Acoustic Instabilities in Aerospace Propulsion
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Lecture - 27
Non-normality, Transient Growth & Triggering Instability-2

Good morning everybody, last class we looked at the non-normality of thermo acoustic system.

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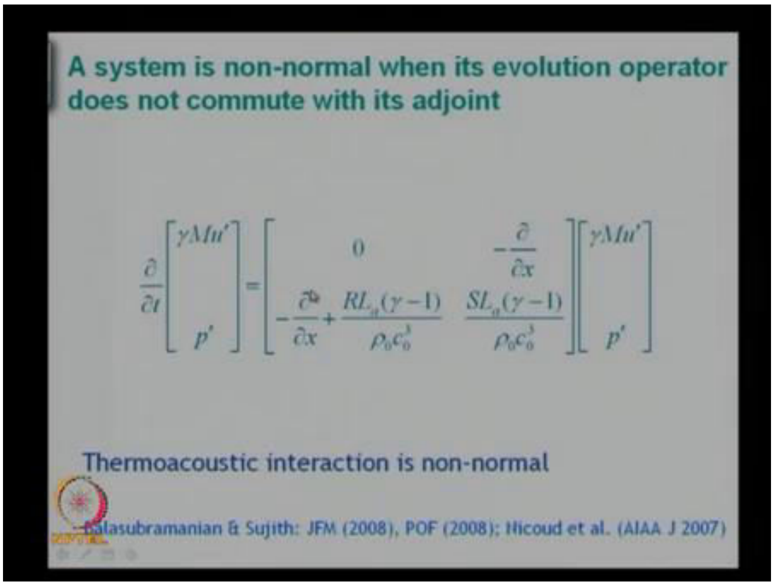

We found that this operator here is actually non-normal.

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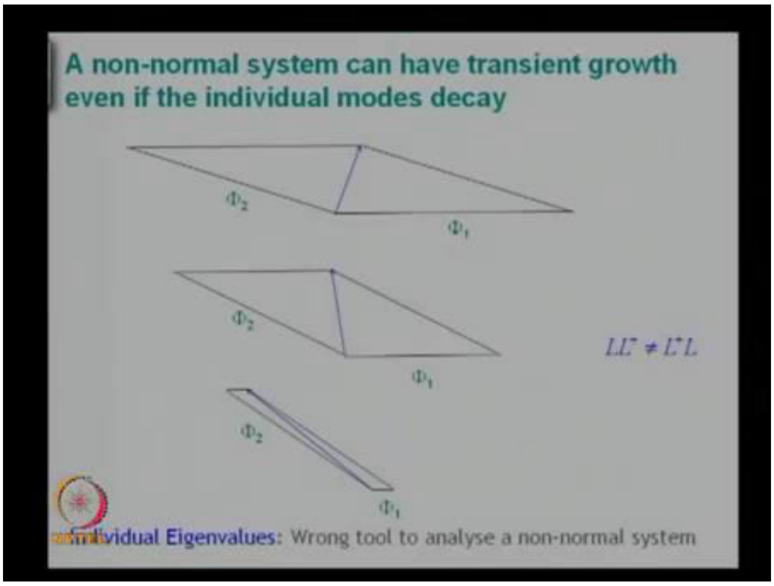

And which results in this non-orthogonal like in modes and we saw that if we have a system where the Eigen modes are decaying, even as the decay you can end up having growth in short term this is called transient growth.

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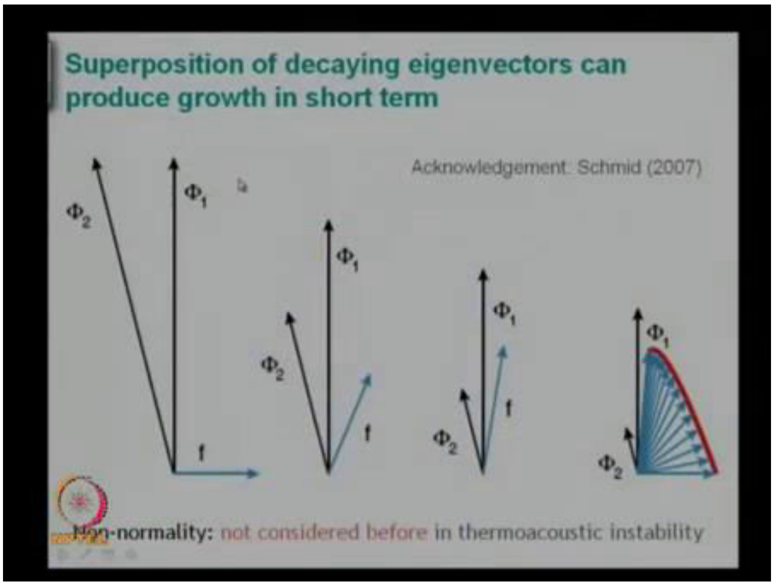

And then we saw that both the parallelogram triangular.

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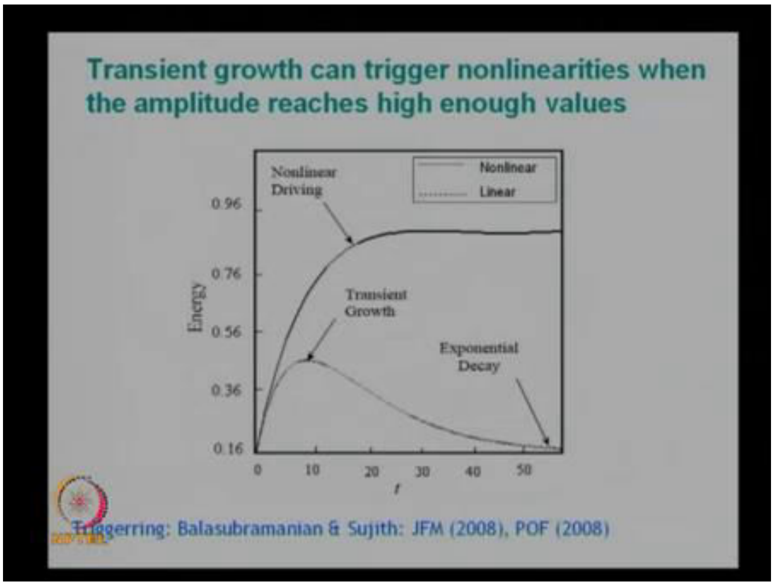

And then we said that if the transient growth is high enough, then non-linear instabilities will become important and then you can have non-linear instabilities. So, this is where we stop and then we are going to look at the Rijke tube model.

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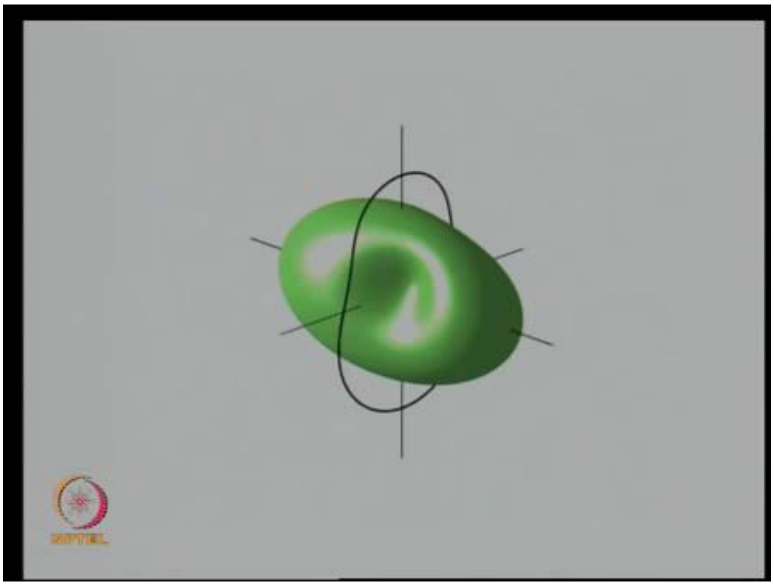

So, one way you can look at it, as from the dynamic system point of you they think of a base in boundary, so you can of this green potato over shaped object as like a base in boundary. And you can see along some direction for example, if you want to go along this access, it takes a large kick to get out and to be outside the base in boundaries, so anything outside this base in boundary will eventually come to this curve, which is symbolic of the limit cycle. But, if you are may be just inside, the dimple which may not be very far from the origin at all, may need just a very small disturbance to get out into this area of dimple and then you can really go out to limit cycle, will look more careful yet how it does. So, this is some many do with the topology of the phase or geometry of the phase and so on.

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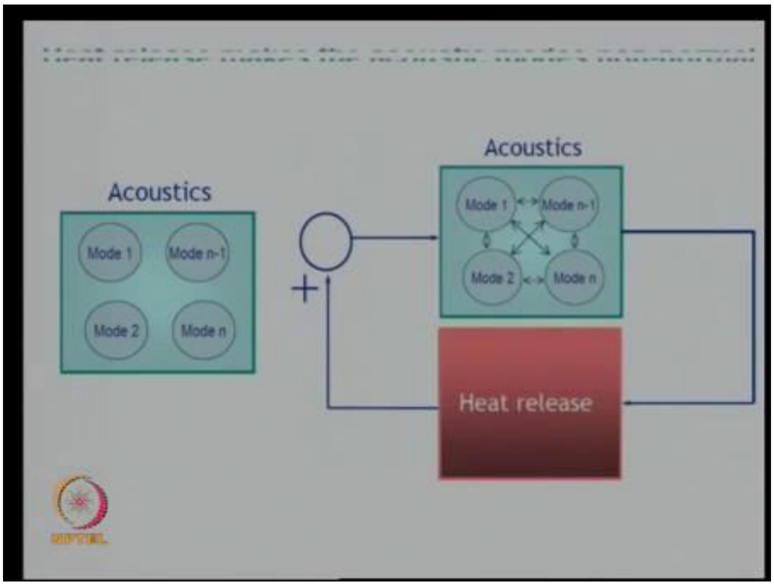

So, this that is a carton which illustrates this things.

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So, now we were looking at the Rijke tube model which we already derived in class simple model.

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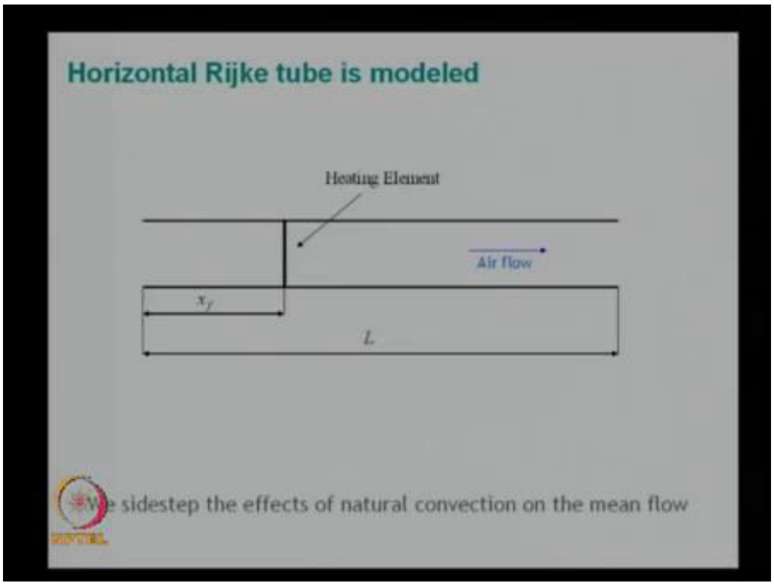

And it just needs representation of the acoustic field and some kind of expression for the heat release rate.

