Acoustic Instabilities in Aerospace Propulsion
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Lecture - 28
Non-normality, Transient Growth and Triggering Instability-3

Good morning everybody, we were looking at how to analyze non normal system and we wanted to, so we showed that thermoacoustic system is non normal. And we show that, there is transient growth and we also showed that, transient growth thus play a role in getting the triggering oscillations of the sub critical transition to instability and we were looking at, how to characterize the transient growth.

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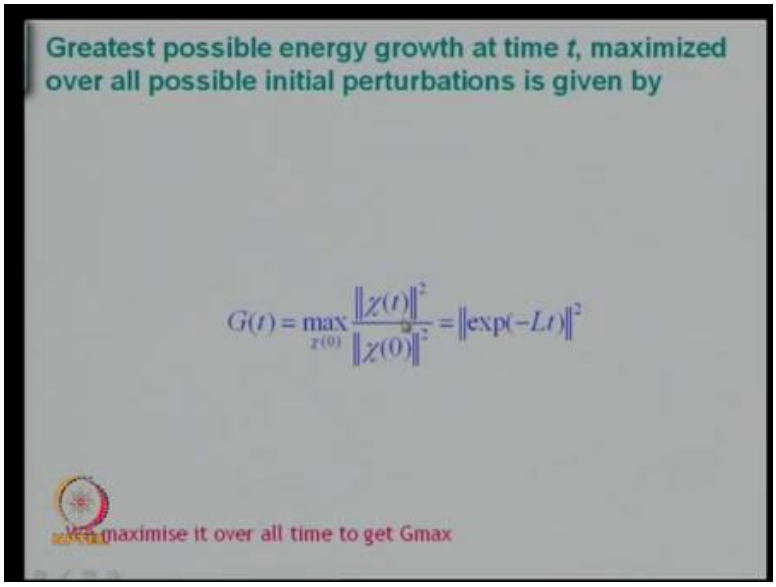

So, we came up with this parameter called G, which is norm of the state vectors at some time t divided by the norm at time t equal to 0, which we showed that, this is equal to the norm of the evolution operator. Then, we said we will maximize this for all initial conditions and then, we maximize it for all times and then, we get the maximum growth factors, other any questions.

Student: Actually I beginning with this principles the norm of the vectors.

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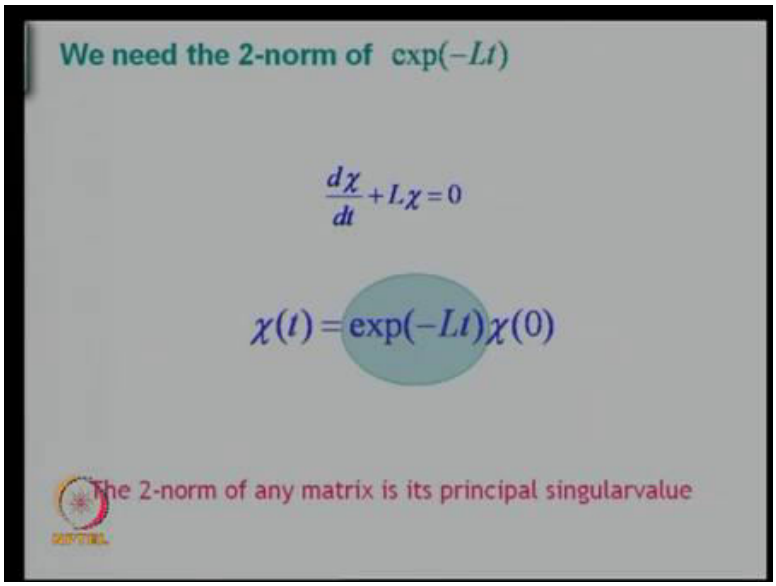

So, we will redo this question, redo this issue, so let us start from the different equation d chi by d t, the chi is the state variables of the vector containing the state variables and you have evaluation operated d chi by d t plus L chi 0. And this is, if chi was just a number then, you can integrate it, the way you integrate a number.

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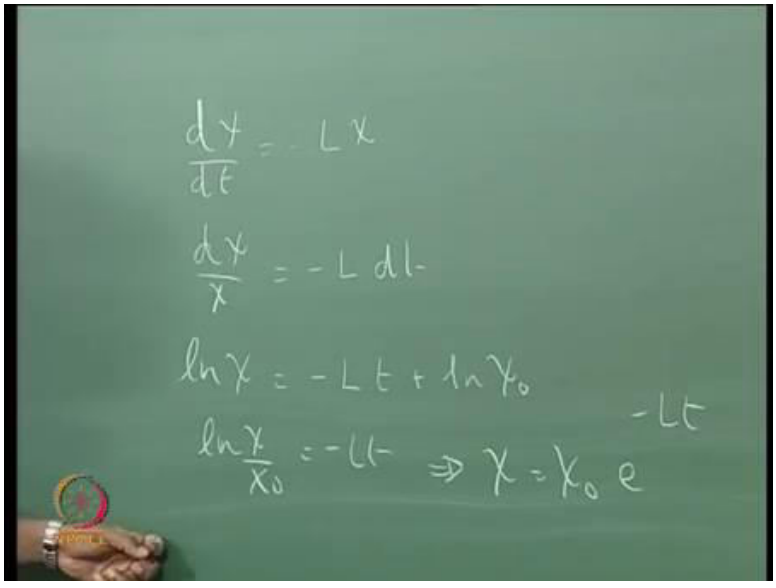

For example, this is what you do if you adjust one variable that is scalar, but you can do the same whether matrix and we saw, what is the meaning of matrix exponential, how to calculate matrix exponential etcetera, yesterday. So now, we say chi of t is e power

minus L t chi naught and we need the norm of this e power minus L t. Now, see we know that, the norm of a vector, you know what is a norm, how did you define a norm.

Student: ((Refer Time: 02:46))

So, let me get to that slide.

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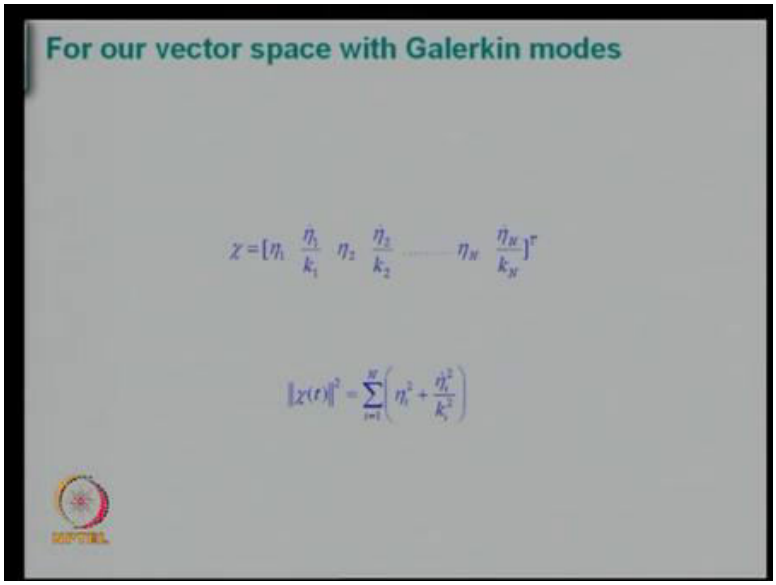

So, if you are a vector, you square them off and then, you get the norm, now this scaling by k 1, it is just the right amount of scaling. For example, you can use if you have x y z and in one direction, you have centimeter, another place you have meter. Then, if you square it and it would not be write, everywhere you have to have meter-meter, then only it will make physical sense, so it something that some scaling to make physical sense.

Now, the question is, so this is ok when you have a vector, but what happens when you have a matrix. Now, who said square them add and you get the norm. I mean, you said this definition at the beginning just before the class started, that square them all and you get the norm and that was ok with you, so who came up with definition of norm.

Student: ((Refer Time: 03:54))

Who defined it.

Student: ((Refer Time: 03:57))

Somebody human being defined it and we define thing such that, so in maths we come up with things and then, we try to interpret real life situation with these things. For example, you say 1 into 5 is 5 that means, if you have 5 chocolates give 1 kid 1 chocolate and then, you have 5 kids, they all got 5 chocolate. So, you set up this maths and then, you see if you can apply to physical, now you say minus 1 into 5 is minus 5, now so this is abstract concept I mean, you cannot prove this, it is just a given thing.

You can interpret it as, you owe me 5 rupees, he owes me 5 rupees, like that 5 people owe me 1 rupee. So then, I call each person owe me minus 1 rupee, 5 times that, I will be get 5 rupee, I have to get, I put minus 5. So, you define something, it may be useful or useless and maths people, you see they do lots and lots of things, only a small fraction of it we use. So, they also do not know and what they doing, see what we are doing, what they did it 100 years or 200 years back.

And what they are doing now, we would not touch till 2200 or 2300, that is the way it is and physicists are one step ahead, what physicist do 50 or 70 years before, engineers are doing now, I mean. And mathematicians are way ahead, but most of the thing mathematicians do will be useless, some will be useful which physicist will take, engineers will wait till the physicist takes the subject to some level then, they will take it and start making things, that is the way it is.

So, the norm was defined that way and you could have define norm in some other way and there are ways of defining norms, what are some other ways of defining now actually.

Student: ((Refer Time: 05:39))

You can also define norms as the maximum value of the set or something, that is also possible.

Student: ((Refer Time: 05:51))

You can define it whichever way, so you define something which is useful, just to give a example, now you have two players, yesterday we saw Suresh Raina play, I saw it actually for a long time I saw cricket match at night and he replace Yusuf Pathan, apparently both have same averages.

So, what they wanted was, they were show that lot of wicket will fall and at the time, they wanted to some guy to stay till the end, so that was the objective. So, if you look at Suresh Raina's average and Yusuf Pathan's averages are same, but standard deviation was very high for Yusuf Pathan, he will someday kill everybody and many other days get killed himself, Raina is kind of steady guy.

So, the average itself as a norm alone then, help in making decision in under that circumstances, you need some guy who is not only has a decent average, but standard deviation is also low so that, he will make the average always, so that is the idea. Now, if we did not care about this situation and we are no problem, if the middle order bated well and the low middle order came and all we have to do was the hit. Then, we need a guy with the high standard deviation, because a guy with a low standard deviation may not attempt these things.

So, why did the standard deviation matter I mean, because the circumstances demanded it, so what we need you have to figure what it is and use it, and that is lots of norms available in maths. So, we have taking two norm for some reason and you can imagine why also, because like a square of something, so you can already seen advance, you have p prime squared plus U prime squared, so that is our hidden agenda, we want to get something similar to that if it all possible ((Refer Time: 07:40)).

