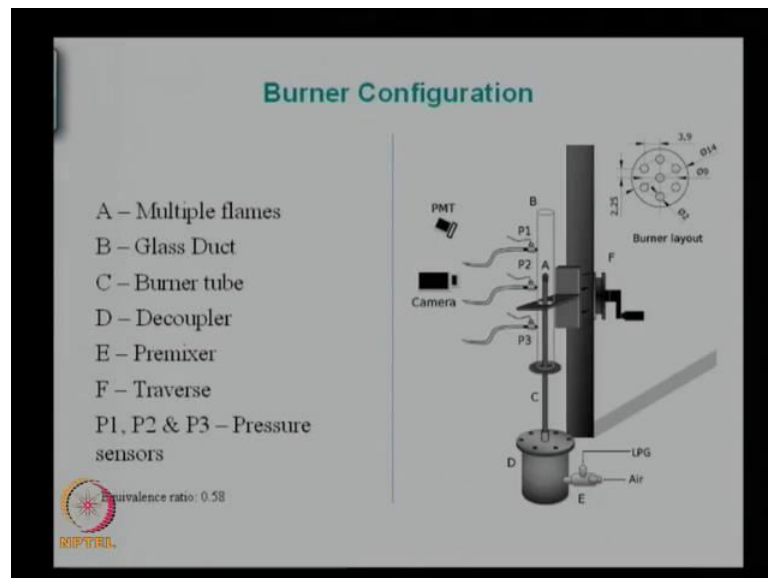


Acoustic Instabilities in Aerospace Propulsion
Prof. R. I. Sujith
Department of Aerospace Engineering
Indian Institute of Technology, Madras

Lecture - 30
Premixed Flame Acoustic Interaction-1

Good morning everybody. We are trying to look that, how an acoustic field and a flame couple together to make the most instabilities. I will show the demonstration of how this happens. First of all, we will see the video of a real experiment.

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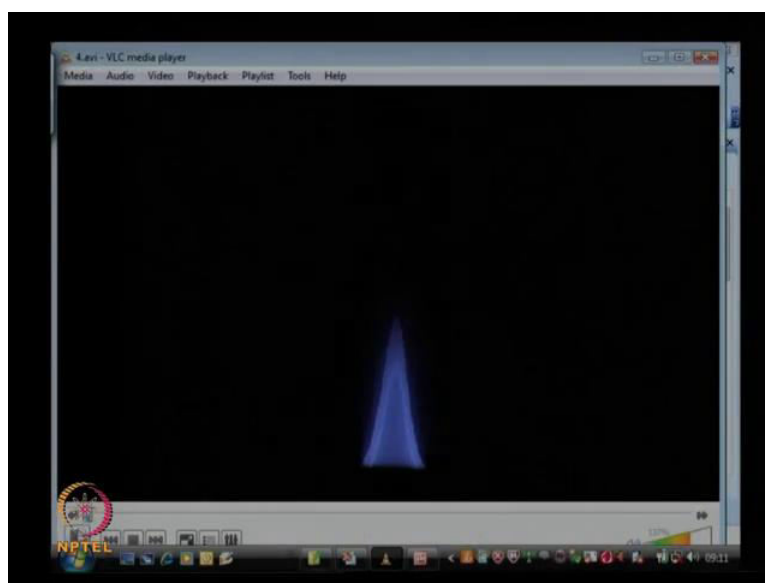
We can have this experiment in a very simple way, and you can do it in home by taking a candle or a Bunsen burner flame, put a glass tube around it, and you can repeat the same kind of thing. We are looking at premixed flame. So we are mixing LPG and air, bring it into a de-coupler here, which is like a large volume to stop the fluctuations or whatever and make everything uniform. Then you sent it out through this tube and there is a burner here, so you can burn here. You can either have a single Bunsen burner type flame or more fancy things like multiple burners, so you can have a multiple flames at each of this holes standing.

Then, you put a tube around it. You can have this tube which has an open end or a closed end. If there is an open end, there will be a draft. If there is a closed end, there will be no draft. So you can simulate a closed-open end or open-open end, whichever you want. You can make the pressure measurements with microphones and can use high speed camera to do imaging or regular camera to get a video.

We can get the chemi-luminescence imaging with photomultiplier tube with appropriate filters. If you use 428 nanometer filters, you will image the blue corresponding to the CH radical, if you use 308 nanometer filters you can image the OH in the flame. So these radicals are present only in the reaction zone. You will see the whole flame to be luminous because there is a black body radiation from the hot gases. So in a regular movie what you will be seeing is the flame luminosity. So you can look at the chemi-luminescence from any of these radicals like CH, CC and OH.

