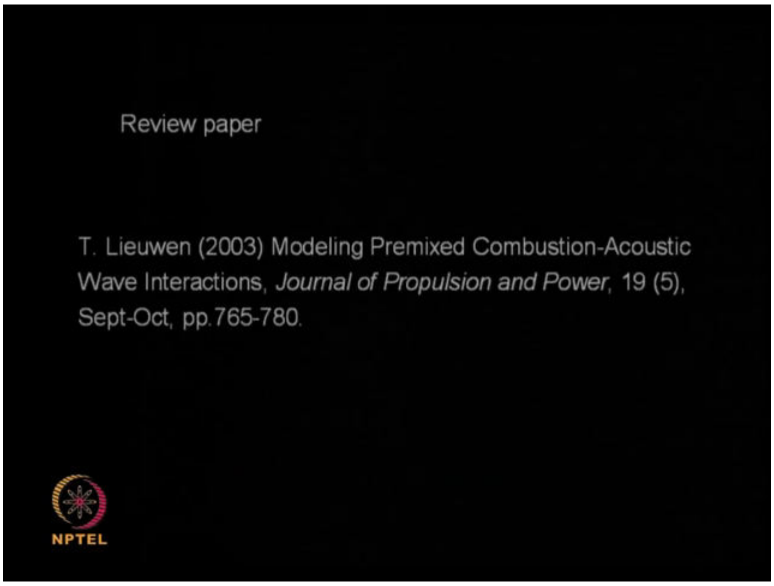
Acoustic Instabilities in Aerospace Propulsion
Prof. R. I. Sujith
Department of Aerospace Engineering
Indian Institute of Technology, Madras

Lecture - 31
Premixed Flame Acoustic Interaction-2

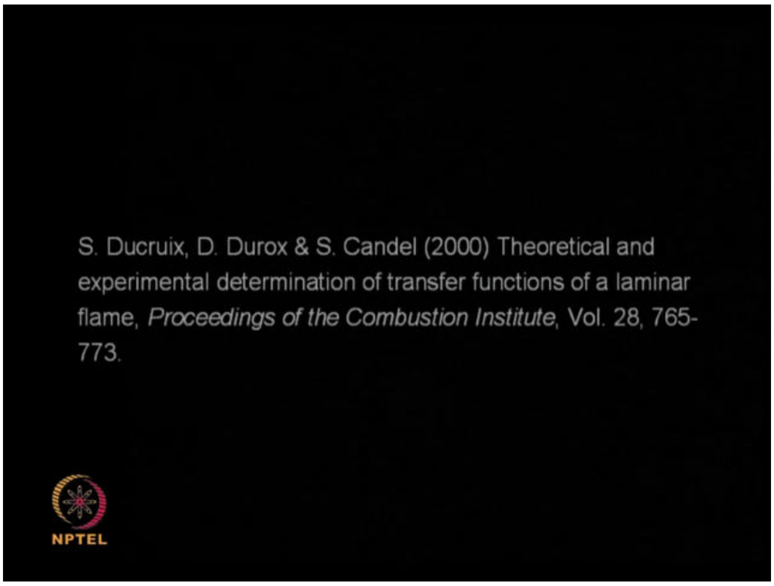
Good morning, everybody we are talking about flame acoustic interaction, we are specifically looking at premixed flame. So, if you consider a premixed flame such as Bunsen burner located in a duct and we are seeing how the thermostatic instability happens. Let me give some references for this topic, the first one is a review paper.

(Refer Slide Time: 00:28)



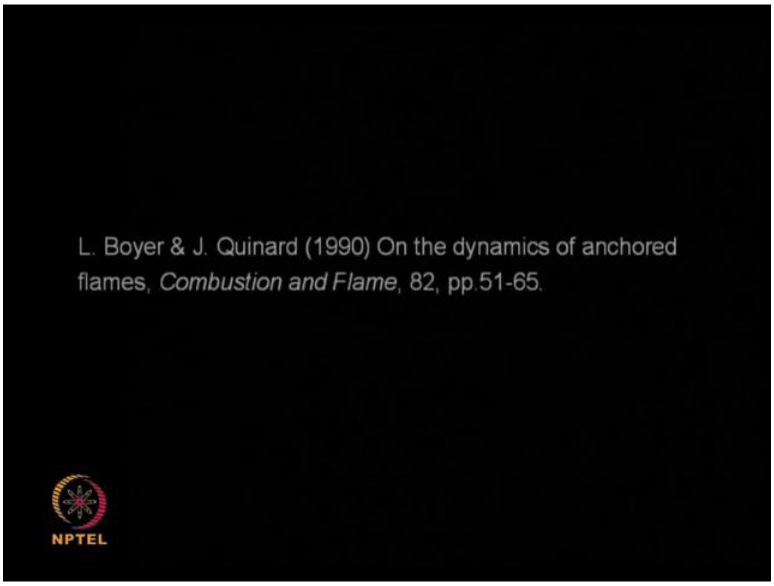
Review paper would mean it talks about all the papers and what they do in certain order and in certain classification. So, this is the paper where T. Lieuwen title modeling premixed combustion acoustic wave interactions, this in general of propulsion and power. They had a special issue on combustion instability so this is that issue it is in 2003 volume 19.

(Refer Slide Time: 00:59)



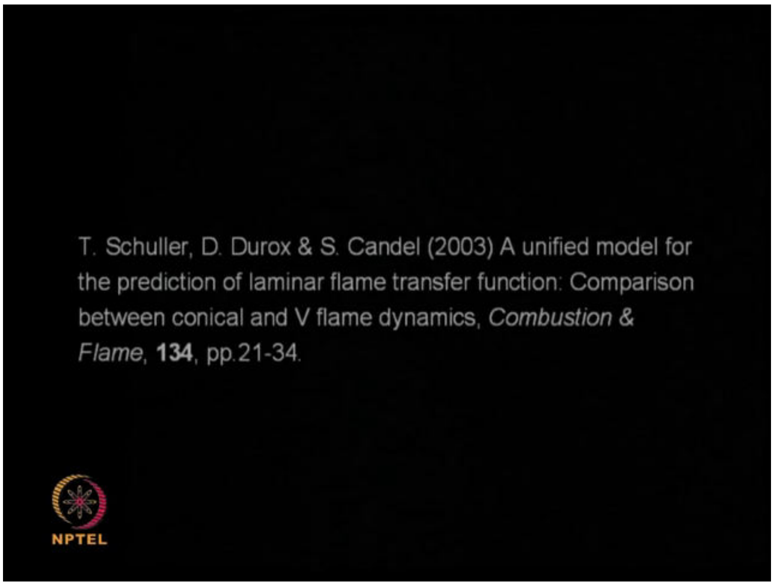
There is another very interesting paper by Ducruix, Sabastian Ducruix then Durox and Candel, theoretical and experimental determination of transfer functions of a laminar flame, proceeding of the combustion institute. The combustion institute has a international combustion symposium every 2 years whose papers are published in this bond volumes so, this is there very prestigious conference. You say short idea of how you can calculate theoretically as well as also determine experimentally I will at the end of the lecture speak very briefly about how to do things experimentally.

(Refer Slide Time: 01:35)



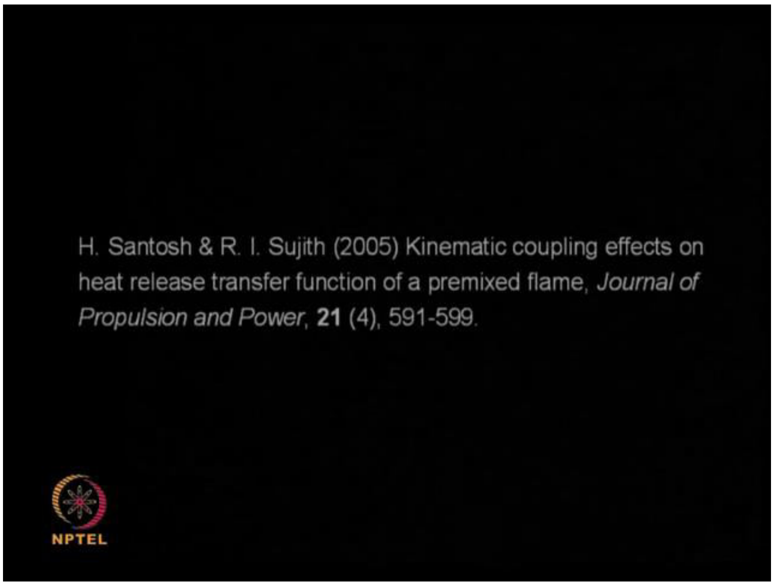
This is a very interesting papers I can give you all these papers I have with me Boyer and Quinard and this is perhaps all the earlier papers on the dynamics of anchored flames in combustion flame. Combustion flame is a very important journal indeed in the easy of combustion.