So now, question comes what is the norm of this exponential operator, so we again look at that is a take the books of linear algebra search through everything. Yes, there is a two norms for that also, which is defined as the principal singular value and so, it does not make sense, because principal singular value some other abstract thing and norm is another abstract thing. So, we have to of course, mathematicians develop abstract things, it is our job to find the right abstract thing and put it in a physical thing.

So now, what we need is, what we can see from the numerical simulations and all the stories that a told earlier that, you can have Eigen vectors are decaying, but some direction you can grow. So, growth has to be measured in some energy norm, so in which case here we say something like p prime square plus U prime square weighted. So now, that is like the two norm of the vectors space, now so if you take two number this side, you take two number other side also. So now, what we need is, some machinery which will find those directions and it is amplifications.

So, that is a idea and it exist a singular value, now if you understand the meaning which are go through against slowly, then you will know why we choose principal singular value, as the or why we choose two norm or why this two norm is appropriate and so on.

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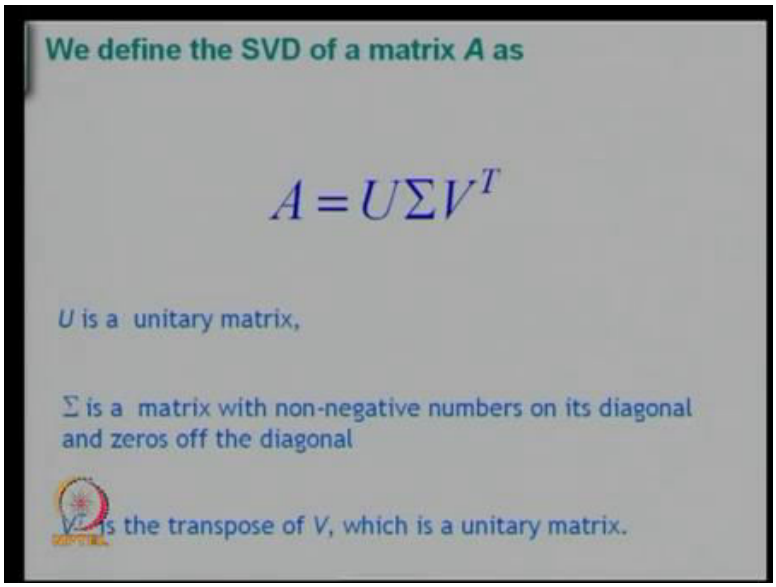

So, this part you understood I guess that singular value decomposition, now there are lots of decompositions matrix algebra, can you come of some name. There are lot of decompositions in linear algebra LU decomposition, QA decomposition, SVD singular value decomposition, Eigen value decomposition, there are lots of decompositions, we have to pick what is necessary.

So, when you look at science, now I say this is what is the thought thing done, when you see how this was discovered, we tried lot of things and this is what works and the guys before me tried everything. And I am sure they would a try 100 things, 99 fail, 1 work and then, we picked up that is my experience and I was trying to. So, I to be honest very I was try to look at this and I was really brought down by this problem, how to find maximum and front of my itself there is fluid mechanics, they use SVD to characterize.

And then I am to trying to understand the meaning of a this SVD and so on, because you they can do whatever they want, but if you are able to interpret that in our circumstance, it looks good. So, in reality this is not the line of thought that happened when this idea has came it is all, but now everything is done, now everything is neatly presented, but that is not the way things are develop.

For example, in a movie when it is shot, they do not shoot in the order of the scenes sometimes may be the last scene will be first time, that is shot in between scenes, may the shot next first scene may be shot last, but the end they edit and put it. So, that is the way finally, it is presented, so that the person who hears a make sense, but that is the probably not the way the guy who made it come up with, so that is some of the difficulty here.

So, we have this decomposition when it is any matrix say and I said you can have many decomposition, here you say U sigma V transpose, U is an unitary matrix, sigma is a matrix with non-negative numbers on the diagonals and zeros of the diagonal and V transpose is the transpose of V, and U and V are both unitary matrix.

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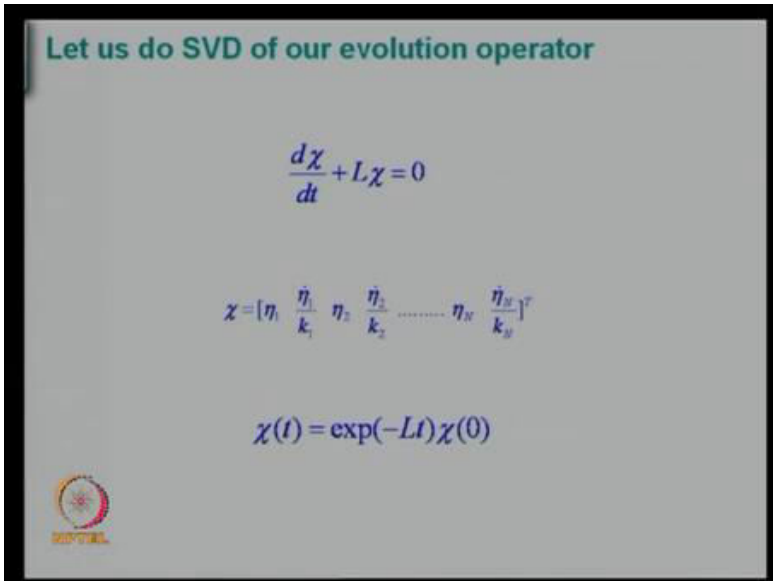

So, now we do SVD of our evolution operator, so we have d chi by d t plus L chi is 0 and chi is this state vectors, eta and eta dots and this is the analytical solution except that e power minus L t involves matrix exponential.

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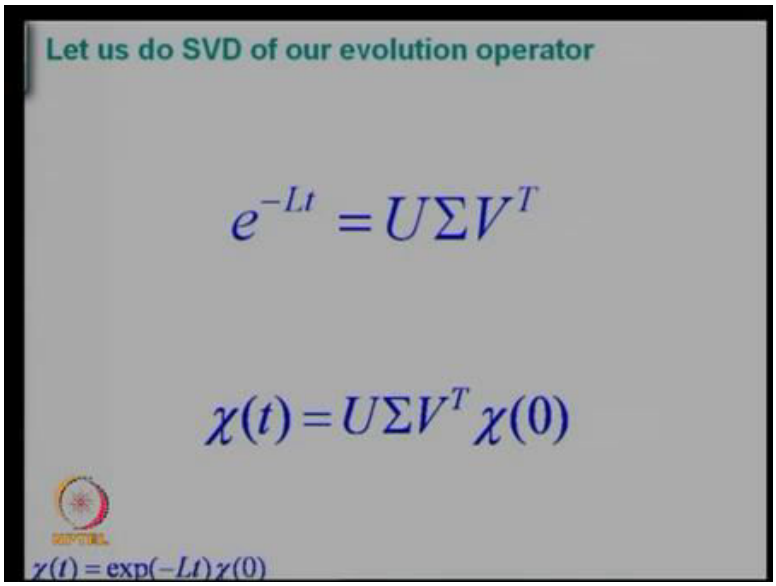

So, we do SVD of this evolution operator, because that is what we are concerned with vector we know how to do SVD, I mean to get the norm, because you have chi naught and chi we know the norm we want to get norm of this. So, we write chi t as U sigma V transpose, I mean we are doing, let us see what happens.

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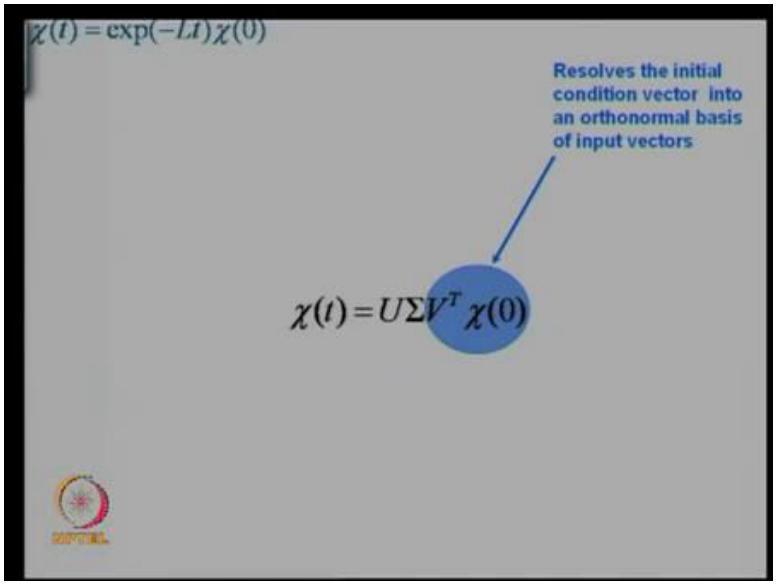

And now , in the physical meaning of this is sense, V transpose it is a unitary matrix, so V transpose chi not is a actually the solving the initial condition vector into orthonormal basis of input vectors. So, just like you have a vector and you can write it as a x plus, I

mean a i plus b j plus c k, or you can write some capitulate times e r plus capital B times e theta plus some other time thing times e pi. So, similarly we have this basis use this V transpose as a basis and it is resolving the initial condition along those basis and they are orthonormal, so that is the idea

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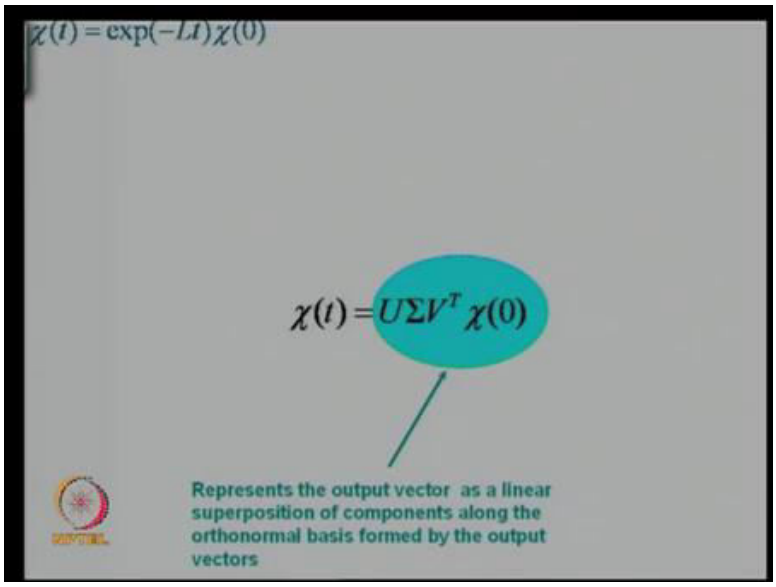

And now U multiplied by sigma V transpose chi naught, this represents the output normal as a linear superposition of components along the orthonormal basis, but the thing is you are having another basis. So, original basis may be like this, but this could be a rotator or something, so there is no guaranty that you will follow the same basis.