These are the things that you can do, and it is a simple experiment. You can set up whatever equivalence ratio you want and then you can track the evolution. So in a mathematical model you have to solve the differential equation to see the evaluation, but in reality, you just have to look and you can see the experiment. So, actually we have is a Bunsen type flame.

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So, there is a flame you can see. There is actually a duct, but as the dark everywhere accept the flame, you cannot see the duct. There is just the luminosity from the flame.

You can see the video as well as hear the audio which is coming. So, you heard that. This is the nice limit oscillations that occur, when a fluctuating heat release from the flame coupled with the acoustic field. The flame looks very steady in this picture. That is because the flame is oscillating at about 200 Hertz or something and our eyes cannot track the 200 Hertz nor can the particular camera.

So, what we do is that we image with a high speed camera, something which can do 2000 frames per second or a few thousand frames per second and then play slowly, so that our eyes can see it. Even if you take 2000 frames per second and play 2000 frames per second you would not see anything. So, what you do in high speed movies is that, you take the video in high speed 5000 frames per second, 6000 or 2000 frames per second at a rate much faster than the frequency involved in the system and then play it slowly.

In cricket match, you would have seen the ball turning very nicely, diving in slow motions and all that. You acquire large number of frames and so that is the meaning of high speed video. Normally it take 30 frames per second or 25 frames per second. You take much larger than that and play it slowly. So that you can sense what happened in the short time, but in that you are a kind of expanding the time. I hope this is clear.

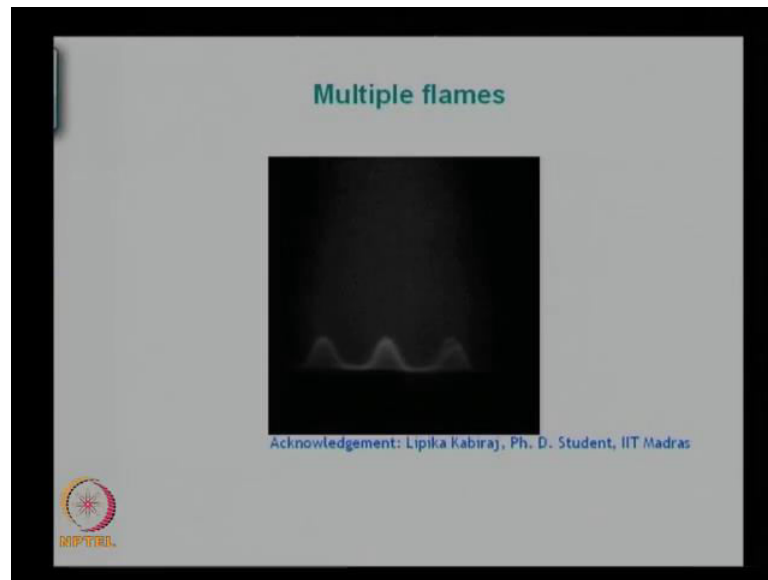
We can see the flame is actually wrinkling. Depending on the frequency and amplitudes that are involved, the wrinkling can look differently. You can have small amount of wrinkling, large amount of wrinkling depending upon the experiments. The important thing to note that is the flame area is fluctuating. You can have a steady flame, or cone or tent like flame. So, at any time the wrinkles are coming the area is changing compared to the steady shape.

When you increase or decrease the area, we are basically altering the amount of fuel that is consumed, because as long as there is a flame everything that goes through a pre-mixed flame is consumed, because we are looking at linear flame. If we have a rich flame, whatever is the balance mixture ratio will be burned there will be excess fuel, but if you are having a lean fuel everything will be burned. So whatever is going through the flame is being consumed.

So, as long as you have flame area, you are consuming everything that will be coming so the flame area is wrinkling. That means, at some instant you are having more area and at

some instant less area. That means, you are consuming more at some instant consuming less at some instant. So that is what the heat release rate is and the acoustic field is setting up velocity fluctuations at the flame which is perturbed in the flame, and we have to look at that.