(Refer Slide Time: 01:56)



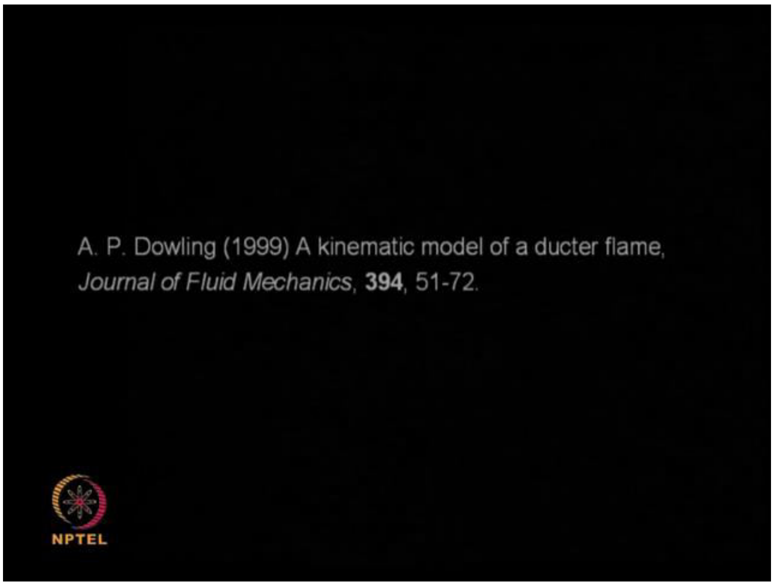
This is really a nice paper by T Schuller, Daniel Durox and Sabastian Candel a unified model for the prediction of laminar flame transfer function comparison between conical and V flame dynamics again in combustion flame this is you must read this.

(Refer Slide Time: 02:15)



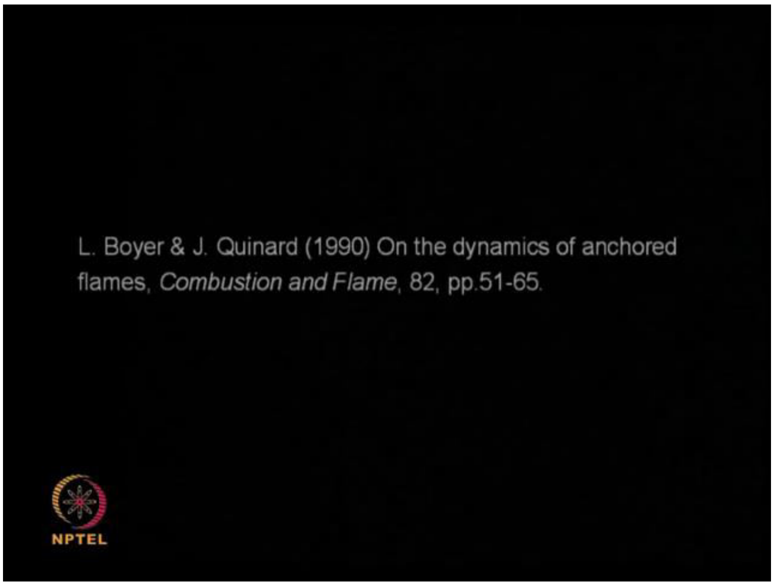
This is my own paper it talked about kinematic coupling effects on heat release transfer function on a premixed flame, speak about 2-dimensional effects. And so again in journal of propulsion and power and volume 21.

(Refer Slide Time: 02:26)



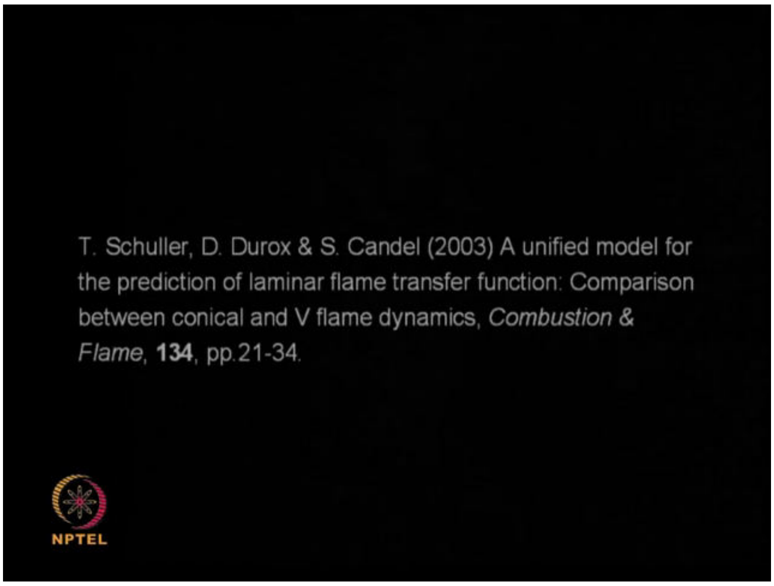
This is Dowling who is quite famous personality, some acoustic she has written a paper a kinematic modeling of a ducter flame, journal of fluid mechanics volume 394. So, I think this should be adequate references there are many more references may be 100s of papers. If you want you can meet me and I can give you these papers I hope you have adequate references and please.

(Refer Slide Time: 03:00)



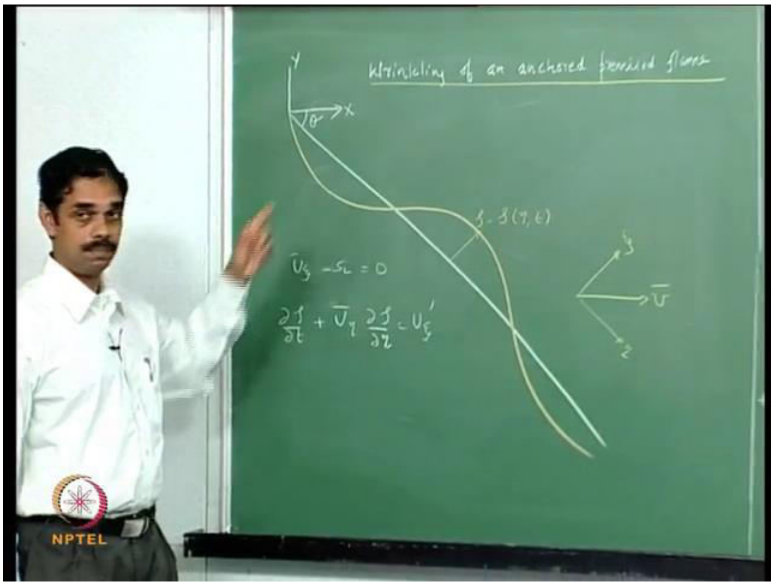
This is very easy to work out this paper it is quite nice, if it is a very first or the very first paper.

(Refer Slide Time: 03:09)



Definitely, go through this papers and I am promising to ask in the exam something from this paper so that is so much for this today.

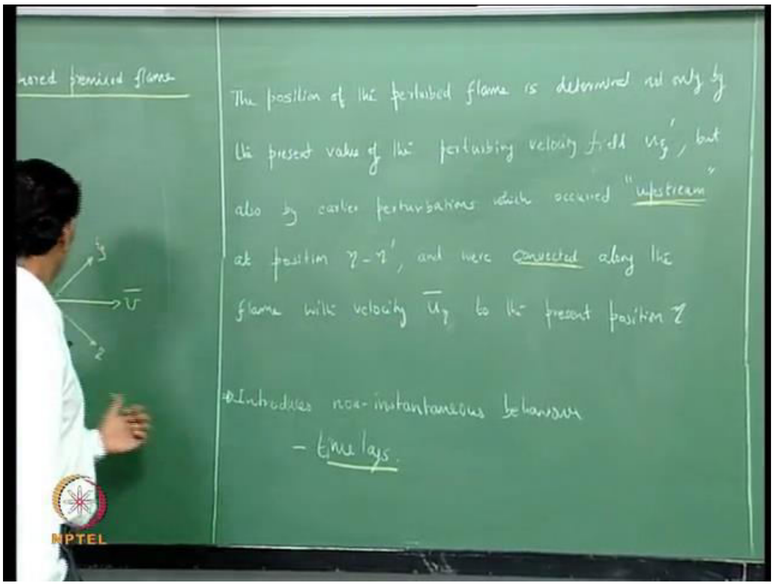
(Refer Slide Time: 03:27)



Now, we will in last class we were looking at trying to work out difference equations, simply differential equations for looking at wrinkling of an anchored premixed flame. And we were looking at a geometry where this white line shows the means flame shape and zeta is like the displacement of the flame with respectively to the mean flame shape. And you have this relation U psi bar minus S L equals to 0, that is a consistent with our idea that the flame will stay if the speed velocity same at the flame propogation velocity.

