Acoustic Instabilities in Aerospace Propulsion
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Lecture - 25
Nonlinear analysis of Thermoacoustic Instability

Good morning everybody, we were trying to derive the time domain equations for a thermoacoustic instability in recative and we are trying to write it in operator matrix form, so that we can do not this numerical simulations, but just analysis of this stability with this equations. So, what I would try to do is to have separate out the linear and non linear term, because we had a delayed differential equation but then, we expanded the term u of t minus tau in terms of this Taylor series. And we kept the linear term and then, or if you have a nonlinearities then, you can have the non-linear terms also. For the small time delays, we can make the assumption that, only the first term in tau is there and then, what we did is to rewrite the equations in such a way that, we can use the dynamical systems approach.

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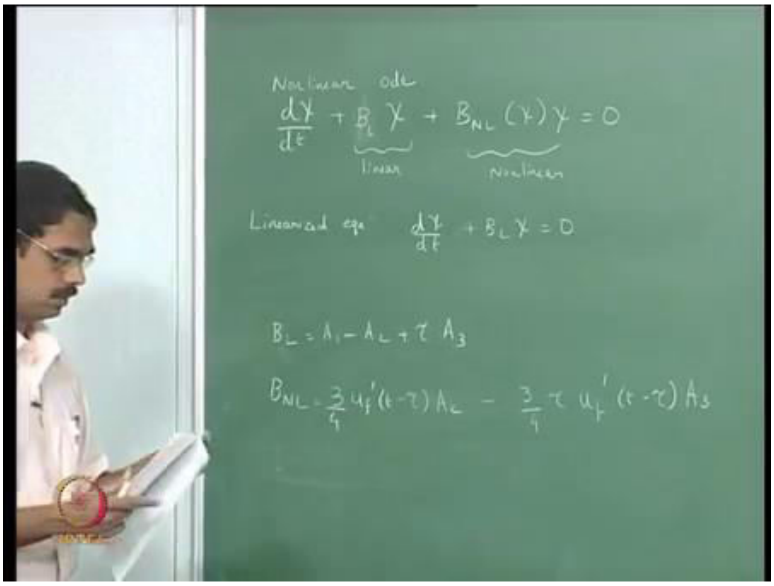

So, what we had in the end was the following, we have d d t plus L times chi plus, this one I call this at last time.

Student: B L

B L and so, this is the non-linear set of ODE and this is the linear part, this is non-linear. So, if we are linearizing the equation then, what we have is just. So, we can look at this equation, I am trying to find the non-linear stability characteristic. But then, non-linear equations are hard to solve and anything non linears much harder to solve, which is the reason why we do linearized approaches.

So, we do have the linearize system here and we can see, whether what we are all found from the earlier analysis is sufficient in the frequency domain or whether we are missing out anything, because this account for the transience as well. So, that is what we will do and I will just assemble the full matrix just for convenience and you can check it I mean, program it in a full matrix, B L we derive the expression for A1, A2, A3 last time if you remember.

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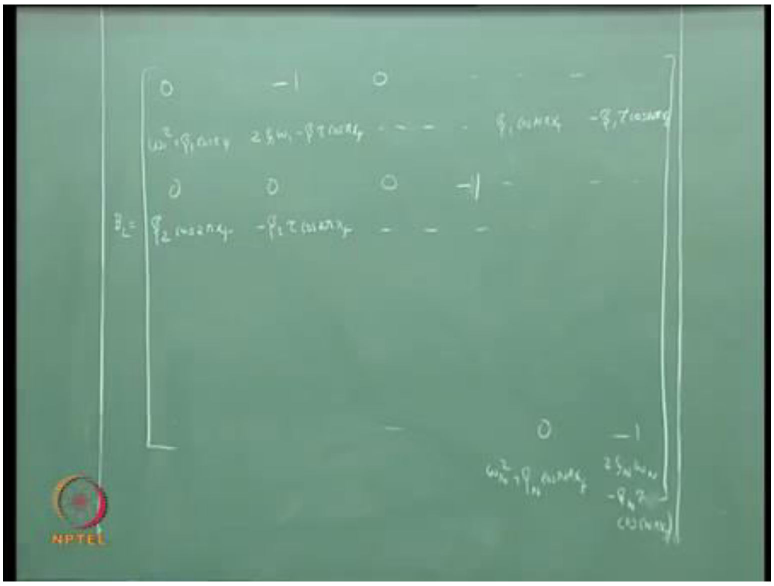

