

**Acoustic Instabilities in Aerospace Propulsion**  
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**Lecture - 37**  
**Solid Propulsion Combustion Instability – 1**

We were looking at actu-control and we looked at the kind of actuators and sensors that can be used and we also looked at very briefly at different kinds of control, we had a model based control and non model based control. And we said that either you have a controller which doesnot need to know about the system, which is known model based control you can do system identification and so on. But then there is no guarantee on the fact that the guarantee on the guarantee on the effectiveness of the control. There is no mathematical proof that you can give that atleast in principle controller should come controller should work.

So, if you have a model based system the controller does need a detailed knowledge of system, but then it can the controller can give mathematical guarantees that the control will at least work on paper. So, which is good which is bad I think that the experience of engineers we can answer this question only after having worked on the subject and controls of oscillations. I have myself not been working on this topic, but I think the answer is not very answer and one needs to have experience and then only can give answers I am trying to do here is to give a favor. And so in the examination not going to ask you whether you use this controller or that controller because I myself do not know the answers so don't worry about it.

Last thing I want to any questions so far

Student: ((Refer Time: 01:39))

So, I used to work in a lab and there was a very old engineer who does not write any equations at all his name was Bob Daniel and there was this young control guy who was trying to set up this new controller. So, this guy lay a challenge he had a pulse combustor he had a combustor which pulses and he said you control this and I will give money, and he controlled it was working at fast mode and he controlled the third mode came up, and

he controlled the third mode, the second mode came up. So, this is kind of water bed effect it is like a water bed you push something down somewhere up it comes up.

So, yes this danger is there and we saw a briefly in that review article by Marc Menase he has written about some conflicting thing you do something to control one mode, but this action may actually end up creating another, another mode itself and that it is there and some solutions to it. But I think let us be on this scope of the level of things that is dealt in the class, but what you are saying is correct.

Student: ((Refer Time: 02:57)) What I was asking acoustic module the whole system.

You mean properties mean properties or some other properties could be possible yes could be possible think that is the challenge.

Student: ((Refer Time: 03:10))

I think we have to ensure that everything is within the operating envelope that is given you shouldn't push the system out of that and come to this performance.

Student: ((Refer Time: 03:27))

I do not know the answer may be I don't know the answer, but I do not know there may be it is a very big topic and I have not worked on it so any anything else. So, in reality the practical the combustors possess a turbulence, I mean the flow is turbulent and we always study laminar flow, but the reality is that the flow is turbulent. And it is very hard to model this turbulence acoustics and combustion accurately and physics based models of the system are unlikely to be sufficiently accurate to use as a basis for controller design.

And we talked about many reasons why what are the difficulties in the modeling and we looked at like the improvements possible and so on. We recent thing is this normality so this means that the controller must be based on system measurements based on possibility although the consequences of instability may be that obtaining, this may not be straight forward in certain operating regime because your acoustic oscillations at the start of the instability may be comparable to turbulent oscillations and so it is quite difficult to make measurements. And there are likely to be several modes of instability

spanning frequency range in which the oscillations can come up and in such cases it is important the controller is able to control more than or more.

So, the system will as we saw will almost involve substantial time delay by factors such as fuel convection and acoustic wave propagation and this could be larger than of the order of period larger than that and we have to estimate them accurately. Nevertheless in spite of all these problems people have succeeded quite a bit in the laboratory with small combustion, but they also installed it in the real combustors in the fields quite sometime back. So, of course, in the lab very sophisticated controllers have been used, but and primarily in the beginning most of these controllers used loud speaker to use this control.

But later on secondary field injection became a very important possibility with the development of good actuators and that was considered better because you can get higher powers to be able to cancel this or to be able to make the system stable, compared to loud speaker. Also secondary field injection is much more easier to deal with compared to loud speaker in a combustor in a class room may be a loud speaker works very well, but in a combustor is not a very good idea we have to have a loud speaker, in this hot environment. Nevertheless in spite of all the problems the people have got a full scale demonstration was done and in 1988 in the after burn of Rolls Royce R B 191 military turbo fine engine by ((Refer Time: 06:38)) the full reference given in the paper by Dowling which I mentioned in annual review.

And actuation was actually achieved by spilling fuel from the engine rather than adding oscillatory fuel in the oscillatory spilled it using high response electro-hydraulically servo valve. And modulated about 5 to 10 percentage of the mean fuel flow rate and so this was a simple time delay controller, and then they got 12 d b reduction of the after burner bus and in 1998 Sumito and Hoffmanitol tried active controller a siemen's heavy duty industrial gas turbine, the industrial gas turbine for power production and so on. So it is a annular combustion with azimuthal modes and they had to measure the pressure at many locations and activation was by modulating fuel to the pilot planes, and they used and again.

It was a simple gain in phase shift controller and this was done first for siemens and they got 17 db reduction and 17 db, 20 db is like one-tenth factor, and this is like one-tenth.

1999 Cohenitor tried in UTRC in united technology research center that is the paternity paternity engines. And they got 16 db in a single combustor and 6.5 db in sector combustor and Noemeerzin tried on I think vesting house engines, adaptive phase shift controller and they got 15 decibel reduction and ever since that. So, this was the this things are about 15 years back or 20 years back. And lot of progress has happened with controllers.

Nevertheless these companies GE, TATA, ELXI and siemen they are not having these active controllers in their engines, neither in the military engines nor in their combustor engines I think siemens ran their combustor engines for quite some time some of them with active control. But then it was quite expensive I think you have to pay substantially more for the version with the controller on and somehow the feel that this thing will work is not there with the engineer for some reason. And so they are not favoring they are not going to the and they are I think they are quite nervous about I mean they expect the combustor to work about several years and so on.

And they do not think actuator will work that long also, the active control people say that look your actuators in the car they work for many years, but for some reason the people in the land based gas turbine industry they are willing to work round it, even military engines if anything they will be they ones first to come to active control. But at the moment they are again working around it by doing passive control and adjusting the operating envelope, such that you do not get into instability. So, that is where the subject is, but overnight things can change with some gain changes coming in and but may be some of you will here to make this practical as used by the industry just like in aeroplane flywheel all are used although its quite intricate technology.

And if something fails its quite a bit of consequence, but people are using it so I guess some day it may work and I think technology prediction is not a very easy thing, you might have heard about predictions have gone wrong. So, I wouldn't make any predictions hope that this may come into existence, I will stop active control with that.

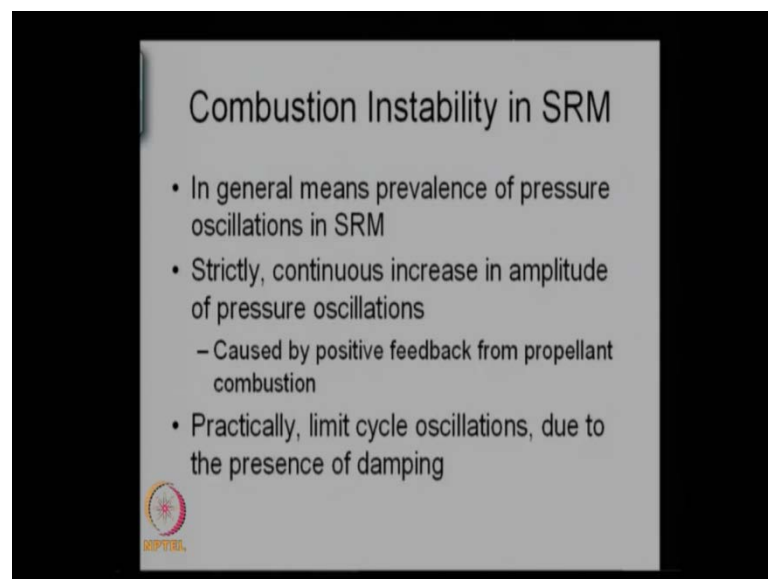
Student: ((Refer Time: 10:20))

There are other type of controls for example, you can change something which sorry so there is tuned passive control. That means if some kind of thing comes you tune to make it disappear, but you can also people would be to I mean I do not know whether you can

call it active control, but if you can something comes on and you can change the equivalence ratio or something, which is happening at the lower scale of acoustic oscillations that the engineers are with and I think that kind of things are used in the industry. But what is traditionally referred to as feedback control is things of the time scale of oscillations that is what the engineers are not very pleased with the existing outcomes of that. But the other one also exists and engineers are happy with that kind of control.