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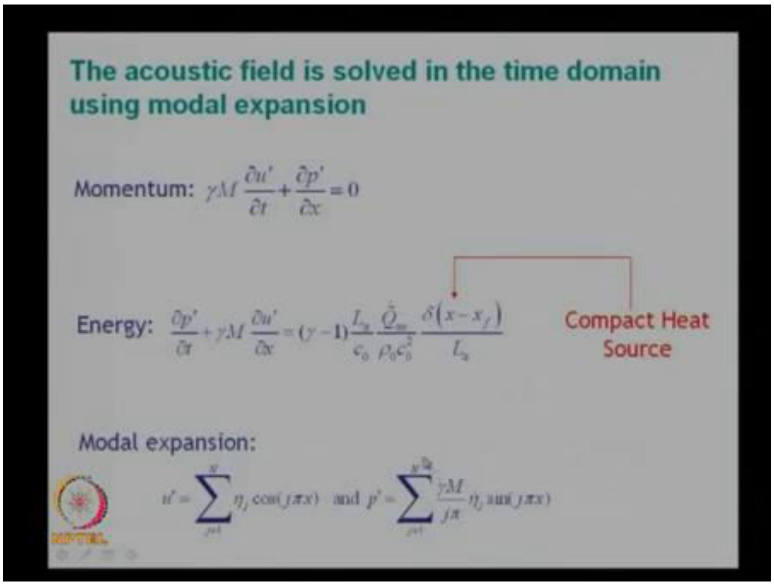

So, we solve whether than momentum energy equations and then we did a model expansion, we already worked is out in the class and we are attempting yet time domain solution, because we are interested in seeing the transient and time domain is a very good way to look, at the transients.

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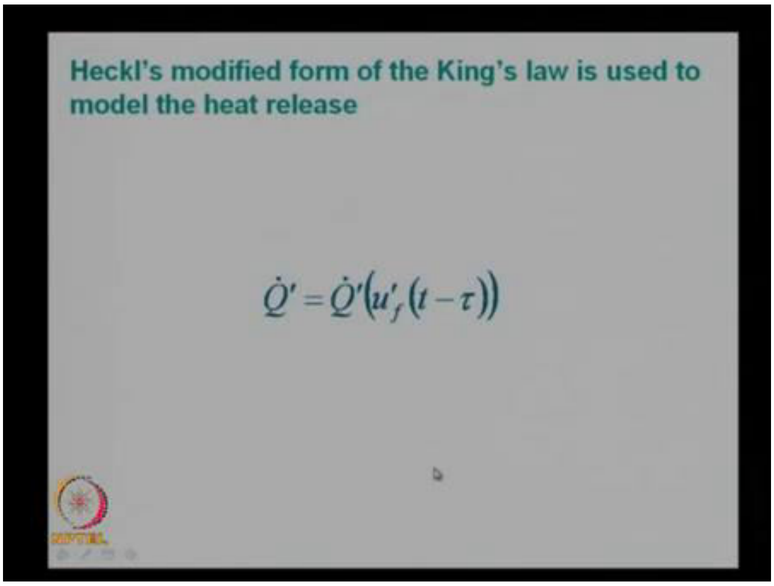

And we need a model for the heat release rate, so we need to express the heat release rate fluctuations as a function of velocity, but then there may be delays in the system and this case the delays, from the boundary thermal and momentum boundary layer of the wire. So, there will be delayed tau and so, we say that the heat release is actually a function of the velocity fluctuation at an earlier time that is t minus tau. Now, you do not have do this you can actually u c of did solve the around the cylinder and calculate the heat release rate from those calculations, which is quite involved, but I wanted to keep the problem as a toy problem, very simple problem that is the reason I take this approach.

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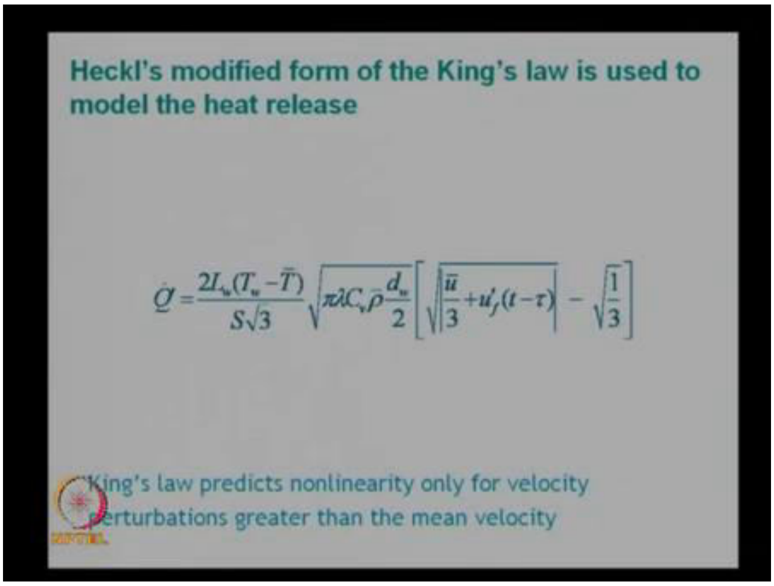

And this is the correlation given by hackle we did this, In fact we even linear rise this in the class and derived the linear rise operator and so on. And I said that this law enables has to get non-linear term important, when u prime over u bar is of the order of one third and this cycle says is what is season experiment debatable, but this is what she says and then we can get something other form like evolution, equation as what people say in dynamical systems theory a d k by d t plus some operate, linear operator acting on kai plus non-linear operator acting on non-linear function of kai is 0.

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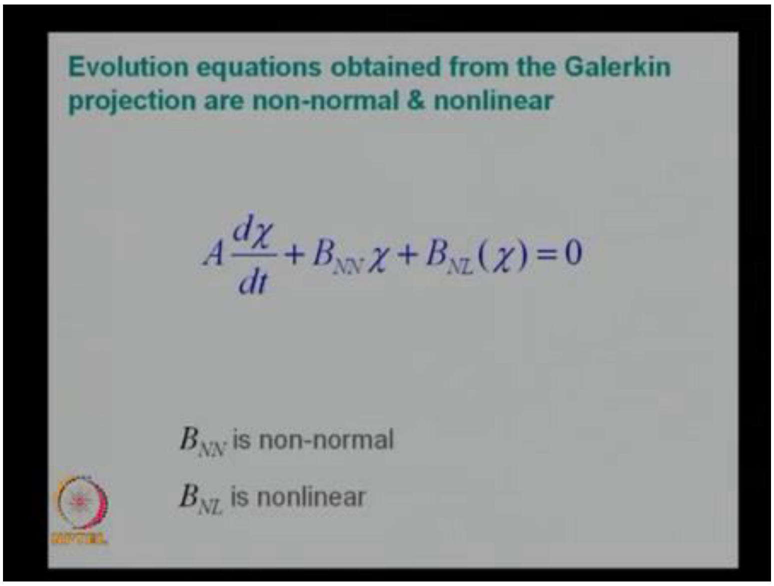

So, the linear rise system will have only the first two terms, so and if there is non-linearity is will have of course, a non-linear function this kind of representation is no what it is called states space representation and dynamical systems theory also in control theory. So, we look at some examples.

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So, we will run a calculation with no damping, because the load of theory is in debates and controversies about damping, so we just stay away we look at the un damp system.

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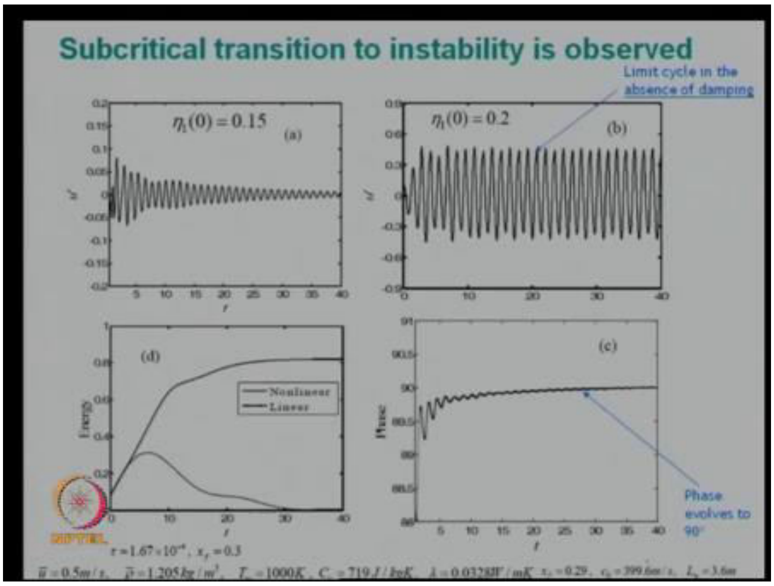

So, I given to parameters at the bottom of the screen at you start with the small excitation and you see some kind of growth and velocity, followed by d k this is like you get, actually if you are in a subcritical zone and you tap make a snap. The fingers near a Rijke tube we can actually hear this, now if you jack up the initial condition just a touch more you actually get a different asymptotic state, you are not actually going to the fix point or not going to equilibrium or died one, but here its actually going to some kind of limit cycle.

Now, if you, so this is subcritical transition to instability which are comb stance ability people call it, what is the name triggering instability. And if you fall look at the energy you can see that the, so energy is a measure of how much fluctuations stays energy is a measure. Now, you see that, so this is the acoustic energy that are used scale, so scale no that is not scale, this is acoustic energy and you see the linear evolutionary it goes up and then we are looking at a subcritical system.

So, eventually everything has to died down to 0 and but the non-linear system actually takes off and goes to high value, where limit cycles is reach and it is interesting look at the phase, we know that the system does not at damping. So, as you reach the instability what should be the value of phase, as your limit side.

Student: ((Refer Time: 06:20))

yeah why.

Student: Because phasing a power ((Refer Time: 06:24))

So, once your limit cycle does not damping, so there is no loss of acoustic energy and limit cycle means you are staying here, you have constant energy, so you do not need any driving modes, this nothing going out, nothing coming in and you already reach some level and you are this is staying there. So, you can actually see in thought or model did not have damping, we see limit cycle in the absence of damping and we can also see that the phase is evolving to from like 90 degree and so on.

Student: in that energy time diagram ((Refer Time: 07:04))

Yes.

Student: those are two different systems non-linearity ((Refer Time: 07:07))

Same system, but I am running a non linear system and linear rise in this system and get a get a linear rise operator and a evolving that, so it is a same system, but you are at it as a fully non-linear system or a linear rise portion of the same system. So, they are in affect the two system, because one is a non-linear system, one is a linear system. But, the basic thermo acoustic system, we are have a non linear model and what happens if you linear rise it, so in that sense they are same.

Student: Insuring non-linearity is come in after some time delta after a amplitude in this beginning or is ((Refer Time: 07:47)).