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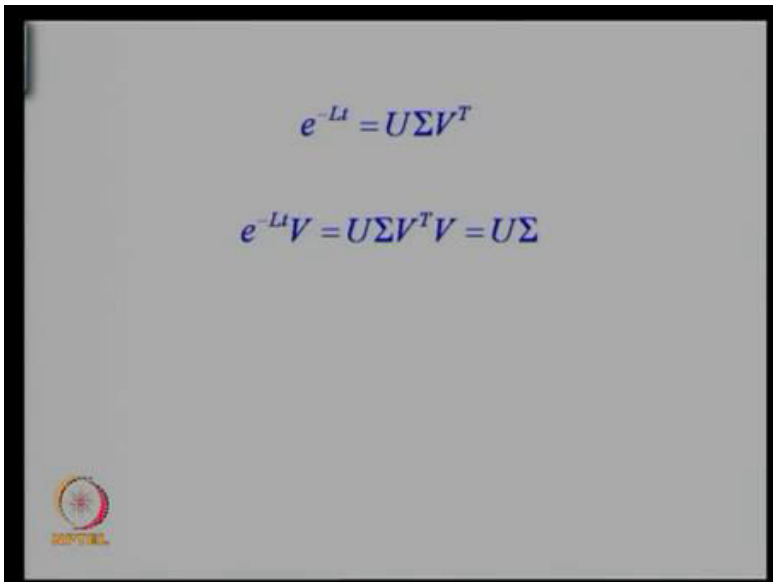

Now, to understand the physical meaning, so the Eigen the singular vector the several vectors in the matrix, so we will have to look at that, so easy way to look at it is we multiply e power minus L t by V from the right side. Now, why do a do this you can do anything you want, whatever you do if it returns some results and it is good, many things would not return the results.

Now for example, whenever the kid I was always wondering why do people bowl outside the off stump, is they want to strike the wicket he should bowl straight. But, later understood that if you bowl outside the off stump people will try to take swing added and then, you will get a catch and will be out, so that is the objective. So, now if you bowl outside the off stump and the guy does not care, he will just a takeover walls of Indian cricket Rahul Dravid just lift the bat up and then, he just wasted a delivery, so something like that. So, you can do anything whether it is useful then it is a, he say wonderful brilliant idea.

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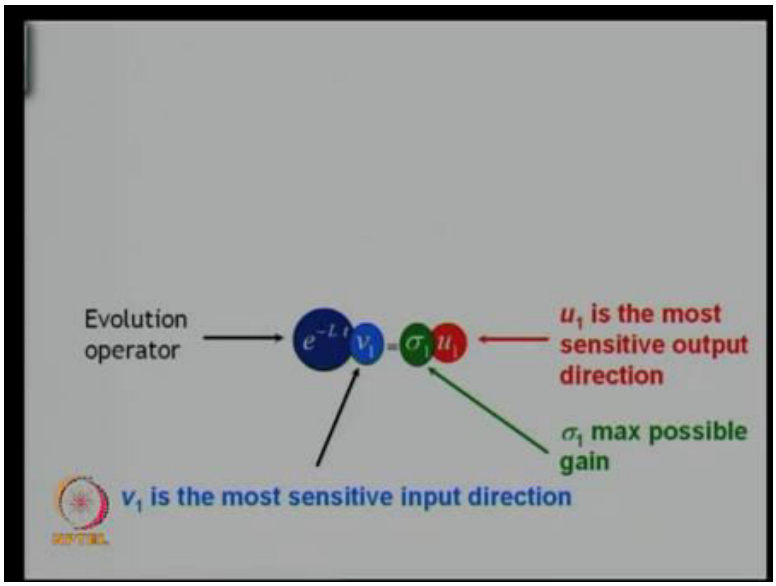

So, but often these things are they just do it not without and then, things work and then, you construct why it is done to be very honest about it, so evolution operator is e power minus L t and V let us see it is act on the most sensitive input condition. So, this will give sigma 1 which is the maximum possible gain multiplied by U 1. Now, the e power L t can not only act on V 1, but can act on V 2, V 3, V 4, V 5, V 10, now why there are these columns, because we are discretize, so the size of the system is now depended on how many modes you have, so you will have that many directions.

So, you have we do not have infinite number of initial conditions you have finite number of combination, determine by the dimension of the system that is be if you take 10 gallery most we will have 10 times, 220 vectors should be there. So, e power L t acts on one of these things and you will get the output U and V are they are unitary, so sigma one will choose show the amplification.

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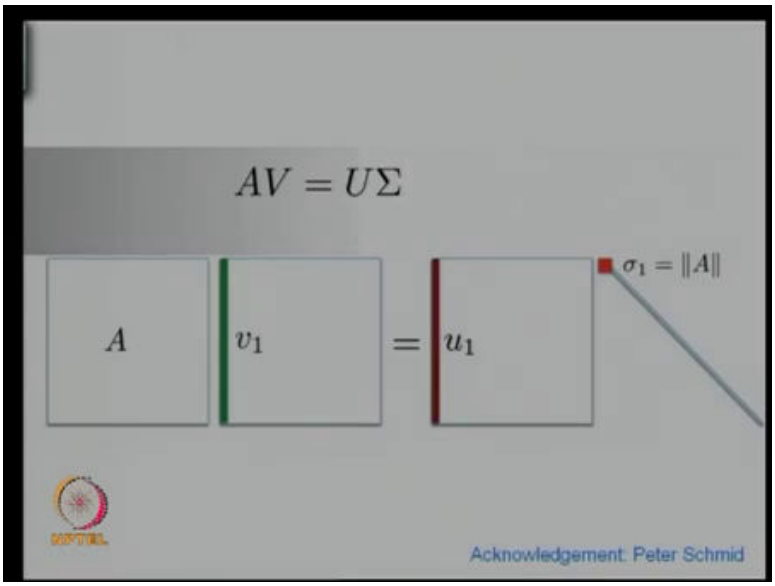

So, to see it in this matrix form, so A if it acts on V 1, you will get U 1, so V 1 is the right or left, here right singular vector or left singular vector, this look at you notes right singular vector. So, the evolution operator acts on the any operator acts on the right singular vector, you will get another basis function which is U 1, which is multiplied by a number.

Now, if it is acting on some vector here, let say you will get another vector here, somewhere here with some other number here, so you can do anything we are interested in the maximum amplification. See generally in instability we have worried about the damage how much it can grow, so if you know the maximum damage we know this is like a bond anything occur will be below this, so here our attempt is to pick the maximum grow.

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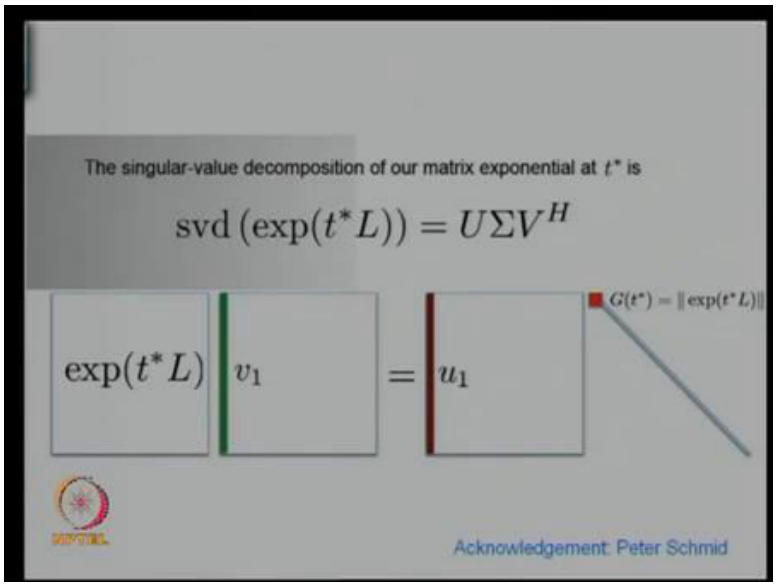

So, if you are if you replace this A with the evolution operator that is e power L t, here it is i mean in our norm this a slide from Peter Schmidt I mean, our L was minus of this, so this operates on V 1, you will get U 1 and a times the sigma which is the norm of this.

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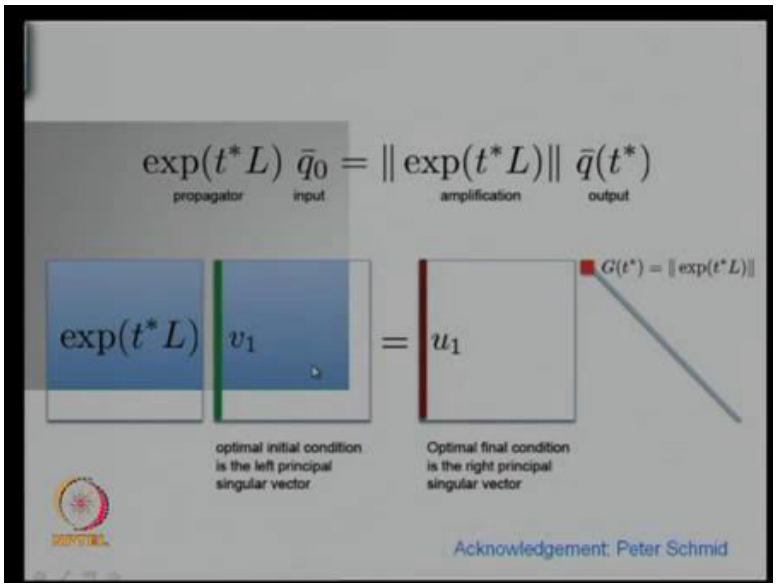

Now, so you have optimal initial condition and U this is wrong entry is it right singular vector and this is you get it amplified and you get the output direction.

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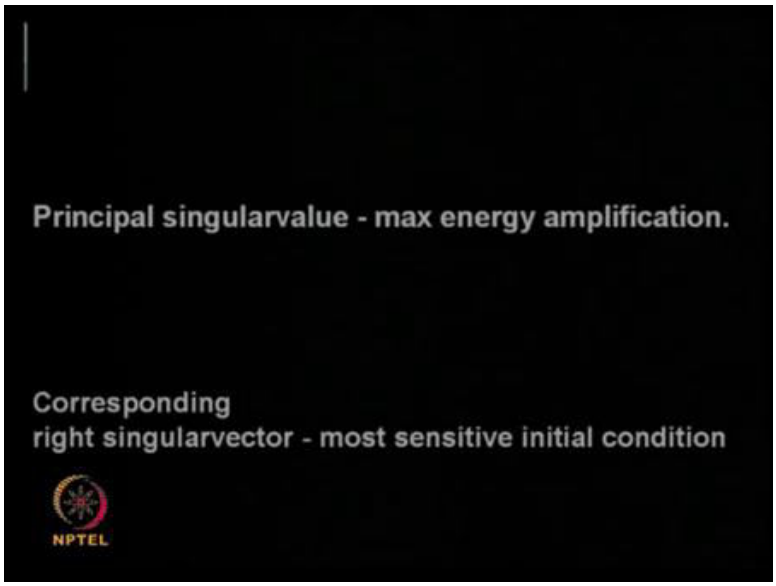

So, principal singular value shows the maximum energy amplification and the corresponding right singular vector shows the most sensitive initial condition, so now I do not know, you have a question.