Anything else so we have one more class left so there isn't time to learn lot of topics that I can cover, but I thought I will speak about solid rocket motor, which is quite important in our country as good solid rocket motor program. This is a very big topic and I have lot of data with me, but I cannot show any of those because I may be put in jail so because of this video recording. So, I will have to I cannot say most of things which I could have normally said so I apologize for that, but I will still try to give a flavor and again copyright for these things. I do not think anybody else will give me I can get for regular, but not for this to put it this calculation I will show myself, but apart from that the data I can give some papers some references you can see in that. But I am not going to put that in the thing which will be recorded and I am really sorry about, but you can come to my room and I cannot tell the names of the motors.

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Combustion Instability in SRM

- In general means prevalence of pressure oscillations in SRM
- Strictly, continuous increase in amplitude of pressure oscillations
  - Caused by positive feedback from propellant combustion
- Practically, limit cycle oscillations, due to the presence of damping

 NPTEL

So, combustion stability in SRM is solid rocket motor and those are those massive things and we are talking about, so far jet engines and combustors and so on. But here you are talking about really massive things which like two hundred tons burning off and ninety seconds or hundred seconds and so on. So, that is the level of intensity the pressure in the gas turbine may be of the order of 20 atmospheres here we are talking about 60, 70 atmospheres and in liquid engine may be even 300 bar I think, I see many of the people here who have familiarity with all these things Rajesh.

So, I think as I said when the performance increases that is when you go crazy, so when performance these are machines which have incredibly high performance and of course, when you push things incredibly high performance, they can go crazy also as human beings do. So, when you say combustion stability in SRM loosely it is meant that the pressure oscillations inside the chamber and strictly again it depends on what context again people use in different sense. You can have continuous increase in amplitude and you have growth rate  $e^{\alpha t}$  and all that has to be we have seen.

And instability means something going unstable and this is caused by positive feedback from the propellant combustion and that is what is driving here. And practically with limit cycle oscillations can exist there is a possibility that also possibility that the oscillation come, and before the oscillation the amplitude stop growing and reach limit cycle, the motor may blow up also. And I suppose you can guess what the level of oscillation when the large instrument is coming or very huge instrument is coming, I do not know what the right term large instrument huge instrument, catastrophic instruments does anybody have any idea.

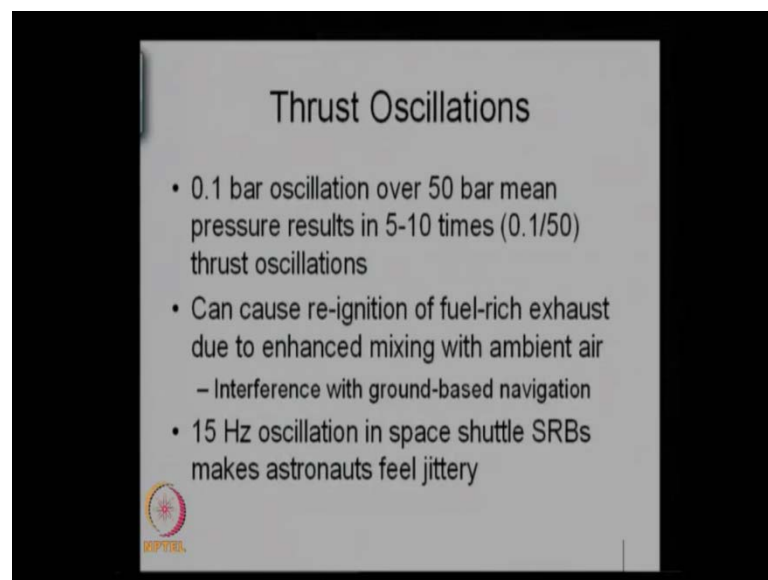
Student: ((Refer Time: 14:33))

I have seen even twenty percent of the mean time or even 25 percentage of the oscillation. So, this is like I seen the oscillation of the order of 20 bar in some where the combustion is actually the rocket melted of that kind of things and the people who were doing the static test, they are standing kilometers away or something. But they sometimes feel that there is a lot of safety regulations before doing srm test this much distance away. I do not know the rules, but they felt many times that they will die or something just hearing the pounding. And the shuttle astronauts, the space shuttle they say it has very feeble oscillations, but it has still its 15 hertz they say that they will feel

very jittery when this oscillation come 15 hertz is like low frequency sound and human body will respond to it.

You might have seen the movie jurassic park in a good theater and you can see when the dinosaur those kind of in a good theatre you will see theater you will feel it and human, I think the natural frequency of 4 hertz if you are coming closer to that you feel more and more jittery. I think 8 hertz some of the internal organs I mean has the natural frequency of low frequency you really feel pounding, and the most of the motors have the instability in some form or the other. The question is only the 20 bar oscillation or its point bar oscillation even if it is 0.1 and 0.1 bar oscillation you still have the feeling, but you can tease somebody to leave the feeling astronaut they can take lot of this.

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So, there is also damping luckily which is why many times we can live with the oscillation and oscillation do not grow exponentially, damping from some of the oscillation can escape out of through the nozzle. The structure or the liner can damp the oscillation and you can have particles aluminum particles in the rocket and they can damp out the oscillation and so on. So, there are many damping phenomena so there is always this driving versus damping and it is always a fine margin and this driving has to be lower than damping throughout the range of operation. Many time what happens is this may be like this in the beginning and then 20 second in to the firing the driving may be become more than the damping and then the things comes out.

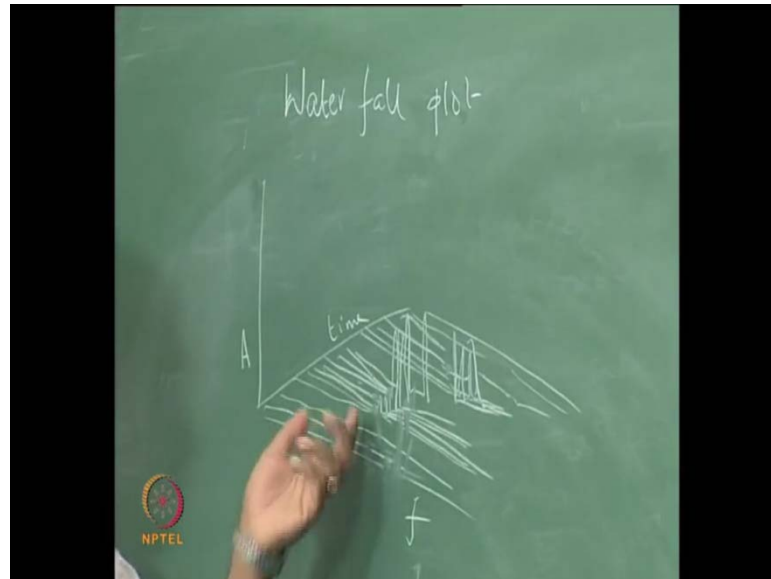
If you do calculations the 0.1 bar oscillations over 50 bar mean pressure results in a typically you know this is a typically like 0.1 divide by 50, but the thrust oscillation will be 5 to 10 times that of the pressure oscillation. So, if you have typically 0.1 percentage pressure oscillation it may translate to a 1 percent trust oscillation I think 1 percent trust oscillation a machine can taken, but more they may not be able to take. It can cause re-ignition of exhaust fuel rich exhaust you know SRM work with solid fuel rocket motor work with fuel rich exhaust. I guess you know why?

No if you have fuel rich product, fuel rich combustion then your molecular weight of the exhaust product will be lower and then you will have higher I S P that the reason not little lower the temperature. In gas turbines you have burnt the excess air to lower the temperature, but this is not the case you want higher I S P that the only technique. And so you have quite a bit of fuel left in the exhaust that can cause that can re-ignited without these oscillation and they can enhance the mixing of the ambient air, and this can cause indifference with ground based navigation systems. And as I mention 15 hertz oscillation in space shuttle SRB's make astronauts feel jittery.

And the typically as I said I will try to draw a picture because I may have copy right violations. So, they express this you know we saw that we can express the oscillation we can look at the oscillation with f of t's, but the problem is as I mentioned there may not instability when the motor starts, but sometime later instability may come out. So, what they do is they acquire signal continuously and split the signal into 0.1 second or 0.3 second and then stack up to f of t's.

