

Acoustic Instabilities in Aerospace Propulsion
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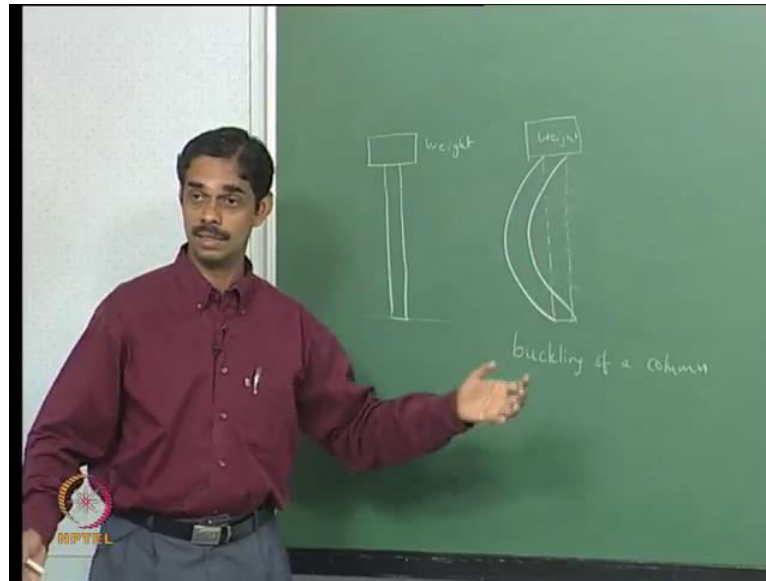
Lecture - 29
Bifurcations

Good morning, everybody we spoke about the linear dynamics of the thermo acoustic system. We in particular attention on the non-normal aspects of the linearized operator. We saw the de linearized operator was non-normal and we talk about transient growth and its consequences and so on. Then we spoke about some ways and means unless transient growth talking about this thing S V D singular value decomposition. Take concept like psdospectral and so on. I think I will stop on this with that, but if you are interested in discussing more you can come to me. I love this topic we are the wants to started this business in thermo acoustic.

Now, I will a wind up discussion with discussion on non-linear dynamics. First I speak in general about non-linear dynamics. Then briefly about a bifurcations or thermo acoustic systems. I just want to remind you that we studied everything based on some model. That I am made some kind of drawing model based on a some acoustic equations use along, with correlation for heat illustrate by the model was developed by Balasubramanyan.

So, this else the general result holds, because there is thermo acoustic system we saw in general non-normal. We are transient growth in general, but the specific things will depend on the specific nature of the system. Same is the non-linear dynamics what kind of bifurcation you get etcetera. Some details may deferred based on the model, but there is some universality behind it. That what I will speak about now. So, I do not know how many of you studied dynamical systems, Ganesh you have studied dynamic took had an five degrees of freedom. So, do you know what the bifurcations, like it may fundamental level. I will give a example, which a you all definitely know, but do not look at it as a bifurcations.

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So, I think in way strength of materials some aircraft structures, you must a studied column engineering. So, you will have a column and there is a weight here and as suppose to a beam, where you apply a load and keeps then in continuously here it will stay like this fine, but then at as you increase the weight. You keep it keep on increasing smoothly at some point it will stop being this and then it will start buckling. So, you keep on increasing everything will be smooth and then suddenly it will buckle there is. We will be smoothly increasing the weight. So, weight is like the weight you applied is like the load on the top just like a parameter for the problem. It is slowly increase it suddenly problem will abruptly buckle.

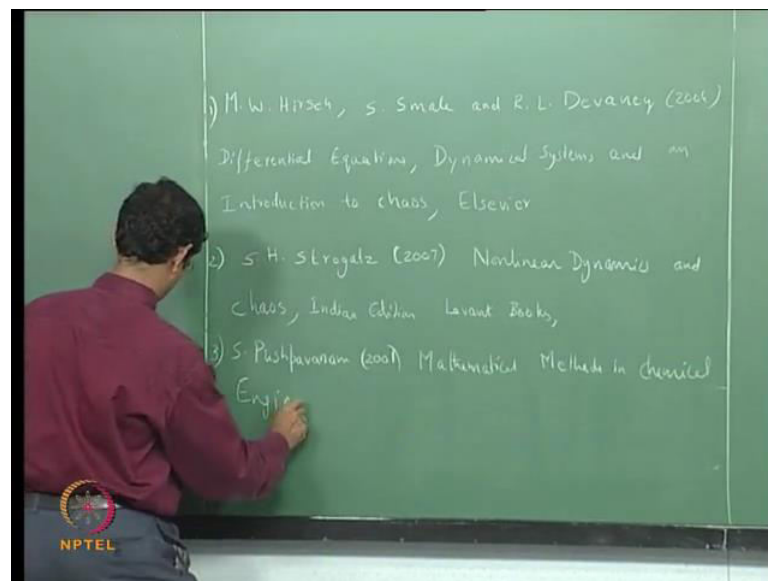
So, can everybody relate to this example yeah everybody in relate. So, this is a case where a you changing the parameter I mean when you speak about engineering. You are changing the weight, which is applied or load, which is applied, but we call that parameter. So, in this problem we are continuously changing the parameter. We are smoothly changing the parameter. It is like not a suddenly put a bang and broke it to something. A smooth change in the parameter suddenly results in a qualitative difference in the solution or the observed behavior, so this is like a bifurcation.

So, that is something we see in rijke tube you keep changing the heat release rate. Slowly, the main heat release rate or the in our rijke tube which we saw the video and so on we keep increasing the heat up power. Suddenly, there is on set up oscillations, so that

is like a everything was study and quite kind of analogs to our nice column, which is standing there. Abruptly the oscillations had came or we are onset of oscillation, like abruptly the column is buckling. So, these are all examples of bifurcations, so there is this subject called dynamical systems. So, which is been quite popular in the last thirty years or so. Here, is a nice reference is there in the library Hirsch male and Devney write it down.

There is another, very nice reference to gets these are Indian additions, they are quite cheap. This another book by Pushpavanam, which is mathematical methods in chemical engineering and this is Pushpavanam is professor in a I I T Madras. So, very inexpensive books, so I would urge to take a look at this, because in modern engineering practice. I think knowing bifurcation theory will greatly help you to describe and analyze and the engineering systems. So, write down this references, which are really nice.