So, in the beginning they amplitudes are small, so the non-linear thing will essentially look like linear, that is what happening so.

Student: some of non-linear ((Refer Time: 07:57))

You take the non linear evolution and integrate forward and see what happens, so it has everything and non linear simulation has linear and non-linear, you are talking about subtracting on from the other, now that is not what is done very good point. So, you can see here at low amplitude the trend of the non-linear simulation is very similar to linear, so that means, the non non-linear terms are kind of they are getting, significant as the a time progress as the amplitude increases.

So, you are absolutely right any other question, so we will run a model including damping we saw a model for damping what are the model to be used.

Student: N tau model ((Refer Time: 08:44))

No, N tau model is the model for heat release rate, we use this specific type of model which was like model damping, it was given by Matveev.

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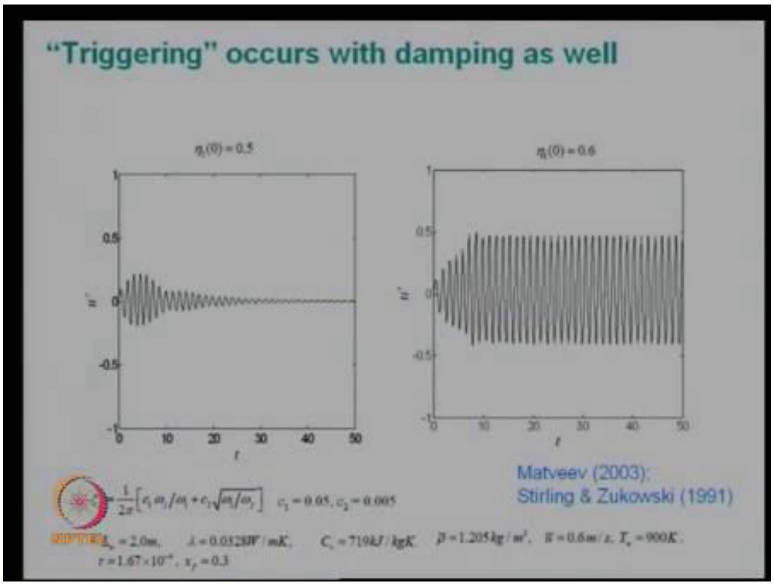

So, of this form for each mode is damp this ((Refer Time: 09:08)) and this is use by Matveev in is this is on Rijke tube and I think originally its propose by Stirling and Zukowski in 1991. So, we use that and we see that same kind of transient growth, followed by d k can be seen and so, you have a small initial condition, you have growth and d k, but a slightly different initial condition leads, it to like a limit cycle. So, you can see that triggering is occurring, even if you choose some model of damping what model of damping to choose is I am sure it is very critical, but this is a model problems I am putting in some model is available. So, even the energy evolution does look very similar.

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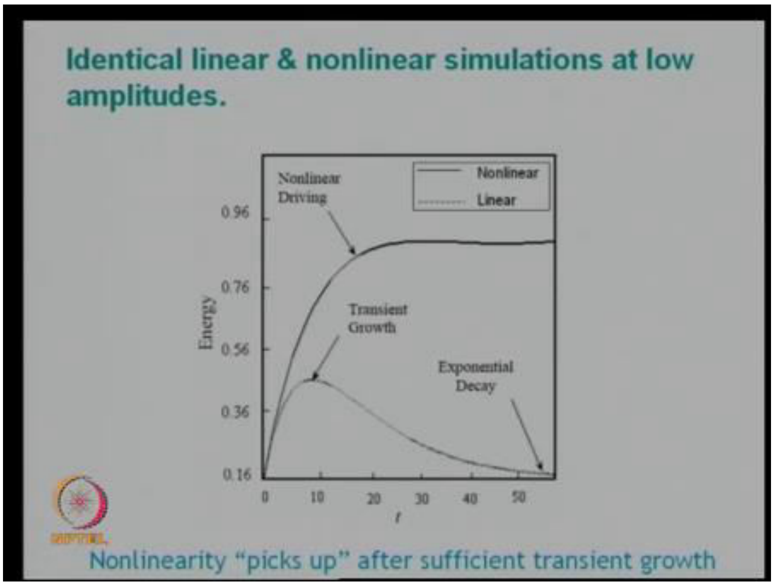

So, you can see the linear evolution it acts to died down, because we are looking at by definition as subcritical system all eigenvectors are d k in, because all Eigen values show negative growth rate. So, it is driving, but you the non-linear system actually kind of fallow the linear system for some time, but then eventually it goes to a limit cycle you do not necessarily have to get curves of the shape, but the linear simulation if the eigenvectors are decaying has to die and the non-linear want. So, the other important thing is if you run the non-linear simulation at low amplitude, that are look very identical to the linear simulation. Because, those non-linearity is which make the system take it out of the base and attraction has not really, kicked it.

Student: Transient growth has ((Refer Time: 10:35))

No.

Student: Over a limit then it will all way go to non linearity ((Refer Time: 10:42))

So, it is a you have to watch did not go back to the picture here.

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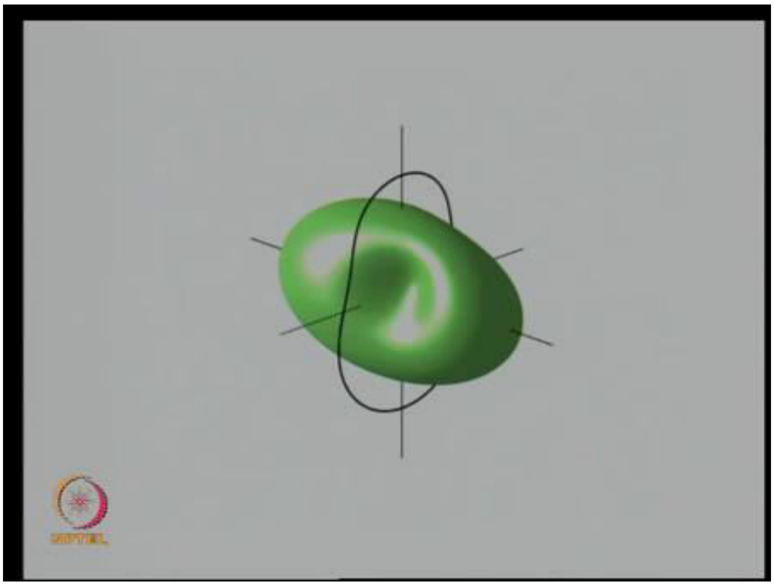

So, you have a basic attraction is here you are out or in like brushes is your with them or at me, so is that your insert base of attraction or outside the base of attraction. But, the linear mechanism does cause some growth, so it does not like the linear mechanism make you jump out of the base of attraction, now is there your outer the base of attraction or inside the base of attraction. But, what the linear thing does is you it, flattens this are it smashes some part of the base of attraction, because why is smash. Because, the linear mechanisms is had been a spear it does not matter where you start with, but because if you linear rise the problem and look at it, you can actually that is look at a 2 d case.

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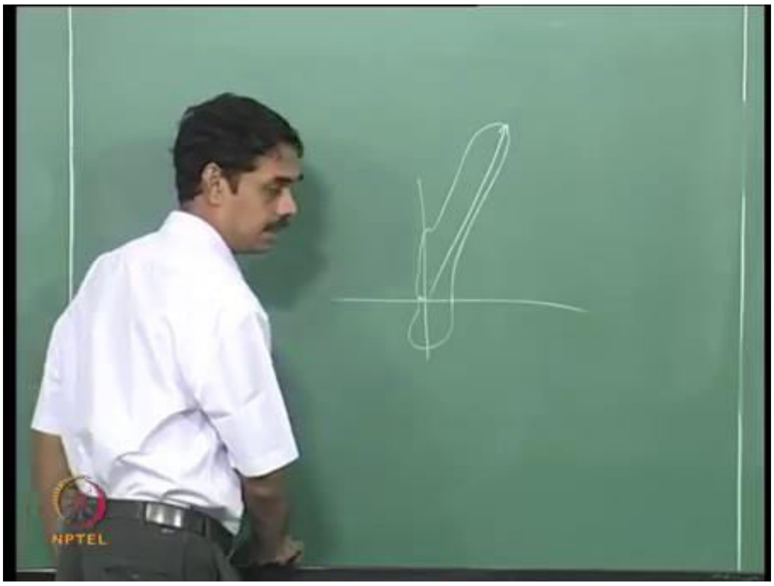

It may be highly distorted thing, so this distance may be very large, but this is very small. So, if you just, so you start of this and you may be able to somehow grow here are something or get out that way, so what this does is to. So, because of the linear mechanism causing growth, the level at which non linearity is become important it different along different direction.

So, along some directions for example, in this punched potato sorry you may need only a small amplitude to be out of the base of attraction, so once you out of the base of attraction non linearity is has to be taken to account, but even. So, along the from here linear mechanism still are it is still operating and it is causing you to grow, so you have to view it in a little imaginative and careful way.

So, we cannot say that the linear mechanisms are making you go out of the base of attraction you always out of the base of attraction, if you have outside if you are in you are always in you cannot get it. But, if non normality is there, the base of attraction would not look uniform. Because, of this linear mechanisms cross in grow is that clear that is one way to look at, the other ways as you said the non normality is can cause growth therefore, and it can you can have very significant growth and therefore, you it may grow to values you must start with very small amplitude 1 percent or 0 point 1 percent, but linear mechanism itself make it grow to 10 percent, 20 percent.

So, just because it is started with small even with linear mechanism we can get very high, so then, but at those amplitudes linearization will not be valid along those direction, so we have to be very careful in figuring out at what amplitudes linearization valid. So, basically non normality reduces, the amplitudes at which linearization valid or a even at small amplitude, we can have linearization fail and non linearity is being important.

Student: ((Refer Time: 13:33))

And then you can in a normal system can change direction with in involve, I mean you can involve any direction any way, but as it changes direction you can have growth is this clear thank you for asking any other question.

Student: what determines the direction on this.

That is our next point, which is the best direction that is what I am going to talk and so I will it will take, half an hour to address and that is what I do not have own, any other question.

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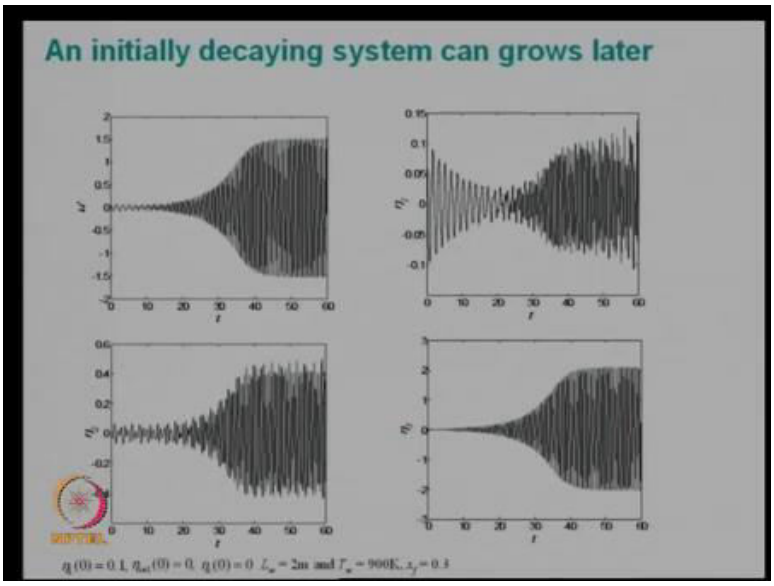

So, before I go which to I the whole idea came to me, because I made a combustor in 90's and it had it was always was a tunable pass combustor, it still there in our lab anything maybe I can some of you are seen. It actually works from very low frequency from 70, 80 hertz to 600, 700 hertz in that range and but, you could never get it work straight away in fundamental mode.