Just for the sake of the completion write this matrix, so B L is equal to 0 minus 1 0 0, this is the first term. The second term would be, let me give some space, let us erase off, I think I will not write this term, not required, space. So, the last term is minus beta 1 tau cos N pi x f and the term before is beta 1. Then, you will have 0s and you will have beta 2 cos 2 pi x f minus beta 2 tau cos. The last term would be here, so now we are able to take these...

Student: ((Refer Time: 06:18))

minus, that will be right, we can take these term and assemble the equation and it is not a big deal I mean, I just wanted to, this is what you would input your computer program when you construct the operation of the matrix what you give and then, you can do all the operation like exponential the matrix, find the Eigen value , some blah blah blah and everything can be done peacefully. Particularly, in MATLAB it is very optimized to do the matrix vector operations. So, this nothing new and it is just be writing this and we are assemble the individual terms yesterday.

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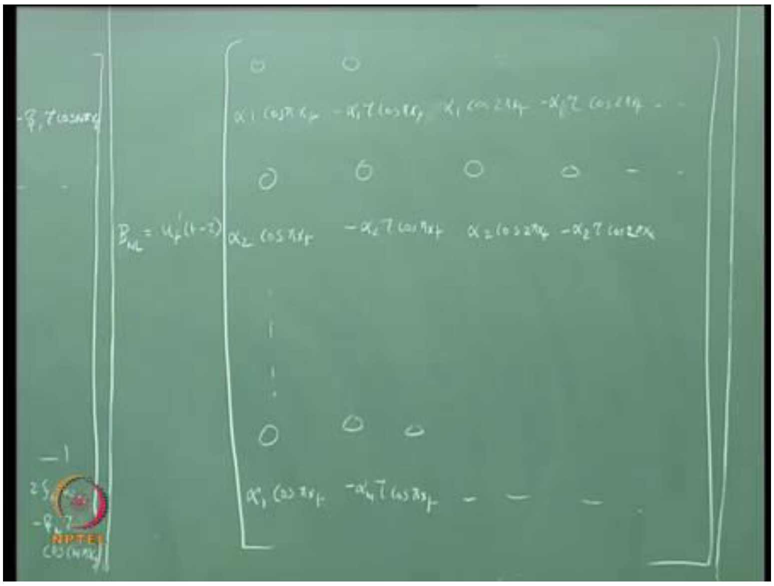

So, there is a mistake please point out, so B NL similarly will be, so I will have one rows of 0 and I will have alpha 1 cos pi x L. So, I will define alpha j is equal to minus three fourth beta j, so this for convenience write it is that is, keep on writing three fourth every time. Then, you will have 0, you will get a big operator like this. Please check one thing, there is a mistake here, that should be ((Refer Time: 09:54)) I made a mistake.

So, this is ((Refer Time: 10:26)) should be pi here and then, the next one you have 2 pi in the N end point, so this is the details, you can check this at home and let me know if I made a mistake. Now, there is one more thing I want to discuss, so this is certain equation and you can scale them anyway. I mean, you have five equations, we can always multiply the first equation by 2, second equation by 3, third equation by 0.1, fourth equation by some other number and so on.

So, you will get different chis as I mean, you can get a different set of chis, so this is not really unique, but what is the way to correctly scale the equations between themselves, what would you think?

Student: If you take normal...

We look for the norm of the system and the norm of the system should represent some kind of physical energy, so that is the idea. So, we should look for the two norm of system here perhaps, so two norm of the system will be, if chi consists of eta 1 dot eta 2 dot, you square all of them and add, that will be the norm. In fact, sometimes that never represent energy, in this case it would not represent energy. But, you will have to divide the eta dot by j pi and then, you square and add, it would actually represent the acoustic energy that means, something physical.