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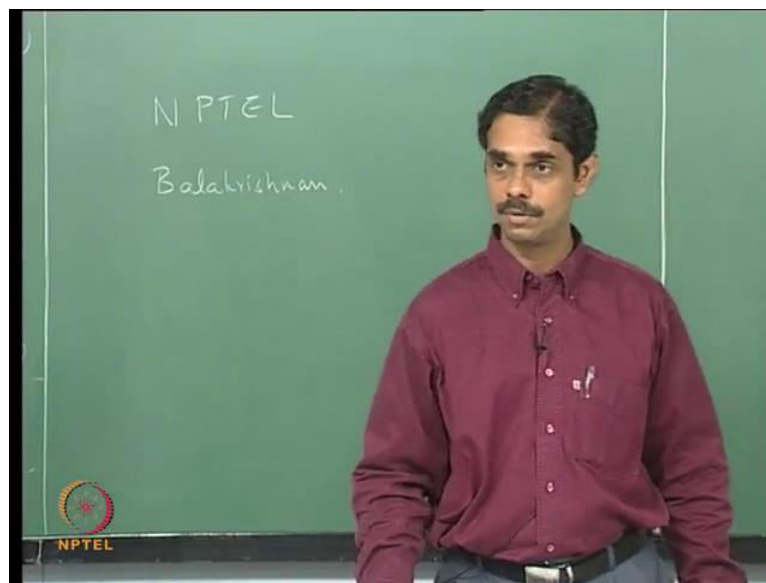


So, this is a really nice book. It also talk about the linear dynamics takes fluency, what is Eigen value, what is the meaning in addition to process a non-linear parameter. This is a really nice book it is available in library and not very expensive. The second book that I would recommend is both books have lot of examples and lot of nice descriptions. So, this is one the flinty of books. Actually, these are the books I have write, which I am prescribe it. I will give you the Indian publishers name Indian editions by a Levant books

2007. The last one other than this three books, I will give you also another very available reference.

Pushpavanam see, which year it is in 2001, so these are really nice book. Really, nice lecture series in NPTEL available by professor Balakrishnam on this subject, which is really charming here thrilling to watch. I have watch that some lectures are watch many times, so nicely he has explain. I would definitely ask in to see NPTEL course by professor Balakrishnam on dynamic resources professor.

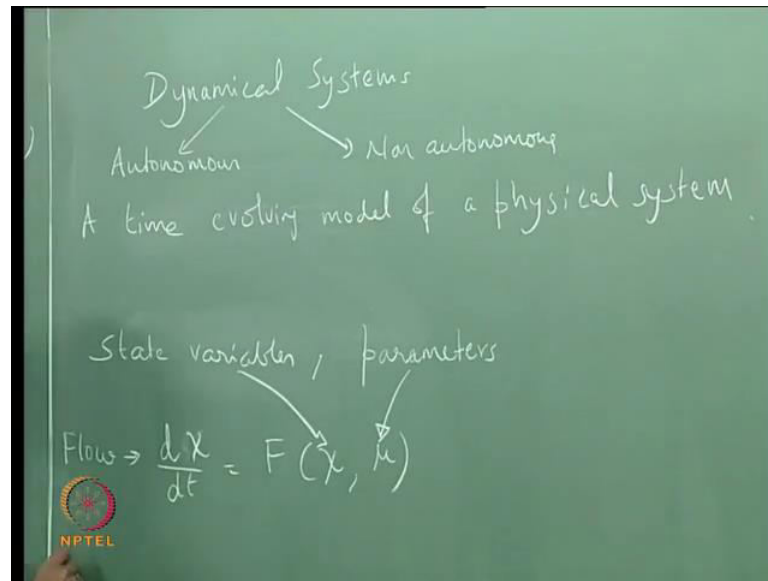
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Balakrishnam is a professor in physics department in I I T Madras. So, having said these things there is a class, which is offered by physics department. I think every other semester this semester is also. It is running by it is very nice, I have taken in class and so on.

So, we call our, so we studied thermo acoustic system. So, generalization that would be a dynamic system dynamic system means a system, which has dynamics very exciting thing not quasi study, as we studied in four years of engineering, everything is steady and stationary. You apply a lower we do not worry about how the bends, but we have a bend shape and so on. Now, here we studied about the dynamics how things evolve and in reality things evolve. So, in dynamical system is a time evolving model of a physical system the physical systems and we have models, in which we attempt to recreate the behavior. So, the dynamic system theory is the steady of such systems.

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So, this is a time evolving model of a. So, this is a dynamic system and you can have autonomous systems and non autonomous system. We have I will restrict my discursion to only to autonomous system that does not been by anywhere non autonomous systems are not important, but we are I mean in this model lecture I am constraining myself to that. So, dynamical system will consist of variables and parameters.

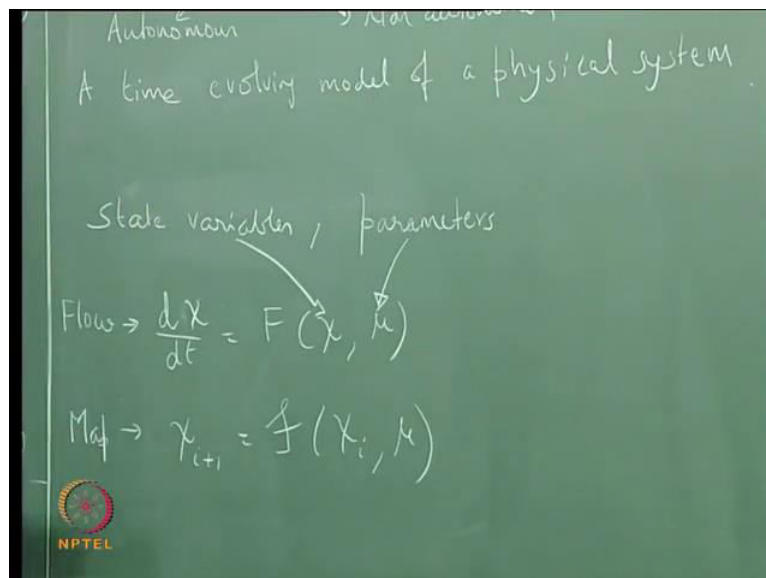
We have the state variables we try to relate everything I am saying in general sense to what we studied in the specific problem. I did not want to start with the lecture on dynamical system. Because, then whole thing will sound abstracts I want to do the problem, then go back and try to put it in the context of the theory. That way I thought you will appreciate better. So, what where our state variables in our model of the rijke tube the kais, which actually correspondent to pressure and velocity, which we express. So, we had a P D E where we are pressure and velocity. Then we are reduce it ordinary differential equation O D E in the form of E T A and E T A dot they are the state variables.

