Acoustic Instabilities in Aerospace Propulsion

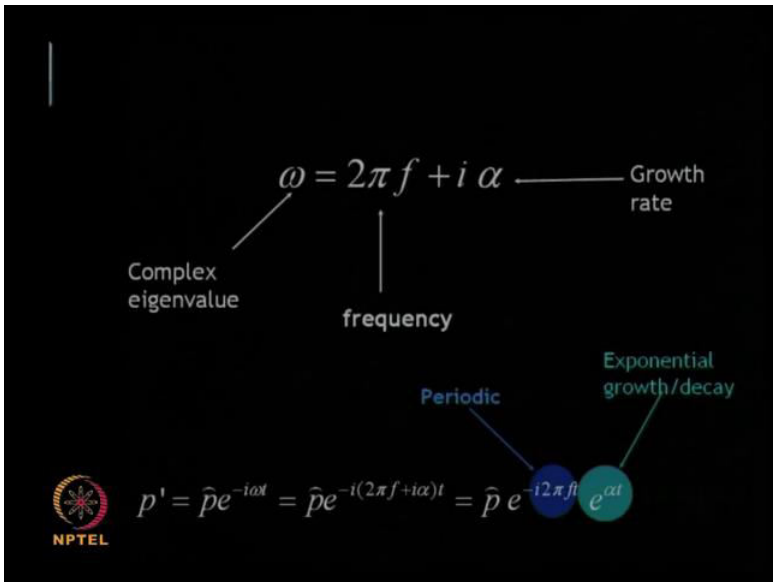
Prof. R. I. Sujith
Department of Aerospace Engineering
Indian Institution of Technology, Madras

Lecture - 26
Non-normality, Transient Growth & Triggering Instability-1

(Refer Slide Time: 00:07)

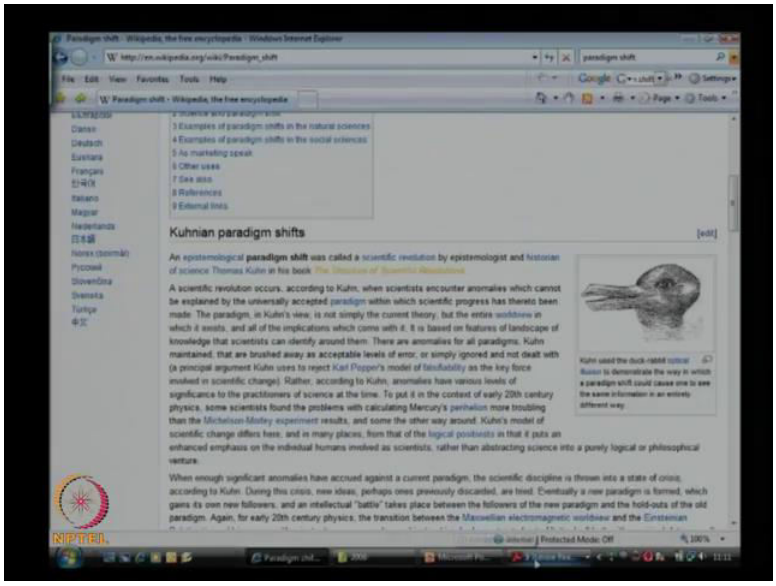
Good morning everybody, we are looking at the stability analysis and we are first we derived the non-linear set of equation for a model. Then, we also had a linearization and we have a linear set of equations of the form, d chi by d t equal to l chi. where l is like a linear operator.

(Refer Slide Time: 00:42)



And going back, we look for Eigen values and elastic stability analysis and the Eigen values are like, go like 2 pi f t plus i alpha. Because, it is complex and this is the periodic path and corresponding to the frequency f and alpha is the growth rate. You can clearly seen this, we have done before. So, you have a e power minus i 2 pi f t. Which is like periodic path and e power alpha t, is the exponential of d k. So, this is where we are and we said that we look at some other type of stability, last time.

(Refer Slide Time: 01:07)



So, lets look at that, before that, I just want to ask you, not able to demonstrated live. Other ways, I could have made this picture big but, the internet is not working. What is this thing? Do not read what is this? What does it appear?

Is it seems (( )).

Yeah.

Sir rabbit;

Rabbit;

(( ))

Duck;

So, some people say rabbit, some people say duck and some says rabbit and duck. So, what is it?

(( ))

It is actually bone.

(( ))

It some kind of; So, you look at something and once you have an idea, I mean if you, if there was none of these things written there. Perhaps you will stick with one of the views. And till somebody point out that. We know that, I mean there were some people said duck, some said rabbit now that both are out. Can you see both? Lots of pictures like that. So, there is a book in our library, I do not know if you know this book. The structure of scientific, there is a book called scientific revolution by Thomas Kuhn. I cannot open these links, because internet is not working, with my computer.

So, what he says I will view his theory in a brief while, in brief. What he says there is a establish view of things. He gives lots of the examples like discovery oxygen, quantum mechanics all kind of things. And several different examples: of how something changed. So, he calls there is something called normal science and Para dimensional. So, normal science, you know there is an answer. Is like what you need from text book. You

know, there you read a a text book and text book and there are problems and you are guaranteed that is solution.

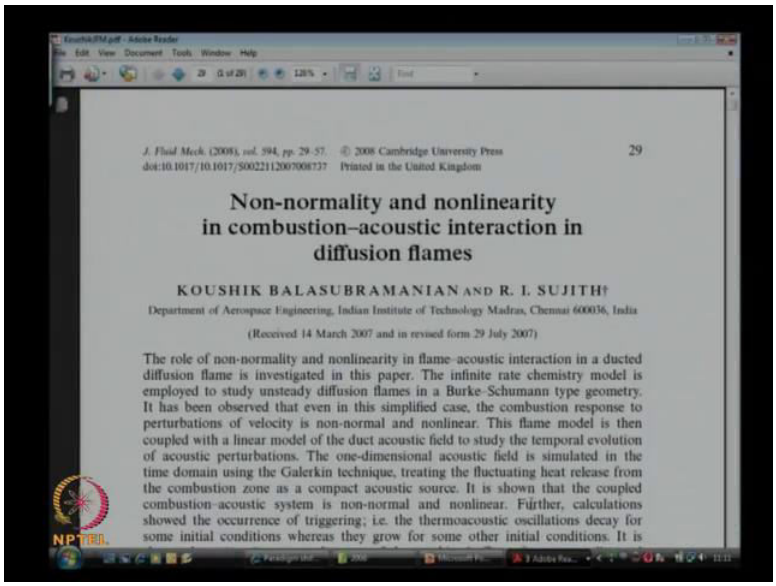
And then you keep on solving that. So, that is like jigs of passing low or something. You know there is an answer. So, you think it is a duck and you try to fix the things to be a duck and then you multiply a top and bottom by something and had a subtracts. Whatever, you do lot of jugglery and so on. But the still are within that frame work and then comes around some diaphragm outside. That is not know, anything. Who has not heard that it is duck? In our science the equivalent is here not bread any papers and is an arrogant, or does not care. And he says it is not a duck, but, it is a rabbit.

And then everybody tries to beat him down, might this happen. It seems the most of the noble discovery is were rejected by the generals first time, ok. And then what happened is the establish get is are upset and then some kind of battles happened. Then in fact: when normal theory came in field mechanics. If you see: in internet traffic there as given a presentation. He said that there such a extent that, I pulled edited out of the TV. But, the groups are not even talking to each other and really going down the trend kill each other and so on.

But, in the end what happens? It there is two in the roof view, a new generation will grow up and eventually the old is down die, or retire. And then, if it is true, the new life will be opened. Completely young guy coming and smiling in the class and then they will come up with this view and given enough time will over through the old one. And then this view is stay for some time, till another set of people come and they come up with a better view. Overthrow this and this will go on forever.

Not not really overthrow, it will say that overed for one particular set of things. But, in general so, new toned in mechanics is one thing is chemical reaction another thing. Lot of reaction as usual: lot of examples. It’s really book worth reading and I found the recommend you to read this. So, we will talk about a little paradigm shifting thermo acoustic, where everything was in the model frame work.