Student: ((Refer Time: 17:15))

But, the thing is the energy is not, so you have several Eigen vectors you asking why it evolves independent evolves.

Student: ((Refer Time: 17:40))

Any matrix can be try, if you do Eigen value decomposition Eigen value decomposition you can do Eigen value decomposition for any matrix

You can have Eigen basis for the basis.

You can have Eigen basis ((Refer Time: 18:11)).

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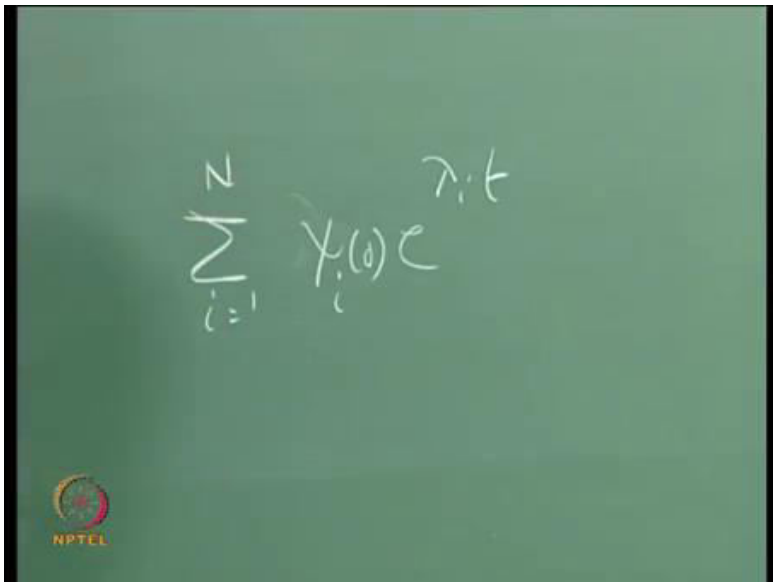

So, the problem is that, so if you have several Eigen vectors, then each of them evolves as, sigma i equal to 1 to n, see each e power lambda 1 t evolves another way, e power lambda 2 e evolves another way, e power lambda 3 evolves another way. And that does not e power lambda 1 t, how it was done depend on e power lambda 2 t, but the problem is you cannot take the energy of the modes and add a and you will not get the energy of the system, so that is the only problem.

Student: ((Refer Time: 19:04))

That is no Eigen vectors independent, but they will have dot product will not be 0.

Student: ((Refer Time: 19:31))

I mean if the repeat it here, if you do not repeat this is, but the problem is you can have modes, but the some of the energy of the modes is not equal to the energy system, that is only problem. So, in that sense they are, so if you have energy of the modes, you add the take the energy in the mod 1, energy in mod 2, energy in mod 3, you add them that will not be a energy system.

There will some interaction terms account from with to account for the balance, so that is the only that is difficulty, so it is in some sense, because of the problem this is useless, because the modes energy of the modes does not give the energy for the system you have account for the interaction, but otherwise you can do this.

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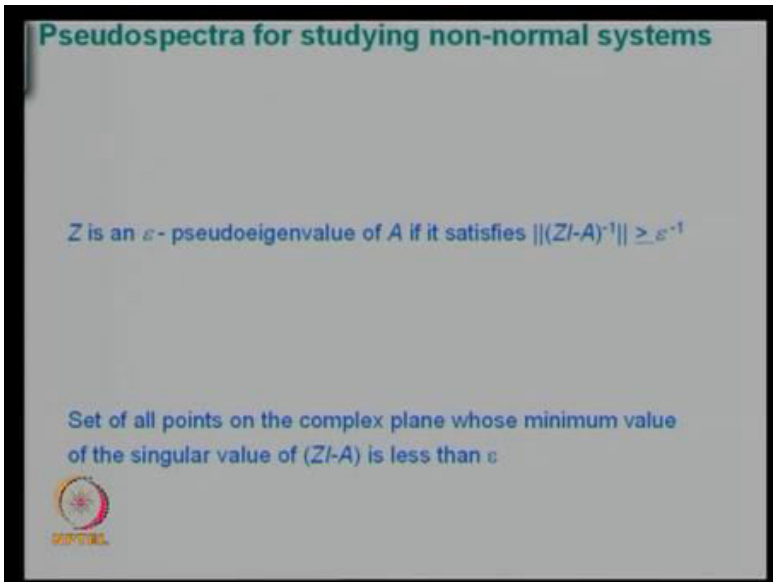

So, next...

Student: ((Refer Time: 20:38))

And the type of excitation, so if you have basis functions basis on first mod, second mod, third mod and all that, you know how much to excite in each of them, so you may not necessarily get the maximum amplification you have excite a combination of modes.

Student: What is the initial condition we are talking ((Refer Time: 21:06))

So, you let say we have several Eigen modes like here, but this initial condition is for the let say it is the projection of the initial condition on each of the Eigen mode. So, you can disturb the first Eigen mode or second Eigen mode or third Eigen mode or hundredth Eigen mode or you can disturb a combination of all these Eigen modes. So, you put the energy distributed in the different Eigen modes and optimization condition means we optimum direction means, in which manner you distribute the energy, a certain distribution will give the optimization condition.

So, this is the Eigen vector decomposition, you can do Eigen vector decomposition or we can do like a orthogonal basis, which is very convenient that is why used orthogonal basis, I use the natural modes of the product, so that is convenient actually. Any other repeated Eigen values you can construct solution.

Student: ((Refer Time: 22:11))

I have not, first you have to define a entropy based on this is in people would do statistical mechanic, they construct the report crystals and all that phone on vibration sound, so could principle do that, but we does not this theory I have not done. So, we can use the concept of pseudospectra for studying normal system, so before that what is spectra, spectra is a set of all Eigen values, how do you get Eigen values a is the operator and lambda is the Eigen value. What is lambda when you say, A minus lambda is 0 or you can say m minus lambda i inverse those like infinity or something like that.

So, Z is a epsilon pseudo Eigen value of a if it satisfies Z I minus A inverse is greater than epsilon 4 minus 1 or we can say Z I minus A is less than epsilon all of these things. That means, you are not looking to satisfy the relation exactly, but we will put small number and say within this number, you will you will satisfy the relationship or if you were looking at a resonant phenomena.

Your amplification goes like inverse to the distance between the forcing frequency and the resonant frequency, is a 1 over distance kind of relationship. And as you approach the resonant you get infinite amplification, but here getting in amplifier application we are looking at a large amplification, so epsilon is 0 and then, it is becomes Eigen value and this inverse gives the amplification. But, instead you say epsilon is small number that is a epsilon inverses 10 or 100 or 10000, this values depends on system itself what is large and what is small.

So, you can actually have large amplifications, even if you are far away from the Eigen value, so non-normal systems and then, we take a look at that pseudo Eigen value. So, they way to compute is to have the set of all points on the complex plane who's minimum value of singular value of Z I minus A is less than epsilon, but in your worry about how to do the calculation, but let us look at the concept.

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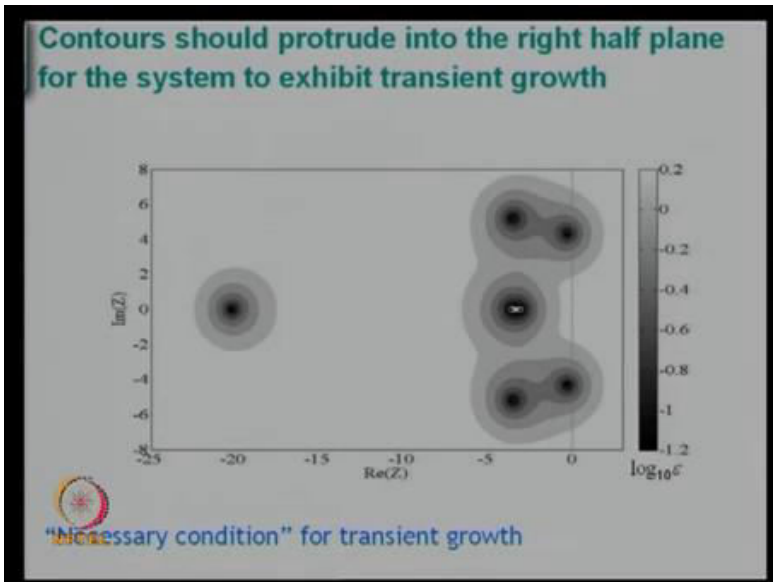

So, if you take a operator and look at the pseudo spectra first you can look at the spectra, so you calculate the Eigen values and this dots here are the Eigen values. Now, you say you do not satisfy this A minus Z I is 0, we do not do that, but A minus Z I is less than some epsilon and you draw take the epsilon contours and then, you actually calculate the couture's for Z and those are this contours.

So, the Eigen values themselves can be on the left half line that is a for the evolution operator, but the pseudo Eigen values can spill to the right apply. So, there is another way to look at it I first said you can think of exciting a system with a frequency and as you come close to the resonant larger amplification, but if you are having instead of you may have large amplification everywhere also.

For example, whatever value you have here for a absolvent you are getting here also, everywhere also, now we think of go back here this operator A. We do not know A exactly, but let us A is perturbed a little bit we have a we have a electrical heater and our experiment and let us a heater power changes slightly or something, so the evolution operator can or the main velocity and change slightly.

So, you do not have exact A, but you have some A is perturb and you look for the Eigen values that perturb matrix, that is another way of interpreting pseudo spectra. So, the crux of the matrix if this pseudo Eigen value cross it is to the right side, then there is a possibility of a transient growth. So, contours should protrude to the right half plane for

the system to exhibit transient growth and this is like a necessary condition for transient growth and you can use this concept of pseudo spectra and we can get bounds on the transient growth and so on, but I would not going to too much detail.