But, the student was working his name is Ram Narayan Balachandan, he is now a professor in university college London, he had some kind of magic with which he could it to work, so he would be working at under third mode 200 and something, hertz something and is a block body combustor. So, it deposes of the block body very important and with the sleight of hand, he will change the position of the block body and this mode will die and the other mode will come up.

So, you can see here this is the evolution velocity, which actually decaying system here some mode is dying, here in this case some low frequency mode is dying and then the high frequency mode came, what he had was the opposite thing. And if you see the projection on the modes you can see that the first mode is dying, but as it is dying it is giving energy to this and this, so this is a subcritical system eigenvectors all decay.

And then this is a border, but then these guys are giving energy back to it is going up like our bollywood movies, the hero was kind when he was young and he help everybody else. But, then he was in a disaster and he was all set to die and then somebody comes and base a mode something, so some kind of thing is happening here, but you do not have to look at that way, but you see, so a system if it is initially decaying it does not make continuing decaying come back that is non linearity is are there. I though it was interesting to mention this.

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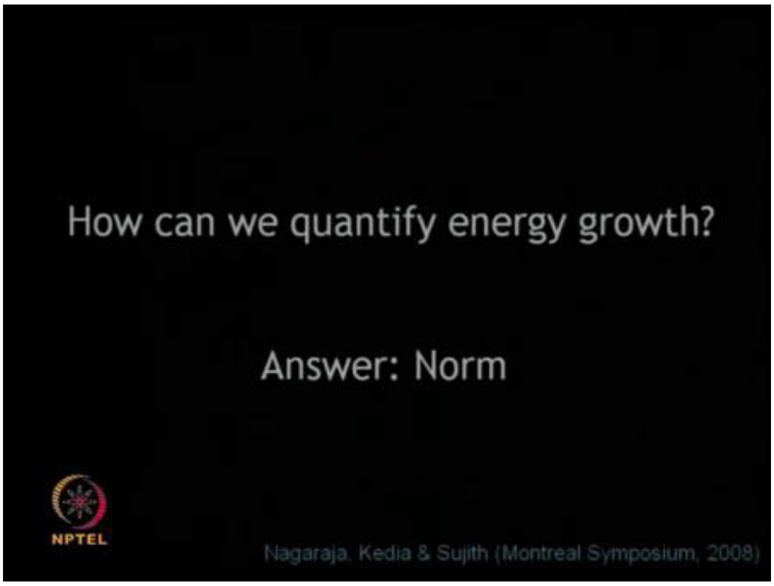

Now, the question is as what he ask how can we quantify energy growth vikram, so that two thing one is the direction and how much it growing and the growth definitely it should depend on the direction, as you are rightly pointed out. Because, I showed a example last time that if you are exciting things around a eigenvector, then what happens will have transient growth, that cannot be transient growth. Because, by definition the eigenvectors decaying and if you do not have contribution along other eigenvectors is just continued to decay.

So, from this example itself it is abundantly clear, that you have to I mean the whole thing is the growth is directional dependent, if you are along some direction you may have transient growth some other direction you may not have transient growth at all. So, the answer is we to quantify you have to measure or a norm and when I heard this first this was.

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My expression because I am not a mathematician or a physicist I can measure quantities, but I did not understand what is a norm. So, I have to an experiment people are do experiment sometimes to get throw discourage when they are here fancy jargon, but soon I found out that does not nothing to worry about, because I will explain to you, because I understood pretty soon what it was.

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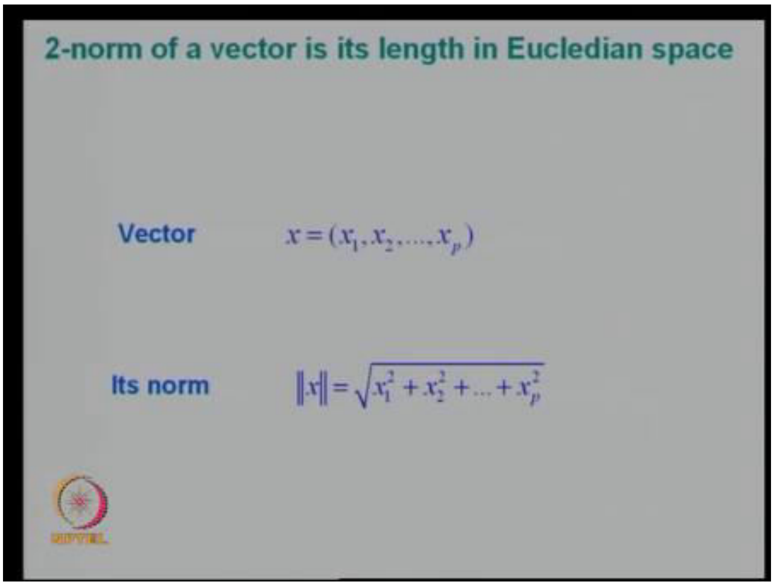

Although the mathematicians scared me with the talk of norm and measure and so on, I understood that first I thought measure was a complicated thing, but then I understood anything to measure you need a measure and it is just like English word, it just at the right landmarks theorems and through you out. So, two norm of a vector is nothing, but its length in the Eucledian space, so we have a pencil or pen and I can measure the length, I do not call it two norm or measure or something like that.

But, all that needed is I have three coordinates and the x 1 squared plus x 2 square plus x 3 or x square plus y square plus z square ((Refer Time: 18:28)) that all this way simple. So, this is geometric space or normal space Eucledian geometry, so you can have a three dimensional vector a x I plus y j plus z k and the length is x squared plus y squared plus z square.

So, we want the extend this to something which has many components, it is a not just I j k, but 100 directions 1000 directions, now I cannot imagine 100 directions and 1000 directions I can imagine 1, 2 and 3, but as long as I, do not have to no I could accept that there could be several directions and you might have read books about four dimensional being five dimensional being, so on.

So, you see the imagination, but assuming we have p dimensions here, then you could by extension we could construct a length which is like x 1 you have a vector with p dimension x 1, x 2, x 3 up to x p then its norm we x 1 squared plus x 2 squared plus up to x p squared. So, as the two norm of a vectors nothing, but its length if it is represented in a Eucledian space that is with this my fear was out and then I could deal with it any questions, if you do not have question I have a question what is two norm of a matrix.

Student: ((Refer Time: 19:57))We will come to the matrix not possible this is a square matrix.

Student: ((Refer Time: 20:07))Heat transfer.

Norm will some number, norm is like g p a one thing to distill everything that you are in the 8 point 8 dot 9 point 3.

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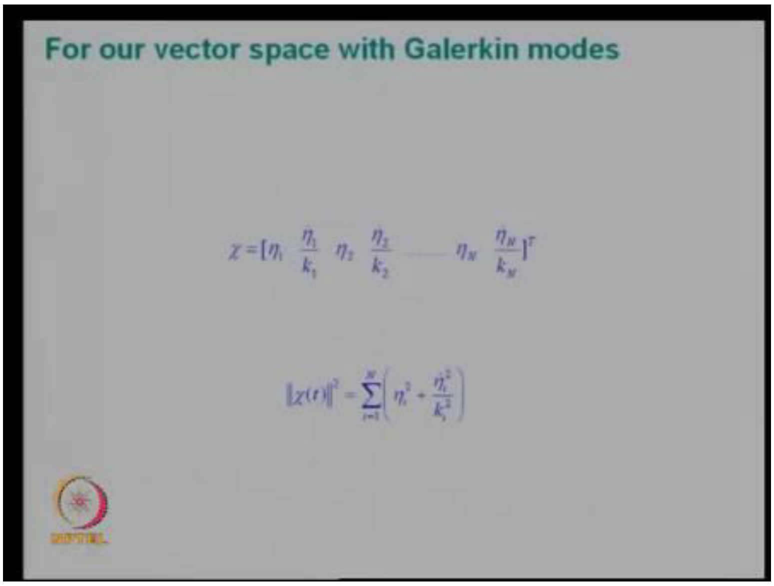

So, we will continue for our vector space with Galerkin modes, so we construct a space with our Galerkin modes to be remember Galerkin mode eta dot everybody no you are, so good. So, we can think of it is like eta 1 COS k plus eta 1 dot I mean the velocity is well, like eta 1 velocity is well a COS sign, yes check mistake pressure was like science, so velocity is well like COS, so it was like velocity was like eta 1 COS pi I x plus eta 2 COS 2 pi I x plus eta 3 COS 3 pi I x.

So, this COS pi I x COS 2, pi I x COS 3, pi I x they are the basis function they are like this I j k in our vector space and the similarly eta 1 dot over k on that is like the cohesion of sign j p x they are also the basis function. So, these things are like the individual vectors, so you have you write the cohesions eta 1 eta 1 dot over k 1 eta 2 eta 2 dot over k 2 up eta n eta n dot over k n, you can write it horizontally and put a transport instead of I think a vector save space. So, if you take the norm it will be eta I square plus eta I over k I square, summation over all the modes, so that is the norm of our vector space with covering this Galerkin modes.

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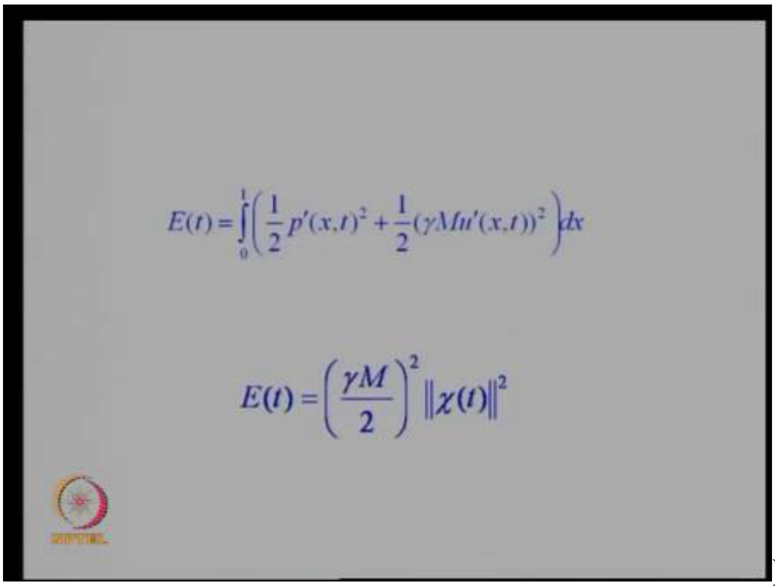

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So, acoustic energy is a non-dimensional one and of course, non dimensionalised with different constant, so the expression you see in papers with change little bit, but it is goes like p prime squared plus u prime squared scale by some factors. Now, the thing is just norm is like a constant times the energy, which is what we need if you scale it some other way you would not get it proportional to energy. So, we have to scale it that way such that the norm is like proportional to energy, then only the measure make sense.