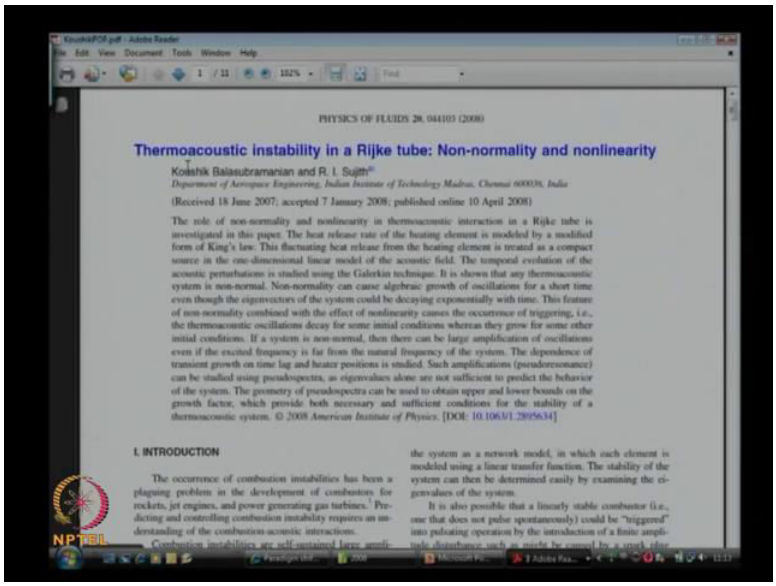
(Refer Slide Time: 05:31)



Till this dude came, what is name? Koushik Balasubramanian, sitting this class B Tech. Then he said that, this normal mode is not right. I mean, in general it is non-normal in fact, there is hardly it is time when it is normal and then what happened is papers were short down. And everybody said that this is nonsense and he did lot of analytically nice things and they said, you can do analytical things in 1920, not in 2005. And I am so honored and analytic things came. But, there was a tooth in it eventually; he overcame everything in goddess things out. So, this is the 1st paper and this is subject and this guy he was a little guy like you.

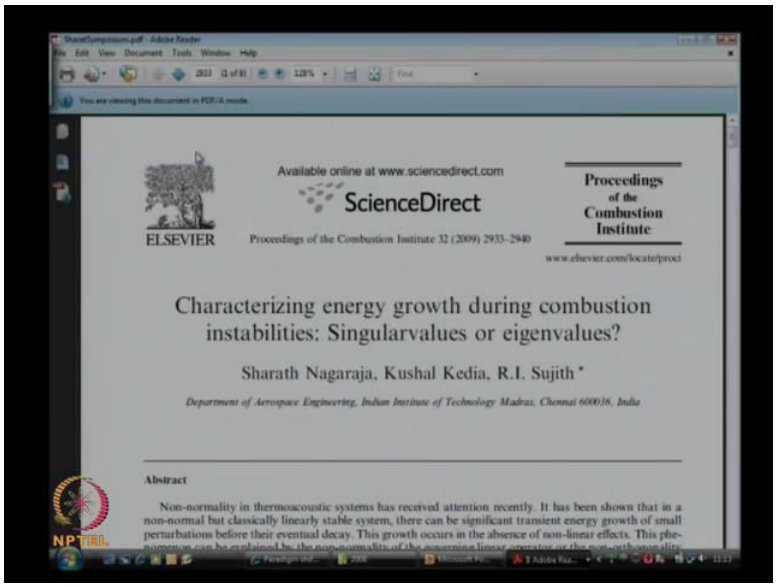
Only difference between you and him is that he behave like twelve standard student, or eleven standard student not like I I T B Tech, which is a disgusting thing, here. I I T B Tech, or M Tech whatever, he behave like nice school kid and that was other ways he was a like you guys. And he was really excited about the life and passion at a what everything he do. Not just science but, ever playing basket ball. You play happily, not I mean not like the B Tech. The things that happiness is a very uncooled thing to use something. Anyway, he did this and he is rolled a series of paper I think. Seven papers values, I mean not even PhD students do this kind of stuff. And so, this is really hard paper to read but, I in write you to read some of you good at solving the equation.

(Refer Slide Time: 06:55)



He wrote another simple paper. So, that people can understand. If they cannot go through the vigorous mathematics and the other paper he constructed a toy of problem. This is really easy going paper and in which he explains things. And even so.

(Refer Slide Time: 07:13)



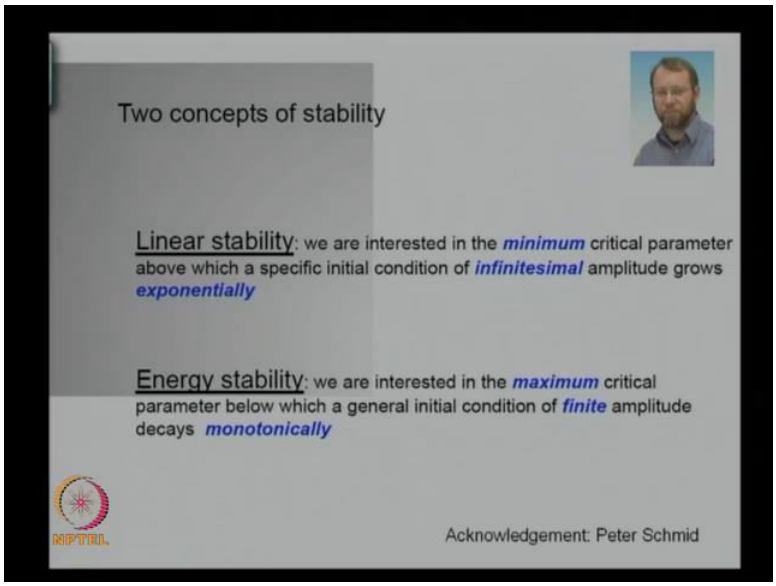
If it is really new, I had and came another set of d d guys. Who are also act like school kids, Sharath and Kushal. They wrote this paper explaining, what is meant by similar, why the similar value characteristics the energy growth and so on. So together with these dudes, I we engineered the at I I T Madras a Para dimension normal mode stability

analysis, or subject was bond here in our aero building actually. So, we will go through this and the this paper is really nice. It is really simple and with the elegantly written, almost like kids write which is really nice. Because: on you they understood. Koushik's paper are little heavy. He everything is obvious to him. But, it is nice but, it will elegantly written and so on. So, I will invite you to read this paper.

Because sometimes, if you see how was subject was born. We should read the paper and the subject to see how would you thinking the processes. Because, here some dude, 19 years old or something taking on a q lake was 8 years old. And saying that you cant saying right and you are and the guy tries to kill him back and so on. So, you know, I think you should see that, I mean eventually 10 years later, 20 years later this will be in text book. And it won't be in this form. It will be ordered very nicely and it will, everything will by the time text book is written. You can be convinced that the subject is over, this no room for you to work.

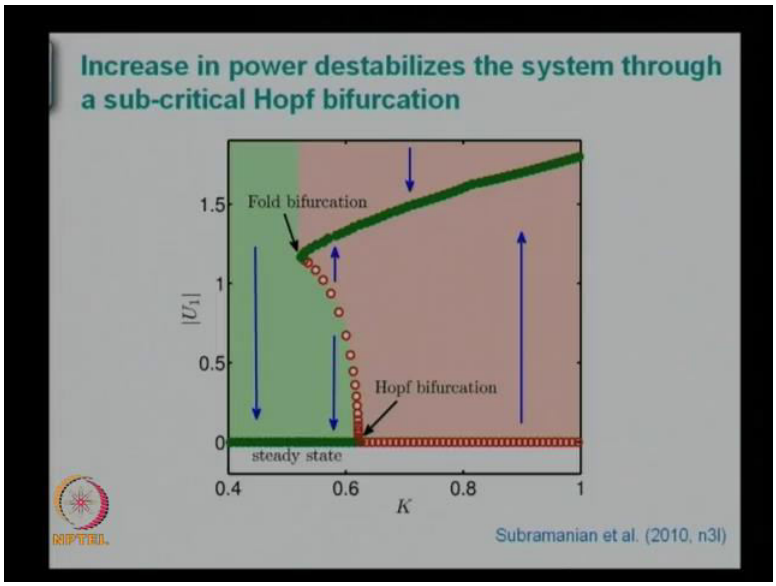
If you working in a subject with a 10 text books. It was solved 10, 100 years back, the gas term everything has done, really. You needs a beautiful subject but, is a 1800 and 1900, if you lived it would have down to exciting. The classic gas another example, its perfect it is a great subject. But, I mean you can't read a fan of flow and rally flow and all that and the unsteady was solved. So, everything is solved, but, so when you moved to the new place where you can make the new things. And then all the initial problems will be solved. Then people do heavy problems, is even heavy things will solve. Then you try to player on with little bit and so on.