So, we can take anything and you can do any scaling that means, scaling what I mean is, relatively weighing different equations differently. If you remember when you are deriving the acoustic energy corollary and also when we derive the Rayleigh criteria, we actually scale the equations, we did not have energy as p prime square plus u prime square, we had u prime square multiplied by half rho bar. If you have half rho bar u prime squared and the p prime square had half p prime squared by rho bar c square or something rather.

So, together they made the, so one was kinetic energy, the term next we will next call the velocity, the term going let us say pressure was the potential energy then, together they made the acoustic energy. So, why did we do that way I mean, that was the appropriate scaling and then, only if you did that scaling, you will get the term del dot p prime u prime, you got p prime d u by d x plus u prime d p d x, together they became d by dx of p prime u prime.

And this kind of nice scaling would not be possible if one term was multiplied by something else and some other was multiplied by something else then, these terms are hanged there, it will be something like terms p prime u prime plus some other things times u prime I mean, something like terms p prime d u prime by dx plus some other number times u prime d p prime by dx, which together wont convert to the flux term like del dot p prime u.

So, we have to scale appropriately and in case of acoustic energy, it the way simple, but some other times it is like the art and some other times it may be possible to construct two norm, which is like energy, sometimes may be not. So, I think it is very deep question, I do not want to get into it, I would show you how to do it in principle, hoe to change the equation.

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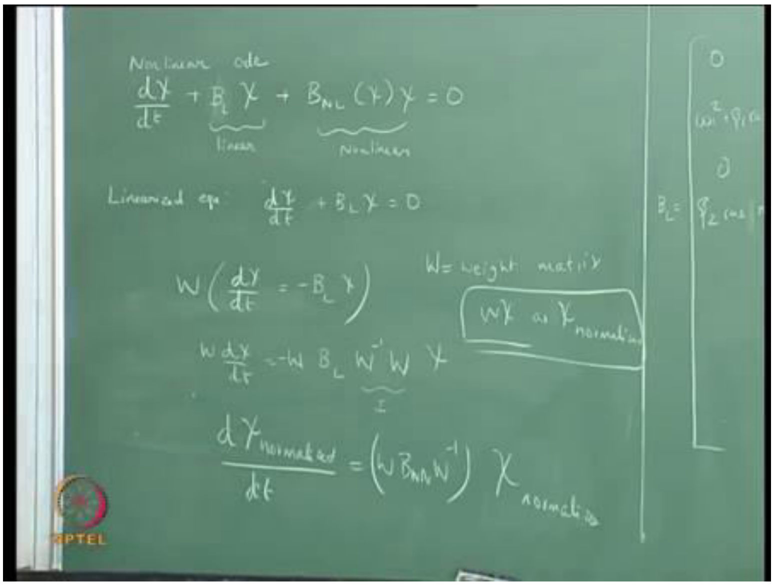

So, we have this equation, so what we can do is, let me rewrite this and I multiply this by W, so I will have W d chi by d t is equal to W B L and I will say, W inverse W. I can always multiply anything by W inverse W, because that is 1, with this together that is identity terms chi. Now, it is peaceful, W is a weight matrix, so it is terms are constants, so we can call W chi as chi normalized, that is it. So, then you will get, so this W chi ((Refer Time: 15:32)).

So, normalized is nothing but W chi, what weight used that goes like the physics of the problem and sometimes it may be obvious, sometimes it is not obvious. But, all of done is to, so if you do this and you are eligible to do this, we can use anyway. But, if the two norm of the dynamic system, if that can measure the energy then, we can use lot of machinery from linear algebra to study lot of things. So, if we aware to, I will give you simple example to be able to convert chi to something, which measures the acoustic energy.