Because, we are looking at the energy of the system see I mean when the scale the G P S they multiplied, the theory courses by four credit and lab courses by two credit from the lab. So, the idea is that the other thing is you put more effort, so you have to scale the great more and scale the other one dash or something like that, so you have to scales as that you get what you want, but a guy may be extremely good at laboratory may be very poor at passing exams.

So, this scaling will not reflect the skills, but there if you way his lab thing by 10 credit and theory by 1 credit he will come out on top, so what you get is what you give in norm. So, there is no perfect answer to what norm you should use, in this problem it is kind obvious use acoustic energy, but it is not at all that obvious for many complicated problems, but the basic thing is what you get is, what you give is what you get or something like that what you make you what you get. But, in this case the two norm of our state vector represents the acoustic energy I hope this is clear acoustic energy goes like pressure square plus velocity squared.

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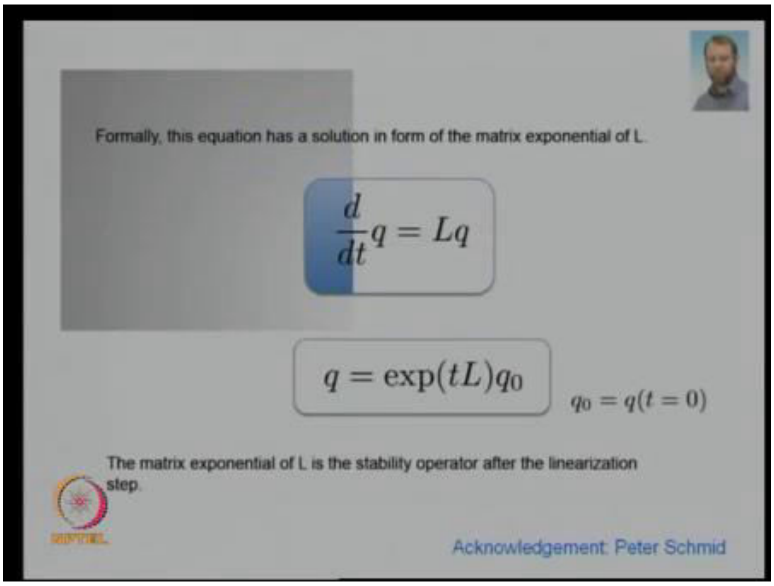

So, got the slides from the Peter Schmid given me permission to use them, so if you we can write this formally as if q is the its nice thing, because sometimes some history is good form some stall word has written it. So, d q or d t is L q, so if it is a ordinarily differential equation how do you what is solution, it will be like e power L t times q naught, the same solution works for matrix. So, we can write e power L t, where L is a matrix times q naught. So, the question is what is meant my matrix exponential I think many of you know this, but I will do this formally here, because some people ask me.

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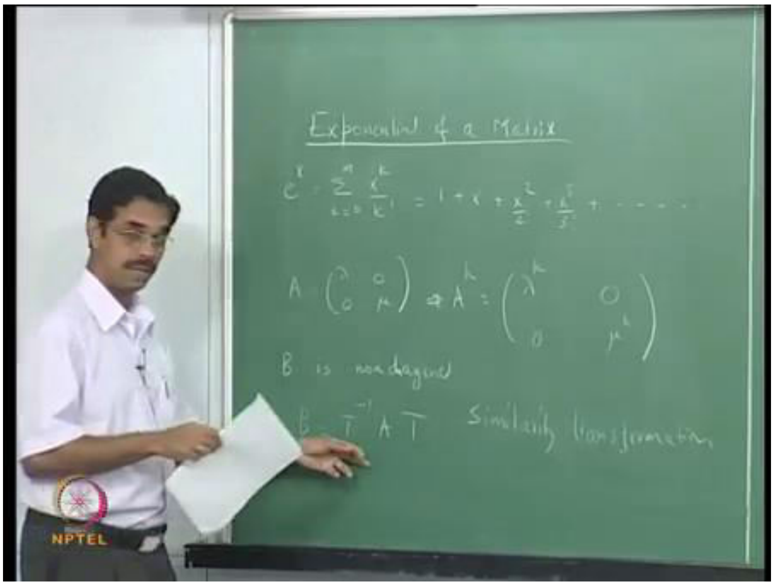

So, we look at a exponential of a matrix we know the expansion e power x equal to sigma k equal to you wrote infinity x power k over k factorial, which is 1 plus x plus x squared over 2 factorial plus x over 3 factorial plus. So, now let us construct this for a matrix, so let say, so assuming that if we can fallow the definition as long as we can calculate x squared, x cube, x 4 then we can actually construct the exponential.

Because, you can make all this powers then add up and construct this exponential of a matrix, so as long as we can solve the power, so if A equal to lambda 0, 0 mu, then A power k would be equal to lambda power k 0, 0 mu power k, this you can multiply it out and see it and then or prove by induction or whichever where you want. So, now the issue is, so then if you have this then you can simply add up all these things there is no problem.

But, issue is what is b is non dying for let say b is non diagonal, so but then you can say we can always make, this transform what is this call transformation, similarity transformation, so how do you find this how do you can be similar.

Student: It converse to matrix into a upper time ((Refer Time: 27:10))

And you look for the Eigen value and then you can calculate the similarities from the Eigen value and the Eigen value so.

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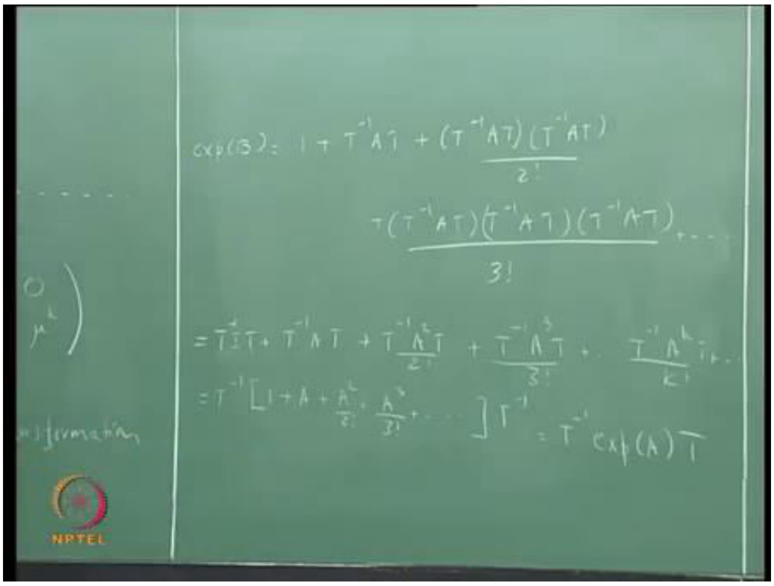

So, you can expand out with this you substitute this for this and then you can expand out this way and now it is quite simple. So, this will become 1 plus T inverse A T plus what would this, become again it will become T inverse A square T plus what would this become T and T inversal growth T and T inversal growth, so it will become A times a times A, so it is again T inverse A cube T plus T inverse A power K t plus.

So, one can be written as like T inverse T, so T inverse I T, I is 1, 1, 1, so this can be recast as T inverse times 1 plus A plus A squared over 2 factorial plus half thank you, A cube by K factorial T inverse. So, this would be equal to T inverse e power A times T. So, this would be may think formal.

Student: ((Refer Time: 29:21)) because the inverse will access

As long as you can do this decomposition we can do this, but you are saying that if you are repeated Eigen values you cannot do this why.

Student: ((Refer Time: 29:49)) Repeated eigenvector i do not eigenvectors.

Then why would not take this.

Student: ((Refer Time: 29:55)) Because, general matrix will be singular the eigenvectors.

You determinant fine, so as long as you can get this transform then you can do this, so

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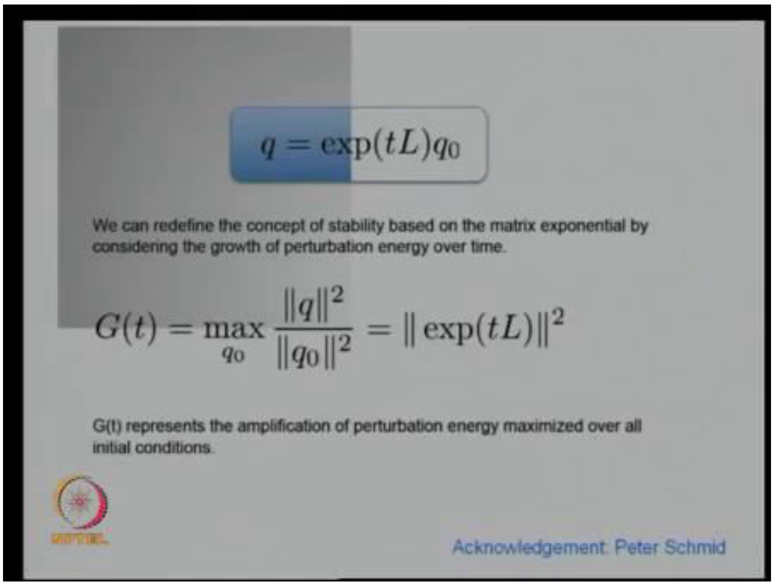

So, we want to look at energy time some time divided by energy at some time, t equal to 0 and that would go like norm of this e power t times L, but L is the linear operator here, so now, this comes back to my question and to how do you get the norm of the matrix.

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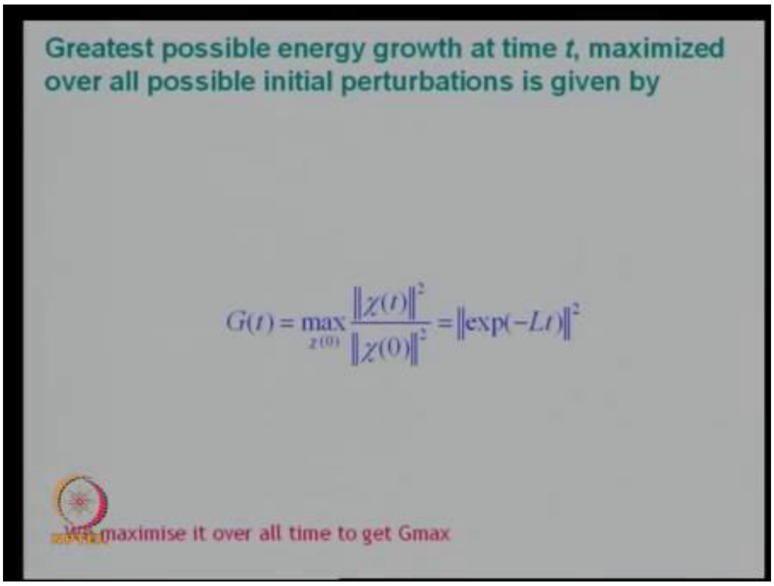

So, before that so in summary, so we want to find in our notation norm of kai I t divided by norm of kai I 0, I mean squared, so you can define norm as square or without square so this is the way I think I seen it use by physicist. So, they put if you have you have to put square then norm squared is like energy, but the other people they sometimes you do not put square. So, just have to be consistent with the notation.