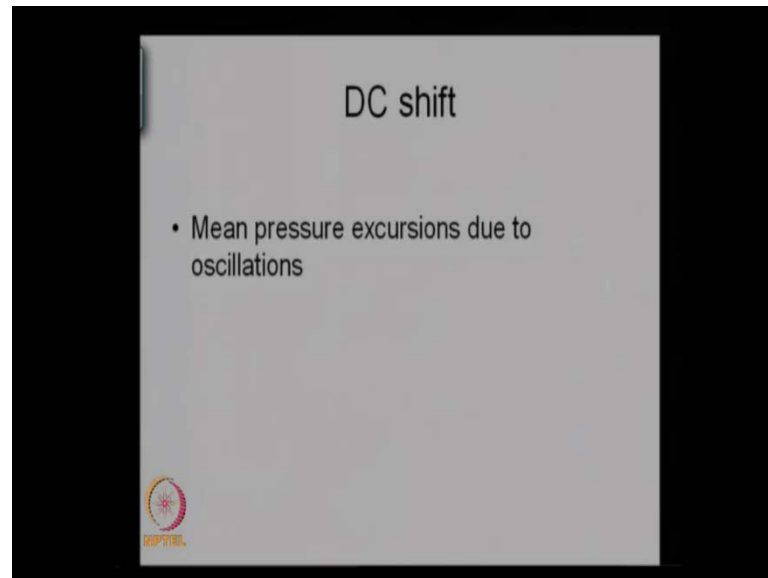


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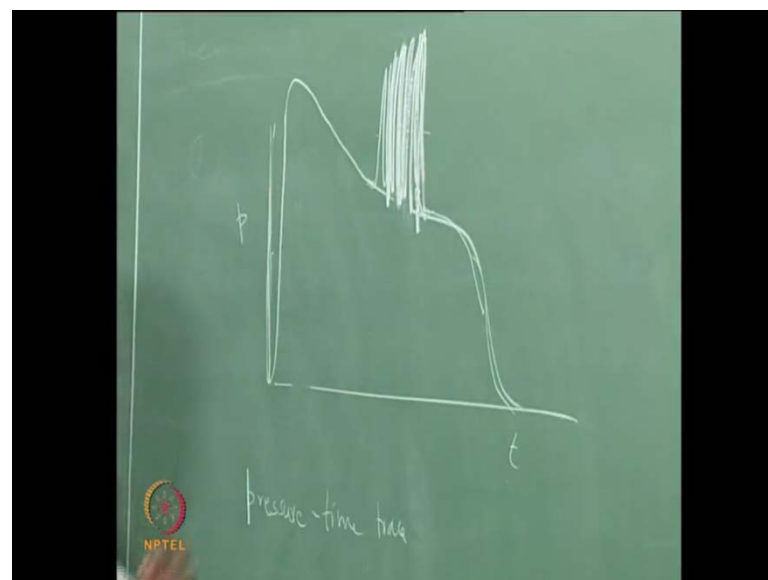
So, let's say it is quiet and then a peak starts appearing and suddenly bank this peak it is big and then what be interesting is some time, the instability changes the frequency and then of course, there died on outside some time. So, this would be frequency this will be frequency this will be amplitude and this will be time and this will be short time accessibility. So, the way this picture look like it looks like a water falls so its called water fall diagram and I am just hand drawing a schematic because I am concern about copy right issues. And security issues this would be called so if you people look at this plot and this spike then there is instability you can make out many things looking at this plot.

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We will also see what is DC shift that means the mean pressure will change may go up called mean pressure excursion will oscillation. So, the typical thrust time trace or pressure time trace vice verse.

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You will have ignition, you will have an ignition transient and then may be if everything goes very smooth you will have burnt this way.

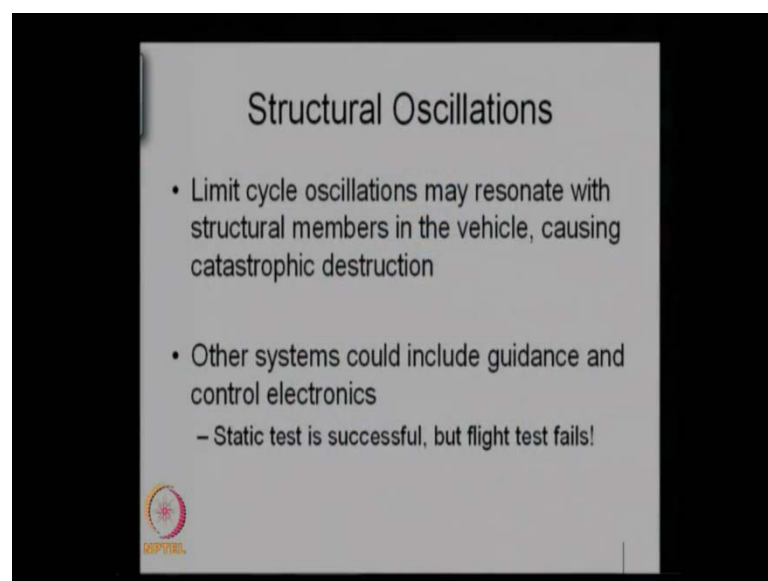
Student: ((Refer Time: 21:17)) Simulative data or what actually.

No its I mean typically they have pressure transient mounted in the solid rocket motor and that is being acquired you can. Of course, simulate and plot, but also I mean the actual life firing static or the flying machine they will acquiring the data they will piezo of the electric I mean piezo of the resistive transmission and they have plotted. So, you have the pressure time trace, you can also get task time trace with accelerator meter or and that will also follow something similar and this is the quite rocket and everything will be happy and if things go wrong, suddenly instability will come on.

And in fact if you look now the mean pressure itself is some where here and then if the motor survives, eventually it may go off and come back or it may the motor may structurally fail. So, this would be like highly non-linear oscillation and mean pressure itself when we said not just you know when we studied earlier, we said that you have mean and you have fluctuation around it and here the mean itself has gone up. You can also have small amplitude fluctuation where you have things like that so that would be like a linear kind of oscillation around the mean, but here the way out I have drawn it as the mean itself has shifted and ou can imagine why this can happen.

Because whenever the oscillation the transport processes get enhanced tremendously and so the heat and mass transfer to from the propellant goes out so such that the mean burn rate itself is very high. And therefore, you can have mean pressure shifts so that is something that we have seen when we have large oscillation.

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The slide is titled "Structural Oscillations" and contains the following text:

- Limit cycle oscillations may resonate with structural members in the vehicle, causing catastrophic destruction
- Other systems could include guidance and control electronics
  - Static test is successful, but flight test fails!

In the bottom left corner, there is a small circular logo with a star and the text "RPVTEL" below it.

Student: ((Refer Time: 23:32))

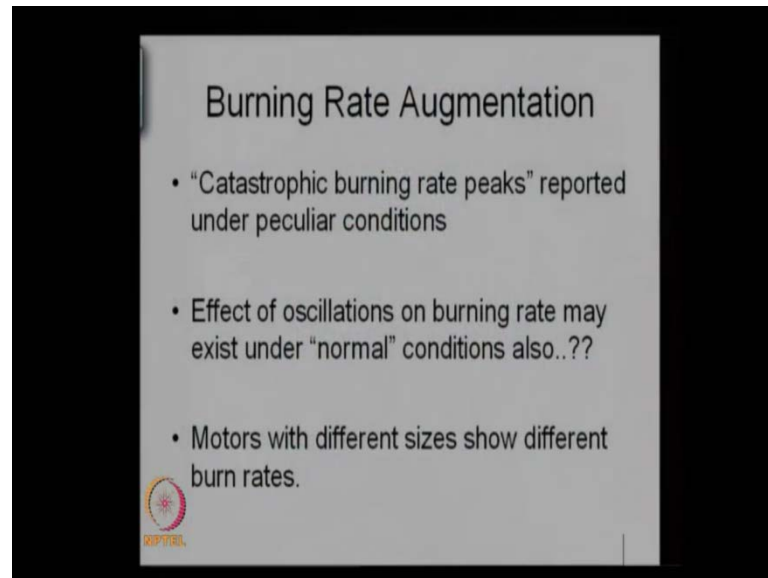
So this will burn up faster this may just follow up like that or if its state that along or it may just structurally fail, you are absolutely right. So, thrust will change and then the vehicle will go somewhere else that I mean even if the vehicle is fine, the machine would have failed because it went somewhere else not where it was suppose to go.

You can have structural oscillations so you can have even you have small amplitude limit cycle, where the oscillation may resonate with the structural member causing catastrophic destruction. Other system would include guidance, guidance and navigational control electronics. So, you may have a motor which is in a static test everything is successful, you have you have the motor staying fine very small amplitude lets say zero point o one percentage of the mean plus oscillation something, but in flight it fails because you have the system this oscillation might have blocked might be corresponding to natural frequency of some of the electronic items electronics are in general sensitive oscillation. But some system may fail because we are simply oscillating at the frequency typically the test for all the possible frequencies in the structure and the systems involved and make sure that none of them resonate.