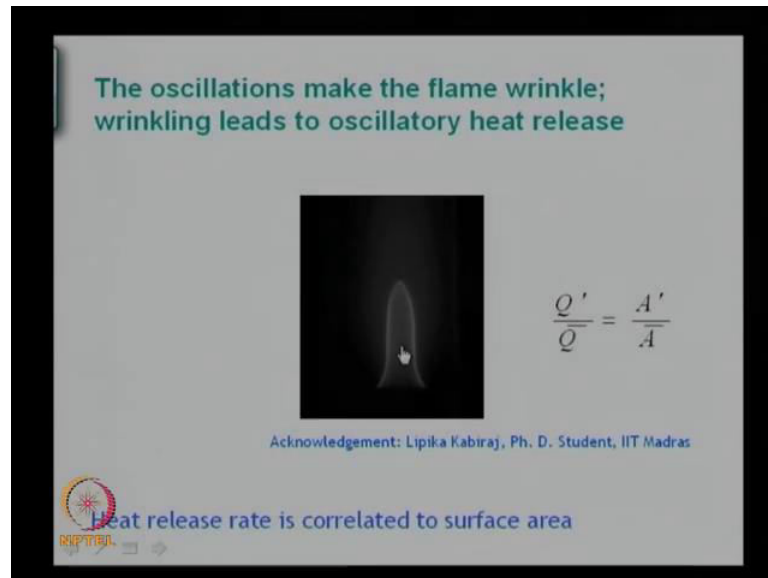
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We can also do multiple flame experiments instead of the Bunsen burner flame. This is a standard configuration which many people study. Let me play the video. Once again the flames are actually oscillating, but we are unable to see because it is oscillating too fast. So we have to do the high speed imaging. This is how the multiple flames look like. Unfortunately, you can see 3 flames on the screen, but there is only 1 because it is too bright. So we can look at the high speed imaging.

You can see they are oscillating. I will play it again. So the flames are oscillating back and forth, the flame surface area is increasing or decreasing. This is what is making the flame consume more mixture at some instant, less at another instant. Therefore, the heat released is more at some instant and less at another instant. So you have fluctuating heat release and we are in a confined environment. So there are acoustic oscillations and we are having a feed back.

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So, the oscillation makes the flame wrinkle; wrinkling leads to oscillatory heat release. Q' prime correlates with area of fluctuation. So, we need to solve the area of fluctuation of the flame. So, given a heat release rate, we know how to deal with the acoustic rate. That is what we did. We wrote the equations. We had a momentum equation and we have energy equation. Energy equation had a heat release term in it. So, we know how to deal with it, provided you put a heat release term in it.

In the earlier example we put a correlation for heat release given by Maria Haeckel. Now, instead we need to put the heat release rate fluctuation that you will get from the premixed flame. So, how do you do that? We need to see what happens to a flame, when there is a velocity fluctuation. So, we have a lag response and so and so for it, and we have to solve for it. Once you solve this, you can input in to the acoustic equations and then solve them in a couple manner, either in the time domain or we can do the analysis in the frequency domain like we did otherwise. Now, the objective of the lesson is to write the appropriate, simplest possible equations for the premixed flame and try to solve the flame shape. Once you know the flame shape you can get the wrinkling and the flame area. Once you know the flame area we can get the heat release rate. Is this clear? Any questions about this?

Student: Sir, Are flame oscillations naturally possible?

Yes, you must have understood from everything you did earlier that at some locations the flame will be stable, at some other locations the flame will be unstable. So, I can keep the burner at some locations, where the flame will be stable. Light the flame and eventually it will be stable. Then generally move it to a location where the instability can come in. You can either move the flame or the tube. In this particular experiment I am actually moving this tube with respect to the burner. It is the relative position that matters.

We have two kinds of instabilities, linear instability and non-linear instability. In a classically stable configuration. It will start off with any arbitrarily small disturbances. And if you are in a bi-stable region, either you can go to limit cycle or you can come back to a fixed point. Then you need appropriate initial conditions which you will have to give through some other means by using a loudspeaker or whatever.

In this picture, we are looking at linearly unstable region. So, it will start spontaneously out of any small disturbance. For example, if you do experiments here, you can hear the hum of the instruments behind, hum of the air conditioning, somebody may move hands or sneeze, and anything will act. When you are in a sub-critical region, then it depends on what kind of initial condition and so on. So, it depends on the type of signal, the type of the initial excitation, some excitations are very good. Even people are like that. Somebody tells you do not ignore, somebody else tells you have to something like that. So I hope it is clear.

So, in this picture we have linear unstable conditions and any perturbation arise. I do not do anything to start. It just spontaneously came out. You can see these kind of things in reacy tubes, thermo acoustic engines and any combustion instabilities. The way the heat released respond will be different in different experiments. For example, in the reacy tube, you saw that it is the cylinder heat transfers, how it oscillates when the flow oscillates. But here it is the flame that is the wrinkling. In the reacy tube there is no wrinkling, it is just the heat transfers in and out when you are blowing back and forth. Whereas here, you are having the wrinkles.

If you are looking at diffusion flame, what would be oscillating the heat release rate? Let us say, we do not oscillate the mass flow rate, so, let us say we choke the incoming flow. It is going to change the mixing field, right. In some other situations like what Mr.