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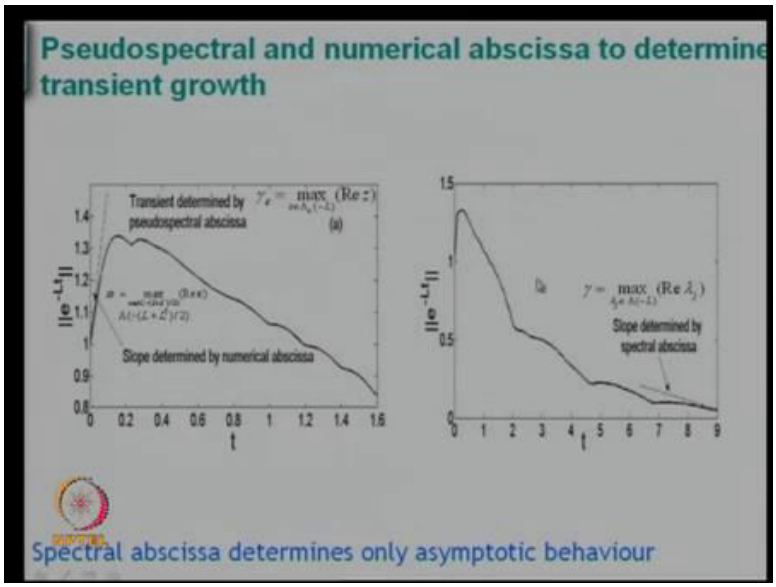

So, if you look at the evolution of this e power L t for a non-normal system, you have a growth and decay, and the decay is determine by the Eigen values which is what is call spectral abscissa. And this peak is determined by how much the pseudo spectral protrude to the other side and this initial slope is determined by the numerical abscissa, because Eigen value of minus L plus L transpose O 2, this you must have studied, I will derive it otherwise classes one second come following parallel here.

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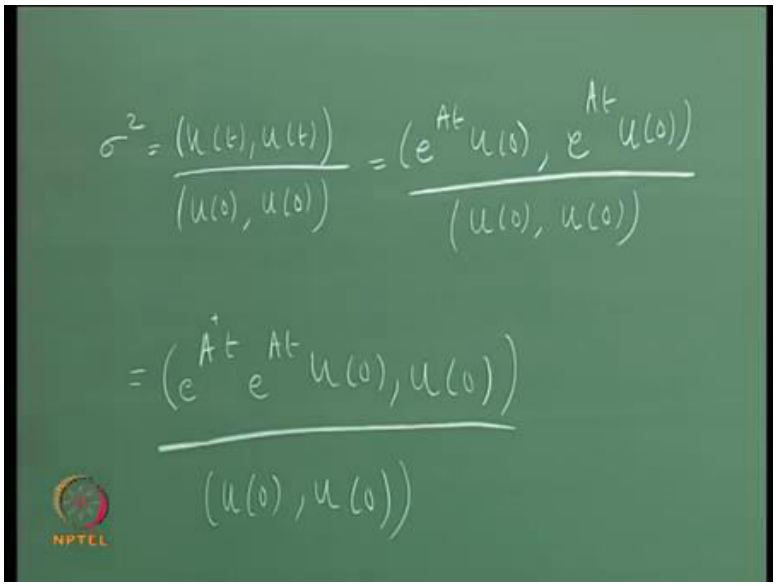

So, this is a missing his notation this 2 so that means, you can read it yourself, so this is like the inner product of the state vector a bit itself and the sometime to divided by inner product come time 0. So, this is like the scholar of the two norm and this we can rewrite this as e power A t U naught in our that is the solution. Now, we can use the definition of a joint, so we have to look at e power A diagonal plus A t, so this depends on this quantity.

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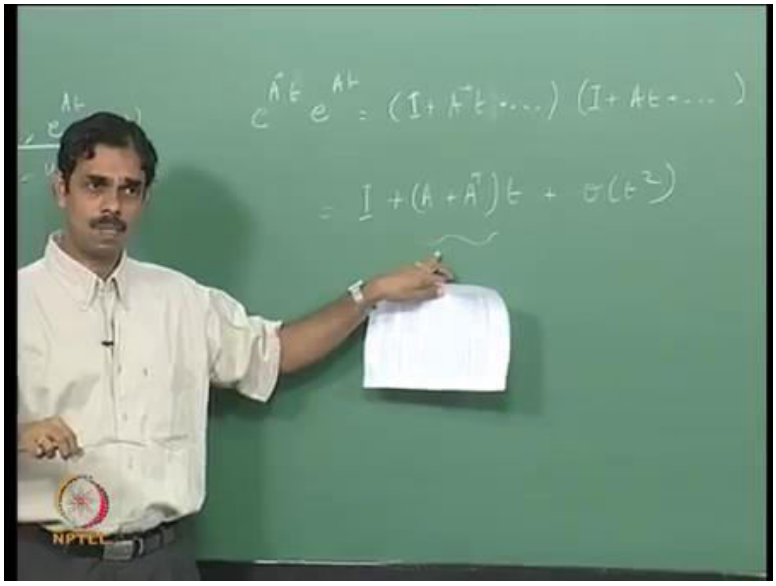

So, strictly speak write I, so this is what kind of matrix, this is symmetric matrix. So, we should be able to look at this Eigen values and tell what is the what is this growing or not, so these things Eigen value actually A plus A transpose divided by 2, that is Eigen value is called ((Refer Time: 30:50)) a numerical abscissa and that will determined how much is the initial growth. And then, how far is grows determines on the pseudo spectral abscissa, how much it protrude to the other side and the asymptotic decay is determined by the Eigen values or the spectral abscissa, that is written here.

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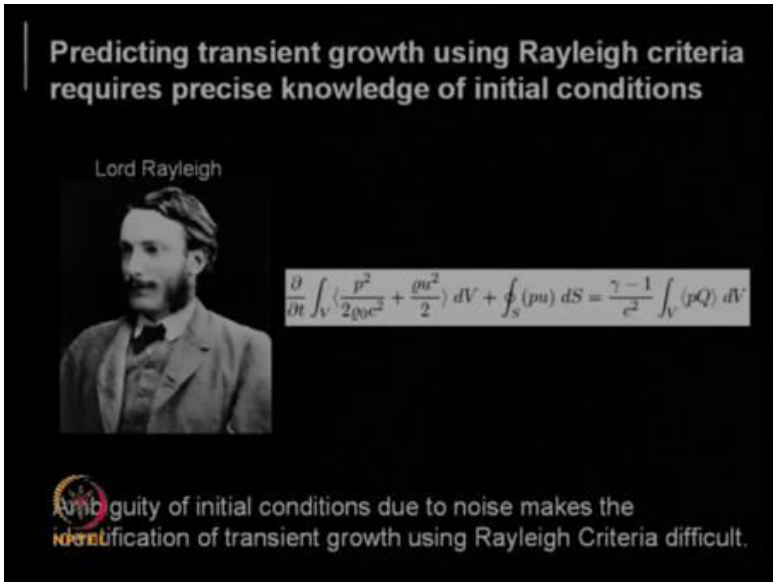

So, you saw this equation, what is this, this is our energy corollary in the presence of heat release given by Lord Rayleigh and a predicting transient growth using Rayleigh criteria requires the precise knowledge of initial conditions. And say ambiguity of initial conditions due to noise, because you cannot precisely know the initial conditions, make the identification of transient growth using Rayleigh criteria very difficult.

This one more problem you see this triangular brackets are normally, we try to average over a cycle to find out a whether energy in this cycle is less or more than energy in the next cycle, but when you are transient we saw the everything is changing. So, we cannot be identify fin point a period, because there many periods available, so what is the period itself is the question.

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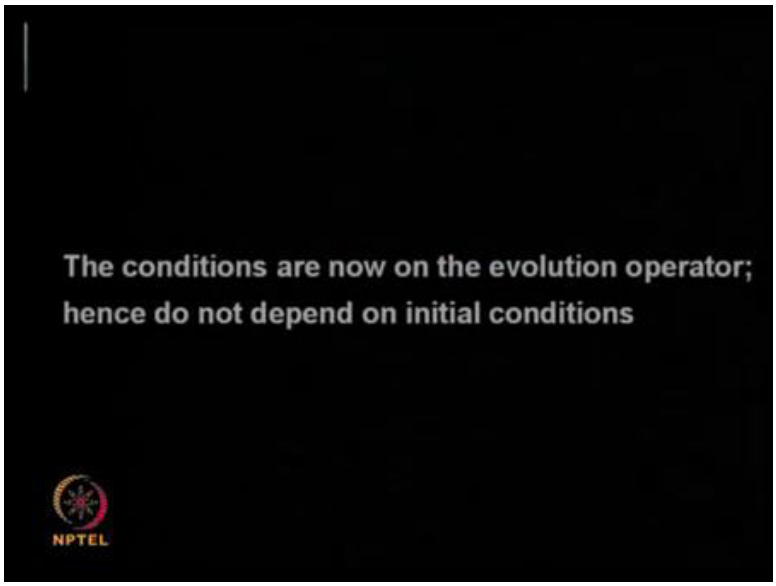

So, we cannot really use this to make any steady of transient growth, but you can look at the operator everything we did. The G max or the pseudo spectral abscissa and so on, and the condition that the pseudo spectral should protrude to the right side, etcetera. They just dependent on evolution operator and there actually giving the maximum possible truth, so we are not able to pin point in study, for this initial condition this is the grow.

For that of course, you have you can do the evolution and get it, but this G backs is like a upper bound that is a maximum transient growth you can see any initial condition together. But, that just the does not dependent on initial condition, because it is maximized our all initial condition it just depends on the evolution operator.

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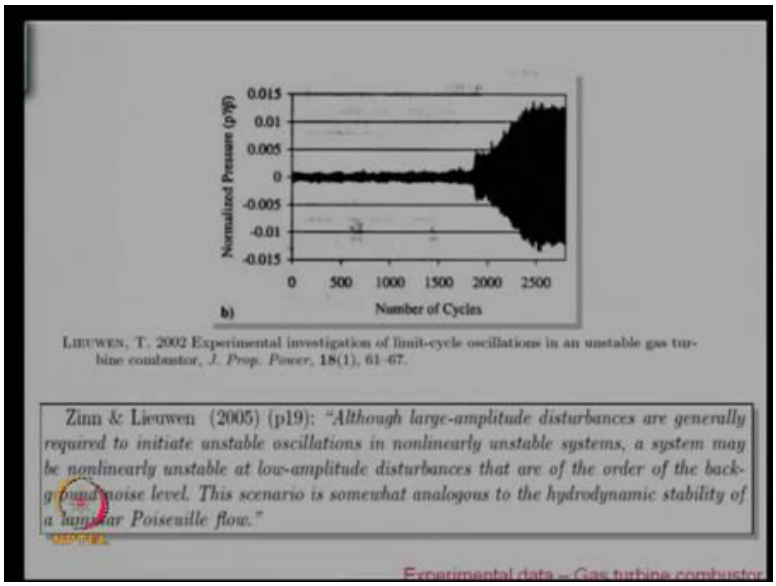

So, let us look at some experimental stuff to see all this is really there fantasy, so this is a you can see this is data taken from Lieuwen, he is a friend of mine colleague. Experimental investigation of limit cycle oscillation unstable gas turbine combustor is real turbulent combustor actually, you see lot of noise here and then, suddenly takes off.