I can tell you a story without naming the motor I have heard in America there was a motor it was a missile working fine, and then all of a sudden in flight test the motor started the missile, the missile started failing its started just blowing up and its thought to be combustion stability. But then in a union piece time to fire missile to be sure that everything worked. Now, they took the missile back to static test I mean the static test and started firing and the missiles are completely fine, but then you fire failing it turned out that some where down the line aluminum supplier changed and something slightly change with aluminum.

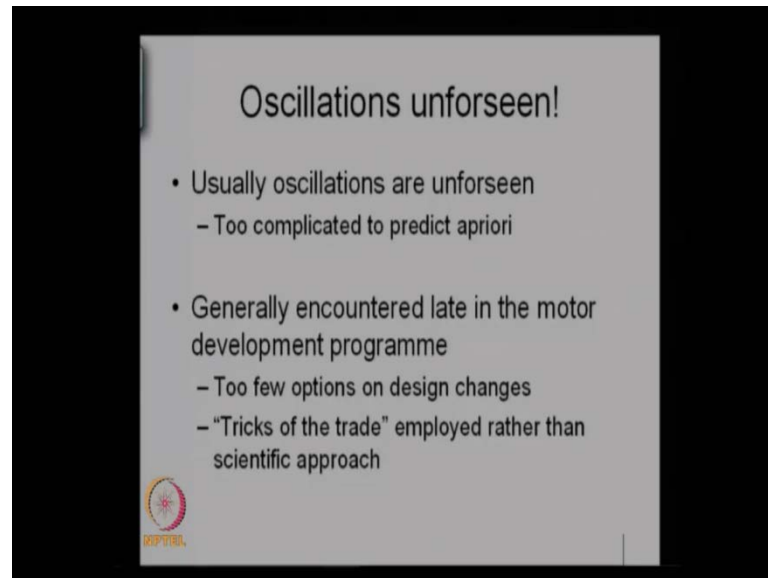
And so they ended up having a different kind of particle damping characteristics even though the aluminum particle apparently looked like the same size, they some slight change was there. And therefore, they had small oscillation they could not get rid of them whatever they did problem. So, what they did was to easy fix to change the natural frequency of the navigation systems and fix the problem. So, I think this kind of things are happening.

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And back to this burn rate and augmentation so this kind of burn rate going up is called catastrophic burn rate peak and they are reported under what people say peculiar conditions. But then effect oscillation is burning this exist in the normal conditions also no one knows no you put it in a diastic evaluation motor and find the characteristics of the motor. But even there they may having oscillations and so what you think is normal way to be under some oscillation. And people also said the same popular when you put it in when you load it in different rockets with different sizes they give different burn rates. All this is not like fancy story if we I speak to people who deal with this profession they will say all this.

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And the most difficult portion is oscillation are unforeseen usually, not usually always and they are they still too complicated to predict a theory inspite of computer programs and all that. Generally they are encountered late in the motor development program because this will not been seen till the firing occurs, and at that time the options are really too few to available. And anything that you propose the program manager will shoot down because he has to have the best missile or the best rocket like this that.

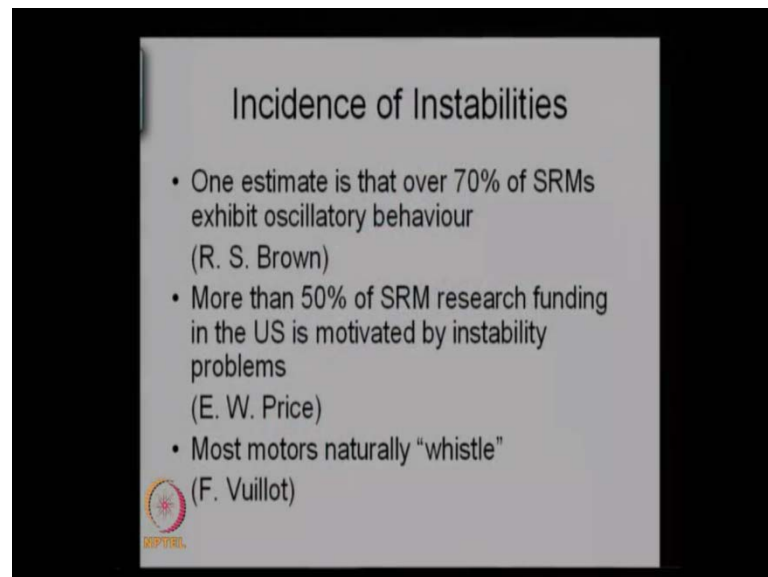
And so anything you say would not be acceptable because they want they do not want to change anything will either mean lot of expenses or it also mean compromise in performance. And nobody will be willing to give you have to have best of everything, but it has to be work also, but it is not working because blowing up. Then the people who have some tricks of the trade what professor plays Professor Price who was an expert at instability called a resident black magician, or something. They will have few tricks of the trade, employ it rather than a scientific approach and in the end I was reading this paper by Bloomshill on lessons learnt in solid rocket combustion instability he says all motors have instability or in general.

Only question is whether they are very large ten bar fifteen bar kind of oscillation or is it 0.1 bar 0.2 bar. And either you did something load instability or abandoned the motor you never have a motor which does not have any oscillations. So, you have you generally have oscillations you can do something to reduce the level and you can leave it some

manageable problems or if it just cannot be undertaken may be you drop the motor and use some other rocket instead for the purpose.

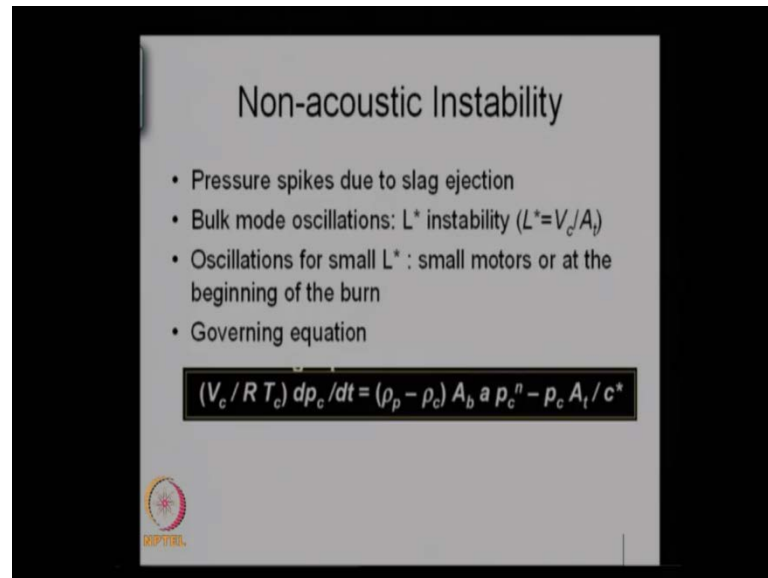
So, I mean the sound very bad, but very un engineering like, but that is the way it is if the rocket works like you can do a performance that is what it is designed to. But it will not perform at all it may just have instability so it is a real thing and you really you sit in this meetings where you try to fix this problem and your hands are tied behind your back and you really you can't change anything, but you still have to solve the problem that is the way it is.

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Just give some codes R S Brown it is incidence of instability said that over 70 present of the SRM sound like motors exhibit oscillatory behavior. The only issue is how much is the problem the problem is always there and professor E W Price in fact he taught me combustion he said more than 50 percent of SRM research funding in the us is motivated by instability and ((Refer Time: 30:27)) from France I think its in onera he said that most motors naturally whistle. So, everybody is saying this so it is unreasonable to expect your rocket motor to be completely silent and so on which is what this guy has said ((Refer Time: 30:45)) it in this a double f paper title lessons learnt in this solid rocket combustion instability. He said that in general all programs had some problem or the other either you can live with the problem or turn down the problem and if you cannot just abandon it.