Kamble said, you have oscillations: fuel flow rate can fluctuate; equivalence ratio can fluctuate, because you have a Δp across in the orifice and Δp increases and decreases. When the pressure goes up the Δp comes down, when the pressure comes down Δp increases. So, you are having an oscillating Δp on the injectors. So, we can have an oscillating flow rate.

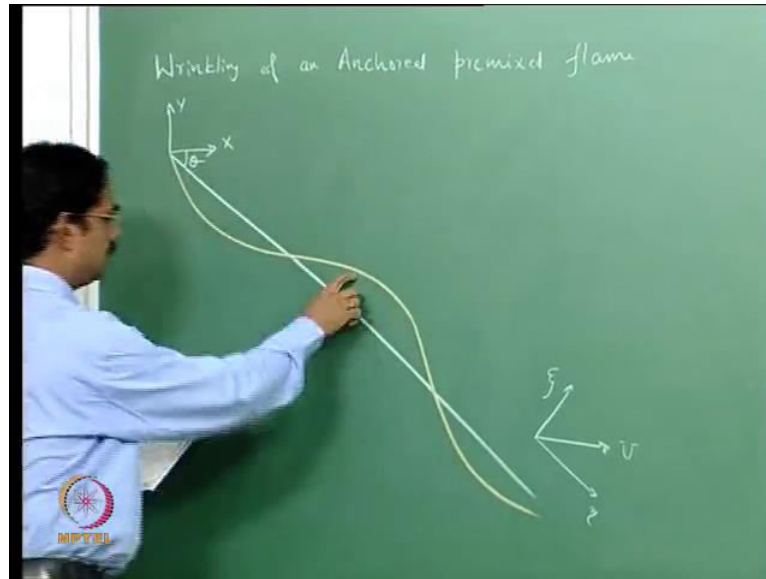
In different combustors, different geometries, different configurations we have different mechanism of the heat release fluctuating. There is no general prescription on how to calculate the heat release rate. For example, in one case you studied flow over a hot wire, in another case we studied wrinkling of a premixed flame, in another case we studied how the mixing field changes in a diffusion flame, in another case we studied how the injection is oscillating.

In each situation, there is a different mechanism as to how the heat release fluctuates. In spite of the fact that there is a commonality as to how the isolations are coming out. This linear stability, non-linear stability, bi-stable region, globally stable, globally unstable; those all are common themes, in spite of the fact that these experiments have quite a bit of differences.

The topic today is, wrinkling of an anchored flame. You saw that, these flames were anchored. That means they were fixed to the burner. These are burner stabilized flames. At the burner you have a local recirculation zone, no matter how small it is, and that will hold the radicals there, and you can have the flames sit on that, so that does not blow. But, we can also have flames which are standing off, because you may not have a flame right at the burner and it may just stand off and lifted. The dynamics of the heat transferred at the burner lip that will control how much the flame is standing off and so on. Those all are complicated and my idea is to reduce everything to toys.

Once we have the formula for heat release; expression, or calculation procedure, computer program, anything for the heat release, then we can go back to all procedure either in time domain or in harmonic domain and then either calculate the stability in time domain, with our numerical integration, linearised operators, normality blablabla. Or we can look at in the harmonic domain, look at the Eigen values, Eigen vectors, stability. So all those procedures will be coming. The only thing that is needed is to get a way to calculate the fluctuated heat release.

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Instead of calculating the heat release rate in the flame, that is a very general objective, I have brought down the objective to wrinkling of an anchored premixed flame. Let us first take a look at the geometry of what we are studying. This is the lab co-ordinate system x and y and let us say this is like a mean flame. What is the concept of the mean flame? Let us say there are no fluctuations and we happened to have a steady flame. How does the steady flame exist for a premixed flame? What is a flame velocity?

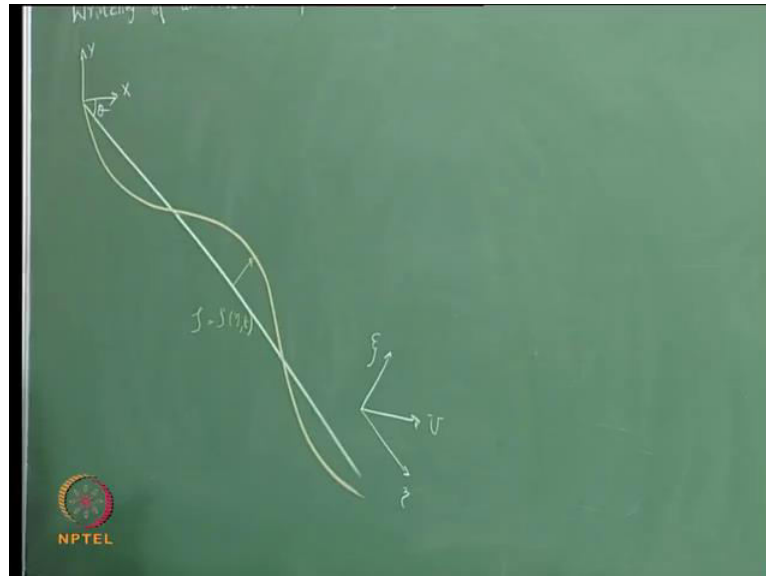
Student: The velocity at which flame will propagate, if the fuel mixture were stationary.