So, what you need to find out is the greatest possible energy growth at time T and we should be able to optimize it over, all initial perturbations, so you have perturbation various direction as he pointed out. Considering all the perturbations available which one gives the maximum growth that is idea to be want to maximize that. So, you amount to optimize our initial condition and find this energy ratio or the ratio of the norm and then we have to optimize it for all times and then you can get the maximum possible transient growth is that clear.

So, you have to each time you have to maximize it over all initial condition and then, so these maxima you maximize it over all time, pick the one which gives a maximum over all time, that is a maximum possible transient growth and the corresponding initial condition, is the one that gives the maximum transient growth. Now, how do you do this that is the idea, would you have to integrate along take all possible initial condition and try to integrate it is that what is to do or is there something available in linear algebra which you can use to address this that is the next question.

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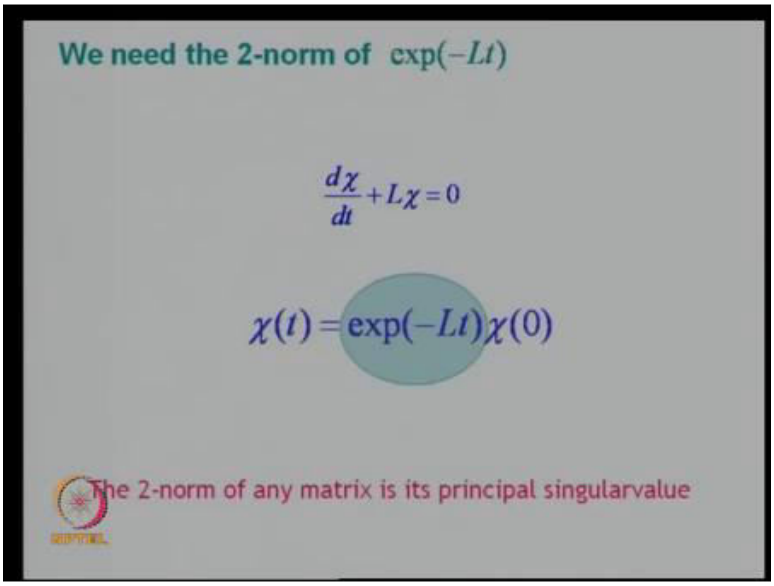

So, we go back again just to recap d kai by d t is L kai, so kai of t is e power minus L t times kai of 0, so I bring this L to the other side it becomes minus L kai. So, we need the two norm of this matrix e power minus L t that is the corrects of the matter, the two norm of any matrix is it principles singular value that is the way it is define. Now, there is a reason why it is defined that which you will see, now I mean to smart and defining this way.

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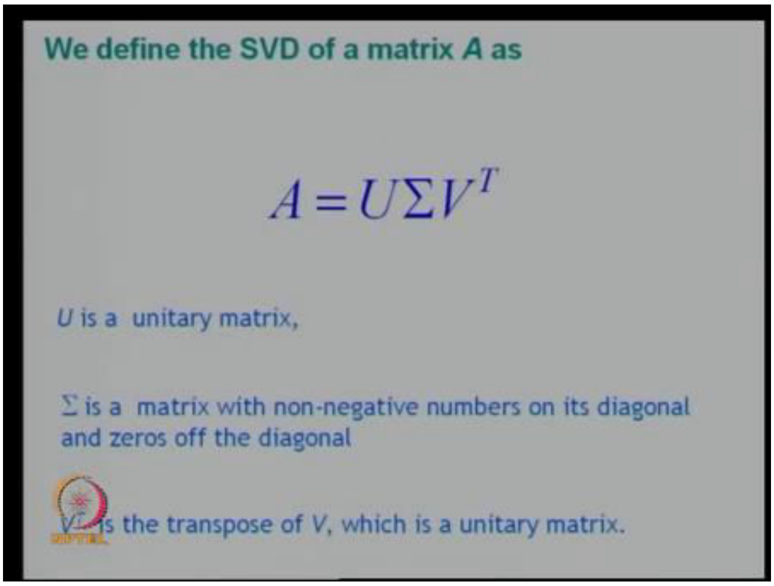

So, we need to do singular value of a matrix way, so I want if you know what a singular value decomposition do you know no, so if you have any matrix A we can write this as U sigma V transpose. Now, so what is big deal about this U is a unitary matrix, V transpose which is the transpose of V, it is also unitary matrix and sigma is the matrix with non negative numbers on its diagonal and zeros off diagram. So, in principle if you have a matrix we can rewrite it as U sigma U transpose, now how to do that I will explain after words I mean how to practically calculate this sigma, but let us assume that we can do this it that, so the idea here is that, let attempt to do a singular value decomposition of our evolution operation.

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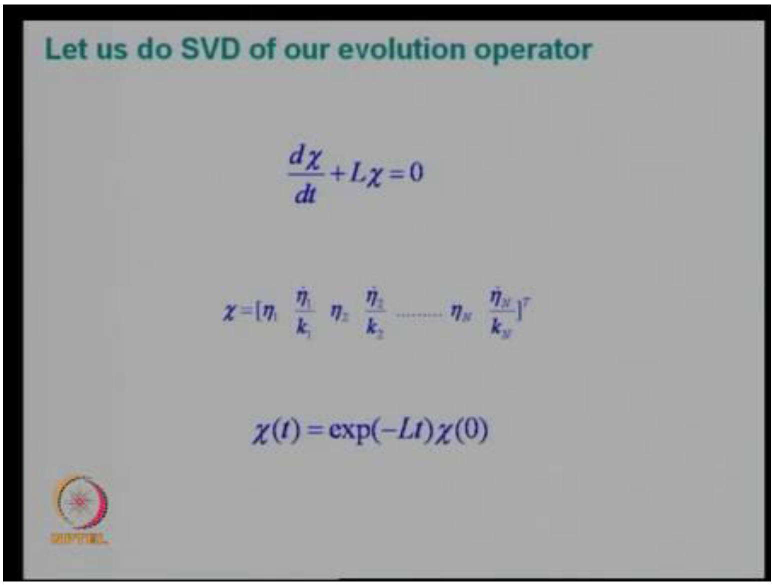

So, back to differential equation d k by d t plus L kai 0 and just to remind that, this is the state vector eta 1 and eta 1 over eta dots and eta dot over j pies, that vector and solution is kai t is e power minus L t times kai naught.

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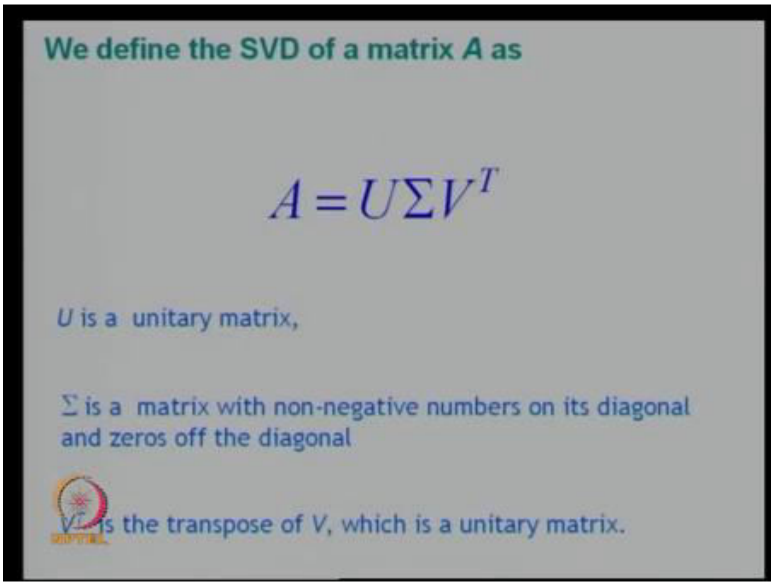

And, so we have to do s v d of e power minus L t, so kai t is, so we replace e power minus L t with U sigma V transpose, assuming we have e power L t minus L t and we do s v d and you get use sigma V transpose and we have just put it back that all, nothing more than that just replace e power minus L t with U sigma V transpose in this equation that all, but this is a profound physical meaning which answers the question vikram asked

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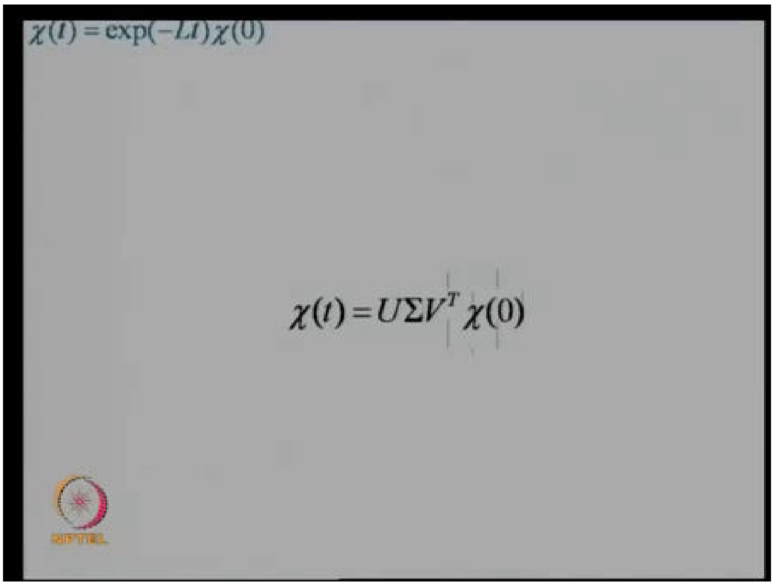

So, let us see what is means I have just rewritten this, so V transpose kai naught actually resolves, the initial condition vector in the Ortho normal basis of Input vectors. We saw that we transpose the unitary matrix, so it this actually solve the initial condition vector into Ortho normal basis of input vectors.

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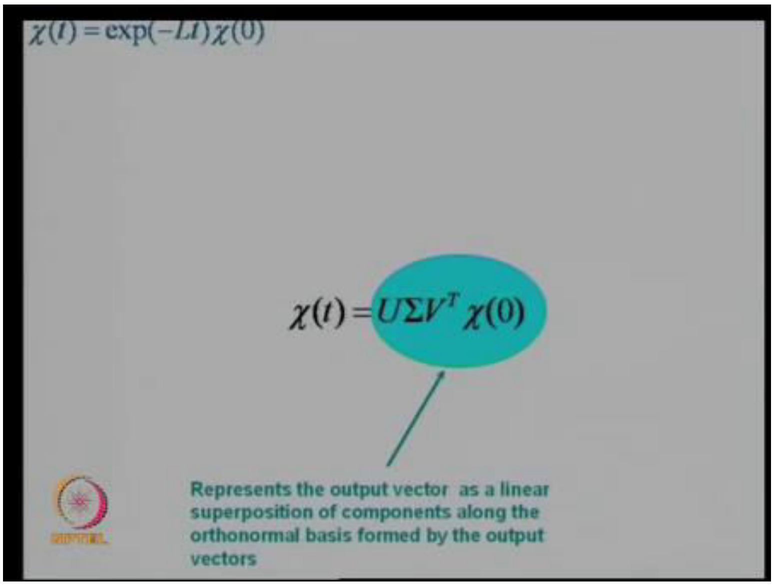

And the whole thing in Representation of the U, U time something it represent the output vector as linear superposition of components along the Ortho normal basis formed by the output vectors.

