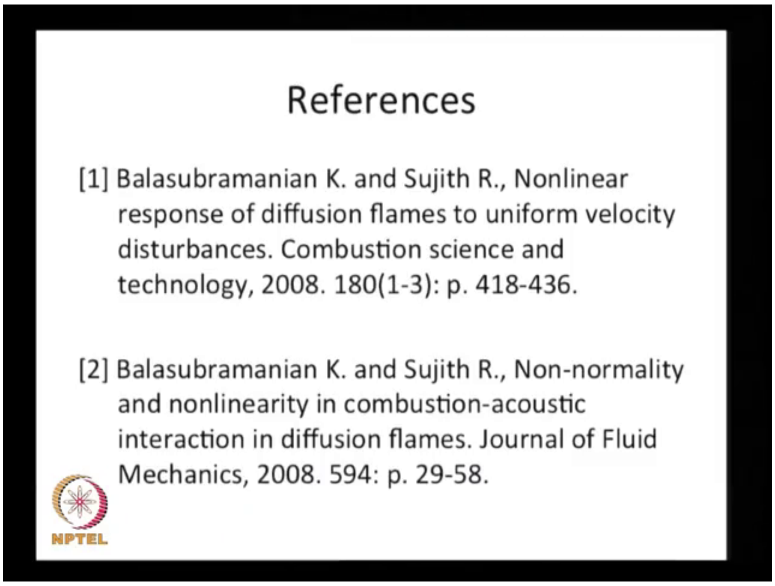

So, first one is non-linear response of diffusion flames to uniform velocity disturbances combustion science and technology 2008, volume 180 page 418 to 436. And the second one is prepared form journal of fluid mechanics on diffusion flames.

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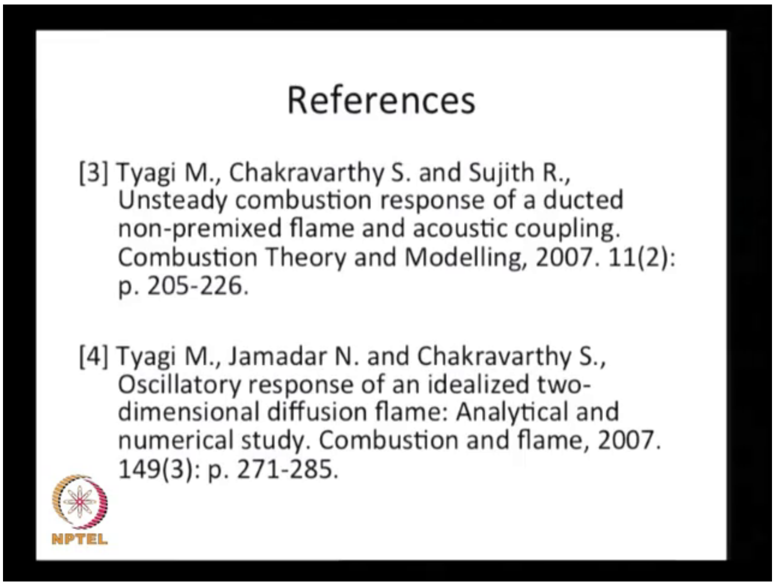

There are two more from our colleagues Chakravarthy's group and that is also an non-premixed flame. Now, compare to premixed flame which has a very large body of literature associated with a it's response, these are the only papers that you will find about diffusion flames. Although now a day's with the reduction of knocks in aero engines which still uses the non premixed flames instability in diffusion flame is becoming a subject of interest again.

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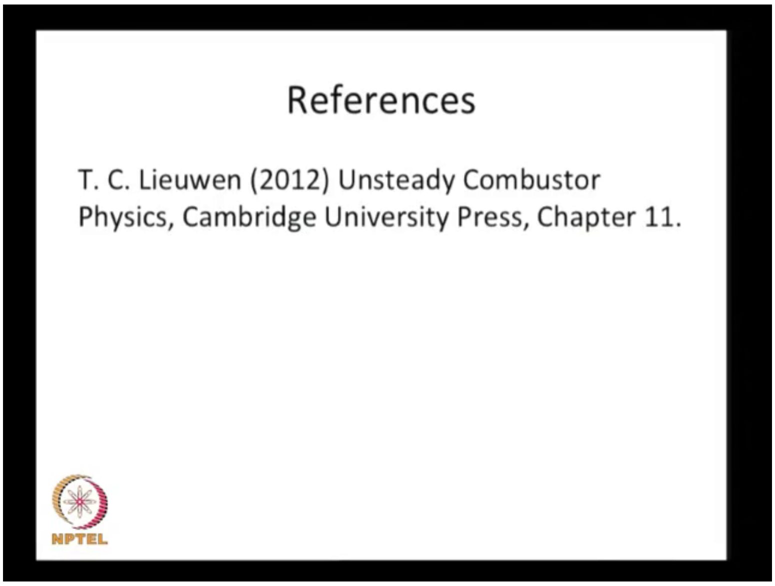

There is a text book that is available by T. C. Lieuwen called unsteady combustor physics, Cambridge university press chapter eleven of this book has a consolatory treatment based on those paper which I mention. Now, this is a very recent text book.