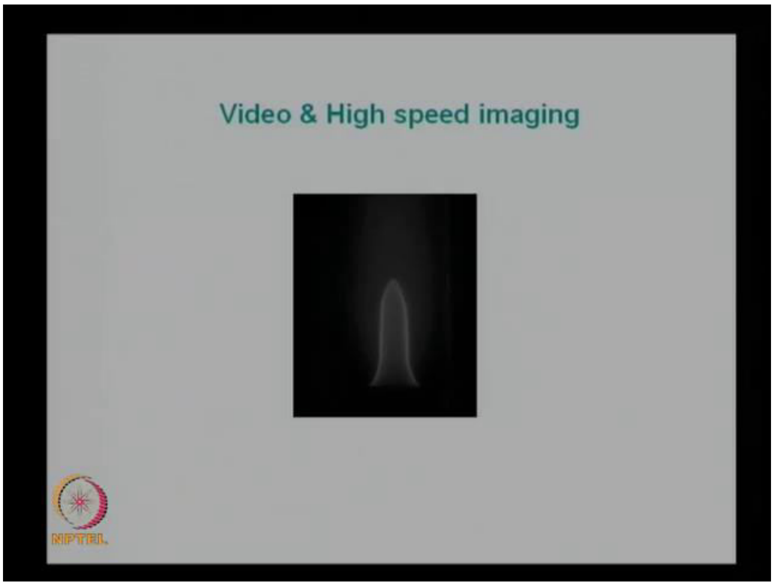
Otherwise, if there is no speed velocity flame will travel at the speed of S L, but if you supply gas at a speed of S L flame will stay there and this is the differential equations we got for the perception for the wrinkling, U zeta by I mean duo zeta by duo T plus U eta bar duo separate duo eta equal U psi prime.

(Refer Slide Time: 04:27)



And this has a very nice physical meaning what does mean is, what does it means is position of the perturbed flame is determined not only by the present value of the perturbing velocity present value of course affects. But it is not just that, but also by earlier perturbations which occurred up stream at positions eta minus eta prime. And convicted along the flame with the velocity U eta bar, that is there is a flow velocity in this directions and that convicts the wrinkles up of the flame to the present position of the eta. And we saw in the movies that I showed that the flames are actually there is a wrinkle and it is going up.

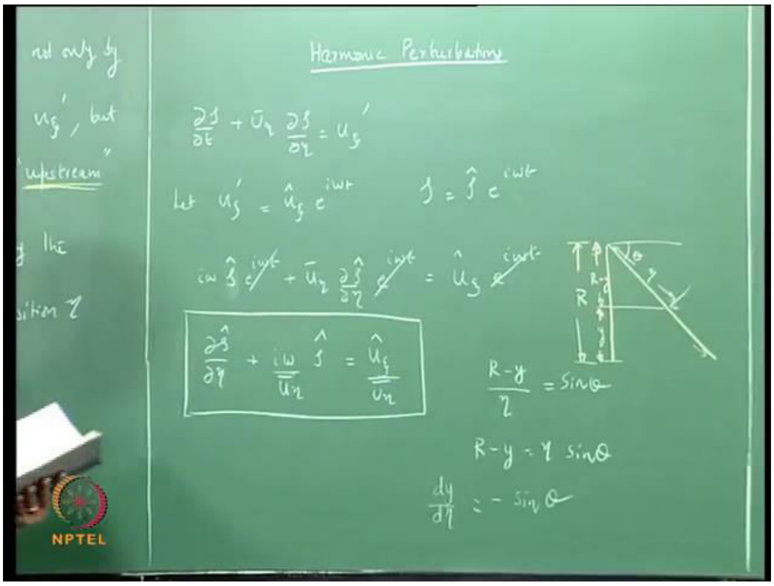
(Refer Slide Time: 05:04)



Let me show that we see the wrinkles that are formed here they are actually moving up along the flame so they are actually convicted, that means by convection. So, our different equation is saying it solution it saying it, so this actually introduces some kind of non-instantaneous behavior which is like a time lag and we know the importance of time lagging thermo system. So, we must pay attention to the system, the convection the flow is what creates this time lag here.

So, we having look at things in time domain will now also try to do things in harmonic domain, which often harmonic domain things are simpler all though. Here, we got exact solution an so on with characteristics we can solve, but in harmonic domain things are much simpler and you can get simpler solution which will also tell you lot of things so, let us do that.

(Refer Slide Time: 06:14)



So, let U psi prime equal to U psi hat e power i mega t, of course we can say that we are take real part of that, but I would not write the explicitly it is implicitly understood that you have to take real part. And we will substituted this here so, you will get i omega U psi hat times e power i omega t plus U eta bar duo eta so, this could be this velocity fluctuation could be imposed by acoustic field. So, we can cancel these thing out and up so we will sorry I put a hat here because so we cross out e power i omega t and then we can write this equation nicely as i omega this is our equation.

I will do sometimes transformation just to get very simple solution otherwise, also you can do it you can see it directly integrate this how to integrate this equation.

Student: Direct.

Direct, how to integrate this equation and close form actually they are how I will do it.

Student: ((Refer Time: 09:25))

It is a OD.

Student: ((Refer Time: 09:35))

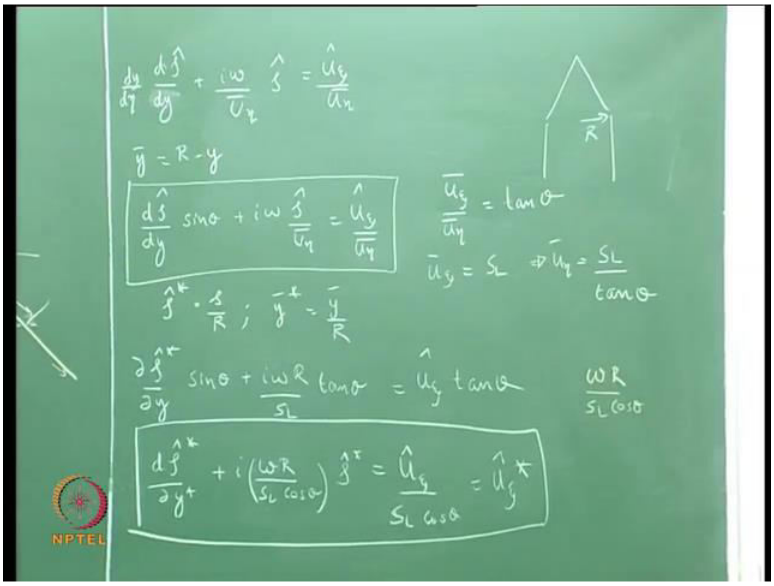
Directly write the solution we do not have to do any recursive.

Student: Integration factor.

Yes, right absolutely integration factor so you can write thank you, but I will just try to do some massaging to get shorter solution same procedure, but before that I want to say that we can notice this term omega over U eta, U eta bar. What kind of non-dimension number is that, let me study in fluid mechanical lot of (( )) number or combustion lot of non-dimensional number in fluid mechanics and combustion, we are studying such as ((Refer Time: 10:16)) number. So, this is also some such number omega over U bar, what it we will see is.

So, let us make a transformation here, let us call this R and we have eta going this way so this is vita eta, this is theta this is y and this is R minus y. So, we can say R minus y over eta, what would this be equal to sine theta right or R minus y equal to eta sine theta. So, we can also, we can non-demonstrate this way and we can get d y by d eta equal to minus sine theta.

(Refer Slide Time: 12:13)



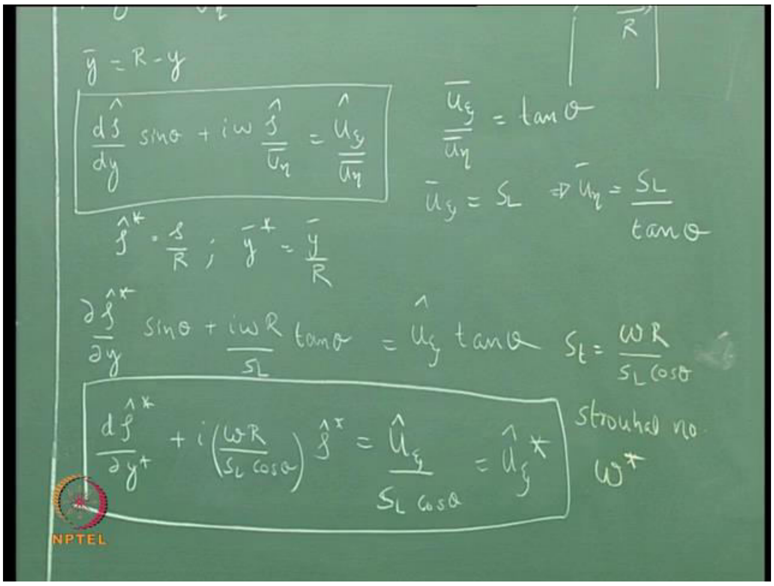
So, if you substitute this things you will get d psi hat over d eta plus i omega u eta bar, this we can write as d y into d y by theta. So, this is a chain rule so, this we can sine theta and we can do one more transformation y bar equal to R minus y just to make things look good just to get rid of the minus sign. So, now we have this equation and we also know that what is this U psi bar by U eta bar, tan theta exactly and we know that U psi bar equal to.

Student: S L.

Yes, absolutely perfect so we know that u eta bar equal to S L over tan theta. So, now we can try to put this and non- dimensionalized so, we can non- dimensionalized the y and zeta by some kind of radius or if you have a burner like a Bunsen burner. And you have flame like this and this is the burner radius that could be used to non- dimensionalized the line that is variation able. So, if you do this you will get put this here or we can club things together and I just yet to sine theta so we have this is non-dimensional, this is non-dimensional and this is also non-dimensional.