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The slide is titled "Non-acoustic Instability" and contains the following content:

- Pressure spikes due to slag ejection
- Bulk mode oscillations:  $L^*$  instability ( $L^* = V_c/A_t$ )
- Oscillations for small  $L^*$  : small motors or at the beginning of the burn
- Governing equation

$$(V_c / R T_c) dp_c / dt = (\rho_p - \rho_c) A_b a p_c^n - p_c A_t / c^*$$

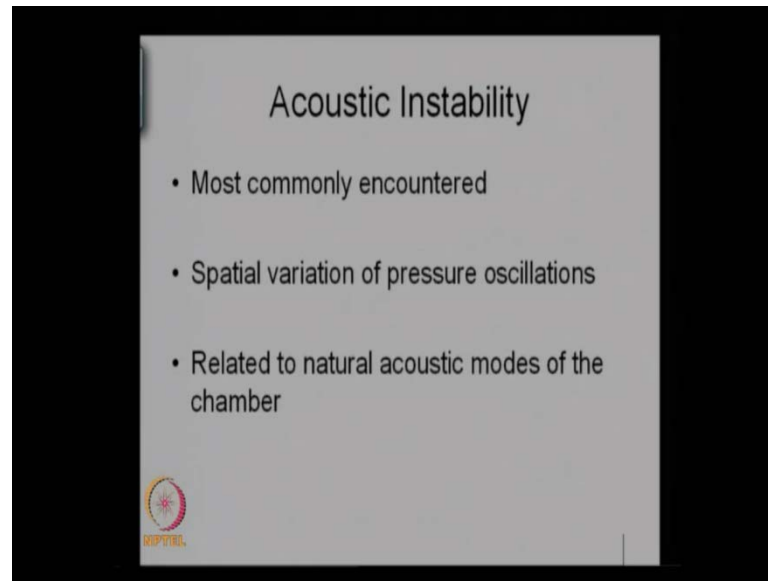
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So, the non-acoustic and the acoustic instabilities, the non-acoustic instability may be because of pressure spikes due to slag ejection or can be because of bulk mode oscillations or what is called 1 star instability, 1 star is volume in motor divided by throat area. And when you are using small 1 star you would not have acoustic mode, but you have the entire pressure in the chamber go up and up. So, normally I am sure we have seen this equation in the this one in the propulsion in one course and you have the rate of change of pressure is going by it is it goes like the difference of amount of gases generated minus amount of gases that is going out of the nozzle.

And we usually say that this equal first term equal to second term and say that this is negligible, the first that is not the case it may not be negligible and when the  $V_c$  is low or the 1 star is low this may not balance. And then the whole pressure in the whole chamber can go up and down and that would be called 1 star instability and so in acoustic instability you will have modes you will have pressure going from a maximum value to a minimum value then back to a maximum value, or a tangential kind of oscillation or radial oscillation. But in this the entire pressure goes up and down that would be the non-acoustic that is why it is called the non-acoustic instability. So, that is a distinct possibility.



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Acoustic stability is of course, most commonly encountered instability in any kind of motor. And when you say acoustic instability there will be spatial variation of pressure and at some places there will be these modes which are often close to the natural acoustic modes of the chamber.

Student:((Refer Time: 33:02)) Is there any situation designing have a particular instability and make sure that instability is causing problem.

Any instability is problematic because simply because you know you are having even a 0.1 percentage of the mean pressure, the mean pressure itself is very large you will cross one thrust oscillations anything more than that cannot be tolerated by the machine people. Plus if you are having a little bit more than that like one percentage of the mean or something, which is actually small in terms of percentages, but it is very large in terms of the structure the structure the way it vibrates. It does not look at the percentage it just looks at the at the vibration field and that is quite large I think it is very hard to say that I will I mean.

I do not think I mean any way of formulation existed this kind of motor which says this much of oscillation, they are coming any way and mostly it will come you do not have to do anything to get it to come. And I think if it comes you try to see how to increase the damping and so on or can you add some additive you can have active control.

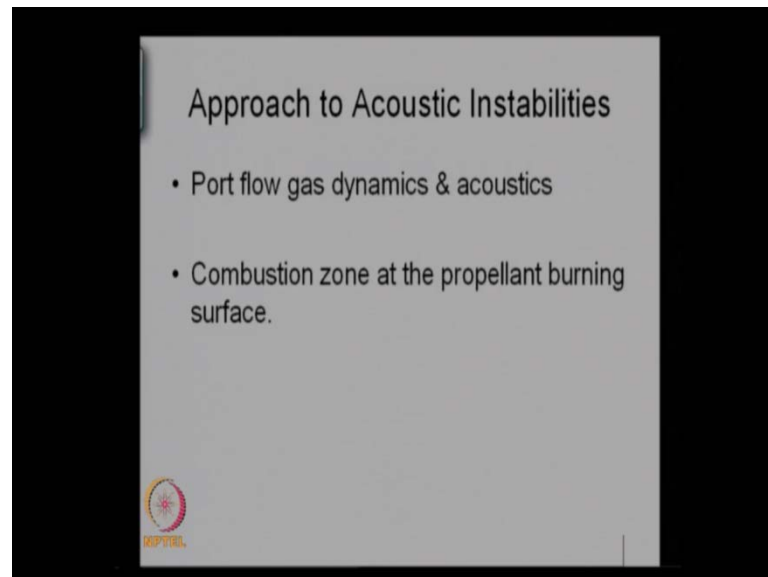
Student: ((Refer Time: 34:20)) When we know the oscillation

I think there is no active control solid rocket motor because you are having two hundred ton rocket with two ton per second and it is about three thousand four hundred degree centigrade and I think there is no way I can imagine a active control at this point. May be some new technology I wouldn't think of it that moment

Student: ((Refer Time: 34:50))

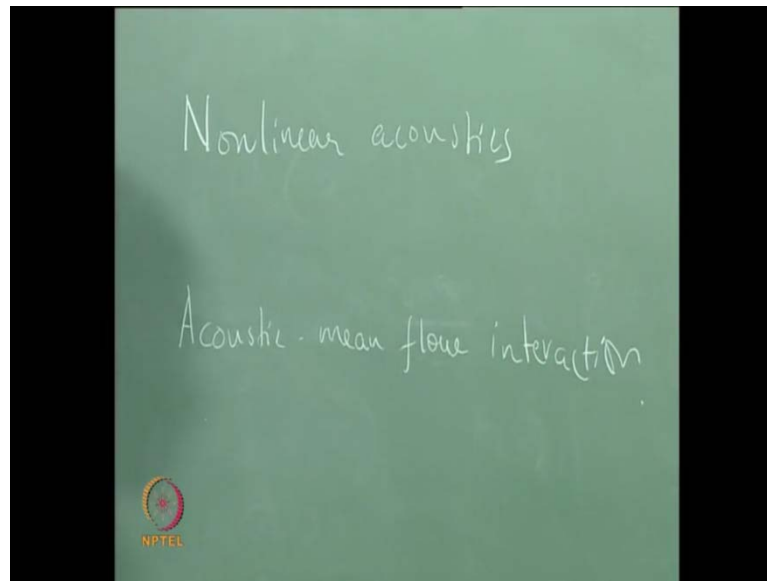
In liquid rockets one could think of it, but there also they would rather no have this they will have lower level instability rather than go with active control.

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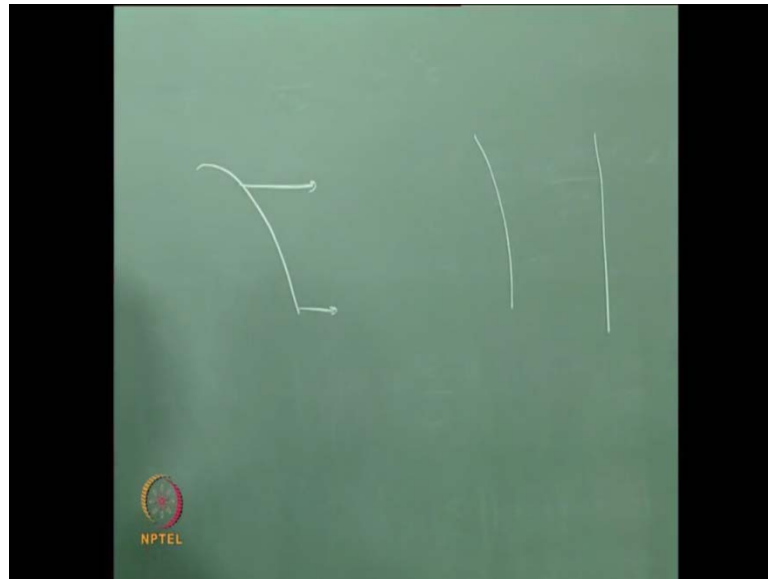
So, there is first acoustic instabilities would be same way as we did for gas turbine kind of combustion a week something. If you have to solve for both flow gas in mix and acoustics and also model the acoustic response. So, the propellant is burning and we have to look at the varied propellant response. And so as supposed to our earlier equations where we non-linear acoustics that means terms that come when the amplitudes are very high, those terms will be quite significant here.

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We have to worry about the non-linear can be gas dynamics unsteady gas dynamics and also worry about acoustic mean flow interaction. When non-linear acoustics is there we will have soft formation and so on wave steepening. So, the if you have a compression wave coming then the as the wave moves into the flow, the wave will heat up the flow. So, the back part of the wave will see a higher temperature. So, then the wave coming from behind will try to travel at a faster speed because simply because it has a higher speed of propagation. And so eventually the front part is going like this the back part is going faster. So, eventually you tend to catch up so you draw diagrammatically.