You have a fuel mixture that is stationary, and if you light up the flame, how does the premixed flame propagate? It is not a combustion class, anyhow we have to deal with it. We have to have some kind of preheating, so we will have the flame going. We need hot radicals to get the reactions going. Hot radicals and heat will diffuse through the flame front. So the flame speed depends upon the conductivity, diffusibility, the amount of heat released and so and so. So this is the crux of the matter.

The speed at which the flame front moves into a quiescent medium of reactants is called flame speed. That means, we are talking about a moving flame front. But if we light up a Bunsen burner flame, we will see the flame standing like this. You have a certain rate at which reactants are consumed. As long as you supply as much as it is consumed, you

will have the flame front standing. That is how we are getting it in a Bunsen burner or something like that.

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So, we will call this angle theta. This is our lab co-ordinates. For convenience we will have another set of co-ordinates, which is eta and psi, v for flow velocity. The orange line is representative of the premixed flame which is wrinkled ok. Zeta equal to zeta of eta t. This co-ordinate is eta and this is psi. Although, it may be advantageous to do things in laboratory co-ordinate itself, sometimes we will do this way to keep things simple. So, this is the geometry and the angle theta is the angle the mean flame makes with respect to the laboratory co-ordinates.

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$\vec{r} = f(t)$


$G(\vec{r}, t) = \vec{r} - f(t) = 0$

A point on this surface will move with a velocity

$V_{flame} = \vec{v} - S_L \hat{n}$ $S_L = \text{Laminar flame speed}$

$\vec{v} = \text{velocity of the fresh mixture}$

$\hat{n} = \frac{\nabla G}{|\nabla G|} = \text{local normal to the flame front}$

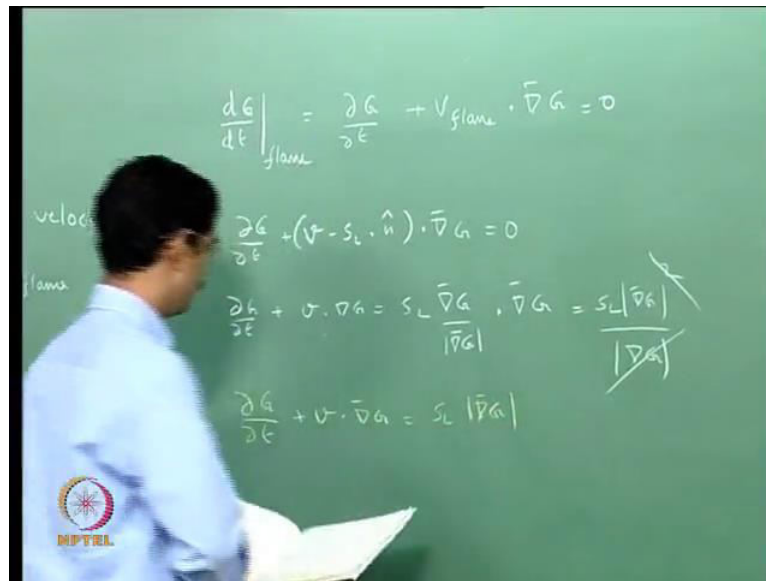


I can say r equal to f of t , where r is the special co-ordinates of the flame. So I can actually say G of r, t equal to r minus f of t . This means that the location of the flame is changing in time. If define a function like this, then it has to be 0 at the flame. We have to track the G equal to 0 surface that is the idea. So, a point on this surface will move with a velocity given by V_{flame} equal to v minus $S_L \cdot n$; where V is the velocity of the local fresh mixture; S_L is the laminar flame speed; n equal to ∇G over modulus of ∇G , which is the local normal to the flame front.

Student: Sir, can we be sure that we can use laminar flame speed?