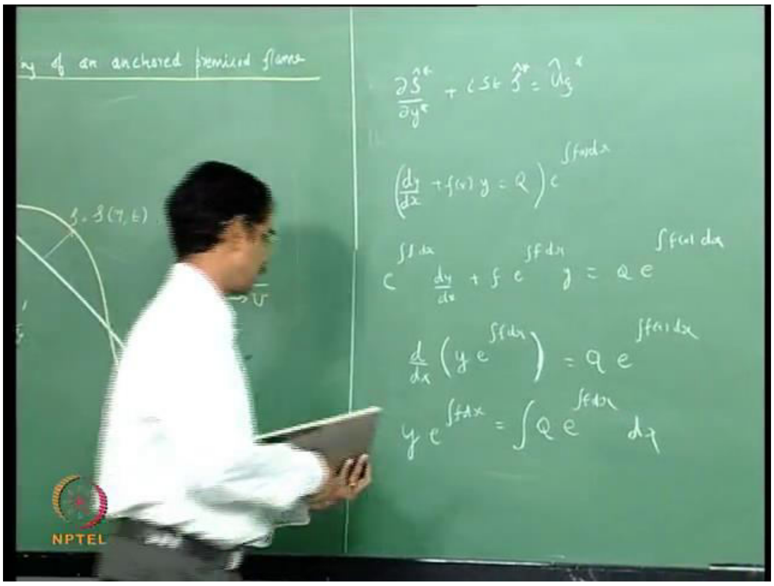
And you have another factor here omega R over S L cos theta what is this, what kind of number is it heard of Strouhal number, what is Strouhal number you have studied fluid mechanics what is definition Strouhal number, what is Strouhal number some F L over U something. This is something like that some omega is like F times R over some velocity. So, it is like the there is a fluctuating it has a frequency and at a time period and the flow also as a time period. So, the flow times scale divided by the times scale of the fluctuation so that is what it is we can so this is like a Strouhal number S T.

(Reference Time Period: 17:00)



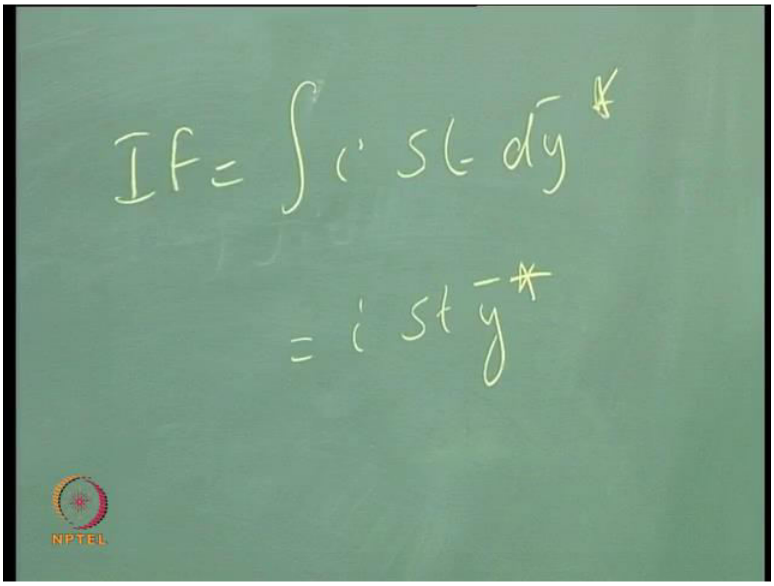
In some papers this term is called omega star, it is like non-dimensional frequency Strouhal number is like a non-dimensional frequency, in non-dimensional like the frequency with another frequency with the characteristic of the flow. So, is this clear to how we got here now we can get the solution right with integration factor?

(Refer Slide Time: 17:35)



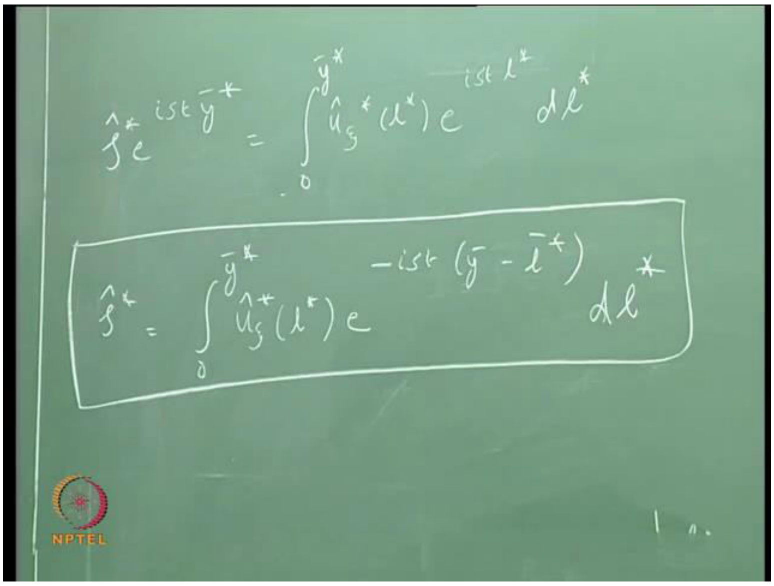
It looks like most people do not know how to integrate this so if we have a differentia equation of this form, what we can do is we can multiply this equation by e power integral f of x d x. And then you will get e power integral f d x d y over d x plus f times e power integral f d x y equal to q power integral f d x. So, this left hand said is what it is d by d x of y e power integral f d h equal to q e power integral f d x. So, integral of this the solution would be y e power integral f d is equal to integral q e power integral f d x d x. This is the solution to linear differential equation the most elemental differential equation which is studied in engineering. So, we can use this solution to solve this so what is the integration factor here.

(Refer Slide Time: 19:22)



I must say that I forgot those bars, there is bar here as well as here so what this so galsarise to well this simple.

(Refer Slide Time: 19:57)



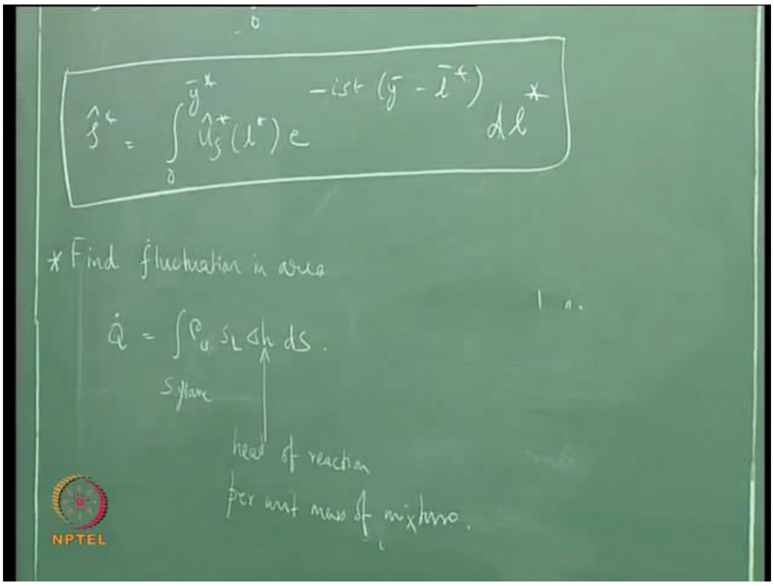
Now, this l star this like a dummy variable I have used for integration you can put time prime anything like that h d f so this is the closed form solution. This is just use this integration factor and we have return down the solution this way very simple. What does this formula physically mean this is linear co-relation between U psi star that varies your acoustic wavelength and e power i s t l star, which varies over convective wavelength. So this co-relation be in two quantities one varies over acoustics length scale and one varies over convective length scale.

Now, we know how to get the flame shape I mean we know how to calculate this perturbation is location that is what it is, this is the formula for that, you are right. Now, how do you proceed to calculate the heat released rate given this. We know in principle the flame shape not in principle in practices you can if we know this quantities, as long as you know this omega R over S L cos theta you can get this, this you can solve.

Student: Sir, you find the area.

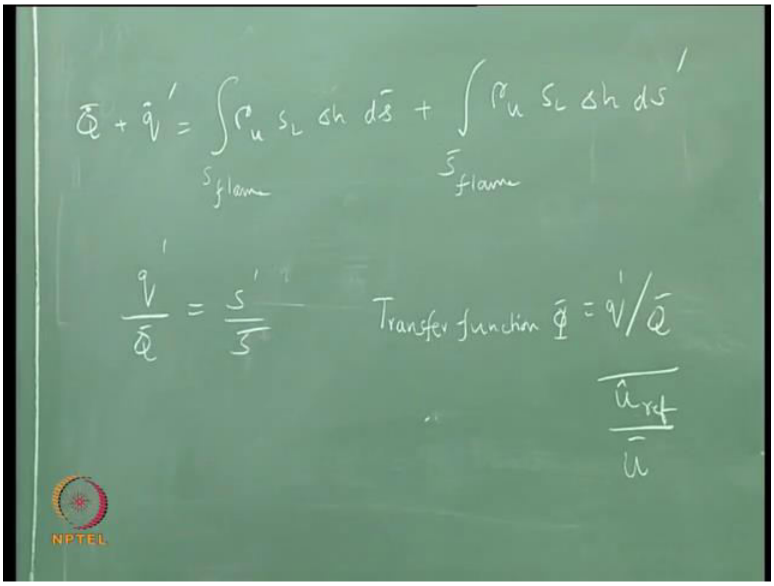
You find the area how do find the area so depends on the flame whether it is a flat flame or a circular flame, if the coned you have to use cone formula. If it is a 2D we just have to add up so, we find the area fluctuation from there how do you do.