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So, let us say we have a front like this guy is moving at a certain speed, but here you are trying to move faster. Eventually you will end up this front steepening to form a shock and this really happened with the kind of oscillation amplitude that are seen in the rockets. So, you are not going to see very nice sine wave on you surely non-linear gas dynamics is important and it will take you have to account for that many time.

We also have to worry about the flame and how it behaves I will speak little more about in the next class. So, you have a flame over the surface and the it is a very complicated flame structure and the flame is responsible for acoustic oscillations and therefore, the mass addition which is coming out, I mean earlier you have a situation where there is no acoustic field steady mass flow. But then that would become unsteady and that is what is driving the acoustic oscillation, we look into it little bit more in the next class.


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Imaginary part of the eigenvalue is growth rate

$$\omega = 2\pi f + i\alpha$$

Complex eigenvalue      frequency      Growth rate

Periodic      Exponential growth/decay

$$p' = \hat{p}e^{-i\alpha t} = \hat{p}e^{-i(2\pi f + i\alpha)t} = \hat{p}e^{-i2\pi f t} e^{\alpha t}$$


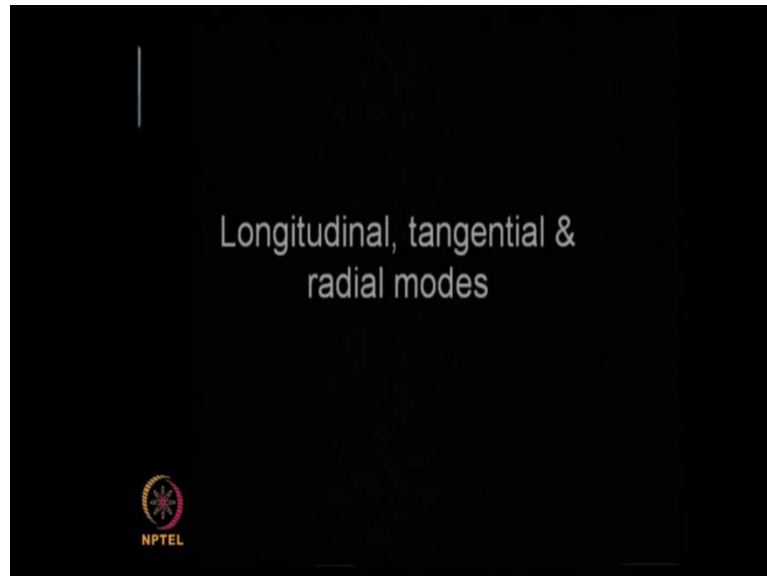
So, there is classical linear analysis which is what we do with this acoustic theory we try to set up the acoustic equation, we modeled the term that are driving just like we did earlier in our class, like our n term model in fact n terms models came from rockets it came from liquid rockets. Coco was working on liquid rockets when he made the model. You have to model the driving terms and you have to model the acoustic field and the propellant, in solid rockets most of the rockets are very long so only acoustic may be fine. But we have to account for variable area because you know the some part may have stars and some other part may have increasing cross-section or decreasing cross-section.

So, if we can model the acoustics and if we can find out what the terms that drive and damp either from the flame on the surface then we should be able to solve for the Eigen values of the problem. And to get the driving you need to model the combustion zone, and damping we will look at the factors that are involved in creating the damping. So, we will be solving for a complex omega which is Eigen value complex, Eigen value which is  $2\pi f$ ,  $f$  is the frequency  $i$  times  $\alpha$   $\alpha$  is the growth rate depending on the sign can be growth rate or decay rate.

So, if we write  $p$  hat as  $p$  power minus  $i$  omega  $t$  and you have  $e$  power minus  $i 2\pi f t$  which is the periodic component. And there is this  $e$  power  $\alpha t$  which is depending on the sign of  $\alpha$  is exponential growth rate or decay rate and so. In summary all the

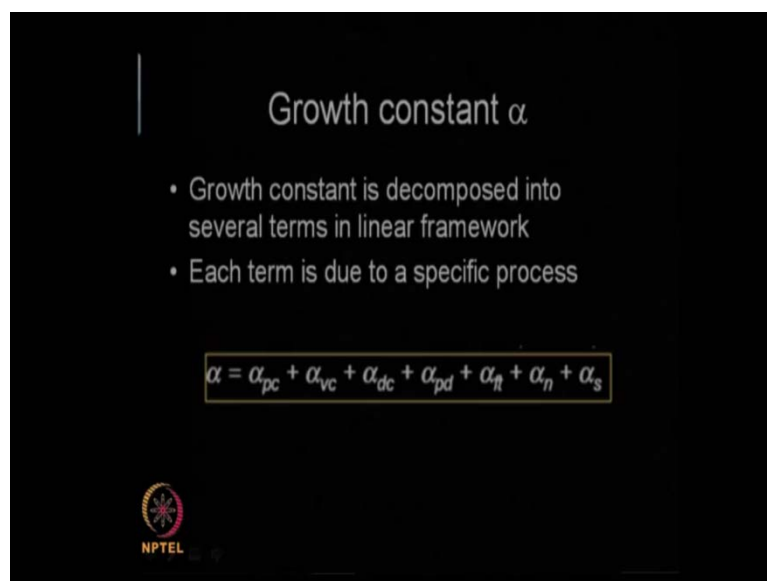
effort is towards finding the imaginary part of the Eigen value, which is called the growth rate or the decay rate.

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So, we have longitudinal or tangential or radial modes radial modes are not that frequently encountered, longitudinal modes are very often encountered because simply because rockets are very long. But sometimes for the rocket for upper stages which are not very long sorry rocket also you can see tangential modes.

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So, we aim for getting the growth the growth constant  $\alpha$  and the growth constant can be decomposed into several terms in the linear framework. So, the way the theory is developed you can actually find growth constant due to individual effects, and then add them all up and you can get the total growth constant that is the way. Am not going to work out the full theory, but give you a brief glimpse of the theory. So, each term so you have a growth rate or decay rate it is consisting of contribution from several different processes.

So, you can have propellant combustion that gives a response which is can be in form of a pressure coupled response or which is what is called  $\alpha_p c$  here or you can have  $\alpha_v c$  which is a velocity coupled response. The pressure coupled response is a term which is non-controversial at the pressure is oscillating the flame above the propellant also oscillates. And therefore, there will be oscillatory mass addition coming in and then and this term will invariably drive the acoustic field. So, that is the pressure coupled response mode. Velocity coupled response is usually kind of controversial term, there you might have heard about erosive burning, what is erosive burning?

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Why does it happen?

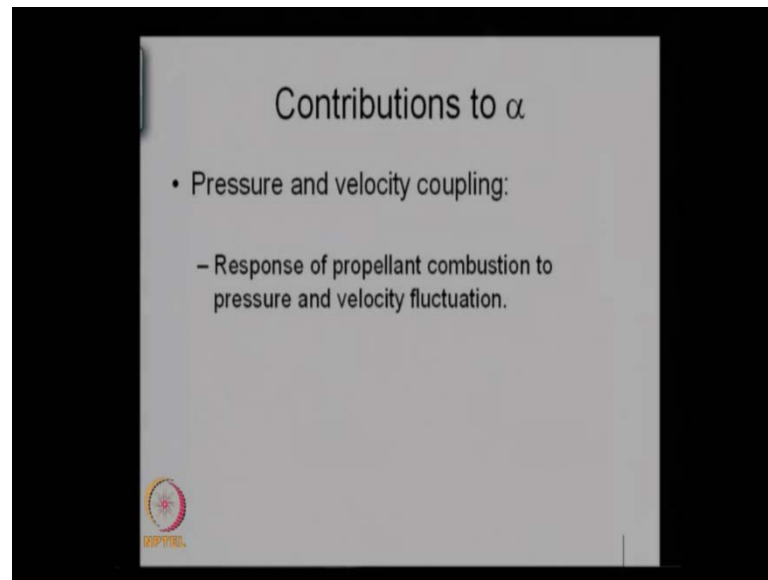
Student: ((Refer Time: 41:44))

So, why does this velocity gradient cause this velocity coupled response if there is flow why will the burn rate change reaction rate. The pyrolysis rate can change heat and mass transfer can change so the propellant the burn rate can change. Now, if there is oscillatory flow also all these things can happen, but I think the way the term is interpreted there's quite a bit of controversy. But everybody accepts that such a contribution is there, but how to model it is what is the problem.