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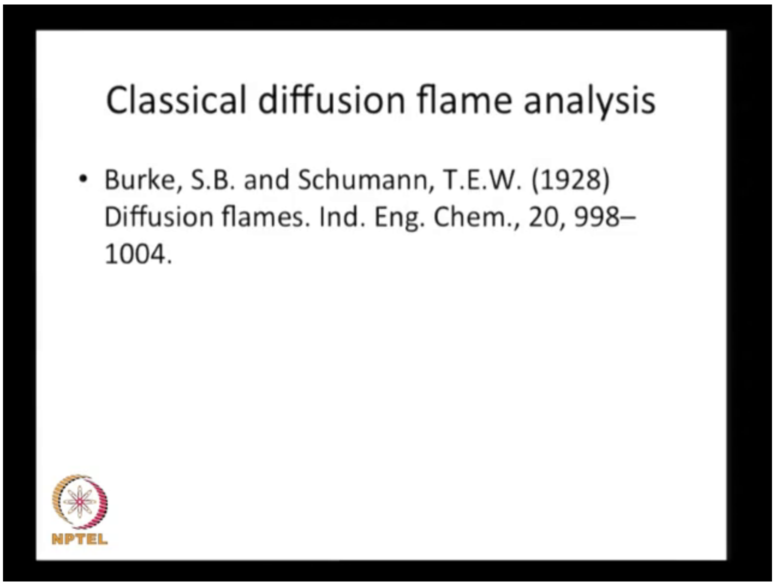

See must we all heard about classical diffusion flames this is the pioneering work by Burke and Schumann by which is by these flames are call Burke Schumann flames, this is a 1928. So, we will follow their approach and now the we are specifically interested in looking at the unsteady problem Burke Schumann studied the steady problem. Now, we are having flow and it has a the flame can get oscillate and wrinkle and so on. So, those are the phenomena, which are very similar to those see would see in premixed flames, but there is some differences. So, let us take a look at the geometry.

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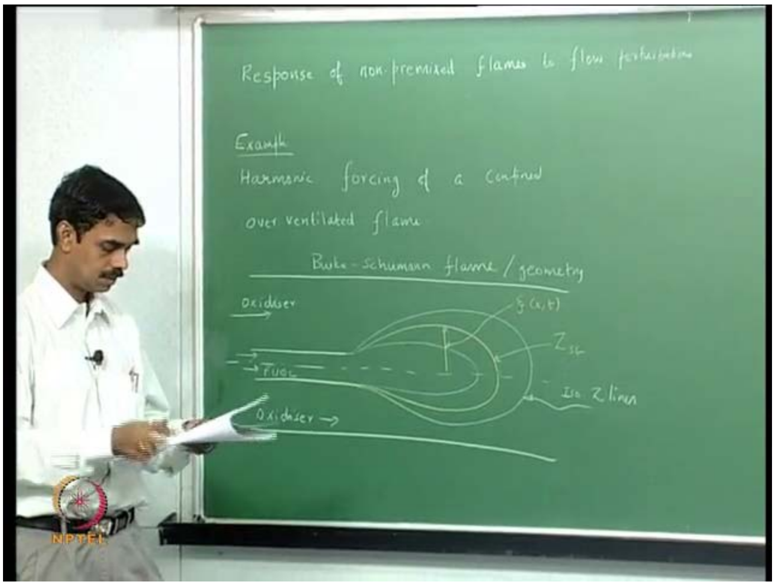

So, this would be a over ventilator flame and you will have this is the central line fuel. So, this would be the classical Burke Schumann configuration or geometry, see you have fuel coming through a slot. And we are constructing two dimension here you can work out that for axis matrix also two-dimension simpler and you have fuel coming inside oxidizer coming outside.

You can work out for the opposite also, but this is the classical configuration and there will be a flame here which is the diffusion flame. And this is usually of course, if you can also have other flame configuration, where the flame coming and attaching on the walls and so on. So, this is a over ventilator we can have a under ventilator flames oxidizer go on like that.

So, how do you analyze this problem typically, so we do this in the frame workout, what is the formulation we use anybody steady in combustor here? So, observe which formulation, so what do you get in the end our oscillation which formulation do we know this or should I do it? I think I will do it, so let us a we have of course, we can actually all both in the case of premixed flame and diffusion flame, we can solve the conservation of ((Refer Time: 06:45)) equation mass, momentum, energy everything and we can solve the full flow fill and get reaction rate out of it, but why did we use the G equation approach because it was very simple. So, the same way we will try to get a simple partial differential equation, which we can solve to get the problem you can of course, do more complicated analysis, but this is the simplest possible way to do it and were some analytical solution possible.

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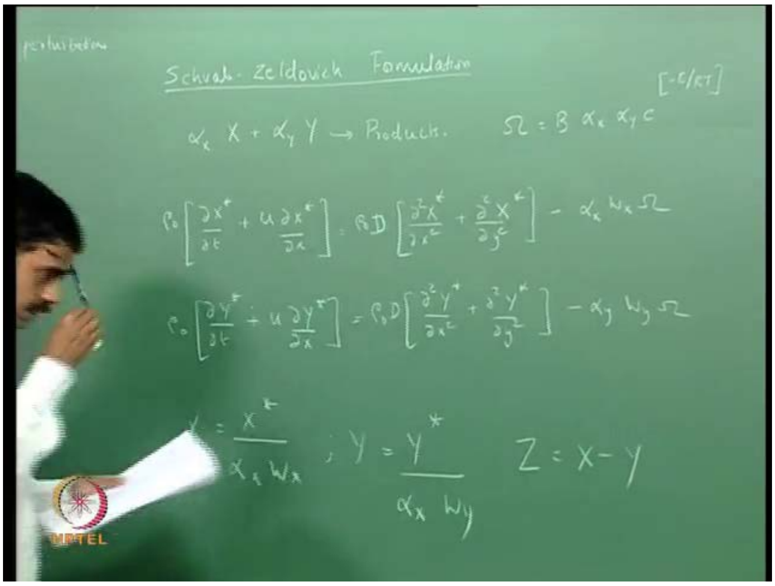

So, let’s say we have some kind of reaction, so we will write the ((Refer Time: 07:46)) equation for x and y star would mean dimensional without star will be non-dimensional, D is feasibility omega is the reaction rate w x is the molecular weight of x, omega can written as B alpha x times x of y E power R T. So, similarly sorry if there is any mistake please point out so similarly, we can write a equation for y also.