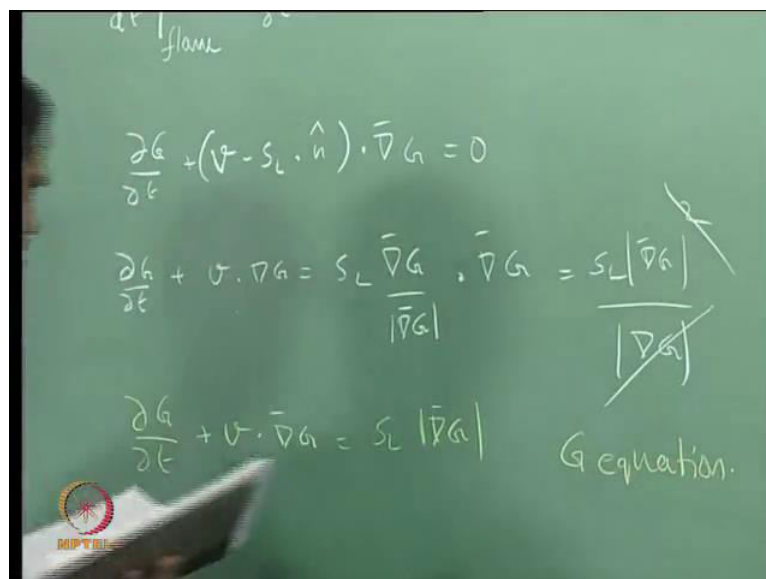
Yeah, it is a very good question. Yes, when you are wrinkling the curvature of the flame changes. So laminar flame speed is a function of many things, including the flame curvature. So S_L itself changes. He has stumped me, but I am a theoretician at this moment. It is just assumption that S_L is constant, the equation is right, but S_L may not be a constant.

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We are tracking the G equal to 0 surface. So, dG over dt at the flame velocity 0, that is we are looking at sitting on G equal 0 surface, dG by dt , that is partial derivative of G with respect to partial t , plus V flame dot ∇G . ok. This is straight forward. Now the question is how to write the expression for G in terms of flame shape, and then solve for the area of fluctuation. Let me elaborate a little bit on this.

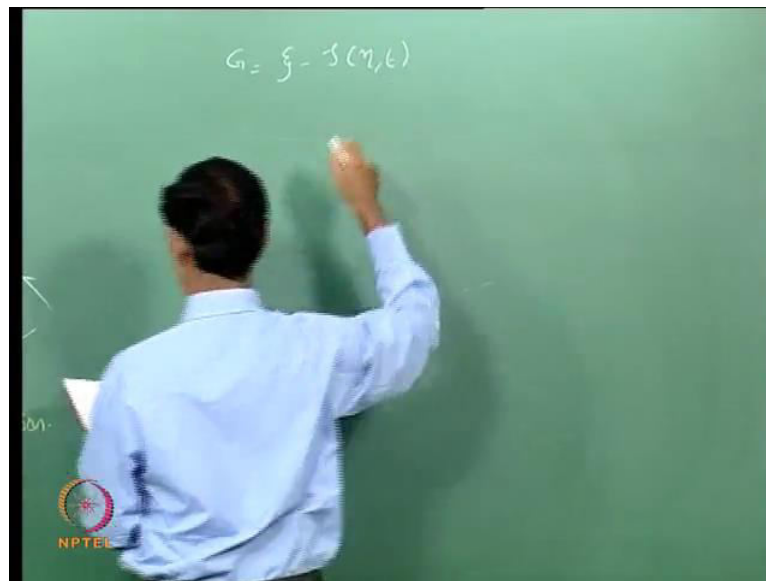
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This equation can be rewritten as dG by dt plus v minus S_L dot n dot ∇G equal to 0. We said that V flame equal to v minus S_L times n , in balance between the mixture

velocity and flame speed. So, that is what I put in here for the V flame. We have the expression for n and thus we can expand as $\text{dow } G \text{ by dow } t \text{ plus } v \cdot \text{del } G \text{ equal to } S L \text{ dot del } G \text{ by mod del } G \text{ dot del } G$. So, $\text{del } G \text{ dot del } G$ is magnitude of $\text{del } G$ squared. So, we can cancel this. Just to write it nicely, $\text{dow } G \text{ by dow } t \text{ plus } v \cdot \text{del } G \text{ equal to } S L \text{ dot mod del } G$. this equation is often referred in combustion literature as G equation. Now, we have to see how we can simplify this for this kind of flames, like cone flame or tent flame or something. Here, we have this as the local flame height.

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Our G can be thought as ψ minus ζ of η and t .

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$$\xi = f(\eta, t)$$

$$\xi - f(\eta, t) = 0$$

ζ

This is the co-ordinate psi. So, we can say psi equal to zeta of eta t. We can say psi minus zeta of eta t equal to 0, and we can say this is G. This is the flame height in this direction. Now, for given G, we have to find the expression for del G. Then our problem is solved.