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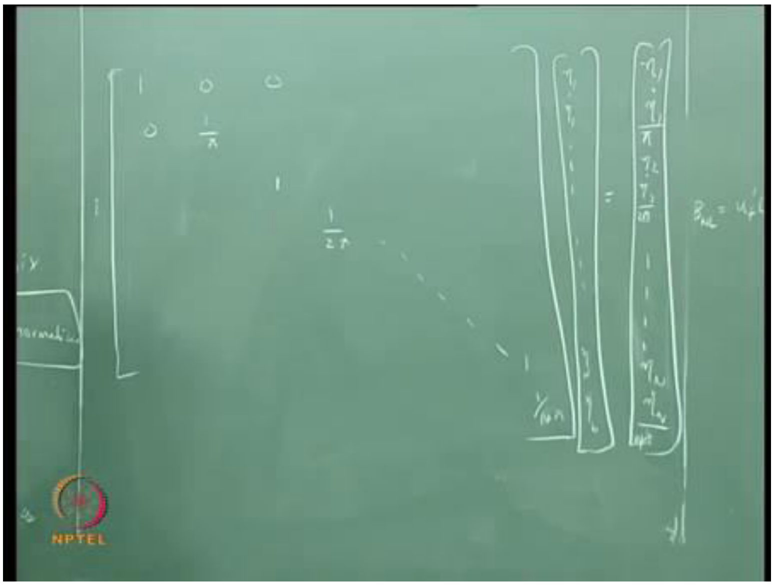

So, what I would do is, my W would be 1 0 and all 0 1 over pi and other over 1 over 2 pi 1 1 over n pi, if you take this and multiplied by eta 1 eta 1 dot and eta n and eta n dot, what I would get would be eta 1 eta 1 dot by pi eta 2 eta 2 dot over pi eta n eta n dot over pi. So, this if you ((Refer Time: 17:15)) this is 2 pi, this id n pi, so if you square all of these things and if you do one minute of algebra, you can show that, it will go like actually as acoustic energy, I will show that in detail another time.

Student: ((Refer Time: 17:28))

There is a minus.

Student: ((Refer Time: 17:33))

B here, linear and so, what really matrix is, this operator normal or non normal ((Refer Time: 17:53)), but leave that, I will use this.

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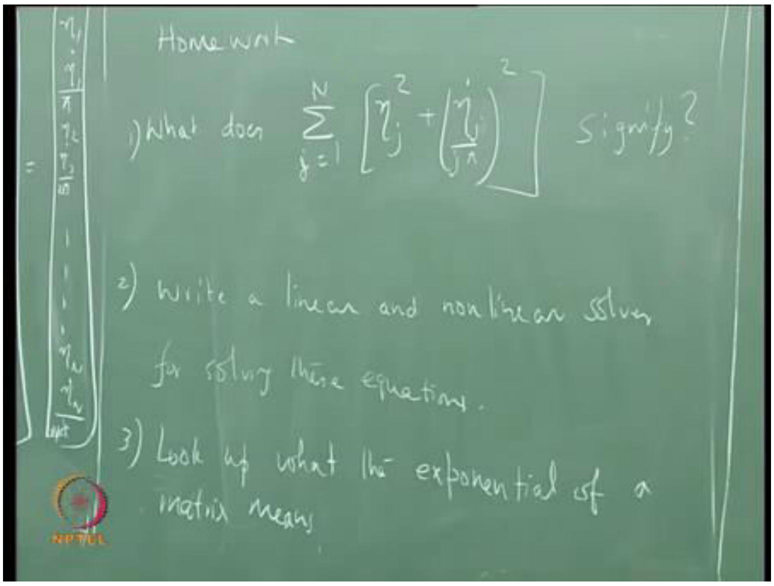

So, you try to do some algebra at home to see, what does this stands for, eta square plus eta dot squared over pi square plus eta 2 square plus eta 2 over 2 pi the whole square ((Refer Time: 18:12)), eta n square plus eta n dot squared over and pi square, what does it mean and the answer is, it is acoustic energy. So, let me write this question, I had j, so this is the first one, the second one write a linear, non linear solver for solving these equations.

So, I think by now you are upto the first assignment, you were should be after solving the standing wave numerically with Runge Kutta, all that and you should be know how to do this. We can use the fourth order Runge Kutta to integrate this equation, both linear and non-linear. But, there is some other way you can actually solve the linear equation, would you know, this equation is the another way of solving it, other than numerical integrity, where chi is a vector.