(Refer Slide Time: 22:39)



We find area fluctuation area then the next step. So, we can say that Q, Q dot equal to integral over the flame, surface of the flame rho u S L, you do not worry about rho u we just say rho u it is the correct thing. And delta h d s, delta h is the heat of reaction per unit mass of the mixture, mixture means ((Refer Time: 23:45)) mixture to so can think of this as mean plus fluctuation.

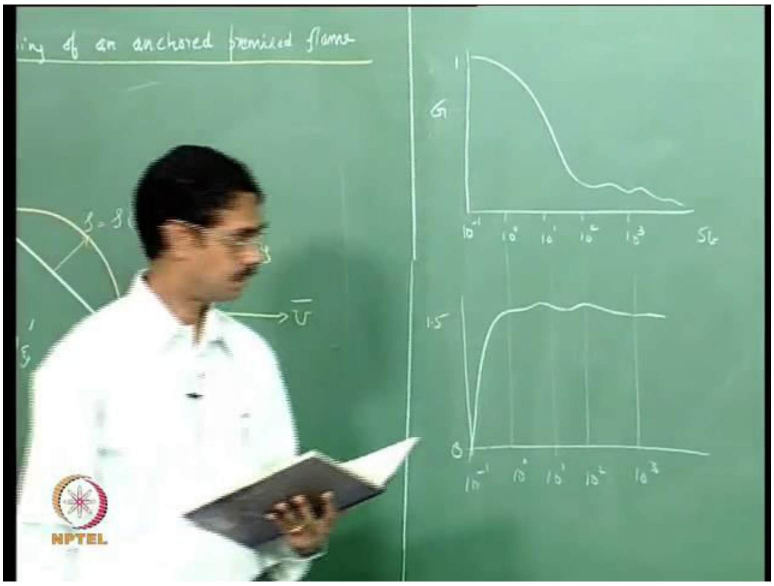
(Refer Slide Time: 23:51)



So, we can say Q bar Q dot bar plus q dot prime equal to omega s flame rho u S L delta h d s bar plus s bar flame rho u S L d s prime. So, q prime by q bar is like s prime by s bar so, we find the s prime as which no point allowed and then we have this Q prime. So, we can now define a transfer function q is equal to q prime divided by q bar divided by u hat reference divided by u bar.

Where, u hat reference as the reference velocity typically it shows velocity at the base of the flame and this could be and the system and the deification languages this is like a flame response, frequency response function transfer function. So, you can use this formula to reproduce the result in surely 2003 in which I told you in the beginning of the lecture and I will sketch how the transfer function will look like and you can calculate it.

(Refer Slide Time: 25:41)



This transfer function is called phi, there in that paper he refers to it as G so I will just use it as G. So it will look like a value going from 1 down to 0 and this would be this a provide number. So, typically it look like 10 power minus 1, 10 power 0 then 10 power 1 10 power 2, 10 power 3. So, the flame response variable low frequency oscillation then cuts of the high frequencies and then the phase will look like this. So, same axis 10 power minus 1, 10 power 0, 10 power 1, 10 power 2, 10 power 3. So, it would look something like this.

Student: Sir, is it compulsory that the flame should behave the same way at low frequency, can you tell me the mechanism.

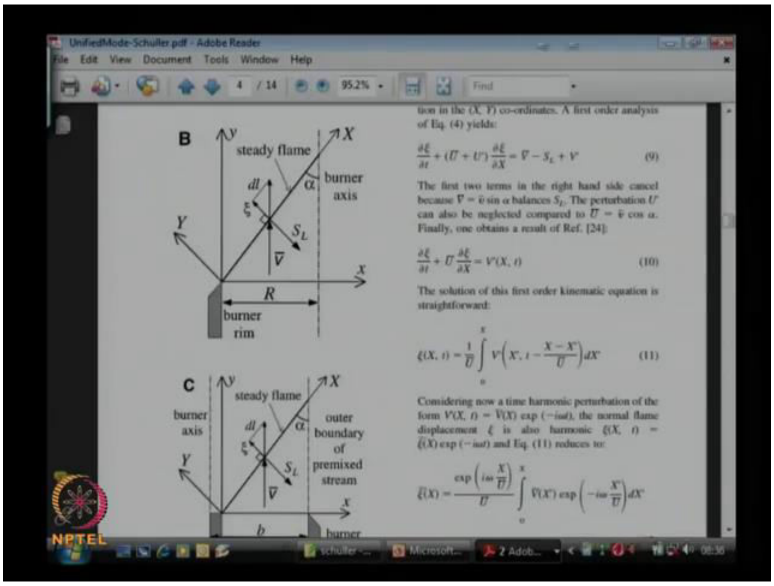
That does not behave right, I mean low frequency it is responding very well high frequency it is.

Student: But the thing is, is the mechanism itself different.

((Refer Time: 27:12))

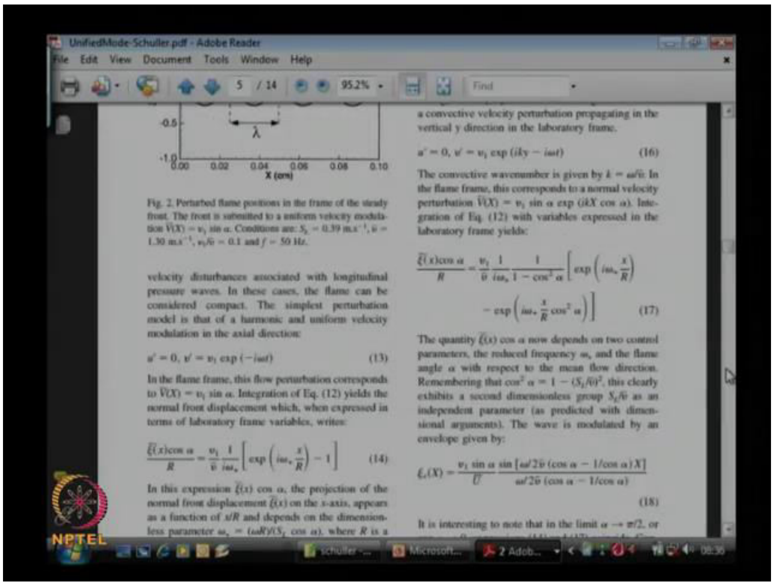
Student: May be we can have (( )).

(Refer Slide Time: 27:24)



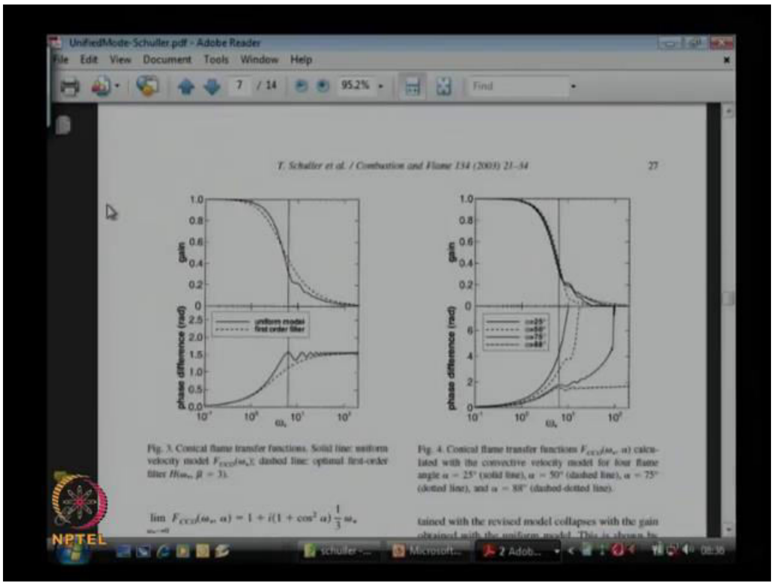
Yes, very good point so Schuller has worked this out, you can say this is the same kind of configurations what all we and this is the solution that we derived in time domain and we just used different variables.