We have parameters can you explain what were the parameters. Our model some other parameters heat a power or by a temperature. Then mesh size area of the I mean in our particular model we did not have length come. Because, we are non dimension everything with L, but wire temperature was a parameter dumping values C 1, C 2. They

were parameters and location of the heater in deduct except was a parameter, time led time led how was a parameter.

So, we had which actually in reality it is serious parameter all though we wonder if you wonder how will change time lack is there some meter with, which you can adjust time lack by changing. So, time lack is a very important parameter, parameter which we actually change by changing the flow. So, these are the parameters, so we have variables and parameters. So, we write in the form $\dot{x} = f(x, \mu)$. So, x is the state variable μ is the parameter. So, this would be call a representation by flow. We can also represent in a another way as a map, whereby we say $x_{i+1} = F(x_i, \mu)$, μ .

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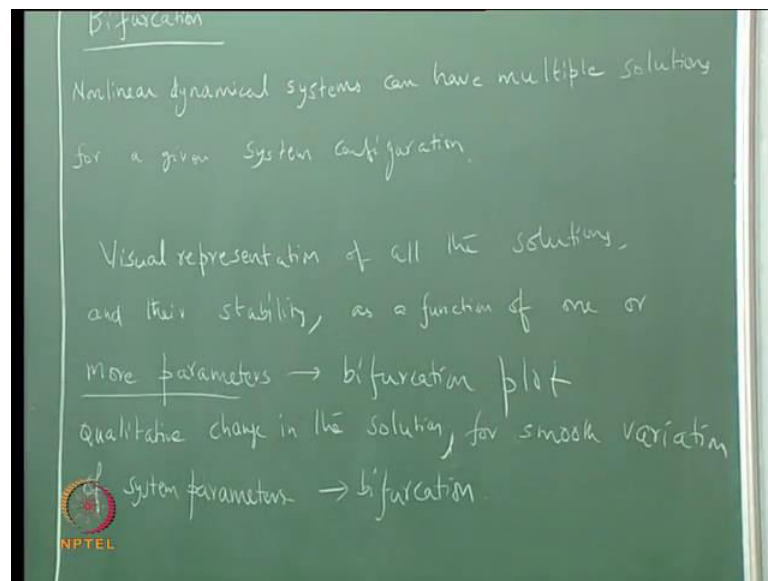
So, this would be map where the value at i plus one instant would be a functional value at the i -th instant and those parameters. So, if you are talking about the digital system if you doing time series and analysis or system mind fugation. This would be the even digital control they would use this kind of this kind of analysis presses use it a lot, but here I will we use this representation flow representation. Because, we wrote a differential equation, but had we suppose a we did not try differential equation behind difference equation. Then we could use a map both are studied very extensively.

So, we just saw that, so such systems can have bifurcation can they will, so qualitative change in the behavior or changes in parameters or this μ 's. So, we saw in this case of

the column that you had a weight. You kept increasing the weight and it suddenly the behavior change. So, this is called this is a kind of bifurcation this is not a only kind of bifurcation. There are several kind of bifurcations for example, sometimes you may have a flame, you may not have a flame in a combustor. So, that is some kind of bifurcation the flame is there the flame is not there. You can have different states the flame may be blone up or it may be holds that would be some other kind of bifurcations.

So, that plenty of examples of bifurcations and bifurcations are quite important scientifically. Because, they provide models for transitions from one state to another or instabilities, which occur some control parameters varied. I mean now our case we saw instability occurring in the rijke tube, but the other cases were some other kind of transitions can take place. That is a possibilities that you can go for a mont steady state to another steady state. That is also very much legitimate bifurcation. So, let me just a write down a formally what is a bifurcation.

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So, non-linear dynamical systems can have multiple solutions for given system configurations. So, want to have a visual representation of all the solutions. That is what we trying to get, but this not enough you want to know if the solutions stable only. Then we will know whether solution can stay in the nature or not and the stability. We want to know this as a function of the parameters this mousier. So, this would be call a bifurcation plot, so we want to there is a dynamical system.

It can have multiple solutions and we want to have visual representation all the solutions. Not just that we want know whether they stable or not we may have two that is a two steady states, but if one is unstable then we want see in the nature. Because, it cannot stay there even if you put it there any side this disturbance. Take it away to out of the state and may go to another state.

Now, so we saw that if we change the weight in this column example slightly you got the behavior change or in thermo acoustic system till power, there were no oscillation and suddenly the oscillations on side. So, you can have a qualitative change in the solution first smooth variation of system parameters, this would be call a bifurcation. So, there are lots of examples of bifurcations in nature, but generally people attack to classify them into a certain number of bifurcations.

Now, dynamical systems theory also deals with we looking a limits cycle oscillations, but they other final states like quasi periodic state cerotic state doubling and so on. So, for I will not go into those things, but privately I am open to discussion. We do see all those things in thermo acoustic, but at the movement I will not those are secondary bifurcations. You have a bifurcations to go to eliminate cycle from there that can break up and then go to cerotic state and so on, but I will not speak much about those things, but we can have private discussions outside the class.

So, I will give a general classification of bifurcations and then in that combustor person were our bifurcation, so that you can see. Because, what we saw was just I mean we saw through a example. We saw through a simulations that I showed the oscillation from the simulation that we saw a certain type of behavior. Now, we should not think that is the only type of behavior like we have the story of frog in a well. It is seen the well and think that the universe is that. So, we should not have such illusions the other things and there. So, scientist are studied a lot of things and sir that there are may be five or six wells. I mean or they would see hundred of wells, but they are classify that is this belongs to this tab this belongs to this tab. This belongs to this tab perhaps some on of you will bring a new type of bifurcation, that is possible also.