So, we can see hints of advection diffusion equation here, which is very famous equation and solve, but we have some un necessary or some other term which we cannot handle. Now, we have to get it off it how do we do this the weight is do this we should divide this alpha x w x down here also alpha y w y and then you will have omega. And if you subtract then omega will go away so that is the key so if you can define our new variables scale variables and we define z as x minus y. So, this is a different scale subtraction so z is call the swap z which variable.

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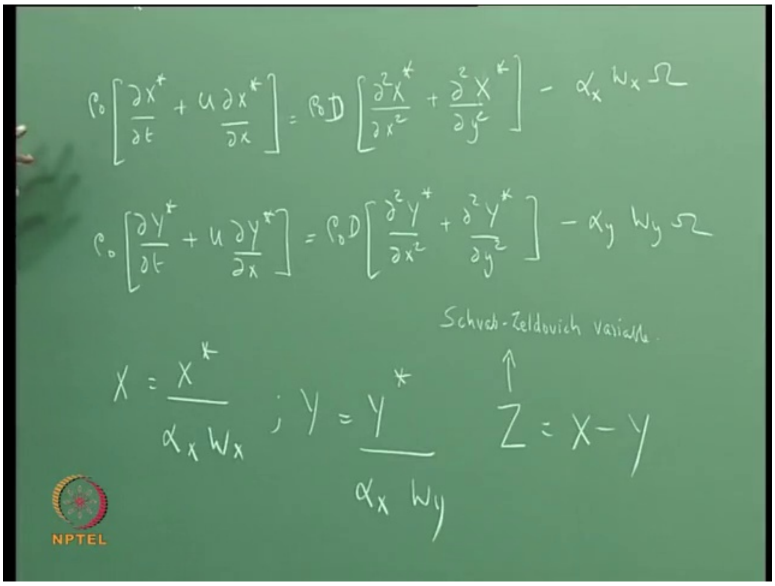

So, if you re-do this you get.

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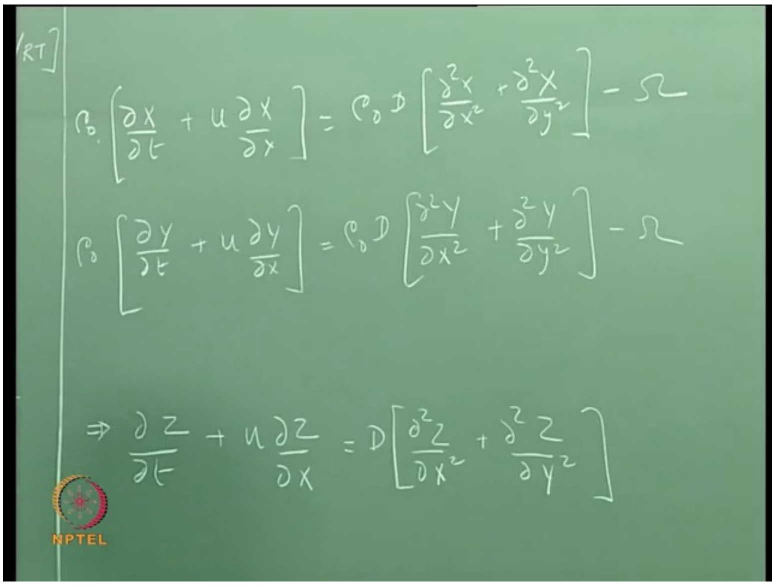

So, I subtract this from this, so I will get x minus y is z and we write the equation and times of this sorry,

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Thank you so if I or there any questions.

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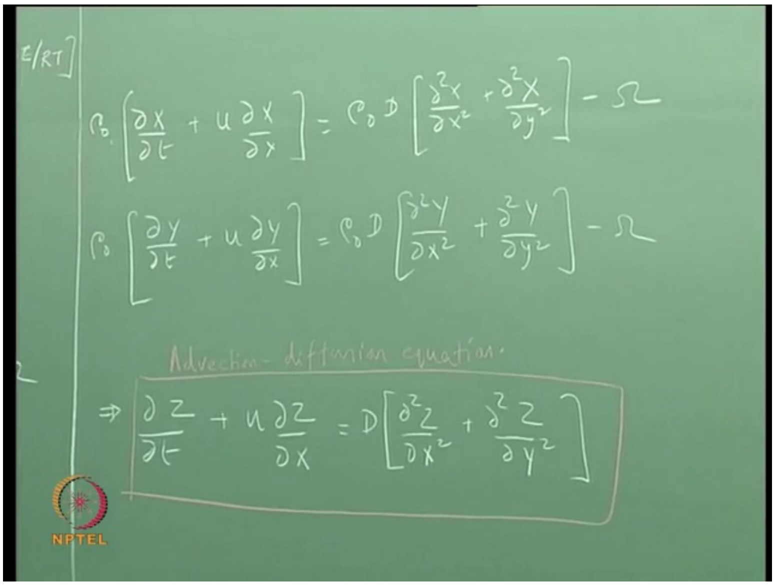

Thank you thanks. So, I made a mistake here that is a why so this is the advection diffusion equation.