(Refer Slide Time: 27:35)



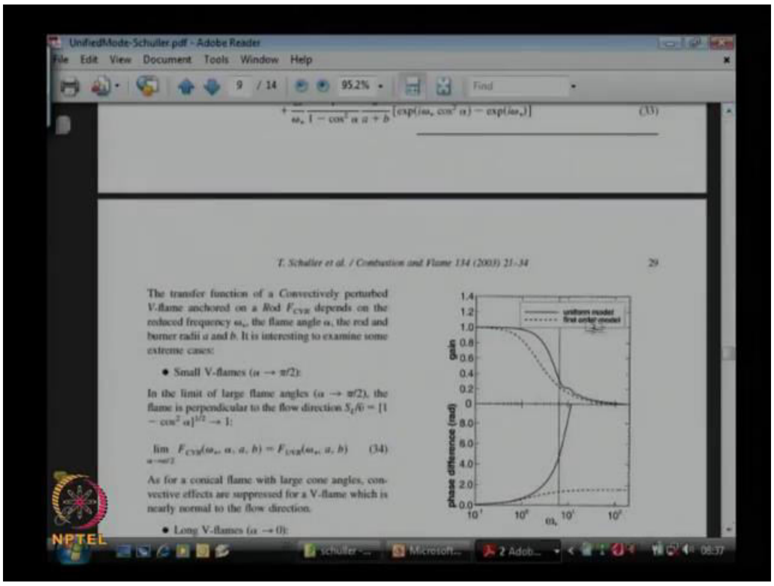
He has found a solution for the equations for this particular problem yes, Schuller has solved for a V flame, V flame and cone flame both flames.

(Refer Slide Time: 27:50)



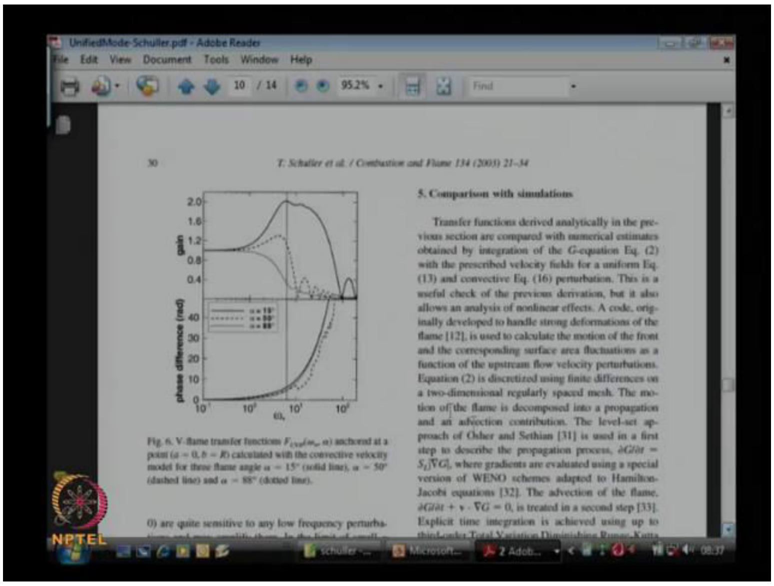
And then has plotted calculated the transfer functions and this is the result that I have showed, but then he also says what happens if velocity is not a constant that there is hydro dynamics and both exceeding and so on. So, it works out for case where here we assume the constant velocity, we can have a velocity like of course omega t minus k y. I mean instead of having a constant value time t power i omega t or which is like some constant term cos omega t or sine omega t that is what we have used. Instead you say some constant times of course, omega t minus k y or something like that and so varying it. And then he has showed that there are some differences for a v flame, but there is dramatic difference for a sorry that is for a cone flame.

(Refer Slide Time: 28:41)



And you would see here this this is the uniform model for a V flame.

(Refer Slide Time: 28:51)



And this is for the V flame with this cellulite or phase difference and you see that the gain actually starts 1 and goes higher than the 1 in all other cases it will did not go than

1. So, there is significant difference, but this is like a some kind of model put in my hand the phase the validity. You will have to solve in a couple manner and solve you have a hydrodynamic solver which you will actually solve for the velocity and then that would go as a input to this, I do not have the differential equation here, but that u psi will not be a constant anymore and then you will have to solve for that and it will surely change.

For example, the simplest case simplest to the model it is not a calculator, in the model itself you can see there is tremendous difference yes, surely you will make a lot of difference.

Student: ((Refer Time: 29:50)) same axis boundary kind of thing.

No, no this one affected the v flame (( )).

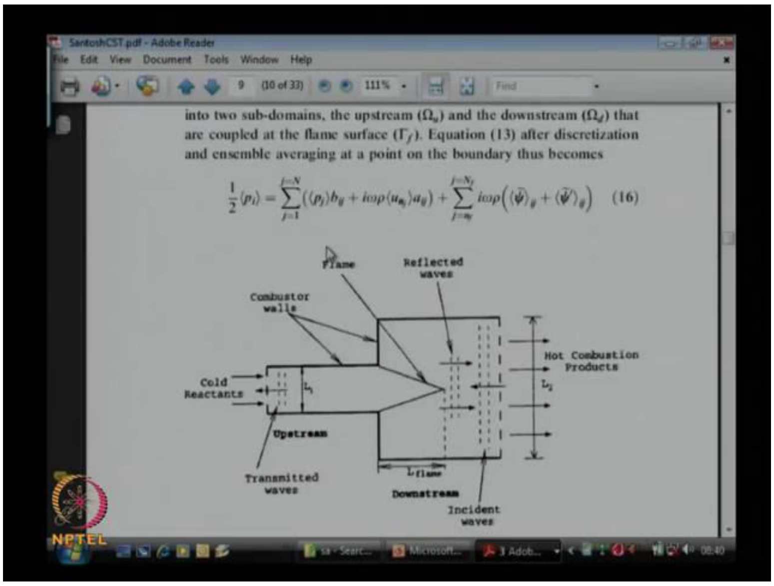
Student: (( )) the conical flame it is not applicable so the vertex (( ))

Well, it change the phase for a conical flame, but did not change the gain grammatically, but in general it does not really a model overtake shedding a just put in some affect in some way Schuler says he pulled a rabbit out of a hat someday he is my friend, but if you do a more serious time domain calculation hydro dynamic zone and you couple it with this. I think there may be even more differences I think you people are not done such detail calculations yet in a couple manner, but it would be webbed way we are trying to do those calculations about the reflections you mean defective sound wave by the imperial discontinuity.

Student: Sound wave or even ((Refer Time: 30:47)).

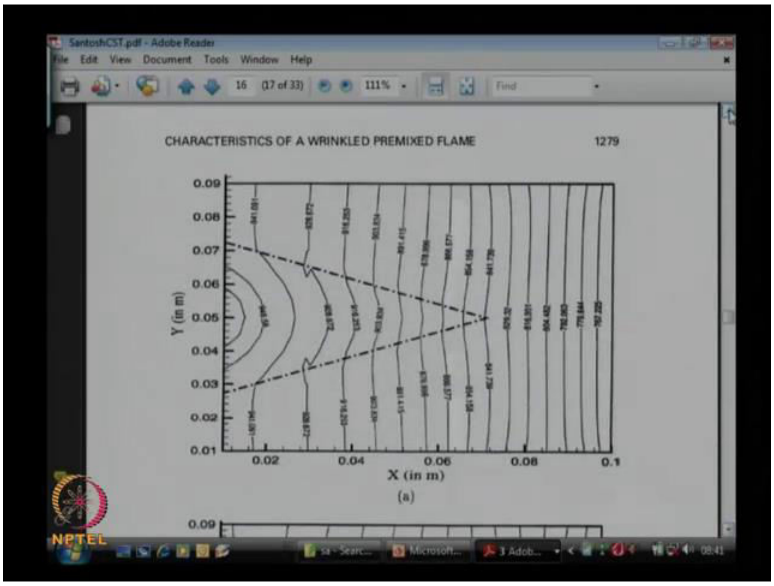
So, it depends on the with the both it says it is a difficult question I do not have an answer, with the sound wave I can show you some results what we did, I had the same question some time back. The way this bound element not that calculation to solve for this.

(Refer Slide Time: 31:35)



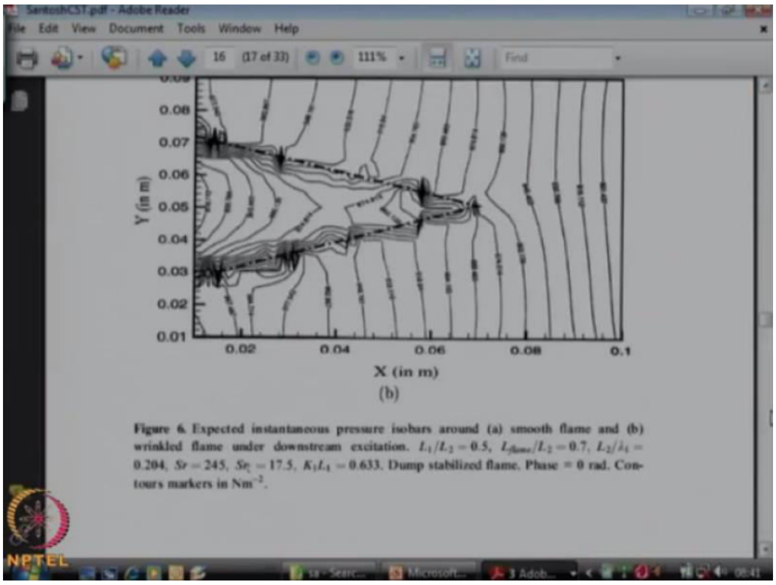
And you looked at how this flame will get reflect from upstream and downstream and so.