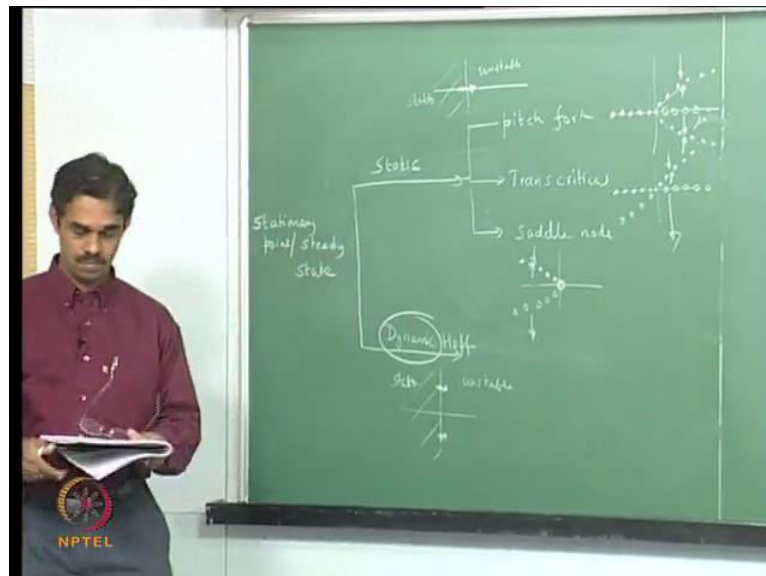
So, is there any questions and anything so for. So, is there any place may be conclude this instability by taking energy from that particular mode and locating some other form of energy like. If you put like a or that is one way or here for example, in this system we

have this walls and walls has this, holes like in a the absorb sound. So, here actually it takes the sound and you convert into fluid mechanics. They sound hit the wholes and set volts in the other sake you are converting energy in the acoustic mode to a fluid mechanics volt city kind of mode. So, I mean that is the way you take out song, where a we take a suppose in many complex and the land being non instabilities.

So, vibrations will be rectangular convert that vibration energy into some other form of energy. In principles yes to do it specifically you have to have specific solutions. I do not have any no experience converting helicopter vibrations anything else, but about thermo acoustic. I can speak about acoustic itself any other example be like you set of one form convert to another.

So, if you have a entropy fluctuation going through a nor or it will create sound. That is you are having energy in the entropy mode give rise to acoustic mode. So, you not only the take from acoustic you can give to acoustic also, but if you ask me about several other subjects. I may not have answer anything else any other questions Rajesh look very amused effort only. So, let us a we are solutions, which are stationary point a fixed point steady state.

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So, we can have two kinds of bifurcation we can have static bifurcation. We can have what is call dynamic or dynamic half bifurcations. So, your solution can be like a steady kind of solution, where you can have some solutions, which satisfy the equation. This

can good 0 f itself going to 0 would be a solution, but you can also have a periodic solution. For example satisfying the equation that be limit cycle that means some kai at some time will be same as kai at that time plus the time period. Or we can have a cerotic solution or quasi periodic solution many solutions are possible.

So, we have a static solution that means you go for a one steady state to some other kind of steady state or we have dynamic half bifurcation, where we go from a steady state to a oscillatory kind of solution, now here we have. So, there are call primary bifurcation and the dynamic half then undergoes secondary bifurcations and then move towards, but we will not speak about this here.

So, here we can have pitch fork bifurcation trans critical and saddle nods. So, all in this cases if you look at idly against the non-linear system. So, you have Eigen values laying on this horizontal axis. When you say Eigen values when the in our thermo acoustics problem. When we study the mac manners problem we were writing everything as e power i omega t, but then where we are studying non normal operators. We shift to the physics people notation e power lambda t. So, lambda is Eigen value i omega is also Eigen value.

It just that mathematicians like to write e power i omega t physicians write e power lambda t or you are free to pick any of them. So, if you are having a e power lambda t the left half would be stable write half would be unstable. Because, Eigen values on the right side such e power lambda t e power real part times t is the own, which exponentially going. If your mathematician all have to do is turn it ninety degree and the upper half will be unstable over half will be stable. Depending on e power i omega t or minus i omega t. So, you are free to choose all that, so here I will follow the physicians notation.

That what is followed in most of the dynamical system book. So, you are here on this real axis. So, this side is stable and the linearize system Eigen value is cross over in all these cases. Then you will put to unstable, but only the linearized system Eigen value is crossing over. That means that the linearized system will exponential blow up, but that does not meant the actual system blow. It will actually go to some other state in all these bifurcations. So, we should not get carried away by our exponential growth or something because, that is for the linearized system. It is varied only immediate vicinity of the that particular point above, which you are linearising not anywhere else.

So, that that something you have to watch out. So, we look at pitch fork bifurcation. So, let us say this is a parameter some any parameter μ . This is some kind of measure or some value. So, you will I will fill circles will be denoting stable solutions. Then now this becomes unstable and then you can have. So, this would be like a like fork that is why, it is call pitch fork. So, you have stable fixed points and you are unstable fixed points, but then you are unstable this way.

Now, trimes critical over would be this you have stable fixed points. You have unstable fixed points, but once you reach some reach here in bifurcation occurs, you get stable solutions here and unstable solutions here. So, derivatives it switches, now saddle note would be write below. So, these are filled circles and these are whole circles. Generally, they show this is half filled and half polo, that would mean it is a neutral stable point. So, these are the static bifurcation from one state it going to another state. That is not what we saw in out thermo acoustic system. We saw oscillations that is actually dynamic systems.

So, it dynamic half bifurcation may occurs you get frequency in this system, where as an static it just goes to another state. How it go we are dealing with a asymptotic state? So, we are it goes to some other state, but whereas, here we are going to a oscillation states. That is the key thing about a dynamic half bifurcation, is it clear? So, we saw that here an Eigen value was crossing, the primary differences is that you will have a payer of Eigen values crossing the ours. So, this would be stable and unstable, so payer of Eigen values will cross over. So, this is the main difference between this and these bifurcation were the cross over happens along the axis actually. So, there will be only one Eigen value cross it, this kind.