And then there is distributed combustion so you have a metallized propellants. So, if you do not have metalized propellants then the combustion may be over right, over the propellant surface. But if you have metal surface this metal may even continue to burn away from the propellant. So, that will can have some contribution and  $p_d$  would mean  $\alpha_p d$  is like particulate damping, particulate damping would I mean when you have two phase flow. That is when aluminium burns to give alumina and that is coming out

through the flow and that is trying to take out energy out of the system. Then  $\alpha_{ft}$  is the flow turning losses that is you are having flow coming out of the propellant, but then the acoustic field is perpendicular to it flow is turning, then  $\alpha_n$  is  $\alpha_{\text{nozzle}}$   $\alpha_s$  is  $\alpha_{\text{structure}}$ .

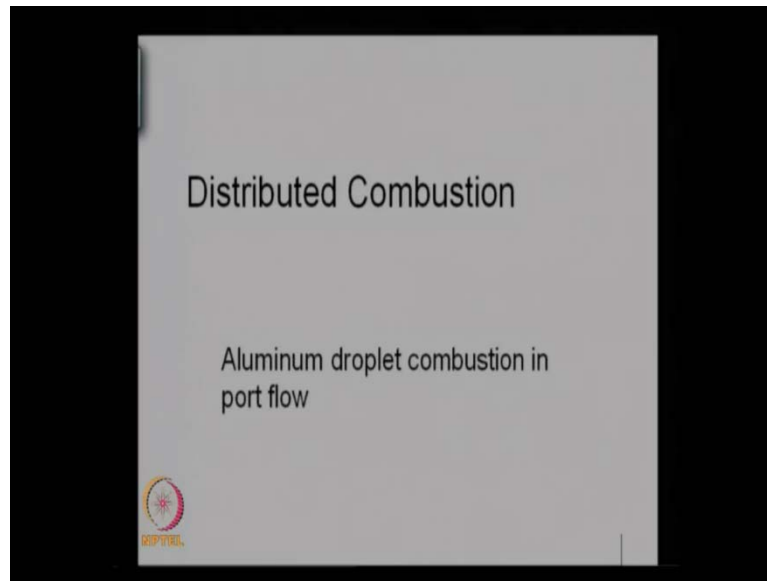
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So, the contribution to  $\alpha$  from pressure and velocity coupling this is the response of propellant combustion to pressure and velocity fluctuation. And in the sense given a velocity fluctuation or pressure fluctuation how the mass flow that is coming into the port how that how that oscillates.

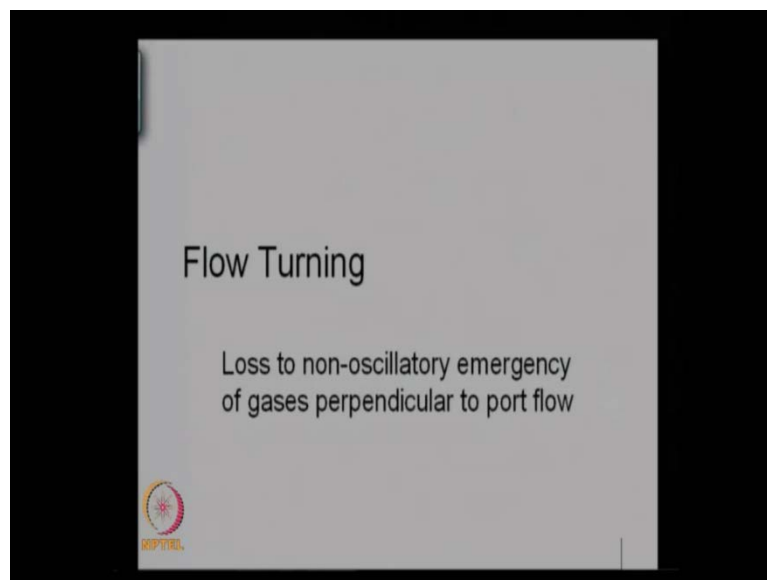


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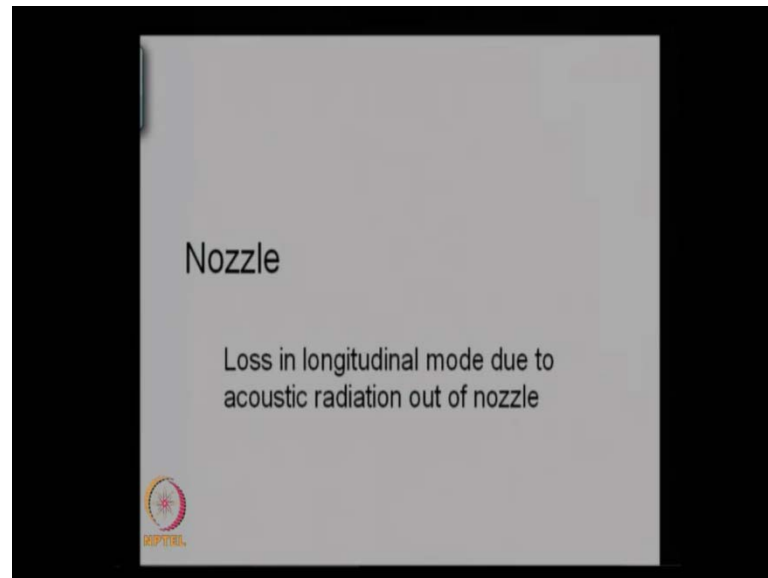
Distributed combustion is when there is aluminium droplet combustion in port flow, port is a hollow thing in the middle. So, when you have aluminium particle there can be other particles also they can also burn.

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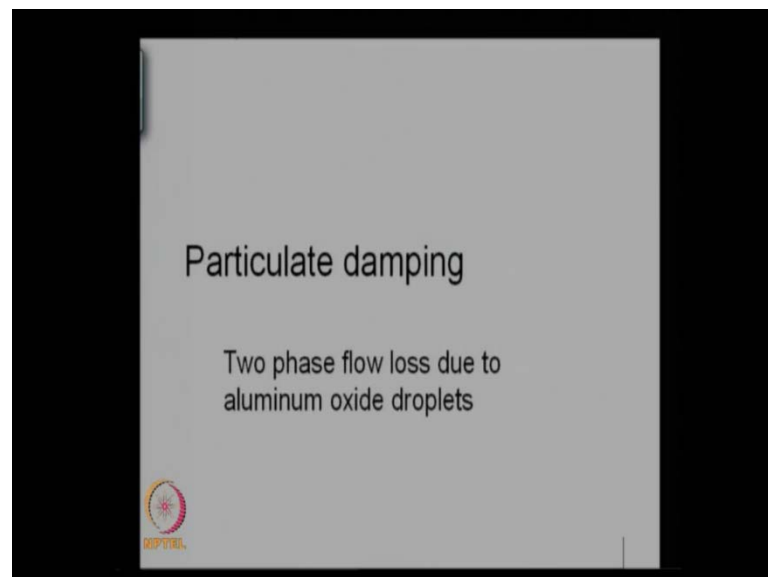
So, turning is the last term is last like you have gasses emerging in a non-oscillatory way out, and then that is turning into the acoustic field. And so in this process it takes some energy from the acoustic field. Gases are come out radially, but then they turn so in this process they take out some energy.

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Nozzle loss is the most dominant loss mechanism so you have acoustic field inside the rocket and nozzle is usually reflects most of the acoustic energy. But some amount of the energy will be leaking out and radiating out as radiation from the nozzle, this is the like nozzle loss factor.

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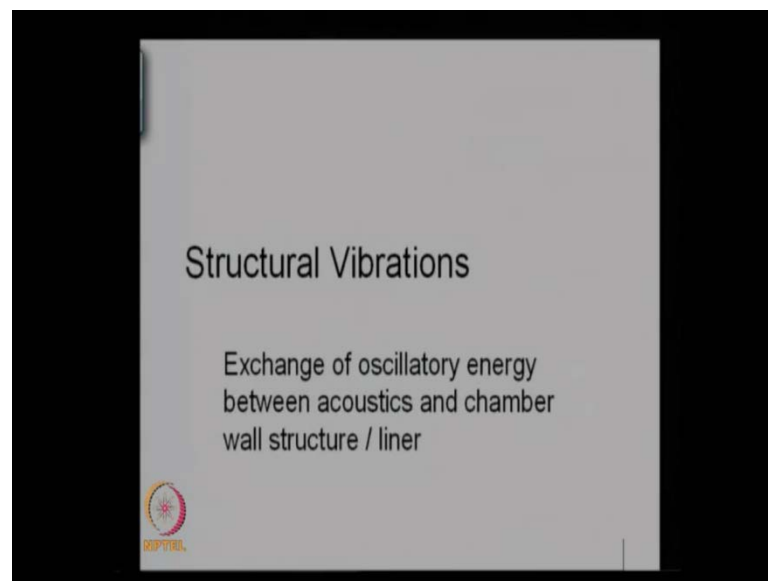


Then you have metal combustion you have aluminium oxide some other oxide. So, there is two phase flow and the droplets they take energy from the propellant, and in this from the acoustic field and start oscillating. So, if you have a droplet in an acoustic field it is

like the acoustic field will drag the droplet back and forth. And in this process now initially all the energy is with acoustic field and now the energy is taken a part of the energy. That means as the droplet gains energy the energy in the acoustic field is coming down so that is the way it is damping. It is because of mainly because of drag it can also be because of is the droplet is taking some heat, the heat transfer and the mass transfer that also can cause losses.