(Refer Slide Time: 09:17)



So, we will try to explain this. So, we have to talk about the two concepts of stability. I took the slide, this is given to me by friend and colleague, Peter Schmid, from ecol polytechnic, he was collaborator. So, he says that there is a something linear stability; we are interested in the minimum critical planet. Above which a specific initial condition of infinite simile amplitude grows exponentially. That means, this key word is infinitesimal. You can the information can be small and if you are condition, still take smaller still take smaller and still take smaller. When something about energy stability, we are understand is that maximum critical parameter. Below which, a general initial condition of finite amplitude decays monotonically. So, that is the other limit.

(Refer Slide Time: 09:59)



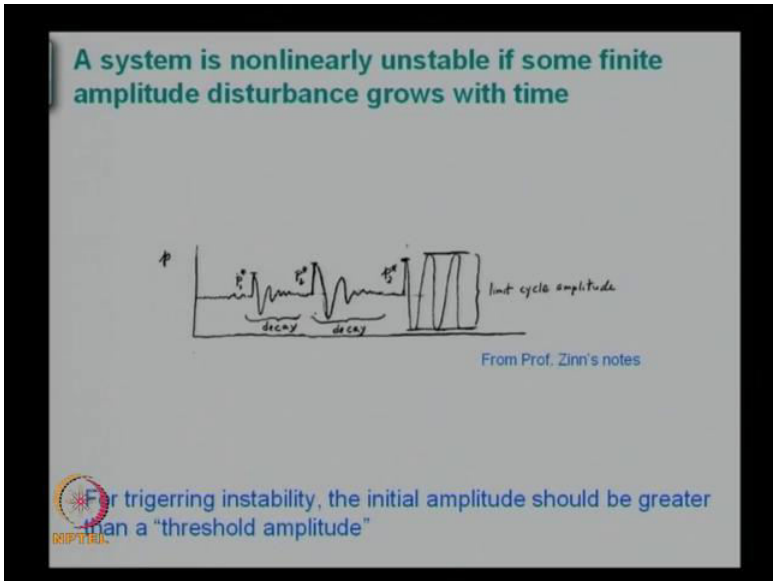
If you go to this graph; which I show you through here; this if you go by the definition above this, or to the right of this any parameter will go up. And to the left is this, anything will die. So, that is the energy stability and this is the linear stability, is that clear. Now we have to link. (Refer Slide Time: 09:17) what so this is a Peter Schmid, is a guy who kind of engineered de fired and shift in fluid mechanics stability analysis and fluid mechanics on toughness and so on, In the 90's, you as student at in 90. Now as a professor in ecol polytechnic and they had very different time. In 90's and establishment came after them and then really hard time.

And I think the subject was started probably, I do not know the history correctly. I think the first pseudo spectrum or perhaps computerized in 1970, I think. And there were, papers from 60's and so on, talking about the transient growth. I think the first do not know probably dealt mathematically, the person called, Chagelishvili. I do not know you met him, when he was here some of you. He was here in January, of about a week or so. Goerge Chagelishvili, you met him.

He did it in astrophysics, a creation disc and so on and Schmid Henningson, ready. They wants in 90's of feudal, late 80's who did it in fluid mechanics and atmospheric science. And now it is major subject in thermo acoustics, we discover it fast and in every subject when this came. The subject faced a lot of opposition from the establishment, but, that is

natural. Anything with this new will face opposite, that is what I understood after reading Thomas kuhn paper.

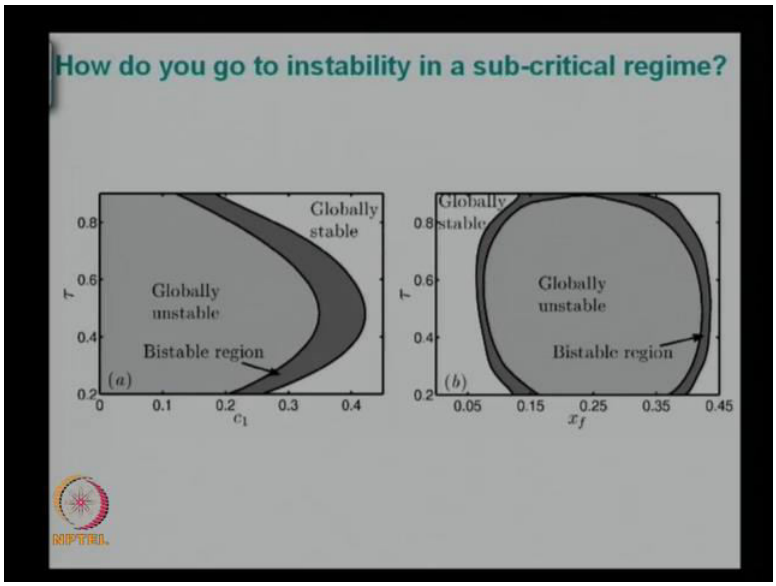
(Refer Slide Time: 11:47)



So, going back to the what to, so, we have to link up the fluid mechanic to what combusted stability people were say. So, in the 60's people talks to what triggering instability, that was is rocket and you look at it. I am its fine twenty first times it glows up and what the do is actually put a bomb inside the rocket, blow it up. And that is like a finite amplitude initial condition, look at it look at whether rocket stable or not. So, this same as subcritical bifurcation and in modern and non-linear dynamics language and that is the same thing is the fluid mechanics what is talking about.

So, we have link what we are seeing here and there? And then try to make pictures or the duck. I mean it will stays a duck, till you start observing something else. So, it is same ways I think you connect the thing and then suddenly new picture. It just thought that it's not like you slowly constructed, or a bit for something. Bang when saw it as a rabbit, it just came you bang it right. So, that is the way blind was.

(Refer Slide Time: 12:51)



So, we saw that there was thus region, where there is globally unstable and globally stable and in between there is a region where scanned of potentially unstable. And that is a globally unstable. You can construct the region or wait as perhaps as. Globally unstable, globally stable then there is like a region where there is potentially stable. So, I give example; with how students behave in class. Last year I had the great fortune to teach 3rd year B Tech's. So, in the class about 80 percent of students were always sleeping. They are like this globally stable region outside, completely sleep. Whatever you do, even before any disturbance put they are sleeping and whatever. Even if you burst the bomb they would not know and there was she shifting in front are always awake.

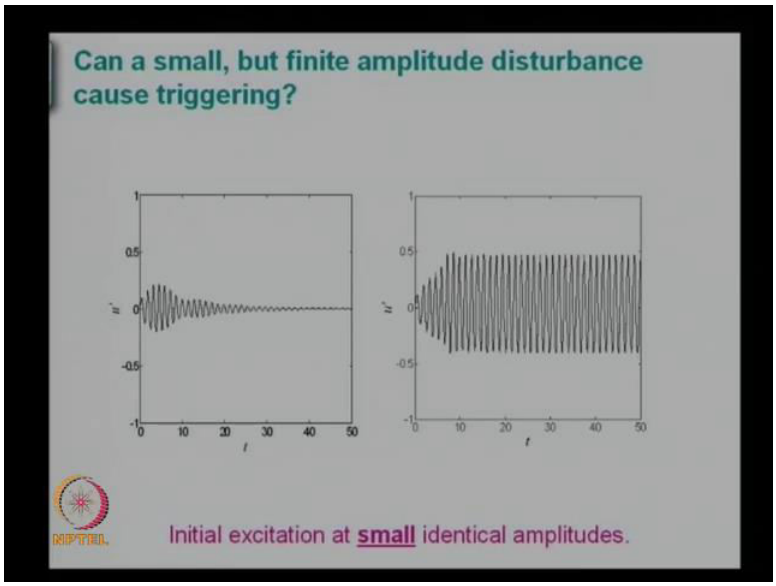
No problem and you do not know matter the teach will be awake and there it is in between guys, not is you are globally stable. This follow I could wake up correct and if I l and scream and so on. Then bang oh all right, shall finish kind of unstable. You are the awake, most of the time, close to the half point. So, this fellow wherever he came, he was very but, many times he was not present. So, this is good I mean that the way I think of this h i stable region. That some people, whatever do they are sleeping. So, they are like stable, you cannot excite them into instability to wake up and all.