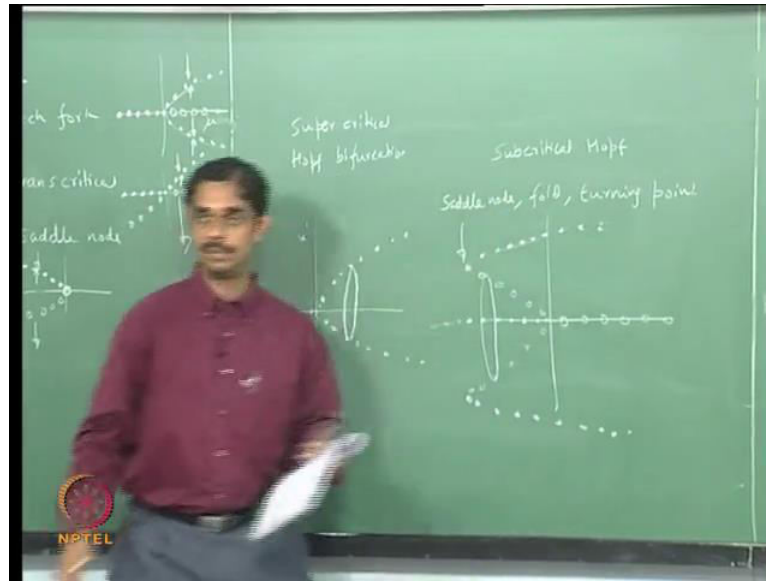
Like, in which bifurcation like just to give example. Like, you may have a combustor whether no flame. So, strictly speaking you can have flame, but it will not stay. So, that like a unstable fixed point. At some other if you vary the temperature role or something as a and them flame be hold. So, still some value of the control parameter flame would not hold and then flame, but hold I think those are all unstable. They are in principle solutions, but you cannot stay at this state. I think there are hundred thousand ten thousand examples, but I think I will not go to I will testing with thermo acoustic. The example you gave is coming under, which of these.

I think this will need much more discussion. So, I would just stick with talk discussing, because you will have to explain each of them much more regure and show of thing. Just to answer the question one shot all these bifurcation any bifurcation something called them normal form, which would be like the very simple differential equation, which would show the basic feature. So, can you reduce you are your system to that normal form, but would be would not way to look at it, but I think it is not going to easy to give answer without eventually of proper lecture, but there are examples. I mean look at these books and you see lots of examples.

Here in pitch for example, you can have solution there a solution here. So, if you are here you get attracted to this if you are here you go here. If you are here you get ripple to here below this you get ripple and get attractive to here, you will get come here. So, here there are two possible things here that is not possible. If you here you will come here, if you are here you will go there. So, here just we blow up, so this will be like a example of flame thing. If there is a flame and if you unstable and you can blow up, but there may be a possibility to get a stable flame. So, those that kind of examples here you will come here, but a here you will blow up.

So, I think I would have to much more elaborate to go especially example of each and so on, do not make any sense, but I just want to say that a reason I brought thing is to say that there are many more things, but where we are here and we are specifically having a dynamic bifurcation. That means oscillations are coming, where as this we do not have this. Now, you have sub critical bifurcation and super critical bifurcation. Again, I am not going to go to those things. They also have the static bifurcation they also have sub critical, super critical. I will speak specifically about a this half bifurcation. I will explain sub critical half bifurcation and super critical half bifurcation. So, first I will explain super critical.

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So, let say we are looking at fluctuating velocity. So, you have stable fixed points or stationary point till here. So, these filled circles and then these happened for degree of system, which have more than one dimension. So, this is actually this is going like this if you look at the another flame, but so you will see a amplitude here and amplitude here. Or if you are just floating the amplitude you can just float the upper half. This would be super critical bifurcation you come here and smoothly the amplitude keeps going up, but there is also sub critical bifurcation, where let say you have fixed point here. Then you have unstable these are stable these are unstable. Then you have these are unstable solution.

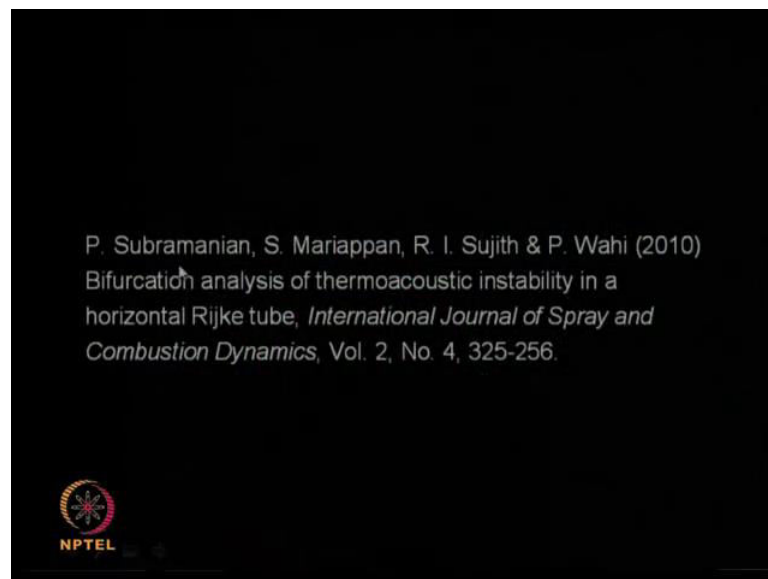
You can have unstable periodic solutions, but they are unstable you cannot really see them in nature, but they are unstable. This is sub critical half bifurcation, you are a question. So, in reality what we see we actually see solutions here stable, we can see stable limits cycle here. So, these are filled circle, so the stable limit cycle. This unstable limit cycle here annihilated this idle node. So, here you can have a sudden node or turning point or a fold bifurcation. So, this point is called the different names saddle node or fold or turning point. So, you can then have these are stable limit cycle solutions that are possible.

So, if you look at a thermo acoustic oscillations. If you have sub critical bifurcation you are increasing the heat of power. Let us say in our rijke tube come here and then you

jump up here, but you can also axis the states. If you are up here and then you reduce the heater power, you will have the hysteresis, you come down come till here. Then you can jump down to this state, so this is what we see. So, there is half bifurcation this part is call the half bifurcation. This is a sub critical bifurcation then it under goes a saddle node bifurcation or a fold or turning point bifurcation. Then you can actually have a stable units. So, you can have a unstable branch here and then turn around to a stable branch, this is what we saw.

So, that is one kind of bifurcating, which is half bifurcation. So, which is in this scheme of things you are like here, I hope you got some idea of this. So, this is not a course on dynamical systems, but a U I I strongly advise you to listen to the N P T E L lectures on dynamical systems, read these books. Now, if you want to get a reference on thermo acoustic.