So, different type of instabilities frequencies need different type of particles. So, you in fact if you have low frequency oscillations, particle which are slightly bigger in diameter they are the best to kill the oscillations. And if we have high frequency oscillations we have to use corresponding smaller particle size because you can imagine, if we are having high frequency oscillation, if you have big particle the particle will not oscillate or anything it will just like it is huge and it won't care. But if you have small particle it will respond to it, for bigger particle it may be for a bigger particle it may be easy to respond to low frequency oscillations that is the scaling.

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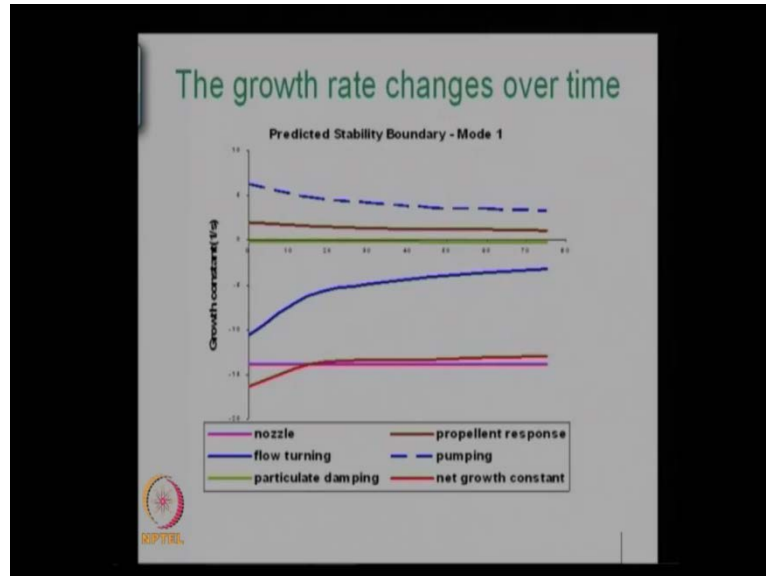


The structural vibrations this is the where it is insignificant actually exchange of oscillatory energy between the acoustics and the chamber wall structure or liner. But perhaps for a big motor this may not be that insignificant and if there may be a big liner and the liner may be able to take some amount of acoustic energy and do the damping.

Student: ((Refer Time: 46:33))

So, the flow itself will take energy from acoustic that is the flow turning losses some amount may be other kind of losses, but so people have listening standard stability program that is the American version the other versions I have some version of my own.

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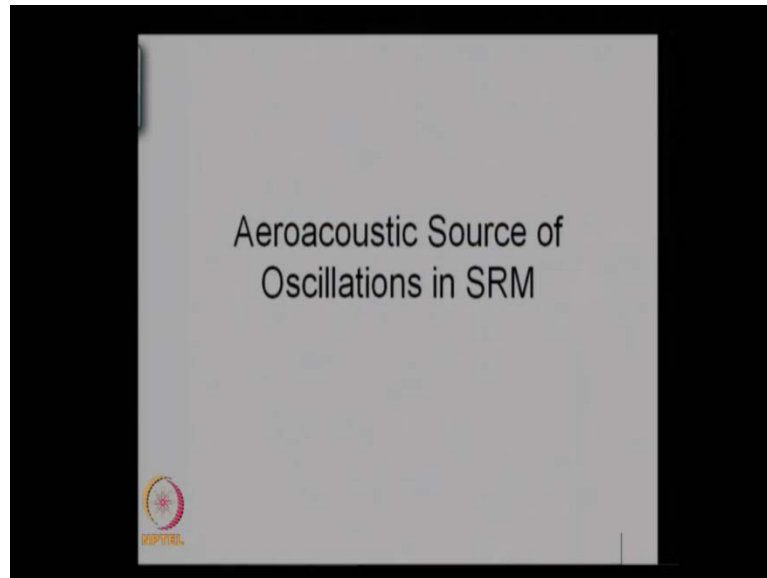
So, you can calculate all these values I think most of you know Priya she wrote this program. So, you at different times the point that does not work, so this is time. So, at different times you are calculating these various alphas, so the pink is nozzle, blue is flow turning, green is particulate damping, black is propellant response, blue broken line is pumping, pumping is because you we look at invisible acoustic field, but actually at the surface the acoustic field comes to rest. And then you are having some oscillatory mass addition there this actually creates some kind of acoustic driving.

So, you add up together so you have a net growth or decay, so in this case red line is the net growth rate and in this case it is negative, it is actually decay. So, you have to evaluate this at all times and then you see what is the sum and when the sum stay negative at all times that is what we are looking for. And if a general rule of thumb if the standard stability, stability program says the motor is unstable surely be unstable. But there is quite a big possibility that your program may stay stable and it may still be unstable.

But generally the opposite doesnot happen the program may stay unstable and the motor may still be stable that is usually not encountered. So, you can predict this for first mode

or second mode or third mode, you will never get very high modes, but the first few modes you should take and look at them and you have to look at the prediction all time.

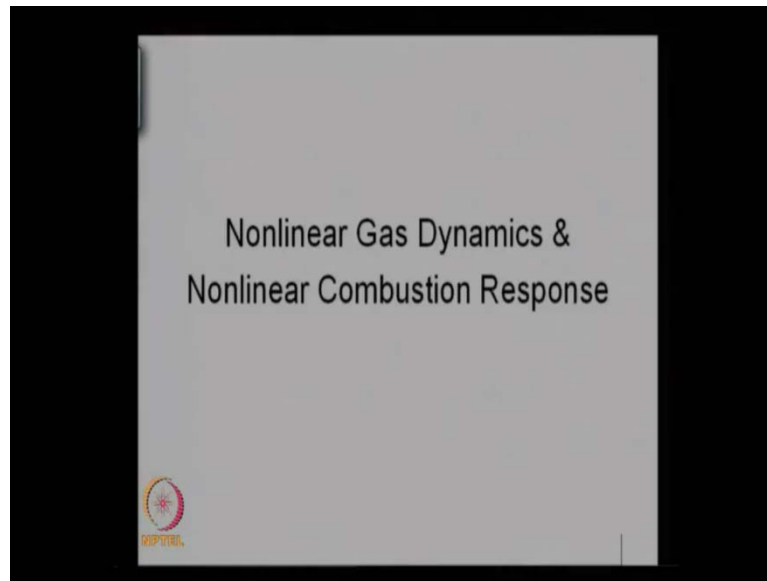
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We also saw earlier vortex shedding we looked at the vortex shedding and it also can cause oscillations in full rocket motor, I think to remember that we said vortex shedding from the inhibitors. So, a vortex can be shed from an inhibitor it can come and interact the next inhibitor this can create sound or the vortex can flow over the nozzle and excite sound over the cavity and the even if the no inhibitor a radially injected flow propellant burning is like a radial flow coming into the port. And that will automatically give sufficient length that will roll up into vortices and solid rocket has sufficient length. So, you will have vortices with all these things that is why ((Refer Time: 49:35)) said all motors whistle so there is an acoustic sound.

Now, this is not a propellant response kind of thing, but if this couples with the propellant response, we can even have the large even more violent response. But even otherwise it can give sufficient problems to make it give sufficiently large amplitude on its own.

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So, the stability program that I showed earlier there we have linearized everything, but in reality there is non-linear instability. So, there is non-linear gas dynamics and non-linear combustion response and their relative importance. So, the non-linearity's have played two roles one is limiting the limit cycle amplitude in fact this non-linear term many times they take energy and they actually won't let that indefinite growth happen, and that is the main effect of the non-linear gas dynamics. There is also the combustion response the acoustic field is non-linear and triggering has been attributed to that. I will of course, some of you must have been thinking this is the analysis what we did for ((Refer Time: 50:46)) can we do it for solid rocket motor. So, if I have tomorrow, I will briefly show how triggering happens in solid rocket motor.