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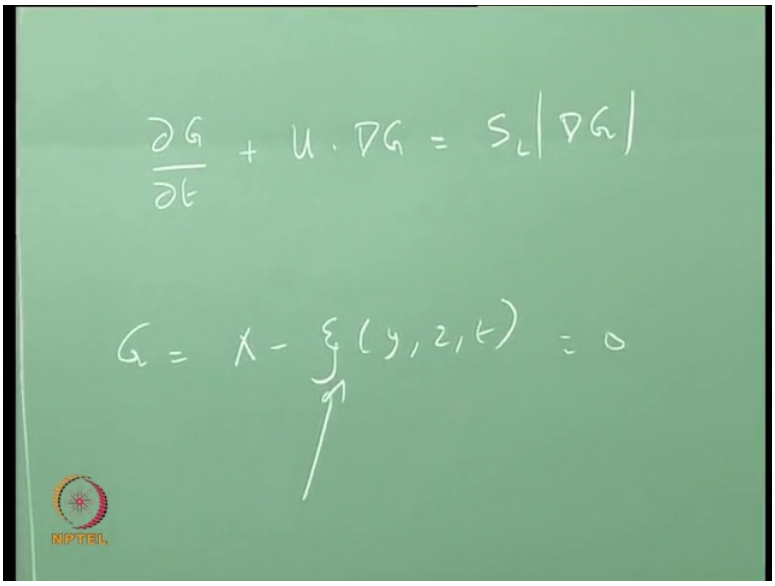

So, if you compare the this equation to G equation there we have so the left hand side of the equation are quite similar, but here the right hand we have S l times a dell G over there, but here we actually have a diffusion operated here. So, that is the that is a different, so there is some similarity, but there is some difference the other, the other similarity is that we have writing it has one variable. But differences is that z is a physically meaning full variable everywhere in the field and you have to solve for it all together. Whereas, G we can G is just defined as G is as one sign one side of the flame another sign another side of the flame and at G equals to 0 there is a flame. So, we can actually say like y minus i is G and so at G equal to 0 y equal to psi write a equations directly for a flame surface in premixed flame right that is what we did.

So, we said that and then we put this equal to 0 and then directly write a equation for the displacement variable. So, we cannot do that here because d z sort of which variable that have at we it is a physical variable everywhere and we have to solve for it everywhere. And then we can actually find the flame surface by how do we define a flame surface. So, the assumption is that the flame stands at the stoichiometric surface. So, we have to find what is the assert stoichiometric is that corresponding to stoichiometric, which is this scaled x star. So, this x minus y corresponding to this stoichiometric and that would be the flame surface. So, from that you can get the flame surface and then we can track the flame movement I hope this is this is clear.

So there is some strong parallel, but there is some strong differences also between a diffusion flames and premixed flame, even in the results in terms of the transient function we will see some strong parallel and some differences. Now, we proceed to solve for it, so we need to apply boundary conditions without which we cannot solve the problem. So, let us examine what are the boundary conditions here.

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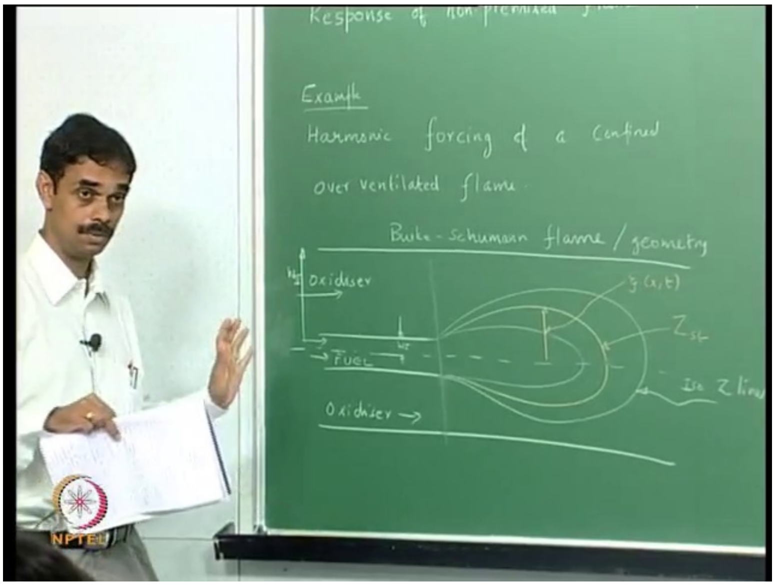

So, here let there is fuel there is oxidizer in reality you can have the oxidizer can diffuse and products can actually diffuse in to the fuel steam. Similarly, fuel can diffuse into oxidizer so we will ignore those effects and we say that here at the inlet flame, in this location we have fuel. So, we will have no oxidizer in this location we are only oxidizer at the inlet flame that is the way, we are going to enforce the boundary conditions, this need one more notation. I am following the symbols from the text by on combustor dynamics. So, this half fit is w 1 this half fit of the outer tube is w 2.