Some other people they are always awake do not have to do anything. But, in between there is this guys like Darik, you speak its small disturbance he will be sleeping. But, you

raise your voice and throw choke at him and all that the. Where I am? I so this is like the all subcritical system. So, these are situations which are, I think teaching 3rd year guys a very great revolution. I think I would have been a lesser human being, if I did not get to teach you guys. These are situations, which are potentially unstable and the issue is how do we go to instability in a subcritical region. See if you teach M Tech, you do not gain a fund wouldn't instability. So, there were always awake.

So, we again go back to features (Refer Slide Time: 09:59) that you increase the parameter and first you come all the way here. You jump up but, you are when you came back. You do not come back straight here; you have to go to all the way back and come back here and then only go here. So, this like I was I mean you mess up but, you cannot just go back straightly and come back here. You have to undo a lot of things, only then you will fixed it. Was just like lies, do one thing and suddenly everything messed up and then you just cannot stop doing that and life would be stable. You would have to go back a lot and clean up your life.

(Refer Slide Time: 15:43)



It is the same way the thermo acoustic. I think that is the very important point talk about machineries, if not in life. For example, (Refer Slide Time: 09:59) if the gas trouble and becomes unstable; usually the controllers would not let it go unstable. They send anything is there and they know the operating point. And they would not let you come and close to that. You what is the problem to this you fit becomes unstable. When you

have to go back all the way, to here (Refer Slide Time: 09:59) you cut back a lot. And that you do not like and generally it could be subcritical bifurcation.

Then you have to come back all the way, now if the instability came on and the things becomes bang and you do not know, what is happening people shut off engines. And shutting of the engine is quite involved, because, even if you shut off, it will still on for half an hour. Because, there are very nice magnetic bearing, with almost no resistance keep guerdoning and then they look if anything is there and not. And then it restarts it will take another half an hour to bring to the r p m so on. So, and the power is lost and this lost, power to the gas surveying company has to pay the power company, for the lost revenue for this hour. So, the lawyers always smart.

So, it is quite important, that you do not approach in instability at all from a commercial point of view. If it is a socket it is unstable then it just blows up, or it does not go vary up wanted to go and so on. It is much more critical, you had thus you can start and bring it back after half an hour and reduce the power something. You can do is rocket gone it's gone; you do not have a mission then. So, they question is, can a small but, finite amplitude cause triggering. So, they looking at triggering and because have the combustors. Which did this triggering that was the motivation when we look at it and you are looking at a how the instability came on?

So, I came over this brilliant ideas, at we had non-linear solvers and non -linear solvers. Showing this kind of thing, will do and came down. So, then but, then at some other condition it came up: So, I would have been if it just came down, for small amplitude but, then before it came down actually went up. Then I came up this brilliant thing that, or nonlinearity, I read some popular books about non-linear dynamics and so on. Non-linear dynamics can do lot of things. Let's write a really linearized solver and then we will know for sure, I mean you can remove all this is things. And the we will see nice d k, for small initial condition.

Now, koushik wrote the linearized the solver the students. And it was going exactly same thing and then non-linear and linear were pertimised identically put them on top the look will same. And but, you jack up the initial condition you would go to limit cycle. So, you are exciting, almost under small identical amplitude and linear theory should work. But, then linear theory is showing this strange behavior. So, the we decided to look

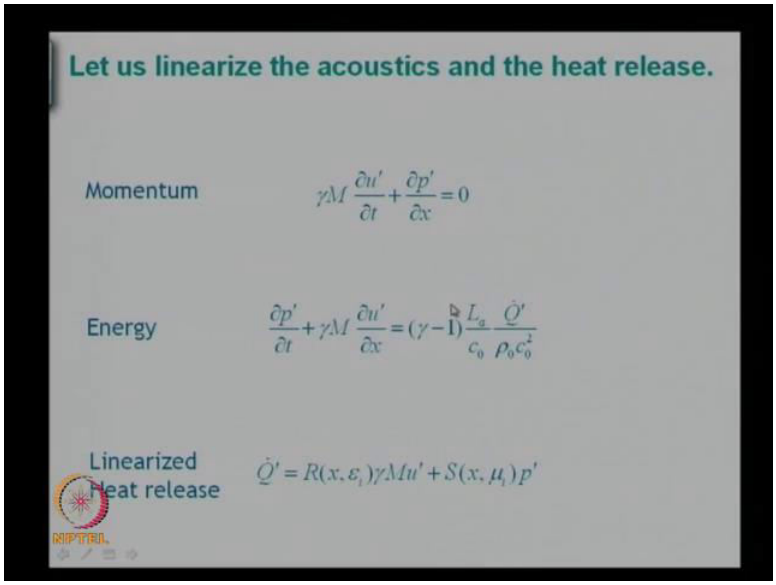
at a acoustic equation and see what is happening. So, we write the equation, you can write it in non-dimensional form or dimensional form.

It is nice do this in dimensional form because, I mean it is general right. So, this is momentum and energy equation. We derived this is class, remember?

You did it but, you did not linearize the heat.

Ya I did not linearized the heat I derived the equation here.

(Refer Slide Time: 19:07)

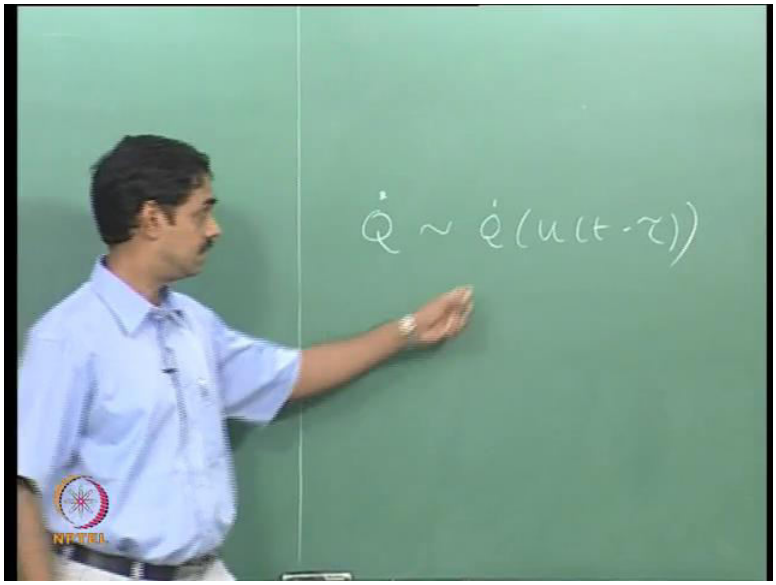


I did not right the last one but, I derived these two right yes. So, I think we should do care other ways it's like, you what is that you do all this Ramayana story and you ask; who is sita? And he would say is a leader or something. Should not be like that. Can you please at least wanting equations acoustic we should do. At least one of them or else shame. So, we have to edit this part out of the T V. So, we have the momentum and energy equation, should re do it, or I will give you this. Oh no I think this is disaster if you are scan thing and you will keep it and bring it in exams and then at that time you will be wanting what is? You will do it.

So, we have the momentum equation and energy equation and then we linearized. Is there anything non-linear in this equation, other than the heat release? In fact in the model we derived, how did heat release go? and we start help me out, I feel very back. In

the model we derived for q dot, heat release rate. We wrote a mode what kind of nonlinearity it was there?

(Refer Slide Time: 20:17)



We had a we said goes like a right. We set this is a function of u of t minus torque. And what thought of function was this? Was if exponential quadratic square root all and all.

Interesting.

In fact what was the co ordination call?

Heckles;

Heckles I dict;

Square root;

Square root, I think we have to read this I think other ways we are not getting anywhere. To confess this whole thing is like I am telling some story and but, you cannot relate to. You can Vishnu, you are suppose know everything?