So, in reality it is not like a system is completely sitting silent and then, bang one initial condition happens and then, you have noise and noise means, all kind of possible states are there. So, eventually hit some right initial condition, because any initial condition is like the thing this is some time t, this is the value and then, how we progress. So, noise can act as like it does same purposes initial condition and what he has written although large-amplitude disturbance are generally required to initiate unstable oscillations.

In nonlinearly unstable system, a systems may be nonlinearly unstable at low amplitude disturbances, that are the order of background noise level, this scenario somewhat analogous to the hydrodynamic instability in laminar Poiseuille flow. This book was published in 2005, before we came up with this theory of non-normality, but a what he has written as he is talking about low-amplitude disturbances.

So, generally triggering was perceived because the high-amplitude disturbances, but they people did experiment notice that you can see a low-amplitude disturbances. Now, in hydrodynamic stability, this revolution happened in 1990s, I think 1990s later, 1980s it is perhaps to started, 1990s the battle happened in 2000 may be people want the value. So,

there the non-linear terms are energy conserving; that means, all the growth this because of linear mechanism, non-linear term is re distributed energy between the modes.

But, in thermo acoustics our terms are not linear, non-linear they are not energy conserving, because you see it adds energy p prime q prime, so we do not have the situation. So, we can actually get triggering without transient growth also by large amplitude exchange station with only one mode analyses, but because of the interaction between modes or the non-normality and the non orthogonal taken of Eigen vectors.

You can get transient growth starting from small amplitude disturbances and then, with the small amplitude disturbances can actually take the system to a another stay limit cycle. So, that is this somewhat, but I asked them he said he did not know really why he wrote divert somewhat, but it is sounds perfect now, I met him recently two month back and I asked them did you really knew what happens this is a no idea, just wrote this turned out we working.

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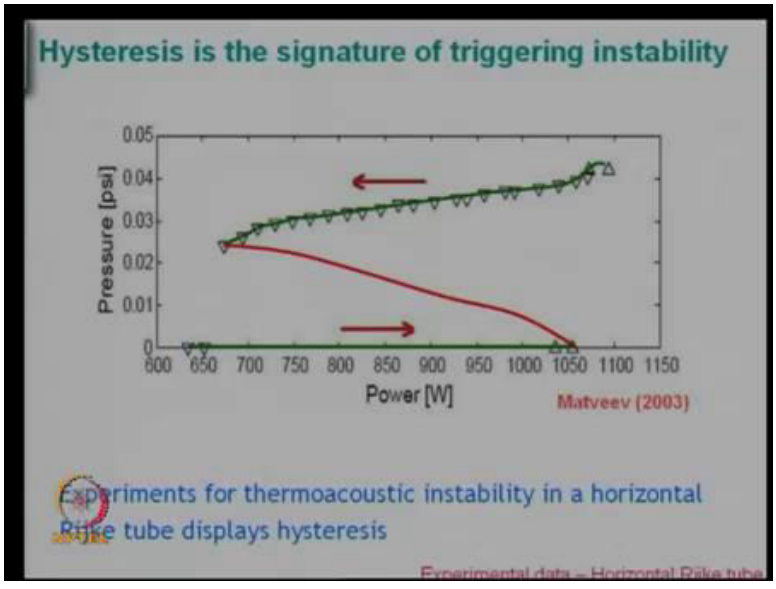

So, I showed you the hysteresis, now you see the same pattern you can see this experimental data from Matveev 2003, this thesis can be downloaded from internet very nice thesis. And it is see you come here you have to come to 1500 watts to go up and you have to go back to 650 watts to come down, so if you just draw a line in between this is very similar to the by frequentation plat I do.

There where this is the threshold line between or which we have to cross, so this is the unstable limits cycle are lying somewhere here and this is the stable limit cycle the green. So, what we our theory you can make any theory, if you and all theories are wrong, because you always make lot of assumption, but can you explain something, so is there any use to your theory that is the idea. So that means, it has to match the a data has to predict something which are observed experimentally.

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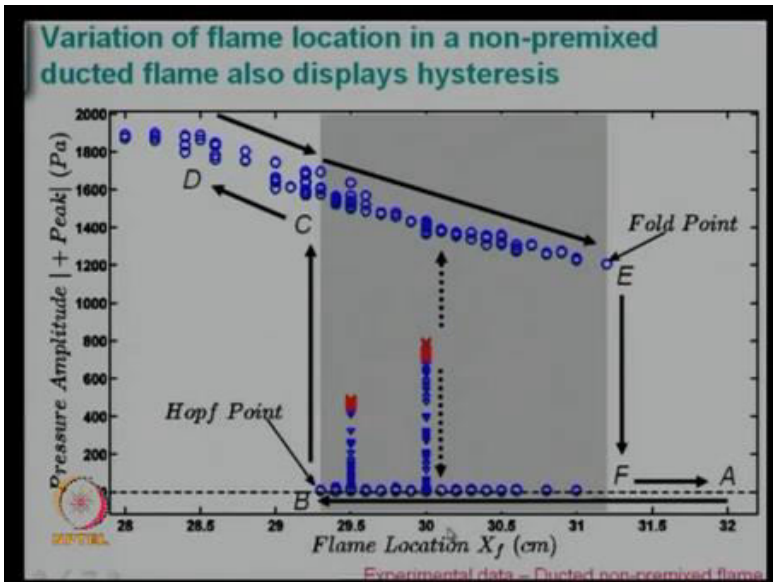

So that was an Rijke tube we see it in a pre mixed flame of you push the flame and it becomes unstable and non way, but when you come back, you come like this and the flame becomes unstable here at that particular location they position of the burner. To come back you cannot just come back right here and become stable you have to go little bit further to make extremely, so this is a diffusion flame this is a premixed flame.

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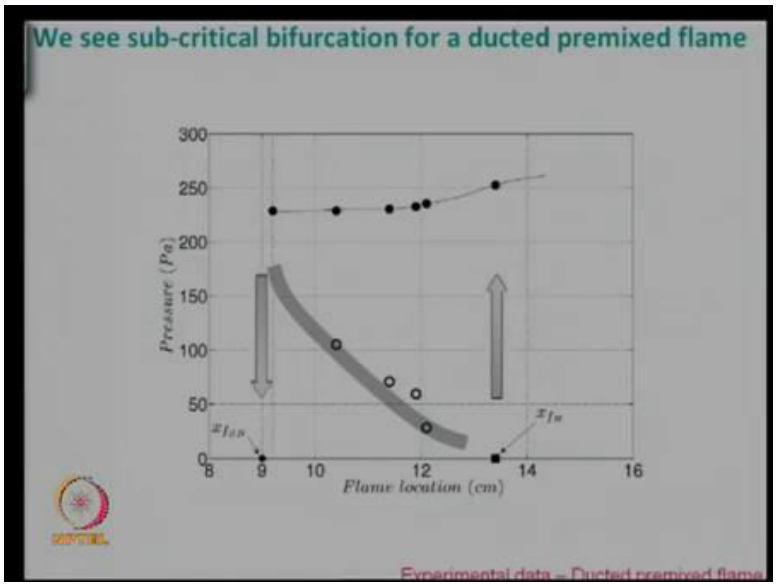

So, again a you have to a come here to go unstable, but you have to go at another place becomes stable, so hysteresis is the order of I mean this is lot of hysteresis to be seen in any kind of experiment, so two of them our experiments on mesh in the somebody else is experiment.

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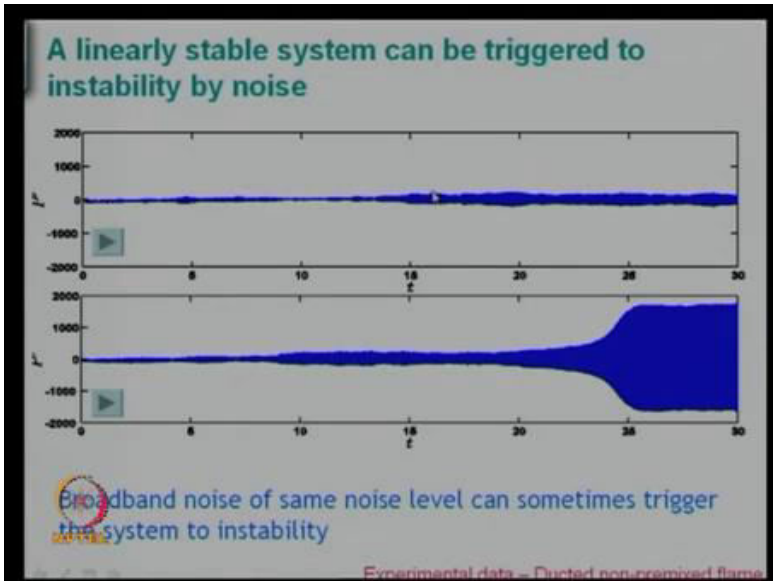

Now, this is the our experiment where unfortunately audio you have plying, this is like a noisily system the noise stay as it is, but some other time the noise suddenly goes to instability.

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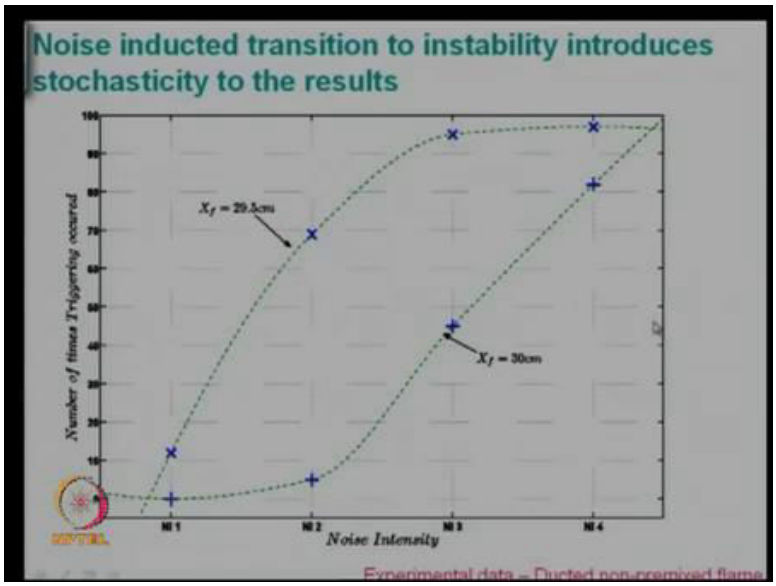

In fact, if you do the experiment running with a different levels of noise for example, this level of noise here for example, if you at certain level of noise, you do the experiment 100 times 70 times it trigger, 30 times it done. So, now, you talk about some kind of probability because noise is stochastic, so you can only talk about probability done I will not speak too much about this.