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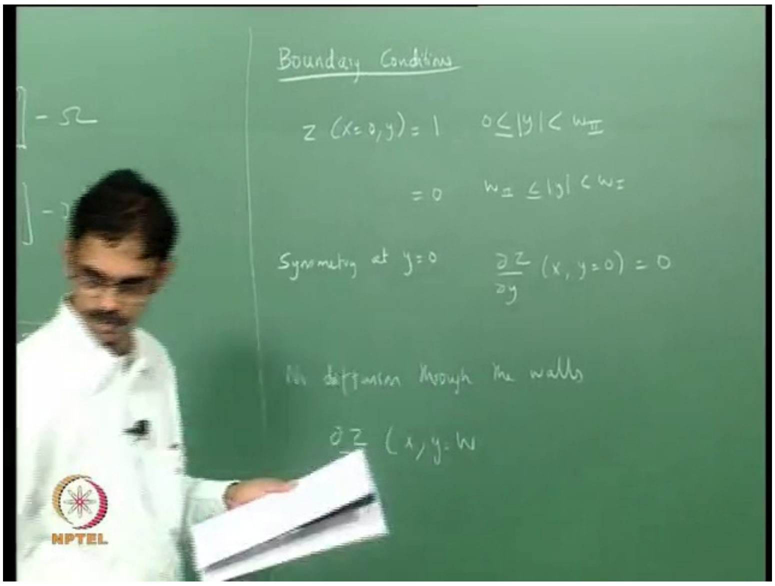

So, this basically means that in the oxidizer tube we have oxidizer and fuel, and enforcing the boundary condition is also enables to obtain a analytical solution. As suppose to the if you enforce the flux boundary condition you would have difficulty in getting analytical solution. So, in reality there is axel diffusion of fuel in to the oxidizer and vice versa, the oxidizer diffusion of oxidation and fuel. So, the solution must be obtained over a large domain that includes the supply lines of fuel oxidizer. So, I think let us correct the figure that is called the bigger outer tube and w 1, this is a w 2. So, this boundary condition implicitly neglect the axel diffusion at x equal to 0 and we can assume symmetry boundary condition at the central line right.

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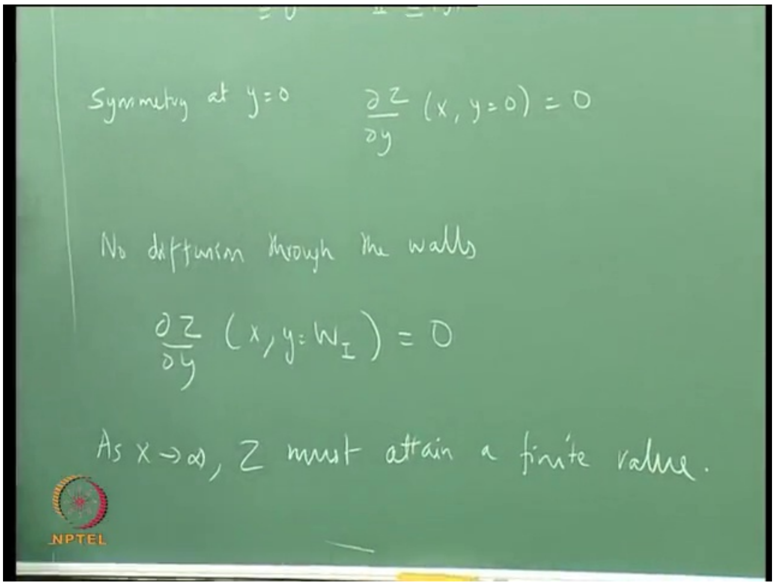

And no diffusion should happened to revolves because you have rigid wall, and as x tends to infinity is a must attain a finite value finite value finally. So, we will derive a solution in the limit of small perturbations, that is we expand everything as on zero total first order, or a base flow plus perturbation. And then we can get a solution, if I do not need to do this because our equation is a linear equation provided u is a input and you can see it is a non-linear equation. So, we do not need to do it, but it is very convenient and will enable us to workout analytical solutions, so we will do that.

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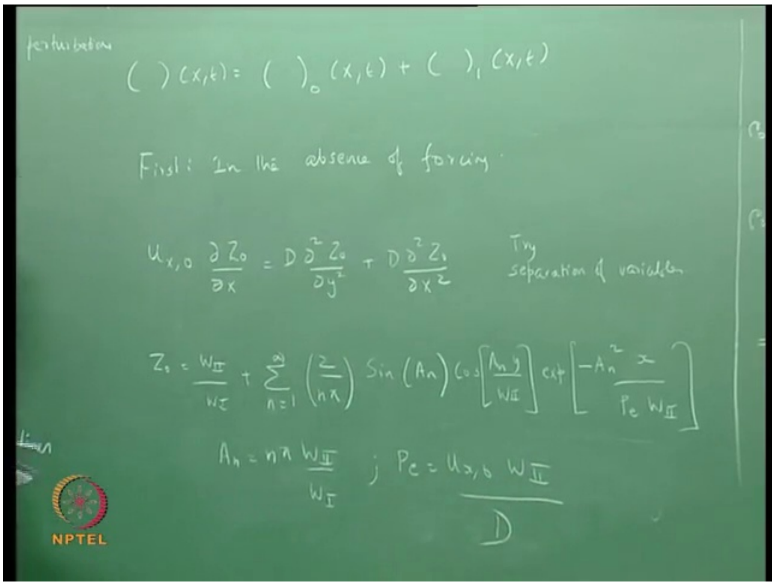

So, any parameter I am following low event notation written as some base flow. So, with this approach will gave to derive a explicit analytical expression for the flame and surface, and this will be very nice in looking at the controlling features of the flame dynamics. So, first we will study the problem in absence of forcing, so we can get the steady solution and then we look at the problem with forcing.

So, in our previous notation this would be z bar and u bar, but I am just following relevant books here, so that you can follow it also, this can be solved with separation of variable, I can work it out next class. But I just want to write a solution here, so just try if you would get this actually you should get some more complicated expression here, but then if you take Peclet number attaining very large then it should reduce to this expression. So, just let you what these all bring it next class and then we will see what you got a what I have.

Now, this quantitative Peclet number may be in this class we have introducing for the first time, so it need some explanation. So, Peclet number physically corresponds to the of course, all numbers corresponds to ratio of time scale or line scale or something other. So here it corresponds to the relative time scale ratio of relative time scale for two phenomena convection and diffusion. So, you want to transfer mass over a distance w 2 that is this and what are time scale involve.