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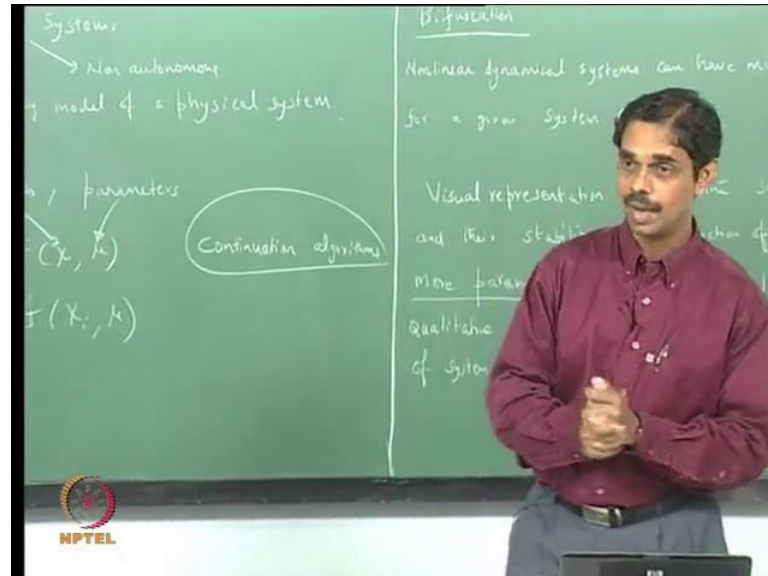


So, you can see this prepared by P Subramanian Mariappan and Sujith and Wahid on, how to calculate bifurcations in a thermo acoustic instability in a horizontal rijke tube references given. Now, one way we did the solutions was to be actually mass in time. You got a solutions right we started from a initial condition. Then I said you can do or some kind of integration delay differential equation and so on and, we can go forward.

Now, if this is a way time constraining first thing, second thing you may will never reach unstable limit cycle, because by definitions they are unstable. So, you cannot time must

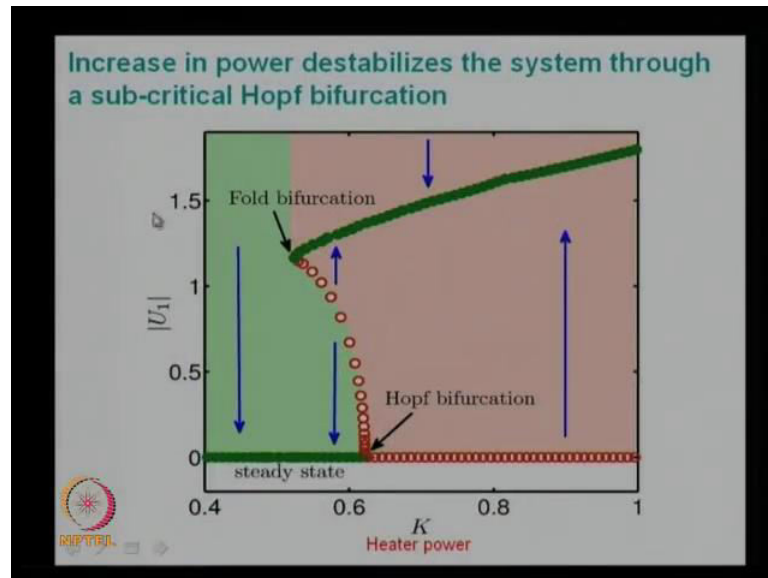
to such a state. So, what people do use thing called continuation algorithm, where are if you find one solution then you can change the μ little bit.

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Then find use this initial use the solution as a approximation for the new solution, we have μ , then is a μ plus $\delta \mu$. So, we have a solution and then you try to use that original solution as a approximation to new solution. Then try to get a new solution numerically without merging in time. So, you can see that in this paper, but I will not a describe this, but I just want to, now go back to a rijke tube bifurcation diagram. See it in the conductor sub critical half bifurcations.

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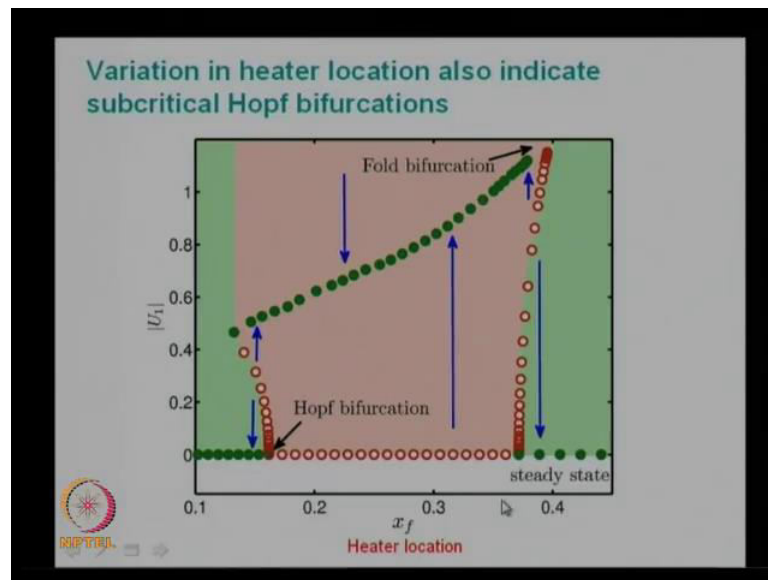


So, this filled circles they are the fixed points. So, the steady state and that is trivial thing there is no, which is the atmospheric pressure deduct. Then you have again the solution exist, but you cannot access it, because it is unstable solution. You come here and you will jump over here, you get to this green filled circles. That is again the limit cycles, but you see this at this point with fold bifurcation occurs, where the limit cycle the stable and the unstable limit cycle, but this annihilate each other of something.

So, you have this is like a globally stable regime, what any disturbance will die on you have bi stable regime, where if you are in the basin of attraction of this limit cycle. You will go there if you are in the basin of attraction this fixed point. You will or this steady state you will fall there. There is a basin boundary in between and the unstable limit cycle is just one line on this basin boundary. Then you have this linearly unstable regime, where our it is always unstable.

You have any disturbance any small disturbance. It will exponentially grow to instability. So, this is the heat of power is the I mean case non dimensional heater power is the parameter, that we are fixing that we are changing. So, this is like the one of the μ . In general μ can be a vector there can be a many parameters, but here heat up powers we look at a we said that the flame location was another parameter.