Student: ((Refer Time: 20:03))

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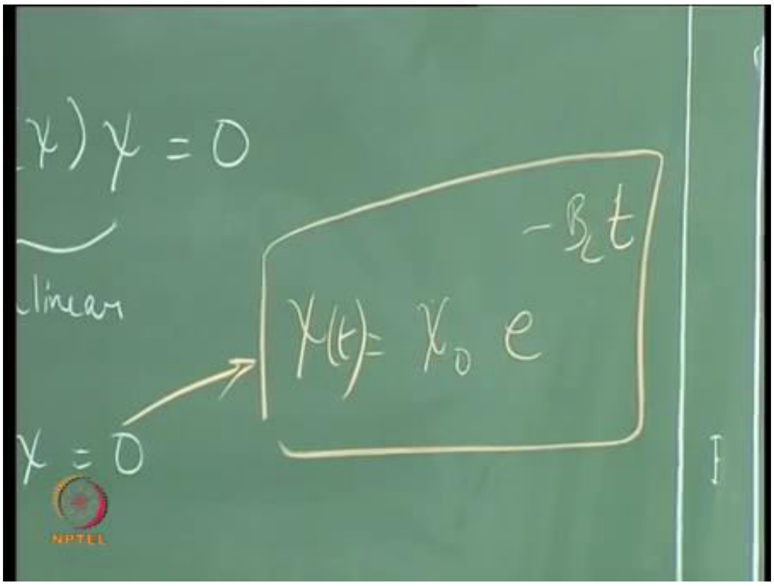

So, you can actually write the solution for this equation as, chi is equal to whatever, chi naught e power minus beta t, so this is a close consolation of that. But, what does that e power minus beta L t mean?

Student: ((Refer Time: 20:33))

So, it is an exponential of the matrix, so third question is, look up what the exponential of a matrix mean, you can look up any linear algebra book or perhaps you can see it in the book and dynamical system or you can probably see it in Wikipedia. So, to summarize, we have our dynamic system written here, so it is in the form d chi by d t plus f of chi equal to 0. And we have a linearized version and we have non linearized version and the non linear version we split into linear and non linear terms and the linear solution has a close form solution of this form based on the matrix exponential.

If you cannot find this, I will answer it in the coming subsequence classes, the last point is, you can normalize the equations by multiplying by a weight factor as shown here. And what weight you choose depend on the physics of the system and to some extent, it may be arc, coming up with, what is the energy of the system. In this case, there is a clean energy definition about an acoustic energy, but in a more complicated system, it may not be that to really obvious, we pause for a minute and answer to the questions.

Student: ((Refer Time: 22:09))

That is the next question, what do you think from that, can you just look at the expression in your book and what do you think. If beta ns were 0 and if psi's were also 0, what will happen, then it will be normal matrix. In fact, if you look at your, it will just be normal matrix without those terms, if there is no damping and if there will be beta 1, beta 2, psi 1, psi 2, they are the ones which makes the matrix normal. So, if you drop those terms, you can A transpose will be A transpose A, but this is no normal, so that I am going to answer here, that is the next set of thing.

What other question you have, to the answer is, in the presence unsteady heat release or in the presence of damping or it is at some other boundary conditions, the operator will be non normal. In the absence of unsteady heat release and with perfect boundary condition, pressure is p prime 0, u prime is 0 then, you would have a normal operator, any other question. So, le us continue, so we wish to look at the linear stability of the system and why is it so important.

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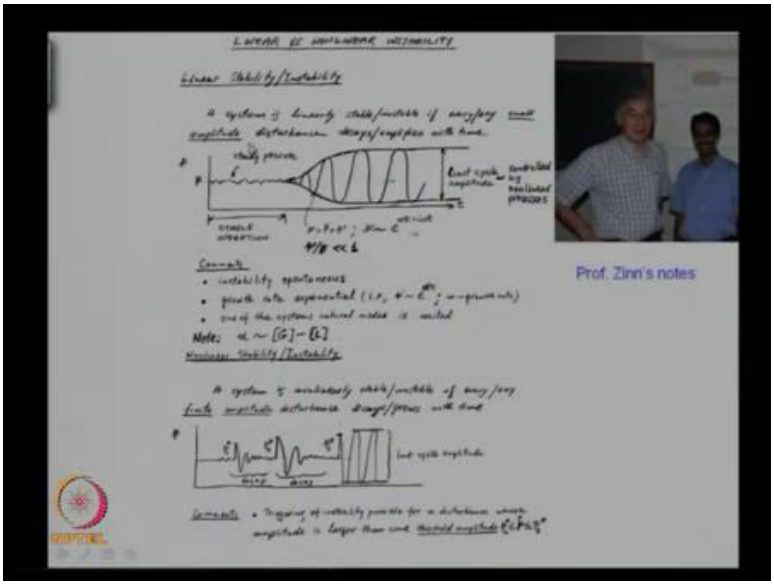