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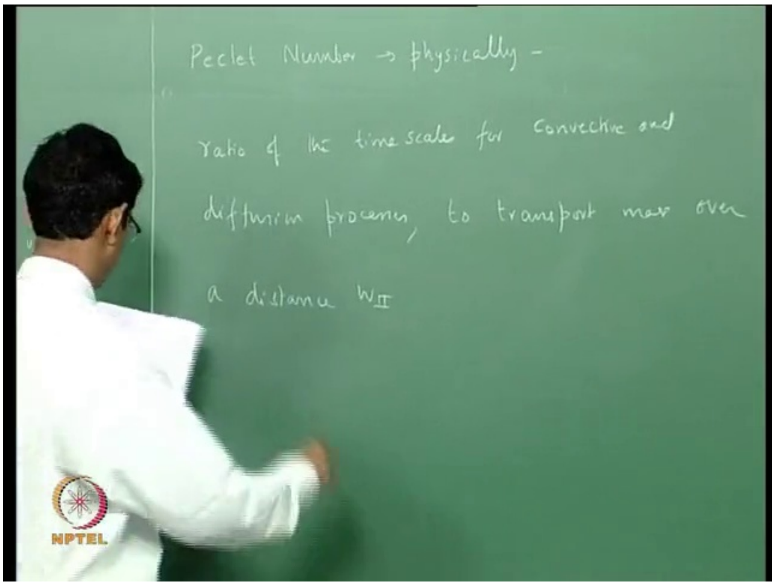

So, basically we want tau diffusion divided by tau convection.

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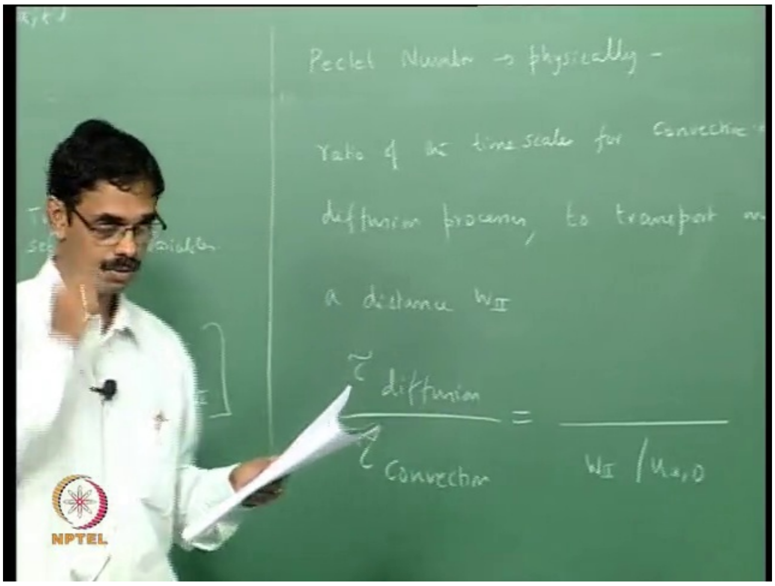

So, tau convection is pretty straight forward the distance divided by the velocity. Now, what is tau diffusion, so let us do a little analysis.

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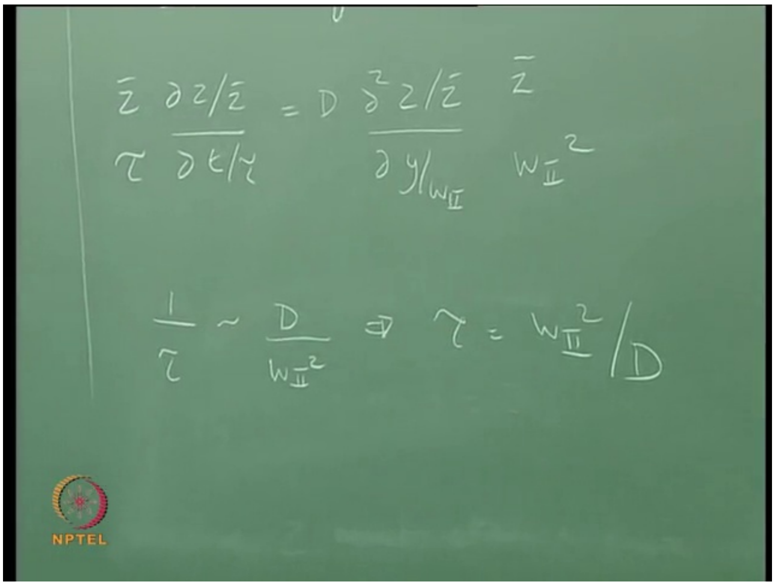

So, this is the heat equation, so if you get the lens case. So, this is the lens squared over diffusion that is the diffusive time scale.