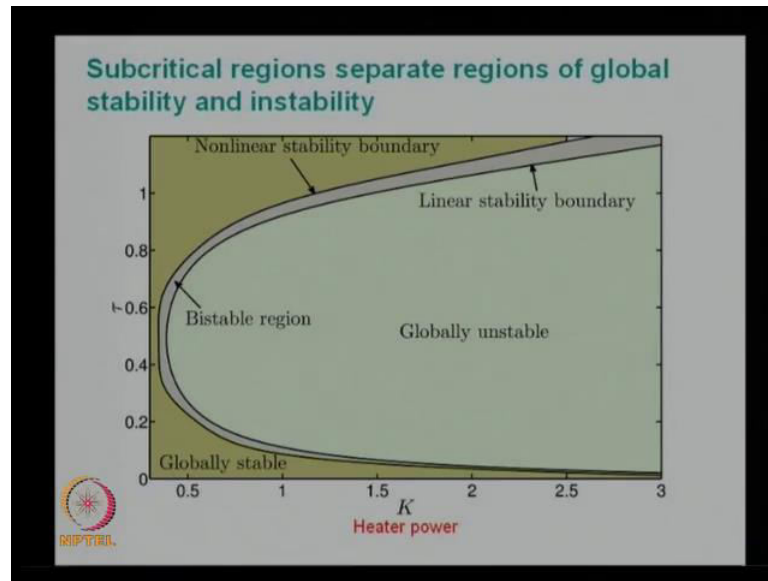
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So, if you change the flame location and heat a location you see this kind of bifurcation. You can see again hysteresis in this model they come till here. Then you jump up, but if you are going back you have to come back here. Then you keep going and again you will have a basin boundary here. Then if you are here then you will follow to here, which is a steady state and in between the this follow circles, which represent the unstable solution. So, this is a completely stable solution a globally stable.

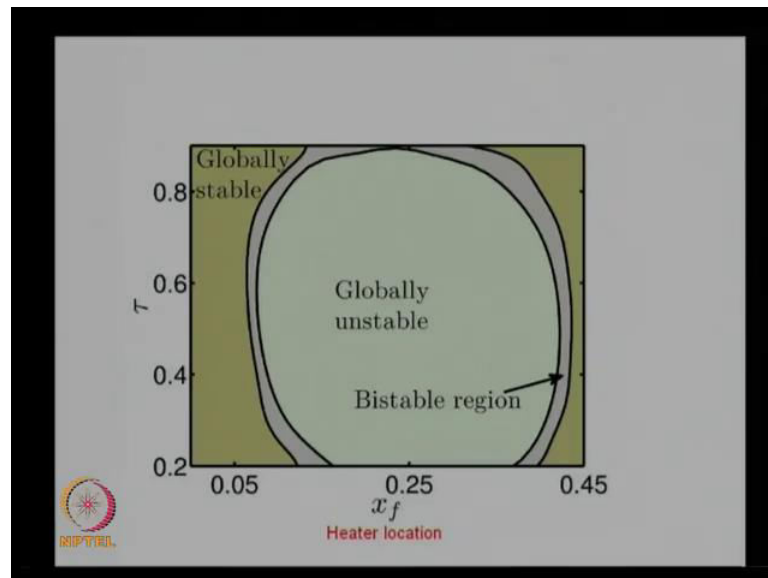
This is by stable depending on whether you are on the basin of attraction of this solution or this periodic solution. You go there here it is a what you say globally unstable and here is a by stable region. It is globally stable region and this globally this bi stable region is corresponding to an example. That gave you students to a half way sleeping, but if you rise you wise will wake up, but if you are having students. You are always sleeping, what we do that is here. Then if you are here they could students always excited.

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So, we see kind of a linear stability boundary and a non-linear stability boundary in between you see this bi stable region. So, here I float it heat up power and tau as the parameters tau is the time lack. So, this is the by stable region that is why the non normality in the linear theory, it plays a role.

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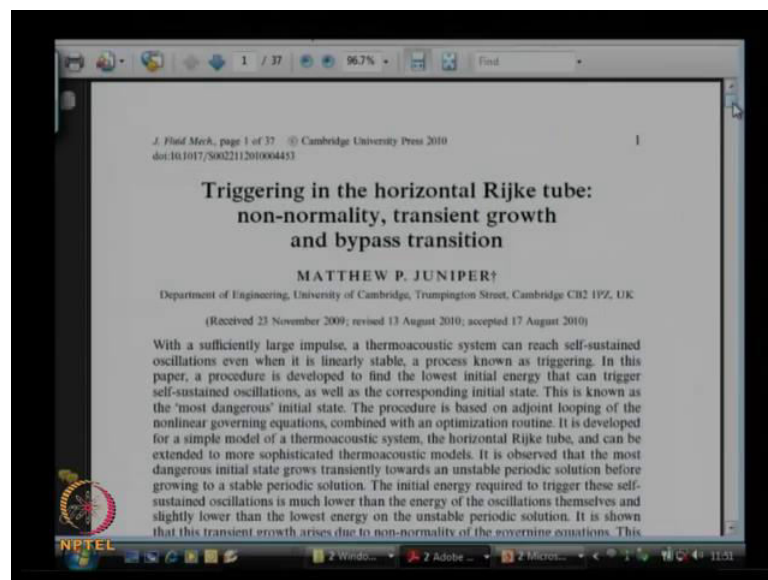


So, we have sub critical regions which separates regions global stability and instability. There is another float of tau verses x_f time lack verses flame location. Here also you see globally stable and globally unstable and in between there is a bi stable region, but you

do not necessarily have to always see this. This is this model predict it is in this case there may be other models, which predict super critical bifurcation and experiments. I think people have seen both sub critical and super critical bifurcation. Although, people have not quite spoken in great detail in this language. I mean this dynamical system approach is only being brought very recently thermo acoustics.

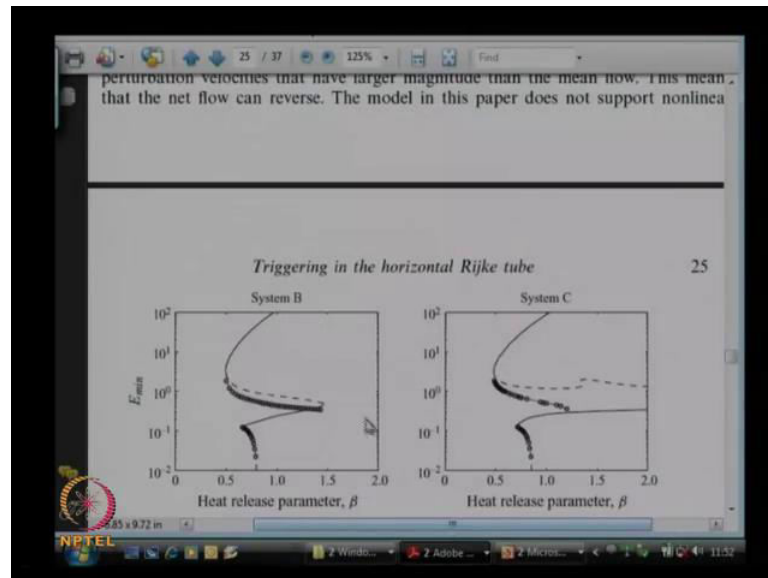
So if you have any questions. That is possible, now that you ask I will get you a reference. So, this is the reference that I spoke to you about earlier by Matthew juniper on in general of fluid mechanics triggering in the horizontal rijke tube, non-normality transient growth, bypass transition.