(( ))

That is enough Vishnu name no. So, what is non-linear here (Refer Slide Time: 19:07) is only q prime right. There is anything else non-linear. The acoustics is that linear or non-linear how does it looks like? These equations;

They are linear;

They are linear, they are linear, ya, because so, you have just operate around u prime and p prime that solve. So, this know you do u prime but, dow t. But, u u dow. Though you prime the dow x write the non-linear

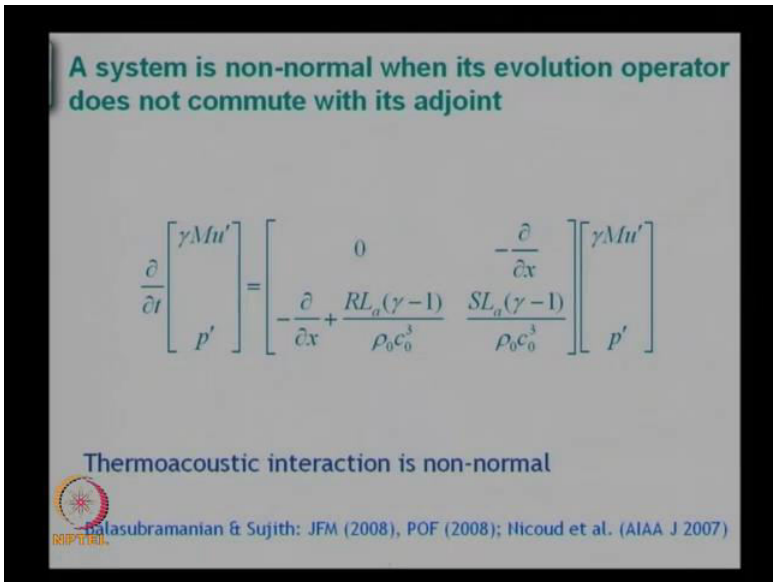
Yes Akshay?

(( ))

Terms which through them on;

Ya m is mean proper amplitude it is a parameter. We did the parameter m is like a q bar. So, it is linear in u bar, linear in u prime in that, we are having pertabrations right on the base flow base flow has given to it. So we linearize the equation, linearizing means (Refer Slide Time: 19:07) you say q dot prime, plus b times p prime. But, to get things look nice. You have the scaling parameter and so on. But, we will not worry about it but, basically you can take the whole thing as a construction time's u prime. Plus another construction times p prime.

(Refer Slide Time: 22:16)



So, I have re cause equation in this form, d by d t of something, equal to operator time something. These is a dynamic system form and of course, the scaling when gum I and u prime, not u prime? That is because this nob of this thing, the state vector, but, goes like some energy which is relevant. So, it is like acoustic energy, now if you look at the operator, assuming r and s were zero. What can you say about it? Akshay, it will be a symmetric matrix, so, it will surely be it will commute this transpose, a a transpose equal to a a. Now, transpose a now because r and s, if you workout. This operator, will commute with it.

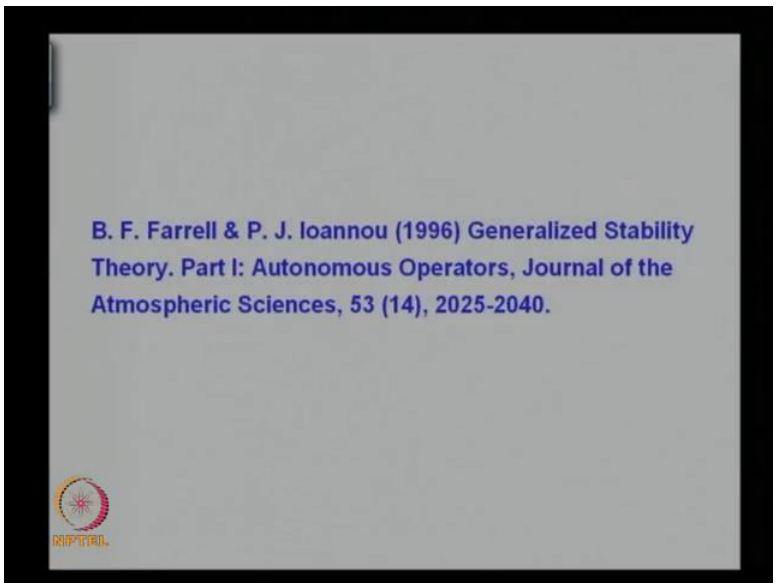
So, see if you take the operator, a a transpose will not be equal to a transpose a. And then you call the operator non normal. If a a transpose equal to, a transpose a, when then we say that the operator is normal. So, a system is non normal. When, the evolution operator does not commute with this adjoint. If the real operator you can take it transpose the matrix and then the d d x will be, a what is the adjoint? its be minus gradient, adjoint of different operation. So, you can what caught in few minutes and you can show that this will not commute with this adjoint.

So, thermo acoustic interaction is actually is general sense non normal. The only time it becomes normal is, when r and s are 0, and even if you complicate more complicated than u prime is 0 orp prime is 0. Then this operator, at the boundary it will change. Will become un symmetric and even that will make some non normality into the system. So, these things you can explain on those references, that I gave you in the beginning of the class. Any questions are there?

(Refer Slide Time: 24:28)

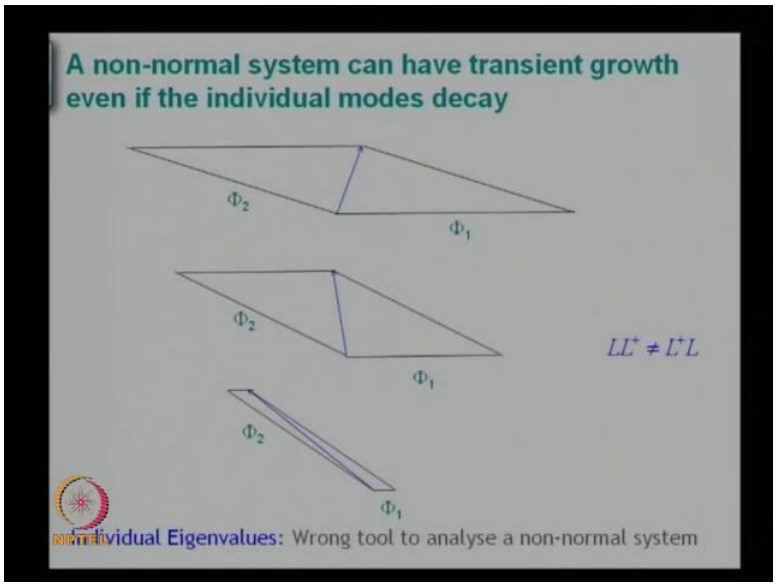
What is this non normality? So when we started I never heard this term to be very honest and later on I found or what it was. Because, the straight behavior or I saw, this growth and d k full linearized the system, I found that it was associated with non normality. But, in the presentation u can explain, what does non normal? And then bring the subject in straight forward way. But, when you do it first time, it's not so straight forward, because you hit on some behavior. You do not what it is, and it is crazy and you think it did it something wrong and later on you find that. This is what they call that.

(Refer Slide Time: 25:00)



I will give you another reference, which is a very nice reference by the Farrell and Ioannou. They are from Harward. I think generalized stability theory, part 1: autonomous operator and part 2 for the non autonomous operator also. Journal of atmospheric science, volume 53, number 14, of page 2025 to 2040. Non autonomous operator means, the operator is not a function of time. Non autonomous means operate itself as a function of time. See you can I can give you the paper.

(Refer Slide Time: 25:33)



So, we can represent Eigen vectors geometrically. Of course, I cannot represent 10 vectors geometrically, in 10 dimensions. But, only 2 dimension I can draw a geometrically. And 3 dimension. I can construct with some sticks and so on and in a you seen a chemistry people construct molecule. So, with that we can do but, at the moment I was stick the 2 vectors, because it's easy to draw. So, fee 1 and fee 2 are 2 Eigen vectors. They are present Eigen vectors. But, the same principle holds for 5000 vector 100,000 vectors.