So, we see in this slide that, if you have Eigen values which are showing growth rate then, system will starts spontaneously from low amplitude and go towards instability. And you will have a exponential growth fast eventually the non-linear terms may be come into play and the amplitude may not be go indefinitely, you reach the kind of the limits cycle. But, you can also have a system which is shown here, this is ((Refer Time: 24:28)) who gave me this notes, he is the legend in this subject, he will also my advisor.

So, if the system is stable and if it will stay stable forever, but if there are no perturbations, but you perturb with the amplitude here he has shown to perturbations, this one p 1 star p 2 star you decade from the system and this p 3 star was high enough that this system went to limits cycle oscillations. So, we want to look at this thing and the issue is, whether this normal operator plays any role in this, although this triggering is non linear phenomena.

Student: ((Refer Time: 25:07))In the limit cycle oscillation, the oscillations will be of the order of the disturbing oscillation.

So, the idea is that, that was the previous idea that the threshold amplitude would be large enough, large would means, something compatible to limits of cycle oscillations. So, but that was right I do not know, that is what we are going to examine, I think that was the previous concept, but I think the recent findings of a group has a overturn this actually.

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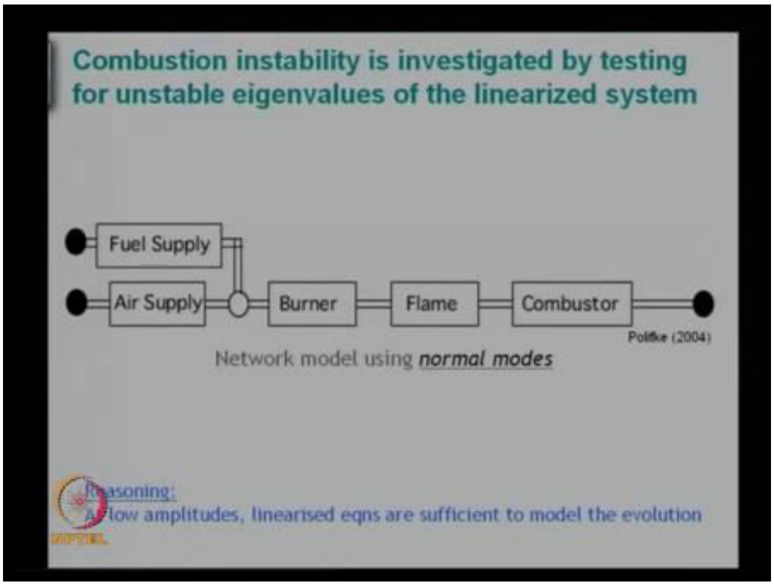

So, we saw in the earlier problem, the example given by Mecmena where we studied things based on Eigen values, so that was the very simple system, where we just had a plain and a dot. So, that was only component, but you can have more things like complicated boundary we have fuel supply, air supply, each of them have it is own impedances and burner flame combust or whatever is. But, each of the system we can write some input output equations or write differential equation for entire system and construct a model.

And this in the combust stability, the ((Refer Time: 26:08)) is called network analysis and we will do it in a similar fashion as we did that example fall off MacManus dot and look at the different types of doing it. But then, it is in principle or it is just a network model using normal modes and we look for the unstable Eigen values or we look for the Eigen values, which show growth rate or decay rate and if it is says, it is decaying, the growth rate is negative, we are happy, we say the system is stable.