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Here, he give such a solution, it is possible you can calculate. So, you have can you see this, here you have a unstable solution here, you go to a stable solution.

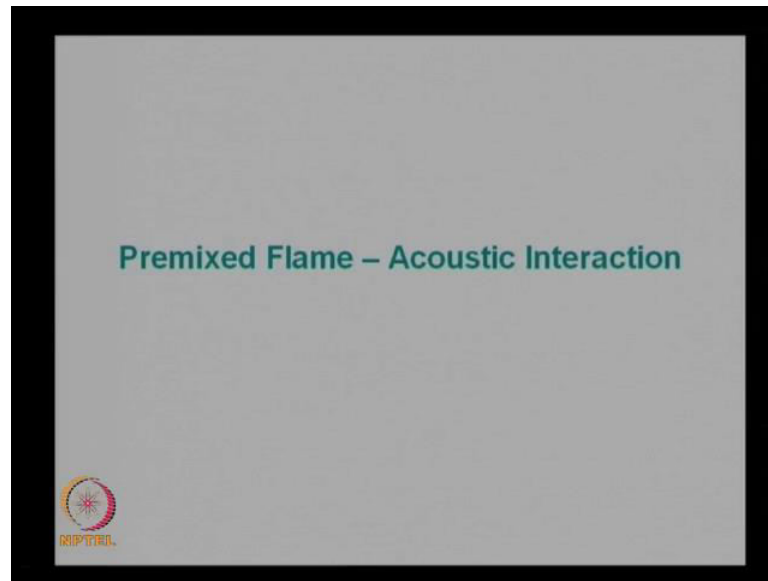
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Then you have another set of unstable solution and you have a stable solution, but I do not know. If this calculation is valid because they have used linear acoustics for this model and this amplitudes are quite large quite large. So, I do not know how much role non-linear acoustics will play. Whether this actually be observed in experiments or not, but now have just model displaces. So, actually there is a stable limit cycle here. This actually higher stable limit cycle here, which is again separated by set of unstable limit cycles. This I think this these are the points which are the points of minimum energy, which is needed to go up to here itself.

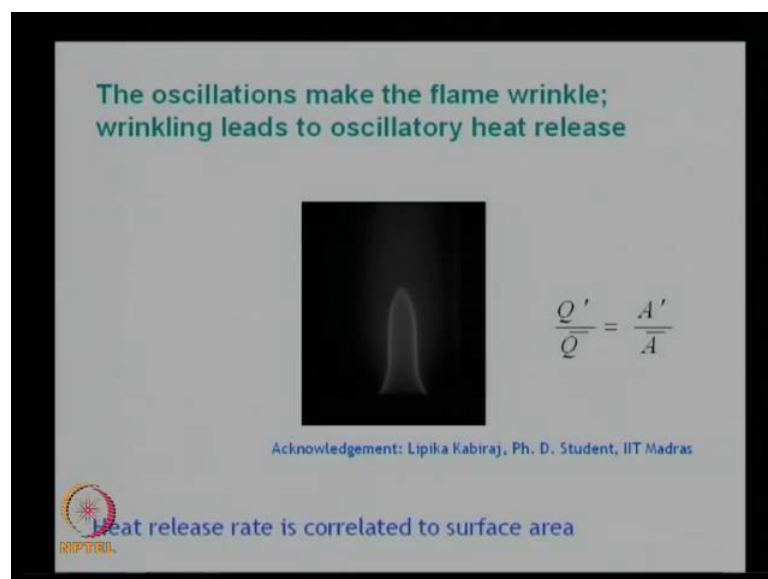
So, it is possible a multiple solutions, I mean this is one example such a calculation any other questions. So, suggest that emphasize that when you say solution we are kind of addicted to stable solutions. We have unstable periodic solutions and solutions with which are unsteady. That is why we are in the business of thermo acoustic. So, I will stop my discussion on non-linear dynamics. Here I hope you got over view of these things. This is of course, this is very insufficient, because just a brief overview, but it is a very fascinating subject read the books go attend the NPTEL lectures. Now, we will go on to back to our thermo acoustics.

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So, the next topic I want to speak about a premixed flame acoustic interaction. So, what we did, so far as a very simple model. I characterize the classical stability analyses as well as the new age or modern stability analyses. This is not normality and non-linear dynamics and so on, but then we had weak point in a model is just a formula or a correlation for hysteresis, but duly have to find out how a flame response to fluctuating velocity. Each system differs in different cases. So, what I would give is a simple example in which a premixed flame will interact with an acoustic field. So, you must have all, so you can do the experiments.

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You can do it at home also is not dangerous. If you take a flame and you put a tube around it. It will sink you can try it out. I will bring a video in next class, but you can try it out yourself no danger at all in take flame and to press a glass tube around it, you should here it sink at the right conditions. So, if you take high speed movies of this you will see a that if flame will be wrinkle and kind of dance like this. Of course, in a premixed flame the heat releases proportional to the area of the flame. Because, everything that goes to the flame is burned and it converted. So, the amount of things that are burned depends on how much react in mixture goes through the flame.

So, as long as there is a flame you have you have hysteresis. So, more flame is there more heater less flame is there less flame is a less heater list more flame is more heater list. So, it can go it can be very strongly correlated to fluctuations in a area. So, we will take a look at how such a system can be studied. Then we will also look at several other systems we look at systems in equivalence ratio fluctuations combustor. We look at solid rocket motors possible like a motors and try to see, how the heat releases correlated to the velocity fluctuations that pressure fluctuations.

So, we know model acoustics we have the equations. Then we set the these are the plug in the correlation or have a have the thing be model by C F D or some simple analytical model or something. So, I will again come do only simple things in class. So, we look at how oscillations effect a premixed flame. Once we know that if you bring in the acoustic equations, we know how the heater list effect a acoustics and there is a feedback. So, that I will be doing. So, stop here now.

Thank you very much.