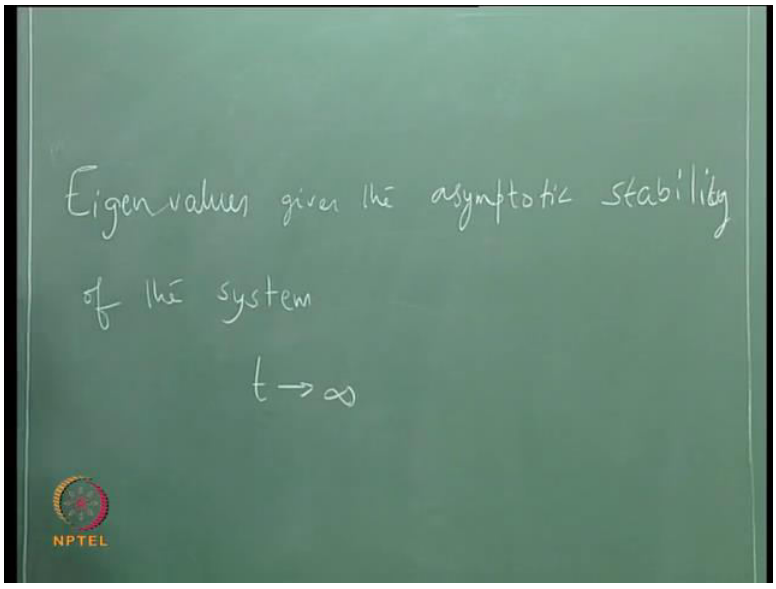
The fee 1 and fee 2 are d k, in fact if you have any doubt. You can bring your ruler and check the, sorry, if you only the actually coming down from top to bottom. Same with fee 2, f ee 2 is also coming down. So, the blue line is the resultant, how do you construct the resultant and vector algebra? You must studied in,

(( ))

Parallelogram law where as another law, triangle law. So, we will draw will parallelogram, now and you see that the vectors are shrinking. But, the diagonal is glowing. Now had the vectors been orthogonal like this and I shrink. My resultant will also shrink. But, now they are like this and you see this is the problem. I mean is the individual vector shrink that does not necessarily means the resultant shrinks. Now yes as you if they are shrinking and these shrinking forever and time tends. To as fee 1 will becomes 0, fee 2 will becomes 0, the resultant also 0, no question about that. So, asymptotically, each of the Eigen vectors, if each of them are shrinking.

The resultant also will shrink and tends to 0. But, that is only asymptotically but, before it dies.It will create some damage and then only it will die. So, in short time it will going to be out. So, as the general rule, if you have more vectors you will get more now as general rule is more vectors will get more transient growth, if you will get 3 or more you can get more transient growth. Then if you have 100 you can get much more transient growth then you can get 2 or 3. If you are 10 1000 you will get even more transient growth and so on. So, under this condition, Eigen values are telling only the asymptotic stability. Asymptotic stability means as time tends to 0.What is the stability?

(Refer Slide Time: 27:59)



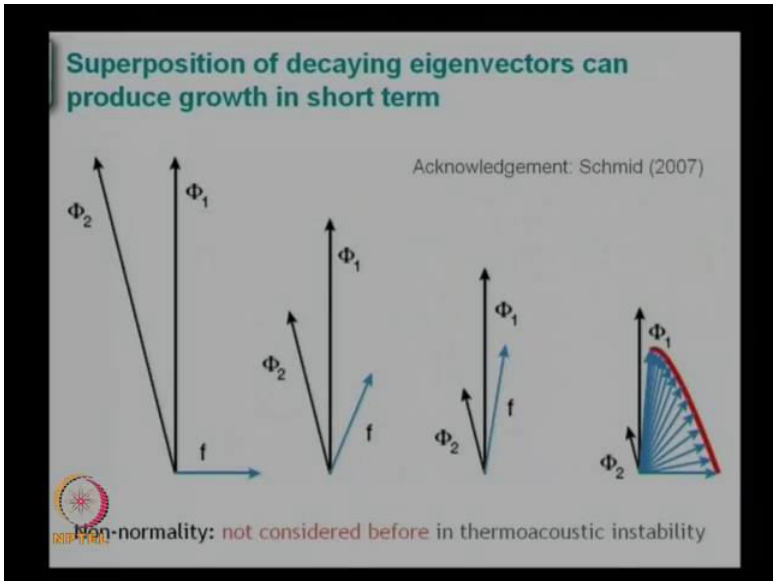
So, that means t tends to infinity, what happens? So, if you see in the mode analysis we did. We were looking at each mode how it is stable, how it is unstable. So, it can have each more stable, you can have all the mode stable. But, you still can have growth. That

is what is the concept of transient growth, you have individually each of the one is d k. All of dying at time of all of dying at a time of time t equal to infinity. But, individually we look and everything looks like stable. But, In a short time on your transient time so that is growth.

Is this concept is clear and can you correlate with the geometrical picture I have to draw. Yes please for a moment, you can tell me why it does? Not so we are condition to the thing that the individually, you use so losses reduce. That needs not these 3 probability. Any other question, draw more picture it is possible. But, even when they are non-orthogonal you can have then reduce. But, and I have set up the example.

(( ))

(Refer Slide Time: 29:45)



Which is a triangle law, this is slider growth from a Peter Schmid again. Fee 1 fee 2, is what you are looking for. That is a and the blue vector shows fee 1 fee 2. So, the clear that fee 1 is coming down, fee 2 is also coming down and in this no. It is not like have trick here, but, the blue is actually growing and you can see it, is really growing. And the direction of the resultant is also changing.

So, super processional k is Eigen vector you can produce growth in short term. Although asymptotically: you died, but, in short term that can be growth and this was not considered, till 2007. In 2008 Koushik came with break-through and he showed that. It

was non normal on the orcancy conditions. Non normality does not necessarily transient growth; under normal operator give a transient growth.

If you start with the right initial condition, because see can you give me case when you do not have transient growth. Given for the non-normal operator, when will non normal operator do not give transient growth?

The decaying that familiar; like similarly like this.

That does not guaranteed depending on direction. But, is one case where it is guaranteed. What if you are exciting along one Eigen vector? Then you continue to go down shrink in Eigen vector. Eigen vector are all independent right. So, I see you have a initial disturbance and then you project one of the Eigen vector. So, at the initial disturbance is along fee 1. That means, draw this.

(Refer Slide Time: 31:36)

So, I have several Eigen vectors, in all the directions and several basic function and along each basic function, I am projecting. So, let say my entire disturbance is along this Eigen vectors. So, eventually it shrinks, shrinks, shrinks. So, I am actually coming down this way, never grow. So, this was not considered before, but the now it seems like you saw from the operator that. I mean as long as heat release you will have non normality. Does not necessarily mean transient growth but, it depends on what initial condition is

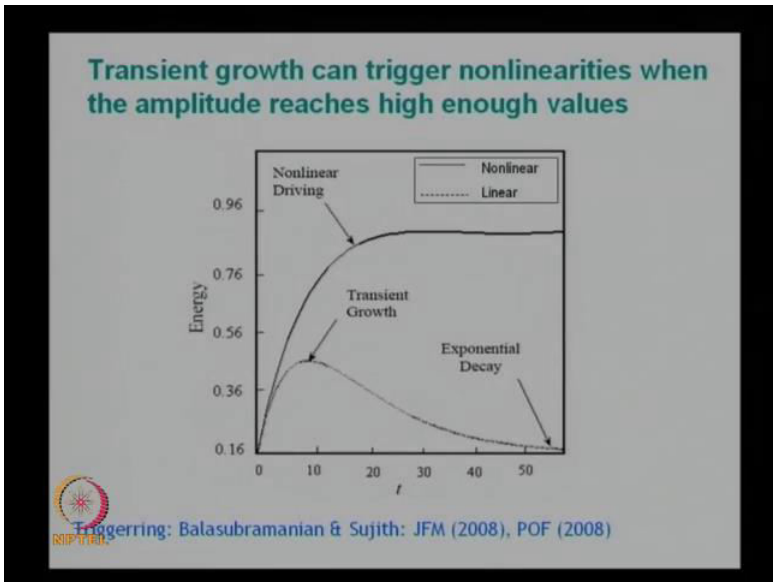
there? Because, that is clearly picked up this picture where, you are initial condition is a along this Eigen vector a and then you are shrinking is the direction.

Whereas I have picture here (Refer Slide Time: 29:45) on the screen, where I have a initial condition where the projection on this fee 1 fee 2. And then there was certain rule and must grow. So, both possibilities depend on what is initial condition, this clear? Actually you have question if this is non normal (( )) even it not guaranteed (( )) generated two individually independent.

You can have, repeated Eigen vectors it is possible. Then you will surely it is also possibility, repeated Eigen like this extreme case (Refer Slide Time: 29:45) would be like de generalized. But, generally my experience, so far I have not got repeated Eigen vectors. Always be I mean, they can coming close, but, not the quite repeated. But, even of it period sends and hold.

It might be a have only 1i think I clearance space is off admission value. Ya in that case you wouldn't have this problem. But, in general, minimum you have two equations. Even if you do not desecrated this, even it is just equation in two vectors. You are I mean principle it is right and practice. I mean you have several modes and so on. We will take a pipe and it has several modes right, natural modes also. Along near that where will be heat release modes of shift, but, that is possible to excite. What he saying is principle true in practice, hard to find the equation? Now it strikes on your face that everything is non-normal, but, at that time it is quite new.