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$$G = \xi - f(\eta, t)$$

$$\nabla G = \frac{\partial G}{\partial \xi} \hat{e}_\xi + \frac{\partial G}{\partial \eta} \hat{e}_\eta$$

$$\nabla G = \hat{e}_\xi - \frac{\partial f}{\partial \eta} \hat{e}_\eta$$

$$|\nabla G| = \sqrt{1 + \left(\frac{\partial f}{\partial \eta}\right)^2}$$

$$\vec{v} \cdot \nabla G = [u_\xi \hat{e}_\xi + u_\eta \hat{e}_\eta] \cdot \left[\hat{e}_\xi - \frac{\partial f}{\partial \eta} \hat{e}_\eta \right]$$

$$\vec{v} \cdot \nabla G = u_\xi - u_\eta \frac{\partial f}{\partial \eta}$$

Remember our co-ordinate system, this is eta and this is psi. del G equal to dow G by dow psi into e psi plus dow g by dow eta into e eta. Dow G by dow psi would be 1. So, del G will be equal to e psi minus dow zeta by dow eta into e eta. This is the expression for the magnitude of del G. So, we need to evaluate the term v dot del G.

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$$-\frac{\partial S}{\partial t} + [u_z - u_\eta \frac{\partial S}{\partial \eta}] = S_L \sqrt{1 + \left(\frac{\partial S}{\partial \eta}\right)^2}$$

$$\frac{\partial S}{\partial t} + u_\eta \frac{\partial S}{\partial \eta} = u_z - S_L \sqrt{1 + \left(\frac{\partial S}{\partial \eta}\right)^2}$$

$$u_z = \bar{u}_z + u_z'$$

$$u_\eta = \bar{u}_\eta + u_\eta'$$

$$\frac{\partial S}{\partial t} + [\bar{u}_\eta + u_\eta'] \frac{\partial S}{\partial \eta} = \bar{u}_z + u_z' - S_L \sqrt{1 + \left(\frac{\partial S}{\partial \eta}\right)^2}$$

$$\bar{u}_z = S_L \quad \text{Mean terms}$$

$$\frac{\partial S}{\partial t} + \bar{u}_\eta \frac{\partial S}{\partial \eta} = u_z' \quad \text{linear}$$

Minus dow zeta divided by dow t plus u psi minus u eta into dow zeta by dow eta will be equal to S L into square root of 1 plus dow zeta by dow eta squared. We can rewrite this as dow zeta by dow t plus eta into dow zeta by dow eta equal to u psi minus S L into square root of 1 plus dow zeta by dow eta squared. So G equation has reduced to this simpler equation.

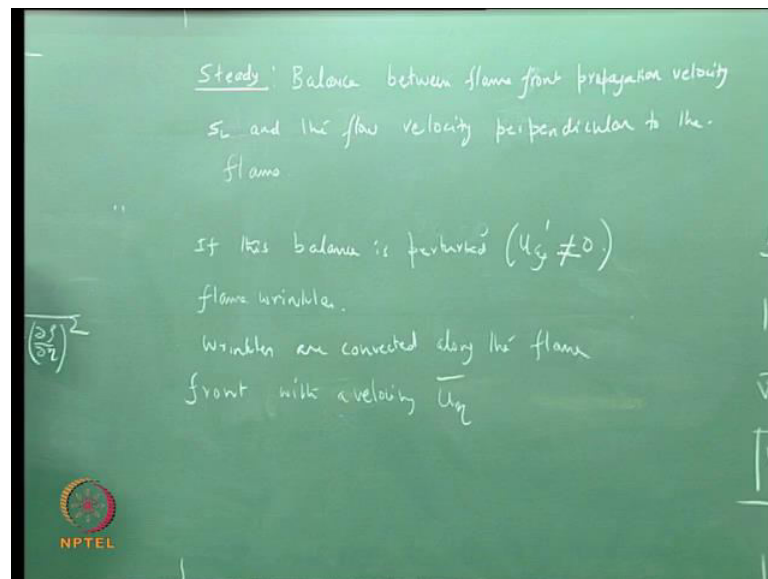
At the moment it has both fluctuating and mean quantities together. We will write all the variables as fluctuating plus mean quantity, and then subtract out the mean part and then we look at the fluctuations. And then we will linearise it also, that you will get linear differential equation, we can very likely solve it. So, you would say u psi is u psi prime plus u psi bar and u eta is u eta prime plus u eta bar. So, if you substitute this here, you will get dow zeta by dow t plus u eta bar plus u eta prime into dow zeta by dow eta equal to u psi bar plus u psi prime minus s L square root of I plus dow zeta by dow eta squared. If you time average these quantities, you will get u psi bar equal to S L.