(Refer Slide Time: 31:48)

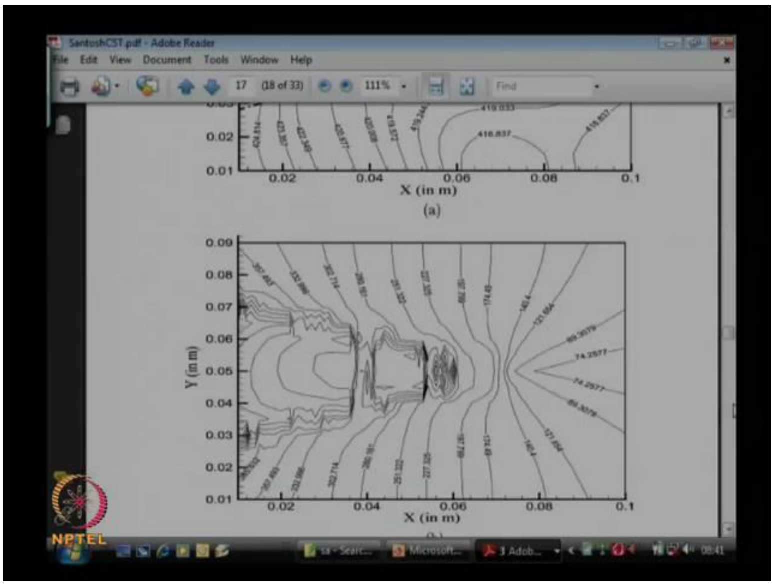


You can see there is some kind of bending and so on and if the flame is quite wrinkled it will be quite a bit iso-bars will be bent quite a bit.

(Refer Slide Time: 32:06)

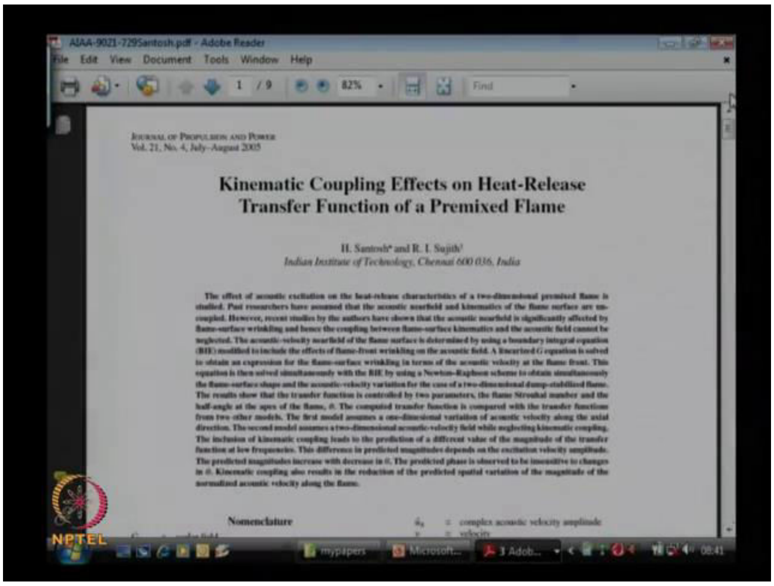


(Refer Slide Time: 32:11)

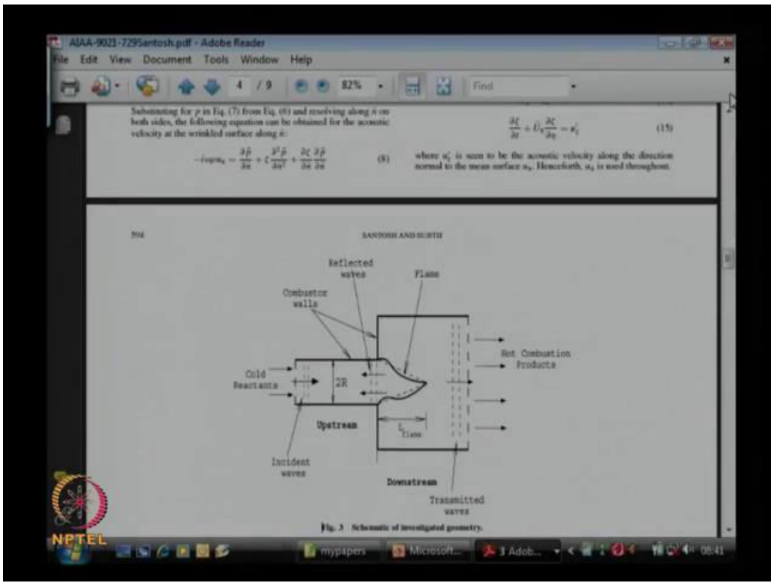


And it also depends on the waves coming from upstream bending will bend up way, waves coming from downstream will bend down way and this will actually affect the transfer function calculation also. It is there in the other paper that sited.

(Refer Slide Time: 32:49)

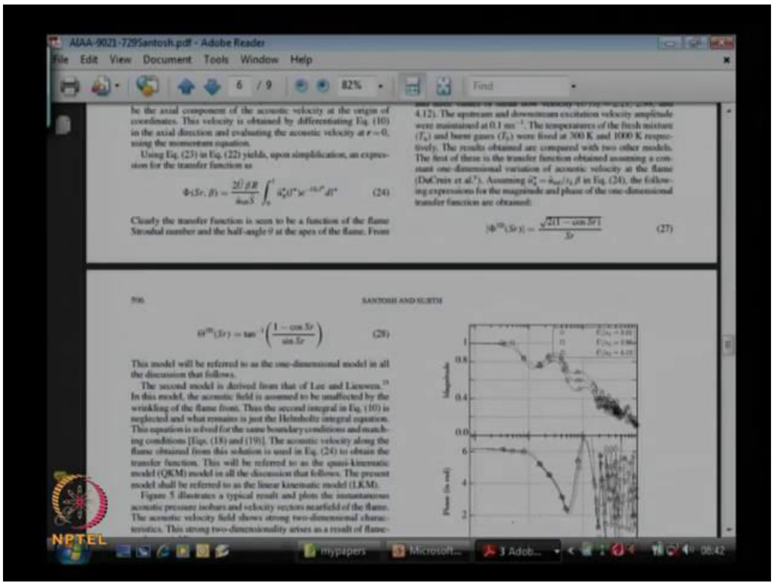


(Refer Slide Time: 32:57)



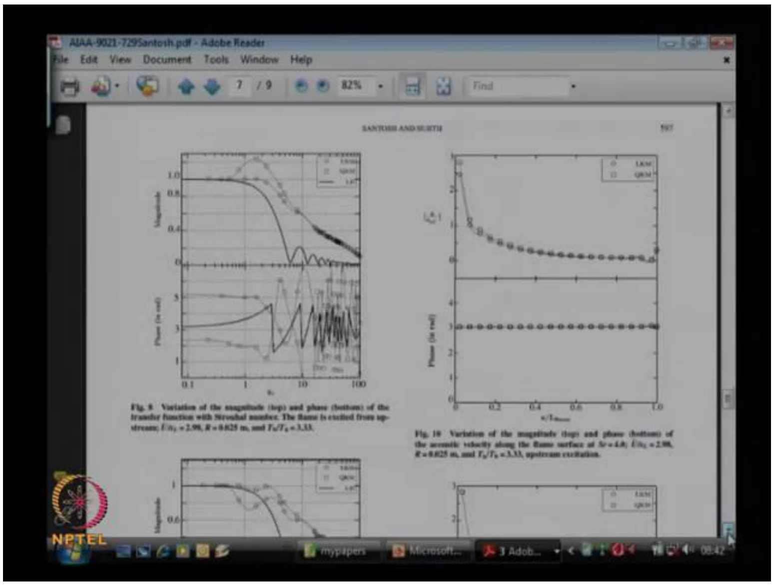
You have to consider the 2D geometry.

(Refer Slide Time: 33:02)



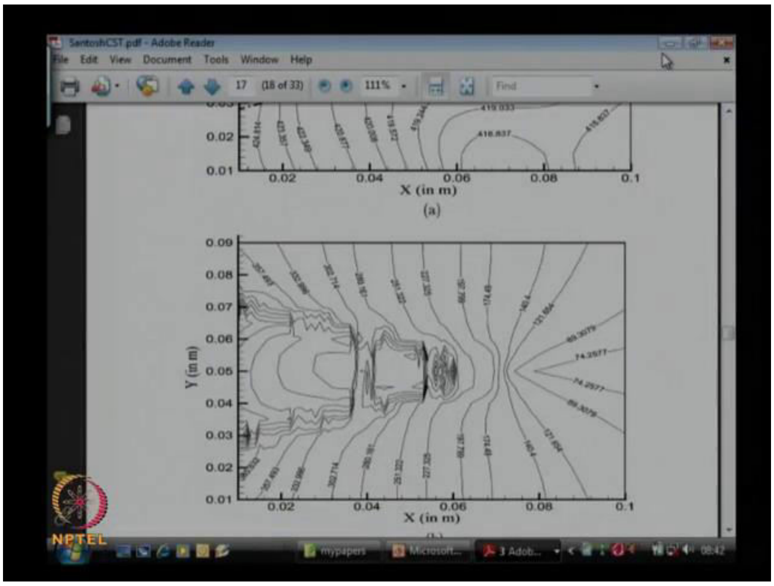
And how the reflection and sound wave happening, and we looked at how the gain changes because of this and how the acoustic field changes and so on.