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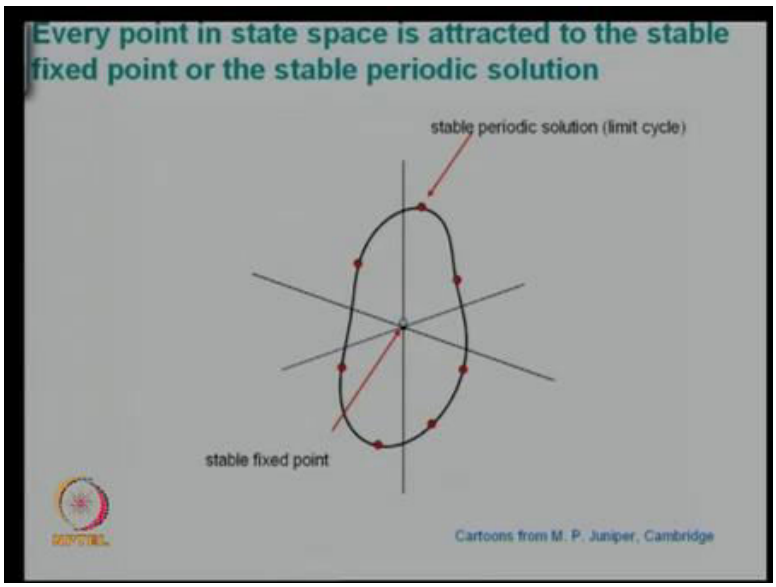

This want to show one last thing, so we construct our state space you can use all the vectors you can construct the state space or there are techniques with which you can

construct reduce our model. So, something call Taken's embedding theorem and you if your time series data from that you can construct state space and so on, I do not want to discuss any of this, but in this state space this is our stable fixed point and this my stable periodic solution, which is also called limit cycle.

Now, the issue is how we are going from which is the easiest way you can or the which is smallest energy which will take you to the limit cycle, that is the question here and this is from Matthew Juniper from Cambridge university is my friend done some follow up on our work.

So, every points in the states space is a attracted to the stable fixed point or the stable periodic solution which is limit cycle, although we say instability is the limit cycle is a stable solution actually, just that the oscillation we do not like it. So, we say that the instability, but in reality the limit cycle is a stable periodic solution, the stable limit cycle, so some points go down here, some points go up here.

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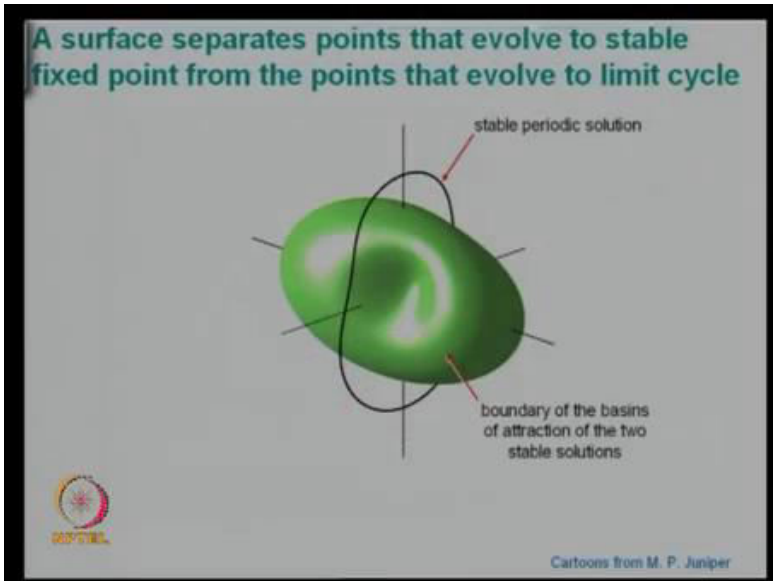

Now, there is really a base in boundary, which is with separates that two basins of attraction and this is basins boundary which separates this basins of attraction. So, the surface separates point at evolve to a stable fixed point, from the points that evolve to a stable limit cycle, so there is the possibility.

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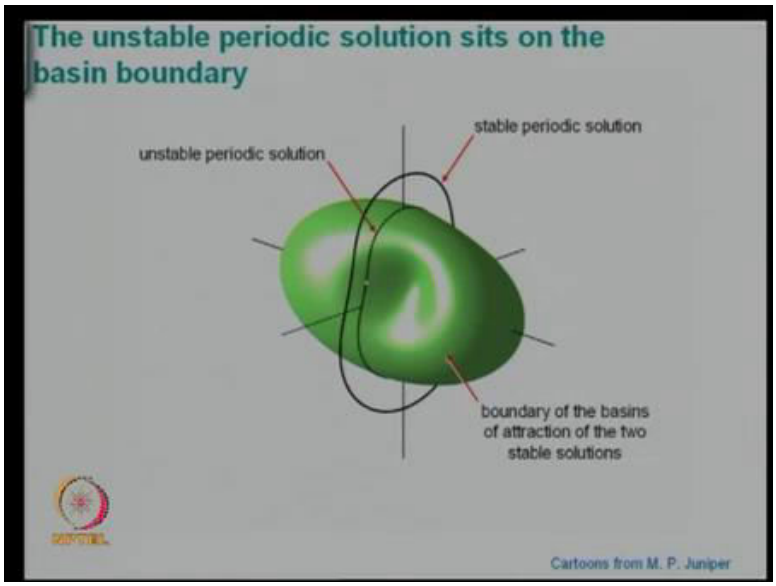

And there is actually you can actually find the unstable periodic solution on this basin boundary; that means, a solution, but you cannot stay there, because any slightest departure will take it out of it. So, in principle there is solution, but you cannot have it there is not stable.

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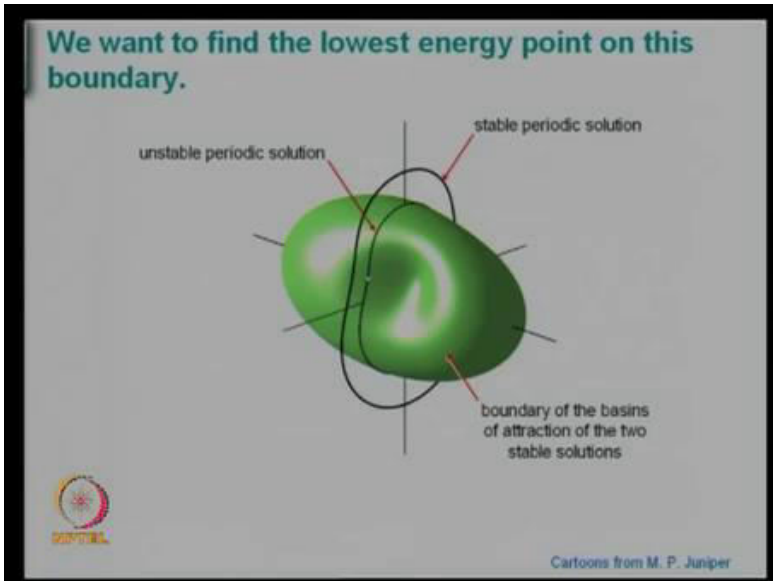

So, any questions please ask me, so we want to find the lowest energy point on this boundary that is the idea, so you have a boundary and the question is about the boundary

is like a foot ball everything round or it is a disorder, from this picture you can imagine that I am hinting at a disorder.

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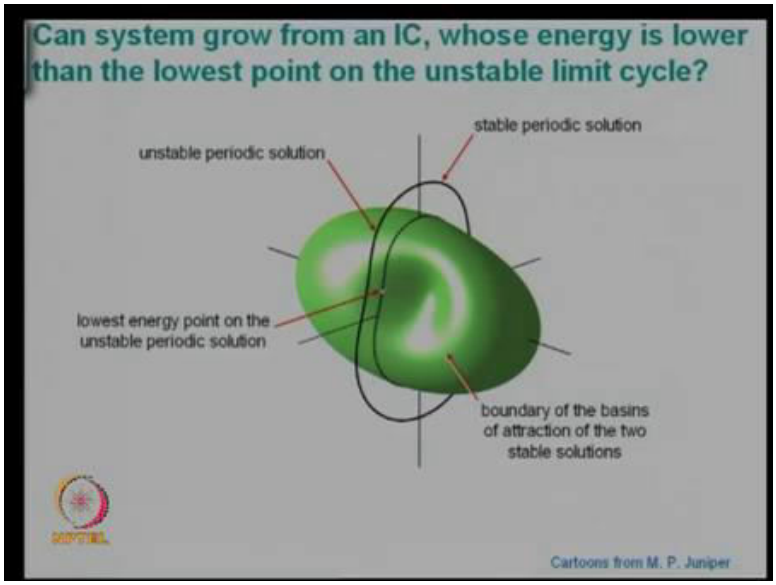

So, let us say you have this limits cycle and let say this is lowest energy point in the unstable periodic solution, do you have to hit the system with at least that energy or can this dimple be dimple such that you can even have lower energy and still go up to the limit cycle that is a question.

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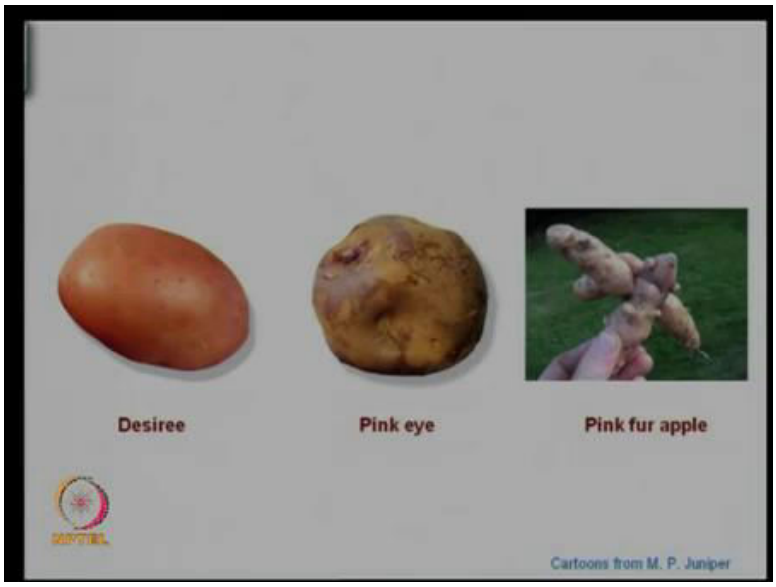

So, we look at different topologies this is a cartoon given by Juniper the different types of potatoes', so see this is does not have any groove inside, but here already you can see the some grooves, where is the in unstable limit cycle is lying on this topology. You could still be inside the split and you can have low energy, you can have this kind of potato also topologies can be quite complicated.