(Refer Slide Time: 34:08)



So, what is consequence? So, I mean so what if it is non normal. So, what is plotted here evolutional acoustic energy for problem. And you see the dotted line shows the evolution of the linear system. So, I have linear simulation and I calculate perceptional laws for every instant and then I calculate acoustic energy a lot. And there is some kind of transient growth and eventually it is dying down and time consequence. But, you see now look at the non-linear simulation. It also starts support this way, but, then it goes off and takes out and just go into a limit cycle. Starting from amplitude itself, you are not really bursting a bomb and starting from a I amplitude disturbance.

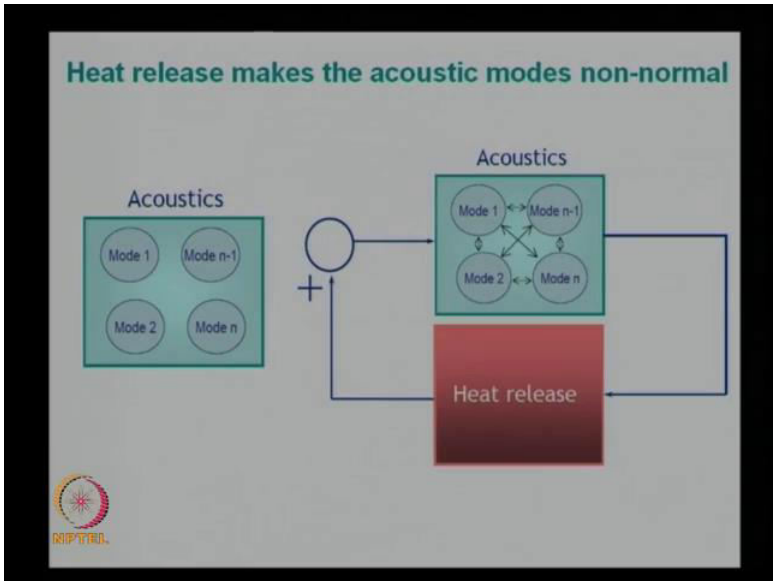
Start amplitude disturbance and going to limit cycle. So, another way you can have a you can think of is, you can have you can think of a base in a vector action. You can imagine, from like football or something, which is very symmetric. And then along any direction you excite, you will have same distance. The cross will be able to get out of the base and all the direction. But, let say if your football is punctured, or one side it is compressed and then along some direction you have is need little bit energy to get out. But, some other direction you need a lot of energy to get out. So, direction of along which u needs only small energy around.

Then there is a possibility you can get out, a easily to out of the base. Now the another way of interpreting is, as engineers were told that 10 percent of this of linear things of something. And some other may be if you studied in the I I T Madras, it is 10 percent.

But, as if you studied in Bombay, the I will say 5 percent is engineers is there. Below that is linear, above that is non-linear. But, really you can't say like that. Because, the very fact that you may start at the direction, which even 1 percent or point 1 percent. But, you can have a linear mechanism have transient growth and take you in time some other pulse some other time it will goes to 5 percent to you something.

And the suddenly you are I mean, you are still having linear system, but, this mechanism says that the range of validity of linear assumption, is not just 5 percent or 10 percent, it is subjected. The non normality actually reduces the range that which, linearization is valid. So, that means you may think linear equation should work, but, actually you have to use non-linear equations. And the it is different ball game or together, it's just clear.

(Refer Slide Time: 36:52)

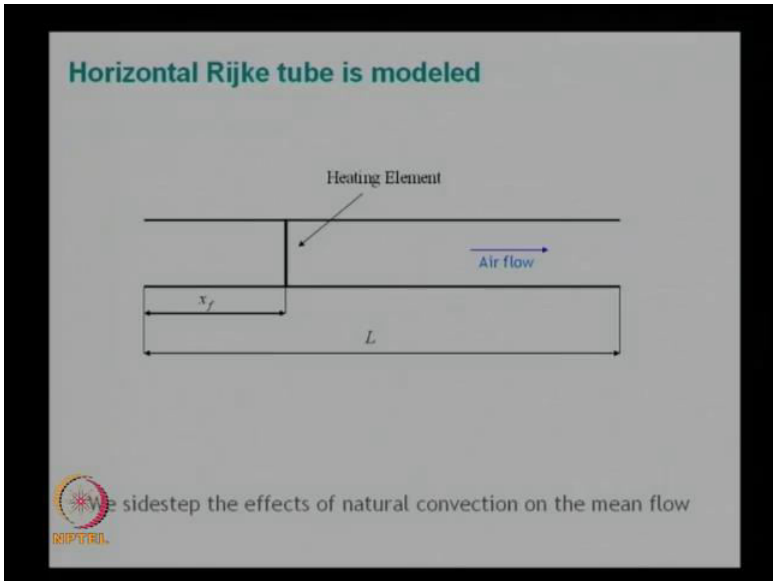


So, that is way I possess, once again classical acoustic is you have all these modes and you all we dependently. They are not they are normal mode, but, moment you have heat release, you really have non normal mode. And then, ya I mean, it makes the acoustic modes on non-normal or non orthogonal. So, you did all this theoretical concepts and so on and so, what are the consequences is? If have any I mean, you can learned all this in perhaps: if you did if linear algebra, you would have learn up this stuff. I learned that 3rd SEM. I have clues if any use, to read learns everything 2, 3 years back.

(Refer Slide Time: 37:40)

So, we look example of a Rijke tube, we saw in a lab, Rijke tube it showed among. And we will make a simple toy model, of the Rijke tube. And this is following (Refer Slide Time: 06:55) this paper. Thermo acoustic instability in a Rijke tube: non-normality and non-normality. We follow this and you can also follow (Refer Slide Time: 07:13) this. This paper also uses the very similar model. May be I mean I think same model, but, may be explained some linear concepts, it will be made in depth.

(Refer Slide Time: 38:31)



So, we saw that the Rijke tube is nothing but, straight piece of pipe. Can or tube or duck, or something squared. They want be as square duct and we have heating element, which was made of wire mesh. Do you remember all this and there was air flowing and how were air flow set up?

Blower;

Blower, ya and how is the classical Rijke tube?

Vertical,

It is vertical and what do use set up the air flow?

Natural convention;

Natural convection, if always they as you do not have to use them go no. If the if the natural convection is there already. Why are we side stepping if? There are side stepping it, because I do not want to model the effect of natural convection. I want, I model, sorry.

(( ))

Ya so, that is one thing. If I have, what I am saying is. If you have horizontal worm, you are using a blower. You can independently vary the heat power, as well as the flow speed. If you have vertical worm, is you do not need a blower but, you flow speed is link to

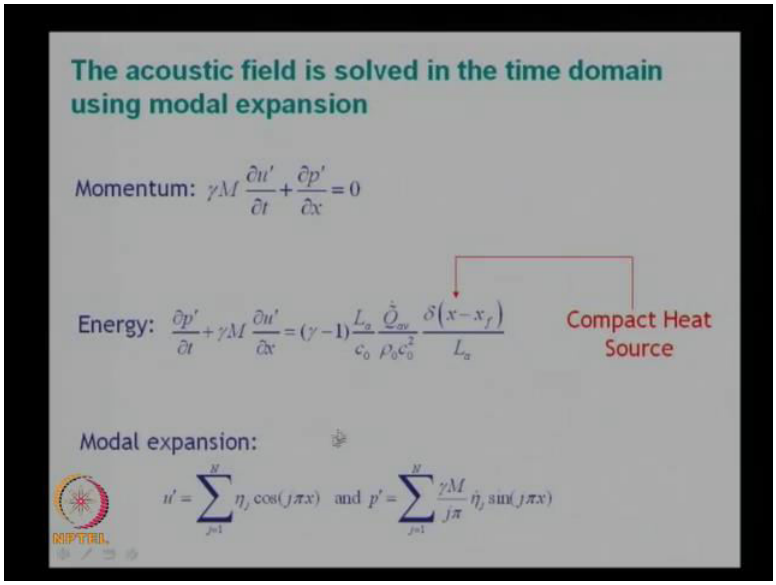
(( ))

Natural convection, of course, you have natural convection, along with that you can blow also. But, then in theory, it is easy to set up air flow rather than, natural convection. Because, if you have natural convection as a model and so, side step, I just do this horizontal thing. Now, if is pretty clear that, if you ever done anything. Anything that easy to do experimentally will be difficult to be model. Anything which is easy to do in theory, it is difficult to do experimentally.