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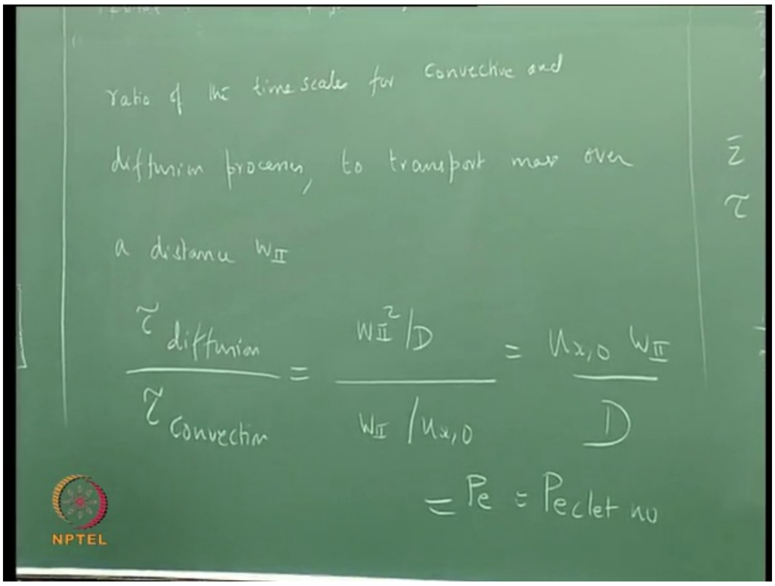

So, this is the formula Peclet number P e the people write Peclet number for convection Peclet number for diffusion. So, both are refer to by this same Peclet number, but here it is we are talking about diffusion. So, how do you having got a expression for the is it not so observed with wherever how do you determining the where the flame sit is what is the assumption in Burke Schumman. So, the flame is like a sheet it means here the inherent assumption in Burke Schumman flame is that you are having infinite reaction right compare to the other processes ,reaction rate is much faster compare to convection diffusion.

So, the flame is really thin sheet and this happened so you solve for the is a treat and where you have stiochiometric contour, where wherever you have. So, you have fuel diffusion out or oxidizing diffusion in so there is a field of x and a field of y right there is a there is a fuel mass fraction of contours oxidation mass fraction of contours. So, somewhere so if you look inside you will have more fuel compare oxidation because naturally fuel is coming out, if you look somewhere here there will be more oxidizer than fuel because fuel is diffusing out, but predominately this oxidizer. But somewhere down in between the concentration would be equal to both fuel and air will be that of stoichiometric mixture and the flame stands there. So, this is the flame sheet assumption.

So, we use that locate the position of the flame so this is the assumption in a real non-premixed flame. For example, other phenomena such a striple flame, flame standoff and so on because the walls may quench. So, reaction may not happened here so there may be a little bit premixed mixture here and so on. So, there are those complicated finite rate chemistry effects that happened, but here we do not have any of that so because we are referring to classical Burke Schumman. So all we have to do is to take this and put is a not a is it and this would be the flame position and we it will be implicit expression, we will have to solve back position iteratively yourself.

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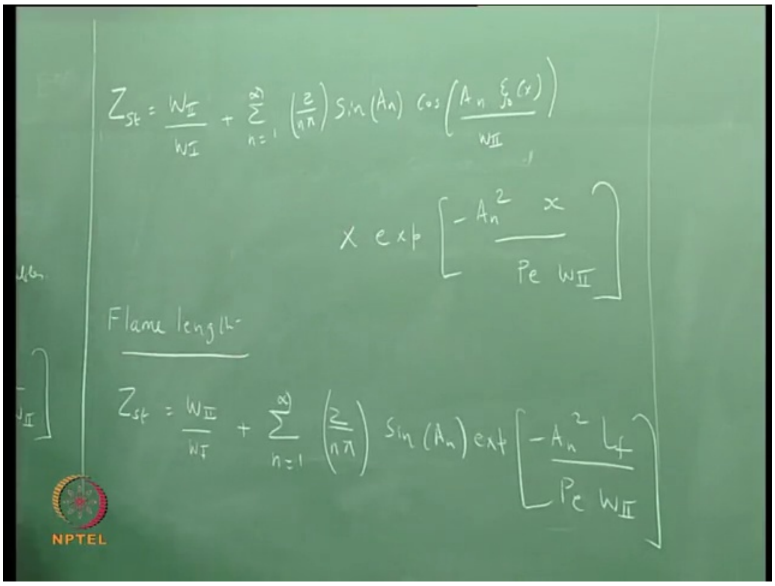

I will work out next class it will be difficult solution, it is there it is there in this solution this try you can take that equation and use separation variables, if does not workout I will do it next class. Now, one remark I want to make is suppose you are solving in a circular geometry then can you imagine how this expression should look like? They would be in terms of basel function, basel functions so that is the difference here will be in sins and cos because we are in the 2-D geometry.

So, next we want get the flame length, so we just have to put this term as one so that way you get the maximum position and that is the flame length. So, in the thermo acoustic problem this product will be long and there will be acoustic field and the there will be a certain pertain velocity fluctuation here, which will affect the mixing here right? I mean which is go and back convection diffusion equation, that would result in the flame fluctuating it will result in the heat release rate fluctuating. Now, that will drive the acoustic field which in term will give the feedback.

So, now we are not studying that problem here I am not working out in the class, but we one of the references in that list talk about that, but here what I am want to do is to impose a velocity fluctuation here and see what happened to the flame. So, the flame dance will it wrinkle will move up and down that kind of thing. So, what we have is a base flow field that is even if there was no fluctuation, you will have fuel and oxidizer and they will diffuse out and they will be a flame.

Now, it with certain velocity fluctuations how can you do a velocity fluctuations you say u is mean plus u bar plus some u prime and u prime goes like sin omega t or in a experiment, you can put a valve and fluctuate the flow rate and something. And then see what happens to equation or advection diffusion equations are unsteady equation we drop the steady term to get the base flow. But now you have to put the term back in and solve for it.