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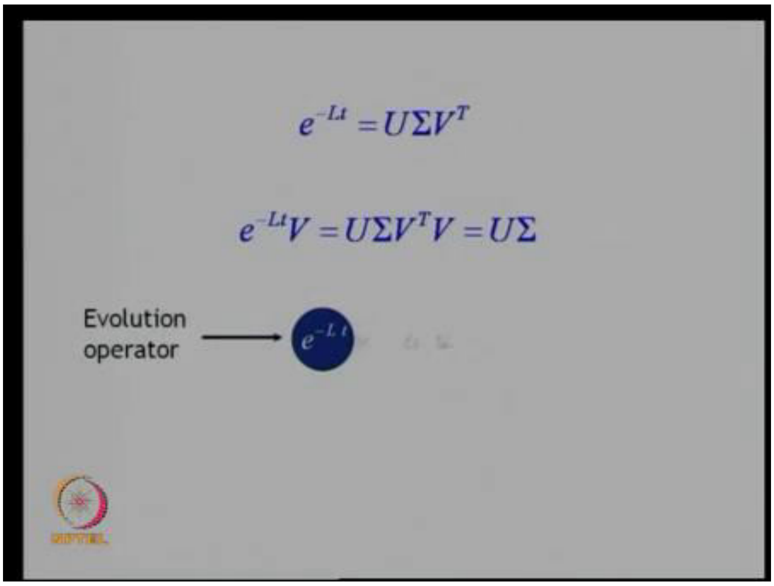

So, we multiply the matrix from the right with V, we have ((Refer Time: 35:41)) l e for minus L t as U sigma V transpose and when you multiply matrix with something, see when you multiply 1 with 2 you can have 1 first or 2 first does not matter, but the matrix you have to multiply either from the left or from the right and both can give different answers and they will give same answers only under very special conditions. So, here I am possessing that I am multiplying from the right side, so when I multiply this V transpose with V, what happens it becomes identity, so, I have e power minus L t times V is U sigma V transpose V, which is U sigma, because V transpose is unity.

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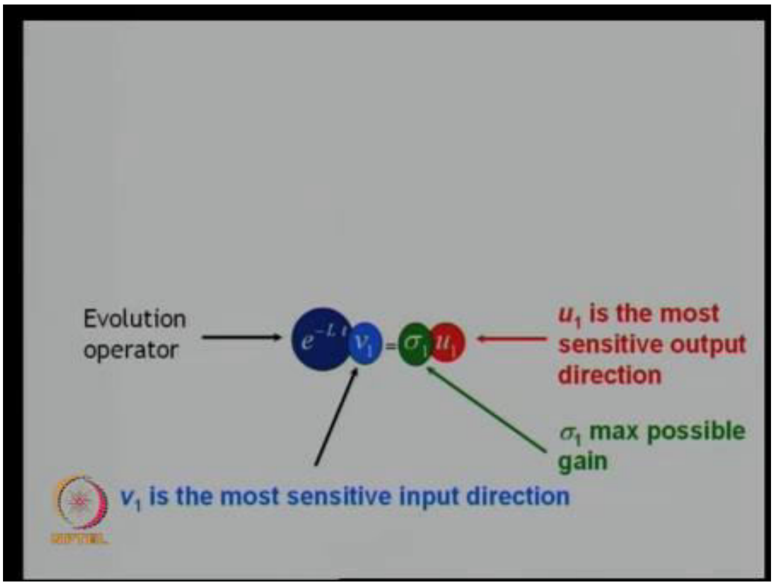

So, if you take the evolution operator and it acts on V 1, which is this column on of the columns with the Eigen of the singular vectors, so this is the most and so e power L t meant acts on the more sensitive input direction, will give a sigma 1 is the maximum possible gain. And the direction will be u 1 will be the more sensitive output directions. So, this would be the a interpretation where sigma 1 is the principles singular value, so single values when they are return their written in the descending order, the highest 1 come first the next highest will be next, the next highest will be next and so on.

So, the top 1 is the principle singular value and that is the own with the maximum magnitude, so this is the machinery in linear algebra available, so when you lean our study linear algebra it just operation. But, is the setup such that or I do not know who said it at which first, but it has a physical meaning in terms of studying energy growth you can use that machinery here, the only critical thing is your norm would mean something physical that you here.

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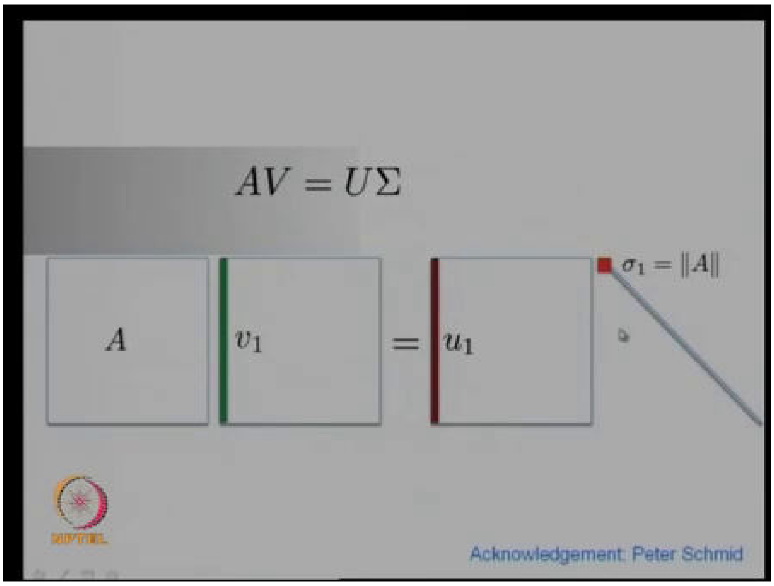

So, let us look at in terms of this matrixes, so A is acting on V and V consist A various columns, so let us take the first column that is corresponding to this one v 1 here. Now, this is equal to u 1 times sigma 1, so this is how it will look in a matrix is that clear, so in a matrix structure this line actually means the first column. So, here it will written the first column and you have to multiply it by this, that that is the meaning of this equation.

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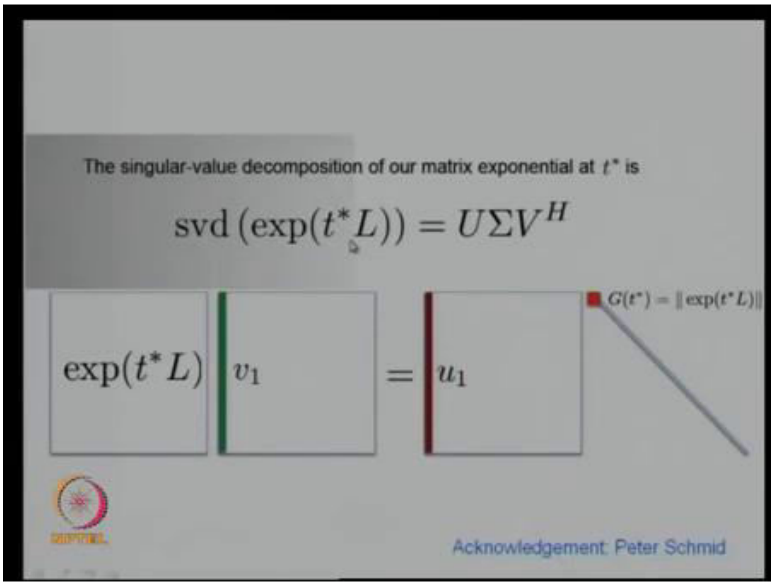

So, we are doing again Peter Schmid for giving this nice illustration to me and giving the permission to use this, so you take s v d of e power L t, which you write as U sigma V transpose or H here means hermitian conjugate, but same as transpose. So, e power L t acts on operate this operates on v 1 and you will get u 1 times norm of the e power L t, so that is the amplificant this is a same as motor out, but you can see kind of pictorially in terms of the columns of the matrix and sorry this line, actually means this is the one non negative and everything else is 0 and this matrix.

So, e power L t operates on input, then you get amplification times e power output, so you have this operator and these are the different direction that are available, so we are discretized the system and some sense. So, actually information dimension system we are discretized into this eta 1 eta dots and etas and so on. So, you have fired numbers, so the fired number directions, but this when you do this s d v this is the optimum initial condition, which is the left principle singular vector and direct would be the gain times the optimal final direction is the sorry right principle singular vector.

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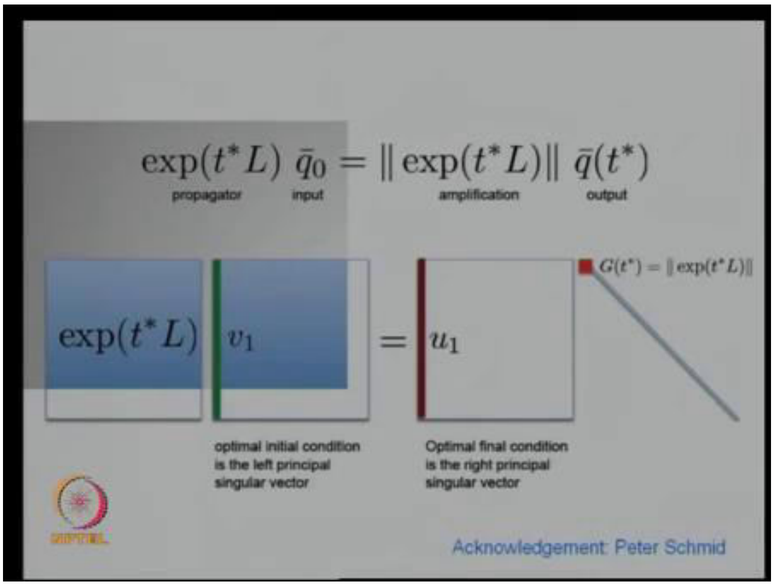

So, the evolution operator operates on the left principle singular vector it'll give the right principle singular vector times the gain.

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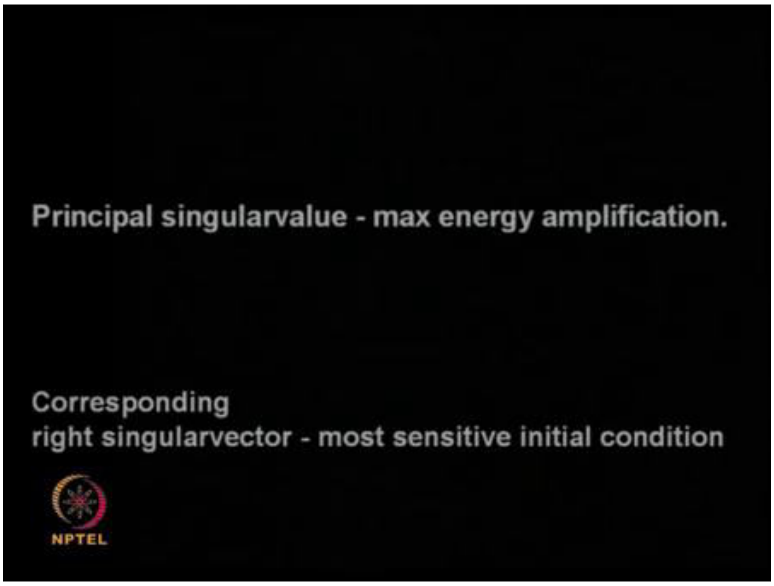

So, Principle singular value in ((Refer Time: 40:29)) the maximum energy amplification and the corresponding singular vector is the most sensitive initial condition.