And if we have a growth rate positive then, we have to worry because instability can happen and this is an approach. And the reasoning was that, we are starting from low amplitude, so linearized equations are sufficient to model the evolution. So, if you are an engineer, we taught to believe that below the amplitude is 10 percent, everything is linear, some magic, some relevant command ((Refer Time: 27:02)) somebody said this somewhere and we are believing it. And there is another crowd of people believe that, 5 percent is when everything is linear, anything more will be, but between these two categories, it includes all the engineers, we have to see whether this is right or not.

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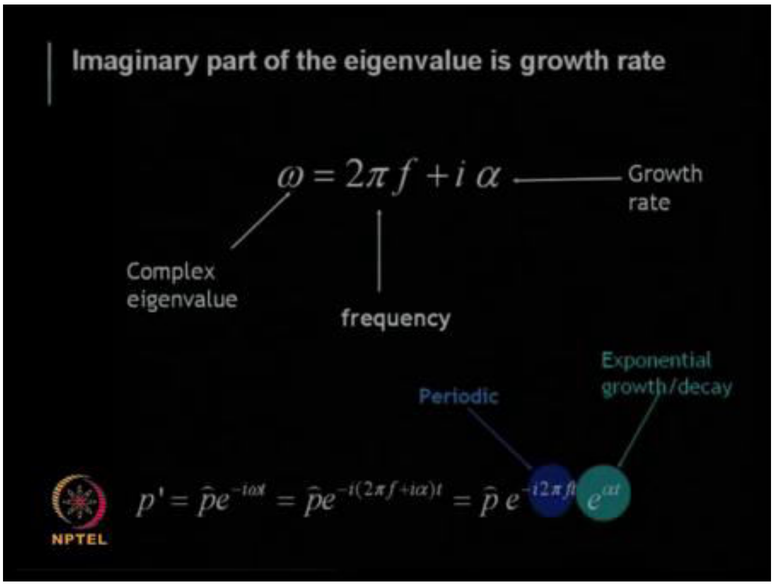

So, we have this equation omega is 2 pi f plus i alpha and this is the frequency f and alpha is a growth rate. So, if you write p prime is p hat e power i omega t, we can substitute this omega in here and we will have a periodic path and a exponential growth of decay, these are some exceptions you have. So, if you have all the alphas, we have several Eigen values, if all of the alphas are negative then, we say they are over something I mean, we say everything is stable and we can sleep in peace, is it the case that there is the issue.

Now, there are this, I think I would told about what is called triggering instability and the term predates the non linear dynamics as the subject. Because, this non linear dynamics formally becomes a subject in this 1970s, 1980s and so on, whereas this term I mean, triggering instability was coined in the 1950s perhaps. So, you had this rockets and they would be fired several times in this round on air and some would be completely fine for 10, 20 firings and may be 21 st one, it just goes unstable and then, again everything is fine.

And then, anyhow when you look at it sometimes, some little bit of propellant broke off and came out through the nozzle, momentarily creating a spike in the pressure and then, bang, the thing went unstable and something like that. And the same kind of thing was seen in the gas turbine industries, where you have combusted which are quite and find nice. But, sometimes they just come out, sometimes they do not depending on the background noise of the system.

So, we are going to look at those system and by linear stability theory fails to account for it and how you can deal with it. So, at the moment with the normal mode, I has as we look at the imaginary part of that Eigen value, call it growth rate. And if it is positive, we have to worry about it, if it is negative we sleep with peace.

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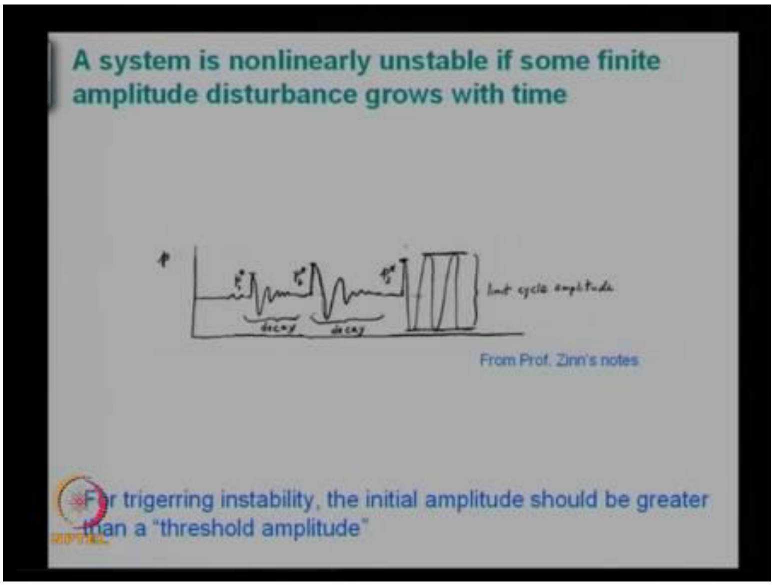