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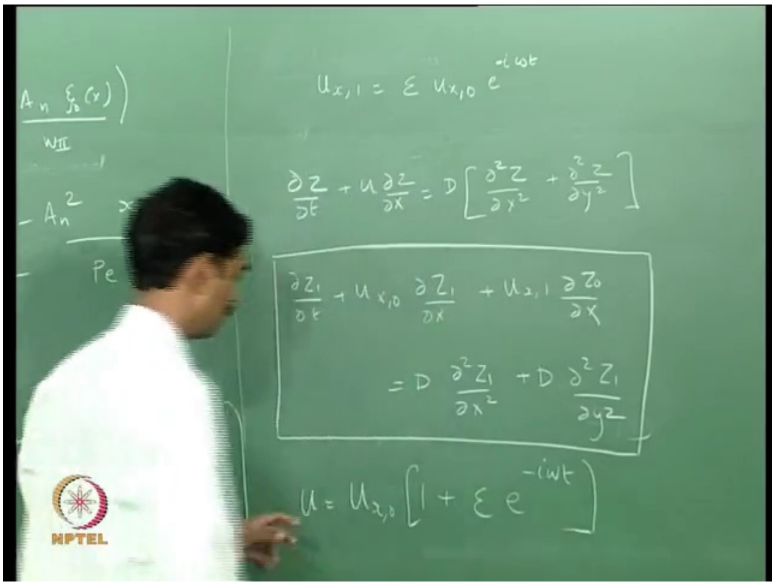

So, what we need to do is to so u x naught was the base flow in this convention u x 1 is the perturbation, I just follow the convention in this text book of flow event and we had. So, if you split it as a is not for exact point what you will get is so this will have mean time fluctuation here plus mean here time fluctuation there. So, that is why the two terms. So, this is the Peclet equation, I will epsilon is like a how much like you have a so the total u equal to convention. So, this is the total velocity field so epsilon is like a what is the like the amplitude of the perturbations as a fraction of the mean velocity. So, again we can solve this try to solve this it will be a interesting exercise, if you do not get it that is completely fine I will solve it here in the class.

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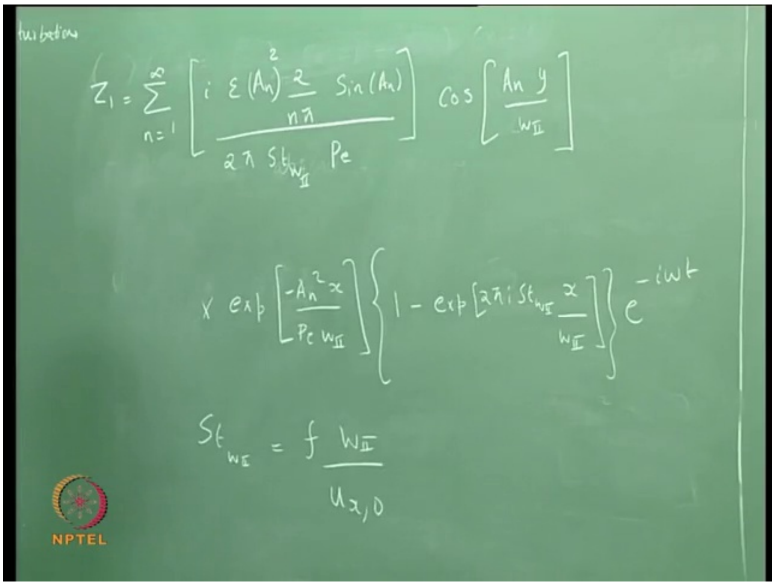

Just try its and interesting thing yeah f is I mean 2 omega is 2 pi f. We can we can simplify the expression because if you take a look at the expression for z naught and if you differentiated, you will get this scaling term sitting in front.

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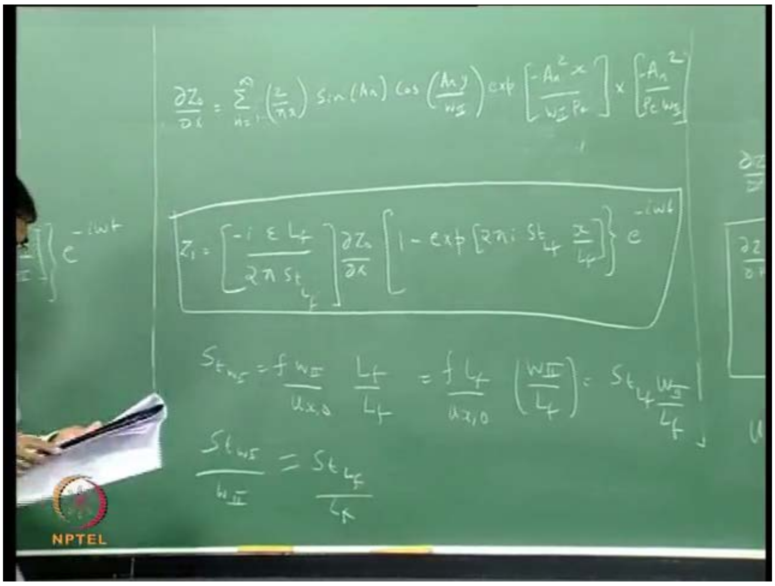

We had the formula for z naught in your book, so if you use this terms are that is here. So, you can write without the sigma as z 1 equal to where a see s t w 2 is equal to f w 2 over u x comma 0. Now, let us multiply top and bottom by L f and L f so this can written as f L f 2 x comma 0 into w 2 by L f this is equal to s t L f w two by L f. So this we have used in here and you get this expression. So, please check on this. So, this is the expression for the perturb is it field so you have base flow plus the perturb is that field.

Now, we are solved for the entire thing although that is not what we are looking for which is want the final what happens to the flame, but there is no option, but solve for this. Now, from this we want to look at what happened to the flame and how the flame oscillates will do that next class. And we will also look at the derivation of these things in case you have having difficulty. So, stop here now.