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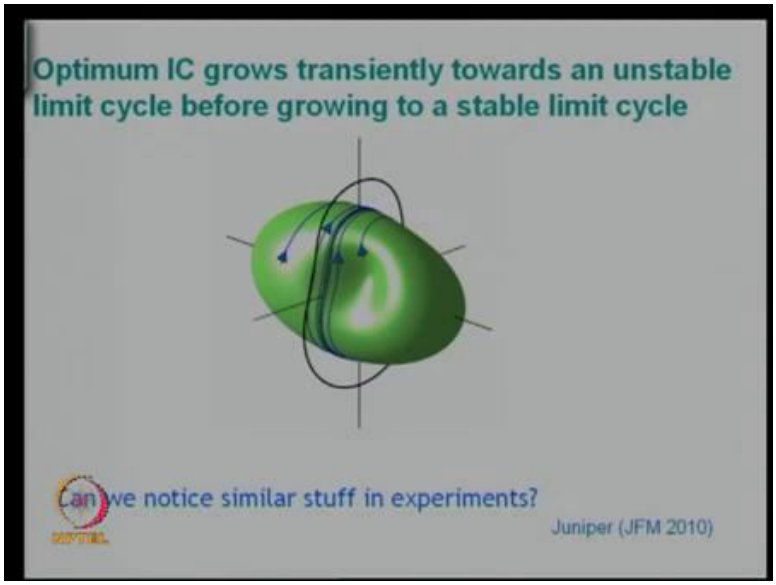

So, the optimal initial condition actually grows transiently towards a unstable limit cycle, so it first start from some low energy point comes goes to a unstable limit cycle this loop surrounded sometimes, because it cannot really stay there. Now, unstable limit cycle will have several attractors which are attracting to the limit cycle, but there will be one neutral direction along the limit cycle and one will be kicking it away. So, eventually it will try to float around here, but then suddenly it will the stake off, it will having sling shot and it will go towards the unstable limit cycle.

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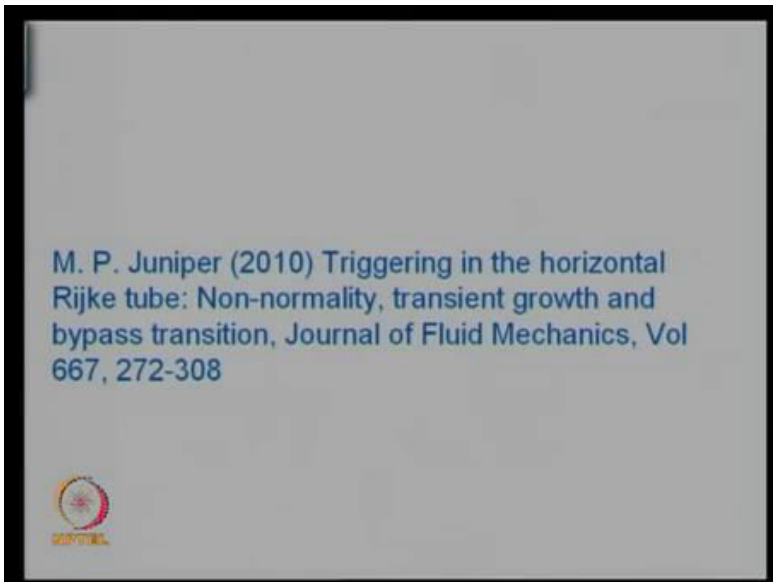

So, you can read about this by reading this article in journal of fluid mechanics by M. P. Juniper, Matthew Juniper, triggering in horizontal Rijke tube non-normality and transient growth and bypass transition and journal of fluid mechanics. Very nice step and you use a calculation choosing our Rijke tube model with that and so on, and use the same solution and try to look at the non-linear dynamics any questions.

Student: ((Refer Time: 42:51))

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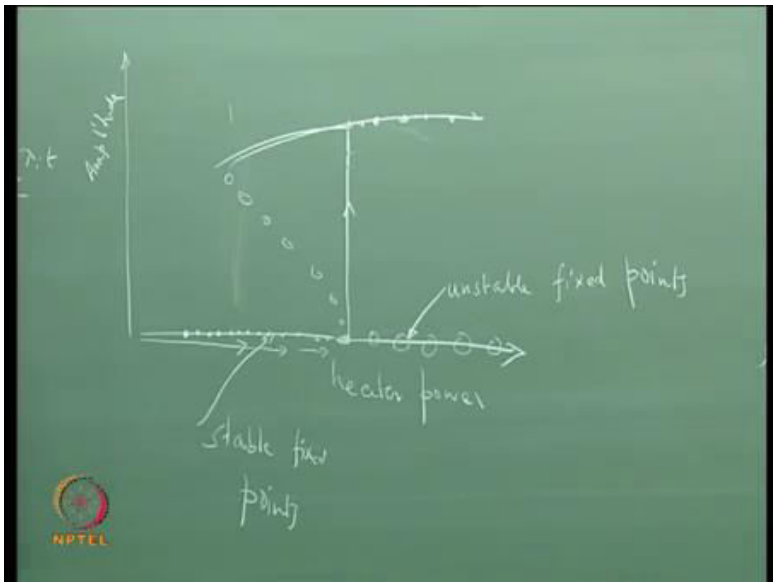

So, let us say this is amplitude and this is heater power, not putting any units, but so let us a you come till here you are increasing heater power, at some point you get to instability. So, these are stable fixed points; that means, you can be here and I am coming this way, but the system is still staying there, here also that is solution, in fact in our Rijke tube first will let us look at what is stable and what is unstable.

You can have the Rijke tube just have the base flow that is the solution to equations, if there is no parturition your solution will be trivially satisfied. If U prime p prime q prime everything is 0, we would not your equation be satisfied it will be, but you cannot have the solution here once the heater power is jacked up. So, there are solutions, but put holo circles; that means, these are unstable fixed points and these are stable fixed points, now I come here and I have solutions that means, even if I disturb the solution it will come back to air.

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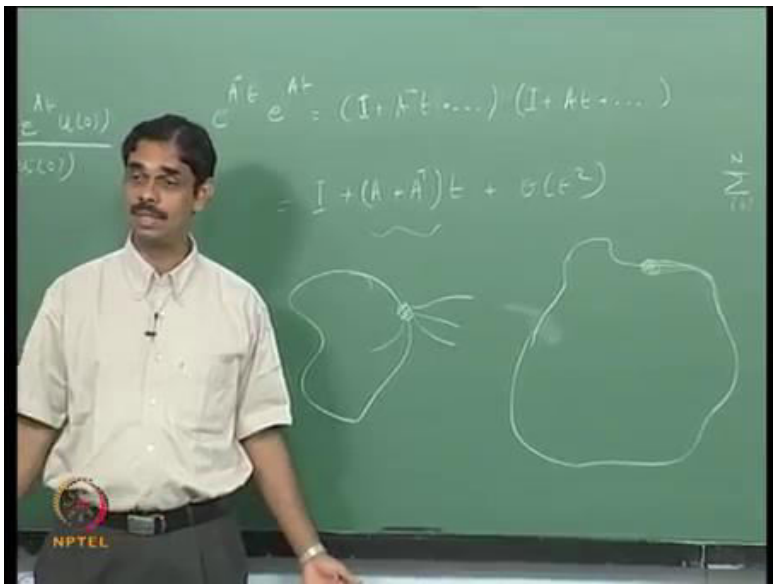

So, if I say this is some kind of curve, this is a limit cycle what it traces, so I push it any of the points inside the envelop, so I am look at point starting from within this gray area how they evolve. Eventually, they will all after sometime converts to this, so that is like a stable limit cycle you can push it, but you will come back, but let us have another scenario where I have another case, now I have a point and I push it instead of conversing it actually diverging away.

So, there is a solution that is possible, but it cannot say there or only time it can stays in principle minus infinity or something, which I mean real time you cannot stay there. So, there is a solution it can be satisfied, but you cannot find it, so that is unstable solution and you cannot find it, because it cannot stay there because any perturbation will be taking the system away from that. So, in this case for example, you can imagine there will be is unstable limit cycles somewhere here.

Student: sir, suppose in the system I am just taking in the gas and not the ((Refer Time: 46:22)) inter power taken in that gas some by the gas is sorted here. So, in that case once is goes to unstable more, it will taking more power because the amplitude vibration here are high. So, front theta the power observed in to the gas will be much more in the unstable mode than in this area. So, like.

You talking about is a heat transfer from the mean energy is taken more, because was trimming and so on.

Student: So, like is that the reason why this hysteresis come because when you have to lot more energy to take it back.

No, hysteresis; that means, once you are here and you are going back first, once you are here and going this way you are already outside the basins attraction, so you cannot find this point here it will just instead stay here. You see you have this unstable basin boundary separating these two solutions and it depends on whether you are inside the boundary or outside the boundary, so if you going this way you are always inside the boundary. So, you will fall back here, but if you are coming this way you already at its loud amplitude, so you cannot go back here because you are already outside, so it is being attracted there.

Student: A speed to the lower energy part of the case, but they is a possibility that you get you do not need to ready; that means, energy to get into the basins that enter to the some other point.

it depends on direction.

Student: That will be in that affect very.

No, this is the see this diagram only indicates the final state that is in asymptotic state, does not talk about what happens at the initial thing, this is float of solution, but in this float here of the face space. You can actually if you have a large energy, but you are in side here, then you can fall back this face, but you are already in this limits cycle and change some parameters, so get different limits, which will very close to this limit cycle, so just light to that higher than come in.

Student: ((Refer Time: 48:58)) should be raise a sample hysterias power should not be take the heater power taken by the yards rather than the heater power you have giving.

That is see we are unable to measure it, so in this case in the theoretical calculation that is the they are the same, but experimental people they cannot measure the heat amount of heat taken by the gas, so they will put.

Student: ((Refer Time: 49:27))

See in experimental you can only measure the current and voltage and all those things, an electrical power, if there is the way to measure the power taken by the gas, so there will some losses and in the cable some amount will be lost by radiation or that. So, if you can account for all that and measure it is possible, but in theory both are same, so in theory in the correlation what we used heat taken by the gas, but a experiment is cannot quite measure it.

And it is in principle possible, but very expensive it to use some there is a something a I mean I would not to you dream up to this, so as much simpler to float input power. So, in a dynamical system we can float anything you want, as the bifurcation parameter as long as a affects the system, that is the good thing about that machinery any other question, so we will stop here.