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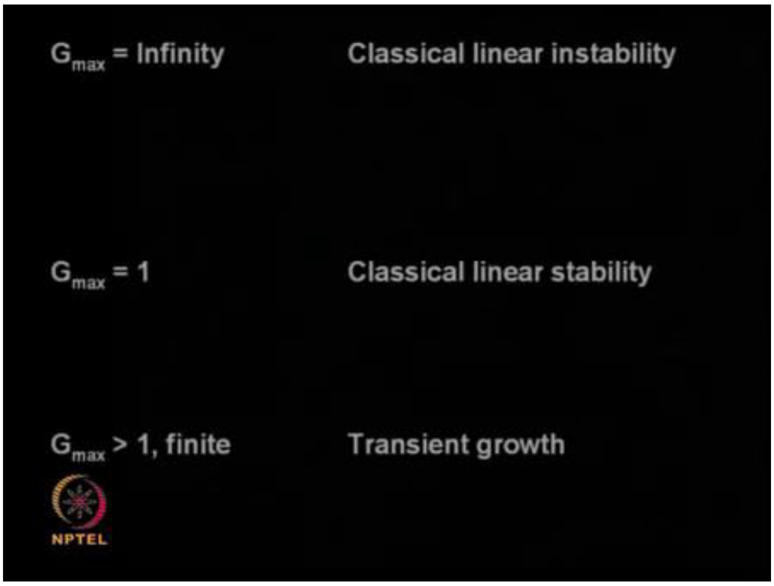

So, let us look at G max is infinity that means, energy is amplified to infinity when does it happen that happens, where classical liner instability G max is 1 that means, you had the maximum energy at time equal to 0 ever seen it is d k . So, that is classical stability I mean you have Eigen let say you have excitation long Eigen mode and it is d k is and that is fine G max greater than 1, but finite. So, this is transient growth.

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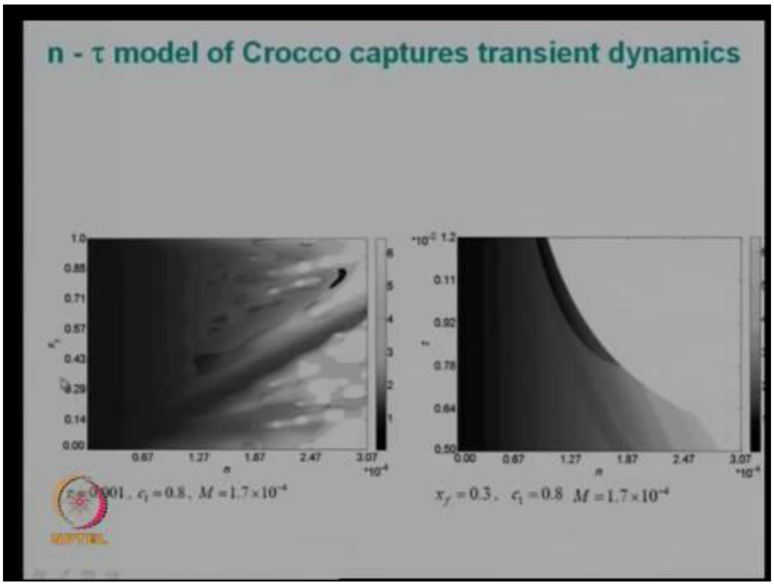

So, even a n tau model of crocco we are done some calculation and this is like in the parametric space, we have floated versus flame location versus n and you can see this scale to read complete, white is G max infinity compete black is G max is 1. So, this pitch white portions are linearly unstable region, complete dark is linearly stable and in between there is gray, which is regions of transient growth same, here in this region there is linear instability in this region, it is classical stability and in between there is a region white is transient growth.

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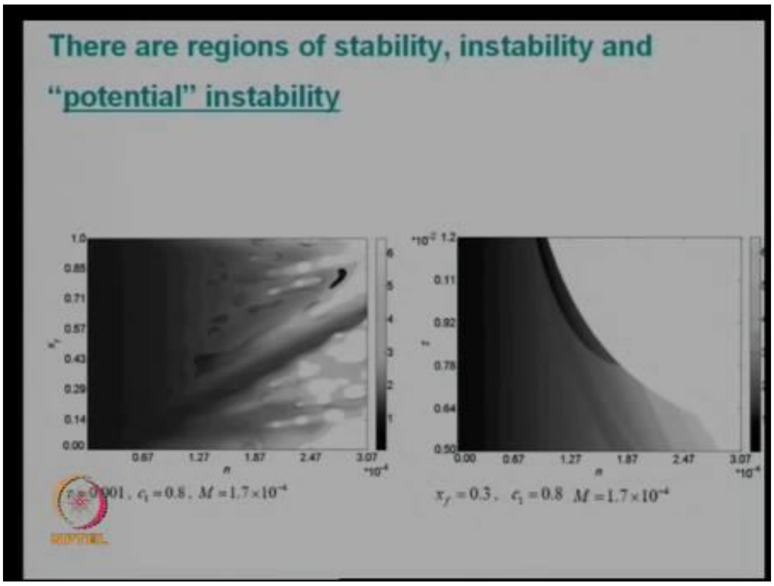

So, the regions of stability, instability and potential instability.

Student: ((Refer Time: 42:24))

So, you have regions here in which, so what are problem does counted max is g max, so when G max is 1 that means, there is no growth orientation., so that is this region and of course, I cannot show infinite gray scale human beings cannot sit and I do not know represent it. So, up to 6 I have gray, but after that I put white, which means and white corresponds to infinity.

So, if you use this model you know we are done this already the regions were take this linearly unstable, so this complete white denotes linearly unstable region here also. And complete dark denotes classical linear stability and in between there shades of gray, which would actually mean the finite G max like G max of 4, 5 on all that for this model for some other model, you may have larger or smaller.

So, I can call this is regions of stability, instability and potential instability, so now location of plane accept.

Student: showing that the kind of ((Refer Time: 43:33)).

So, it not only the amount of acoustic driving depends on the n and tau or heat release rate, but see q is a function of u, but then u and p are also related and what you want is a phase between q and p so.

Student: particular ((Refer Time: 43:58))

just a.

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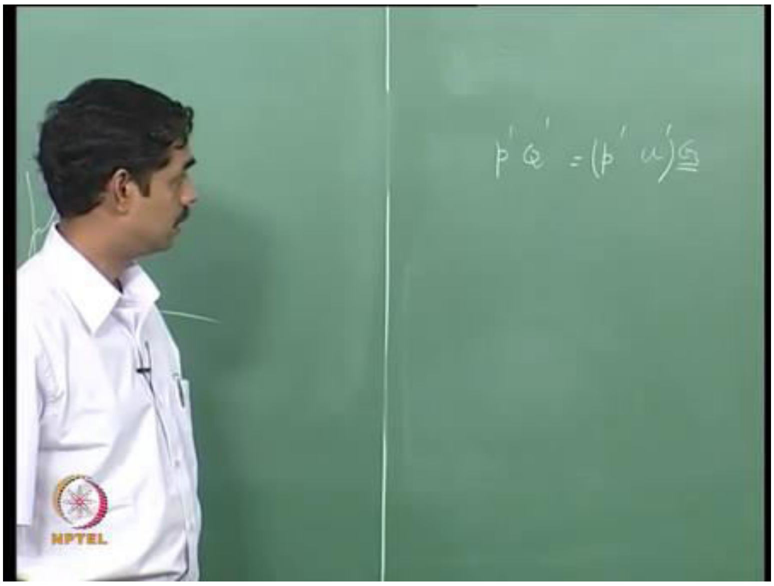

So, let me explain this, so We want p prime q prime, so q prime is let say goes like u prime time g, so we just does not depend only on g, but it also depends on the phase between p prime and u prime or if I it depends on the phase between p prime and u prime they mode to driving. So, this critically affects, so that is a reason why x f is a important parameter, so it is not just n being larger smaller, but what is the phase between pressure and velocity and then velocity and its gain, when the heat gain or heat release.

Student: Asking me should in it be there should be some its values where even it low time it goes to instability.

It is possible.

Student: So, but we is Refer Time: ((45:17))

I mean, so this is like a gray is increasing right and so I mean it is not like flat contours.

Student: ((Refer Time: 45:32))

It in very complex shape I cannot interpret, but is possible, so the I just want to stop with explaining how to obtain singular values, how to compute singular values.

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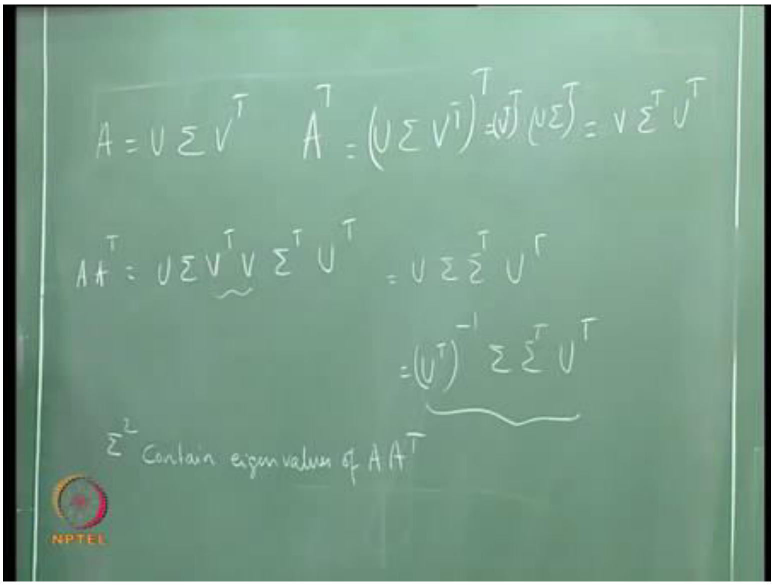

So, A times should be U sigma V transpose this would be U sigma transpose no V transpose, which is V times U sigma transpose this is equal to V sigma transpose U transpose is that right probably right. So, let us take A transpose this is U sigma V transpose multiplied by V sigma transpose U transpose, so what is this.

Student: ((Refer Time: 47:10))

So, this would be U sigma, sigma transpose U transpose, so this can be rewritten as U transpose inverse sigma, sigma transpose U transpose, so what transformation is this of A transpose is this that is a similarity transform. So, that means this is the this will contain the Eigen values A transpose, so if it is square matrix sigma square will contain Eigen values of A A transpose. So, the way to get singular values of a matrix A is to multiplied by its transpose and obtain the Eigen value that will give the square of the singular value very simple.

And if you using mat lab, we can have this commands to do is I am out of time, so stop here. So, we looked at the evolution of the operator and we looked at the linear evolution of a dynamical system and we saw this analytical solution and then, we try s v d and then we found that we could, find the optimal initial condition and the maximum energy amplification using s v d. So, will stop here next.