If you expand this binomial series, then your S L will come out here, and this would be the mean equation. So, this is what we started with. At the beginning of the class I asked you how does the flame stay, and we said that the mixture velocity should balance the flame speed and then you can have the flame staying steady, and that is coming out of our analysis. So it is consistent so far.

We will subtract the mean, and drop the non linear terms, and then you can get, $\frac{\partial \zeta}{\partial t} + \bar{u} \frac{\partial \zeta}{\partial x} = u' \psi'$. So, what is $u' \psi'$? It is the direction normal to the mean flame shape. If we have an acoustic field, it has a certain velocity and pressure. The velocity has a component normal to the flame shape and that is working as a force η .

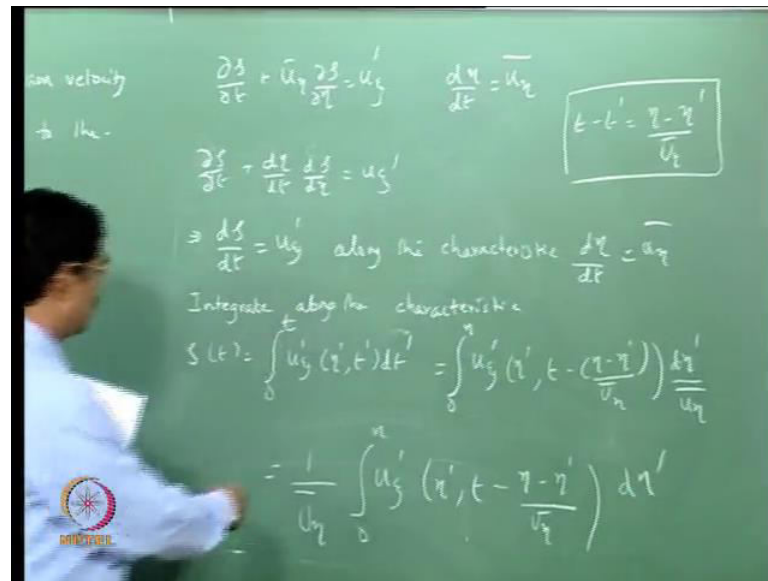
What is the physical meaning of this equation? In steady state there is no fluctuation, and there is a balance between the reactant speed and the flame speed. If this is perturbed, there will be an imbalance, and the flame will either move in or out, depending upon which way it is imbalanced. So that causes a wrinkle. These wrinkles are actually propagating in this direction by this component of velocity. I will write down this interpretation.

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For the steady flame, there is a balance between the flame front propagation velocity and the flow velocity is perpendicular to the flame. If this balance is perturbed, that would mean $u' \psi'$ not equal to 0. That is what is creating the balance, and then the flame wrinkles. The wrinkles are convected along the flame front with the velocity \bar{u} . So, let us rewrite this equation here.

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Dow zeta by dow t plus u eta bar into dow zeta by dow t equals to u psi prime. So, if you define a characteristic speed as d eta by d t equal to u eta bar, then you can rewrite this equation as d zeta by d t equal to u psi prime along this characteristic direction, d eta by dt equal to u eta bar only, not in any other direction. So, along this direction you can integrate it because this is a full derivative. So, integrate along the characteristics. We will use primes as the dummy variables. Given this we can say t minus t prime equal to eta minus eta prime divided by u eta bar. So, this is the solution term for the characteristic.

More importantly there is a physical meaning to this. The position of the flame front is not determined by the percent value of the perturbation velocity but, also by the earlier perturbations which happened up stream of the position eta minus eta prime and which are now convicted along the wave front with a convective velocity u eta bar.

So, in this case the integral is not just integral but, integral over the history of all perturbation and which will and introduce some kind of non instantaneous or time lag behavior, because you are having a wrinkle and the wrinkle was originated here, that will move up here. So, what is happening here, not just depends on the local velocity there, but also the local wrinkle which actually depends on whatever happens to this flame at some time t earlier. So, you can clearly see it is bringing a time lag into the problem.

We already learned that time lags are very critical, and here there is the time lag coming because of this wrinkles which is propagating due η bar and this will play a key role in determining the phase. The will be very important in determining whether the flame is driving or damping.

The flame angle determines what is the velocity along with the flame, and what is the velocity across the flame. So clearly time lag depends on this. So, we will do this in a harmonic domain and there perhaps the interpretation of this may be simpler. From the time domain solution it is very clear that you do not have just instantaneous response, but there is a time lag response. Also by looking at this differential equation, you see this wrinkles have been convected. So, when something is convecting, everything is the past will affect what is happening at this time. So stop here now.