(Refer Slide Time: 33:11)



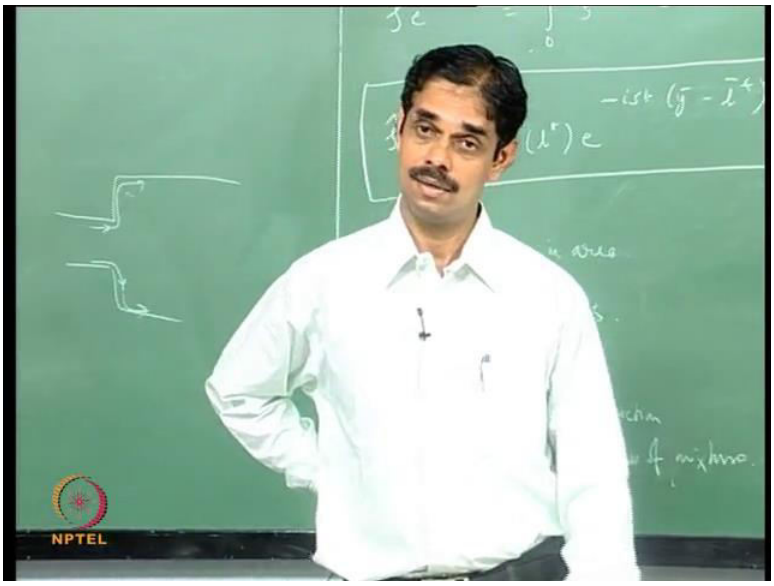
In fact, this plot is not ((Refer Slide Time: 33:15)). We can see that this calculation which I did and you see a quite a bit of change in the gain because this I mean accounting for the reflection and the I mean does not periodic change and modifies the how the acoustic wave are affected and so on. I mean you can read these two papers and get some answer.

(Refer Slide Time: 33:52)



But again this is simplest model for the acoustic wave that can be put in, but in reality we have complex 2-dimension, multi-dimensional acoustic field. And when the flame is more wrinkled like you said at high frequency it or it is turbulence flow it would kind of like a fractal and that is what we calculated, how that kind of surplus would affect the acoustic field locally. And even a smooth flame it will scatter and so on quite a bit and near the flame, the velocity field will be highly 2-dimensional because the you know the even if there is no flame the acoustic lines will bend know.

(Refer Slide Time: 34:38)



If you have typically geometry of this form, the acoustic stream lines will actually won't separate just go like potential flow lines. So, the velocity field will be highly 2-dimensional, but pressure will be quite 1-dimensional. So, it is a quite a bit of interesting things here.

Student: Sir, U prime field.

U prime field, from the acoustic which is actually we did not actually considering acoustic at all, we just imposed the U prime on the frame and see what happened this has to be coupled with the acoustic calculations. So, may I have looked at this very element aspect of the question you ask, but I think yes you are trying to look at bit more seriously also, but deep question what we ask, any other question.

So, the next a few questions I want the answer before stopping, how to measure Q prime in the experiment, can we put some meter which will sense

Student: If we measure the flow rate of the Q prime.

Q prime, fluctuating it is right how can you measure, you have a theory how do you check if it is right.

Student: Fluctuating, there will be fluctuation ((Refer Time: 35:52)).

No, we have considering a case where a constant velocity and we are not having that I will do next right next to this, but how will you measure the heat releasing fluctuation rate.

Student: Measure the fluctuation.

No, I am asking how to measure the heat releasing fluctuation.

Absolutely, that is what is a mark and with a fully premixed flame with no entrainment CH and O H are very good indicators of the reaction rate. So, we can measure the O H or C H. And that will be a direct indication of heat release so that would be, how do you measure it actually.

Student: Photo detector.

Yes, you can use a photo detector or a photo multiplier tube or we can use a high speed camera and measure this one, we should be able to or camera should fast to respond to this. Now, you have to actually if you were to so, you can measure this and compare with the experiments, compare with the calculations that is what in the references which I have given to you. Now, you can couple this Q prime which you obtain from this calculations so, we have obtained A prime and then we can get Q prime.

And then how do you couple the acoustic field, we have Q prime with our calculation which we just described today. Now, how do you go ahead and calculate the stability.

Student: ((Refer Time: 37:34)).

So, we did this N tau formulation so here, you have Q prime has a function of U prime. So, in terms of function it is like that it has a amplitude and phase that is like the m and tau. That we have put it into minus time analysis and then we can get the hygen values and hygen vectors and you can look for the flame stability. Alternatively, we can have this Q prime to be solved in time domain, we had a time domain solution also or we can use some other time domain like a solution. And then couple them together and march in time step by step, the acoustic field and the flame.

The flame is wrinkling that is affecting the acoustic field, acoustic field is getting modified that is further affecting the wrinkling you can do step by step marching of a fully tightly coupled system. That is one way, other way is put the transfer function into entire model that is what we got in entire representation. So, in our model we completely neglected certain serious issues which was the criticism yesterday from Vishnu, one was SL changes, how can SL change.

Student: ((Refer Time: 39:00)).

Consider a laminar flame, how will SL change Rajesh.

Student: Flame shape.

Flame shape curvature is one thing, how else can S L change. Suppose, there are equivalence ratio fluctuations what happens.

Student: Reaction will change.

Reaction will change so, SL will change and delta it will also start fluctuating because equi ratio fluctuating so heat released per unit mass of the mixture will change. So, those things we did not consider, but if you had to look at we had set up a problem or experiment such that there are no equivalence rate fluctuation, it is possible to have equivalence rate fluctuation and then you have to take those things into account. Now, if you do mention that you can couple this and so on in time domain and then what happens is your amplitude will blow up right, I mean if you are having instability the oscillations Q prime, U prime, p prime everything will be going to infinity.

But in our experiments I showed some experiment in which it made noise and I mean noise was not infinitely large speakers will blow off, you saw the experiments in lab also so, what happens there.

Student: ((Refer Time: 40:21))

What happens in the flame so, now the flame wrinkles will start stepping they will start becoming shock wave kind of things. Now, linearity and flame response will come into play and the phase will start drifting so, the linear whatever we derived in linear so we have to do a non-linear solution to this equation which is not revealed, you have to use some more complicated CFD schemes, which are, which can kind of schemes which can capital shocks like we know waited essential non-oscillatory schemes those have to be used to do this problem. It is a difficult problem with CFD, but people have done that and that is what you have to solve.

Student: ((Refer Time: 41: 25)).

So, first thing is this can actually steep and form shock wave like structures, if you see those papers they will show that. The flames will form curves and then the flame can flash back that is one thing.

(Refer Slide Time: 41:49)

So, if the flame is standing like this actually come down also and it can blow up also and it can stand up need not, it need not be anchored anymore I mean if the amplitude is keep going it can standoff so, many affects there are considering. And for standoff I mean we assumed that the flame is anchored, but this will work only when the flame is anchored, but the amplitude goes up the anchoring will go away. And then you have to solve the heat transfer problem there and flame can if you are near the linear limit or something there can be lean blow or the flame can flash back, lift up it can do periodic lift up a lot of complications we can do, such a very.

Even in case of laminar flame there is a very exciting subject and turbulence jet flame would be even more exciting and I think people are studying it how to modulate it, it is very complicated I will get to it, but I think this is to say welcome to the subject. So, in the first part if you go back to few classes or few weeks we did not worry too much about how the n and tau came. We just said there is a n and tau and put it in there or Maria Hegedus said there is a co-relation put in there into the acoustic field and we learnt how to solve for the acoustics in both in the time domain and in the frequency domain. First, we did frequency domain then we did time domain, but really we need to find n and tau nobody will give n and tau and that depends on the specific problem at hand.

So, we now learnt how to do a linear model, a pre mixed flame how the flame gets wrinkled and how it gives rise to heat released rate and so on that is one thing. We solved the problem fully for a linear problem and of course, transfer function I told you to work out the Schuler's paper, all the details are given very easy reading paper, nice paper. Now, we can also solve the, we have a we can derive the non-linear equations we just threw away the non-linear term, but we can in principle derive the non-linear equation and we can solve for them using whatever scheme it is.

And then we can couple it with the acoustics here, the non-linearity's will come in combustion much earlier than the non-linear and acoustic because the non-linearity's acoustic to happen the fluctuating velocity should get very large comparable to speed of sound, but for the non-linearity's and fluid mechanism for the combustion to come the fluctuation should be comparable to the mean flow velocity which will happen much earlier. So, we do not really need to consider non-linear acoustics, but we could consider it. We must consider the non-linearity's in the flame response which will happen at much lower amplitude. Again if you are talking about amplitude of 100 Pascal you would not really worry about non-linearity's acoustics or gas dynamics, but you will have to worry about non-linearity's flame. So, stop here.