So, we you see the Rijke tube it has so, much accessories. To de couplers on each side blower and so on and you and you had a separate power supply. Which one burns, lots of

things. But, if you do horizontal tube all linear was a piece of pipe and that solve. You did not need any of these accessories. That is the way it is.

(Refer Slide Time: 40:38)



So, we take this equations and do you remember, we did model analysis. And what happened after we did model analysis. Why do you do model analysis? Ya I know model analysis so I would. So, there same here, if you find a hammer finding nails everywhere. So, surely studied model expansion, so, wherever this is do model expansion. Yes that may be true. If you do that what happened, to drive a nail of nail grows and you can hang calendar of something. Here what happens directly? at this model expansion, it is like I ask you where is Anna university say it is in front of I I T. And then where is I I T? you will say in front of Anna university, clear. You do not form the equation, but, ya it is towards solving. What does it depends?

(( ))

What are these equations? What are these equation?

Partial

And mainly do this what happens?

(( ))

You got od. You want to convert td to od and why are we doing this?

Easier to change;

Easier to solve: much easier to solve. So, we want to solve the acoustic field. I am very very keen that I solve in time domain. Why? Because, I got craze because: simple frequency domain solution available. Why do not I wanted to do this? So, the you had the frequency domain solution weigh nice and elegant manners, I had worked it out the way nicely. So, why am I insisting on transience?

(( ))

Such a solution in the race, each (( )) is in the,

Solution configure we are basis like a linear (( ))

Why do you want to study in time?

(( ))

Key in the word transient,

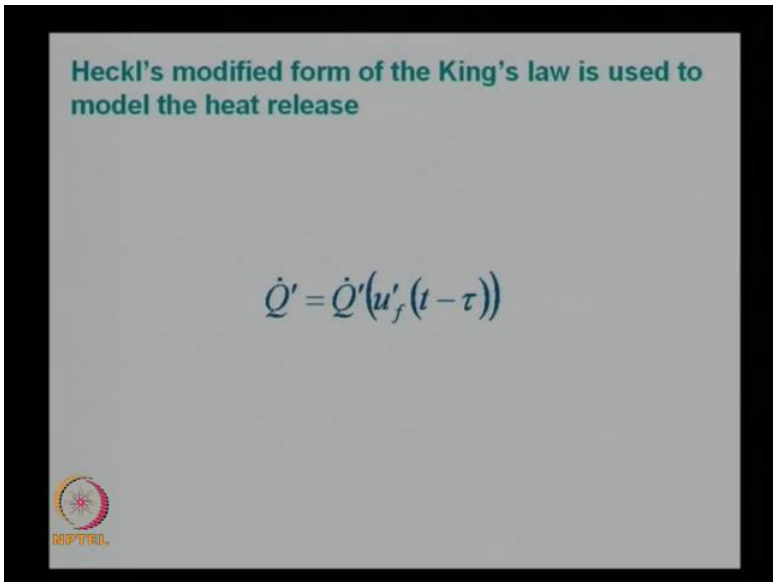
See the variation in time larger than,

Why I am studying in time domain it to see the variation in time. It will back to Anna University? Where is I I T? Said why should is study in the time. The key word is in the transient.

(( ))

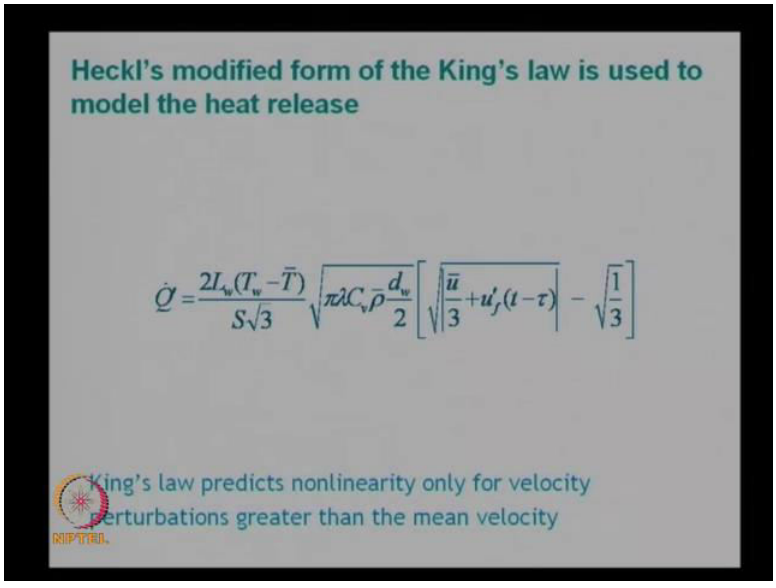
In frequency domain you can't get transient, we want transient. So, we have to do time domain. Only in time domain you can see this. Ya, where you can do Laplace standard form and you may be able to say. But, in the Fourier transform, it is non cause write this how. Well you can roundabout way do it, but, not I the way we did. So, in summary we take the momentum and energy equation, we assume a compact heat source. Going by exact same derivation that I did and why did we use a delta function here? It is compact very we had a small heater and then we write the expansion.

(Refer Slide Time: 43:53)



And the we use this Heckals model I fate the form of king's law, to model the heat release. That is heat release in a function of velocity, at a delayed time. And I mentioned that on reality, you will have to solve set of pd's for actually getting thus. But, we next stepping it make a toy problem. So, that you can; play with it.

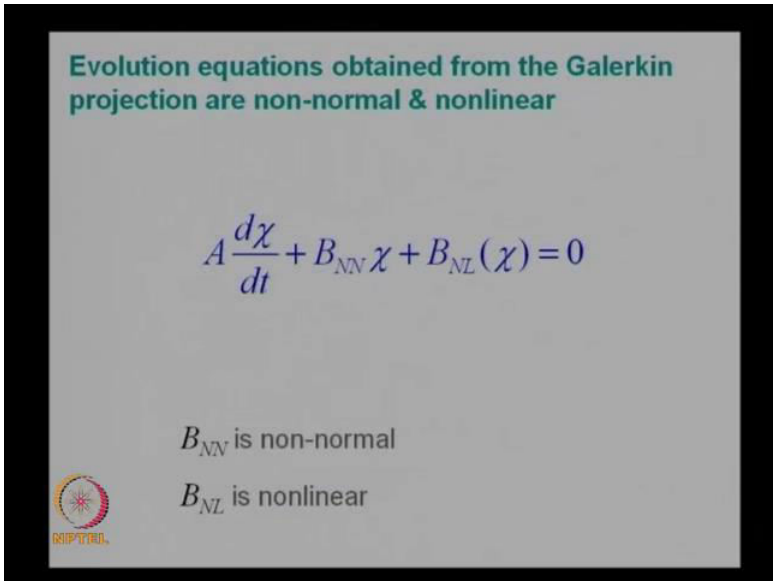
(Refer Slide Time: 44:13)



So, this is correlation and 3 is put in by hand, to match some experiment. With this make any same? You can see in this. Ya still what ask you. So, kings law predicts non-linearity only for velocity pertabration. Near or around the mean velocity, but, if use this

correlation. You will have non-linear become important that 1 3-rd itself, u prime by u bayler something.

(Refer Slide Time: 44:39)



So, you do this algebra and then ya this is the matrix you about. You get the equation of the form, talking a d k over d t plus. Operator working on the chi: plus under the non-linear function of chi equal to 0. So, if you had only linear system you will have only the first 2 terms and last one is the non-linear system. So, we get evolution equation and this is the language of dynamic system. No in the earlier types analysis people wrote everything in form of 2nd all equation. But, then you get it in this form, you use the machinery from the dynamical system straight away.

(( ))

A chi is the eta eta dot.

if you the linear question mark.

Ya,

so should have nearer I into some chi function of chi. Then only it will give non linear path.

No this used it different language that the. The whole thing represents that. Ya I had 2 terms in that, ya. So, the together I say it is a non-linear function of chi. So, in summary

so, we explained what is non normal modes. And what is mean by non normal operator. We saw that the thermo acoustic system was non normal in general, as long as there was a heat release you the system will definitely be non normal. In fact even non tribune prompty condition will make if non normal.

And then there is a possibility that, you can have subcritical system. That all the Eigen vectors are dk. But, that is resultants can grow, if they are non normal; or not 90 degree, or not orthogonal. And how will look at the consequences of this, of this calculation in the Rijke tube for other system. And then we will I explained the machineries that we can use to study this things, ok.

Thank you.