So, but we have to face this non linear instability, so system is non linear and unstable if some fired amplitude does not necessarily mean, all amplitude, but some finite amplitude. Linear stability means, any amplitude can be arbitrarily small, it was still grow, but here means some of the amplitude with some of the disturbances it can grow, it need not grow with all disturbances. And the general idea was that, for triggering instability, the instability amplitude should be greater than a threshold amplitude.

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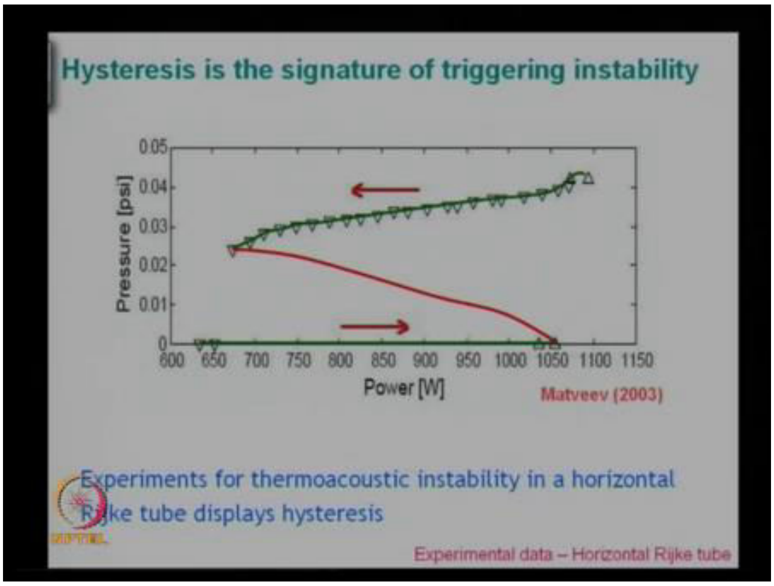

So, this is a data which I have shown from Matveev thesis, which have been produced from Matveev thesis. So, here we are having a Rijke tube like the horizontal Rijke tube which are showed that, all are Rijke tube, but not we have similar Rijke tube and we increase the power given to the heater. So, we are measuring the input power and you come to 650 Watts till then, everything is quite and fine and then, you increase the next power setting, it will follow the green line.

Upto 10, 50 Watts, it is quite, you are quite here and then, at 10, 50 Watts if you increase the power above that, you go to instability here and then, you keep on staying stability, but you cannot heat up too much, because the wire mesh will burned eventually if this is too much heat till ((Refer Time: 30:44)). And then, you want to come back, so you would think that, if you come to 10, 50 Watts, it would come back, no it does not, you have come back all the way.

And only when you come to 650 Watts, you are coming down, like if you mess up, do some mess things up, if you stop doing it, you would not clean up, we have to undo lot of damage, so that is what it is. So, this is like the forward path and this is the return path, so this is called a hysteresis and if you stick to people working in non linear dynamics, this hysteresis is signature of sub critical bifurcations. Then, in such system, the triggering instability can happen, so there is some kind of threshold line here, again what is a line and what is a threshold, we will debate later on.

But, in this region, heated with small disturbances, they will died out, but in this region, they will grow and the amount of disturbance depends on, where you are in this region. So, we can see that, there is a big sub critical zone with 40 percent of the power at the half point and there it is possible that, you can trigger this system to instability. I give the example of a family at 8' O clock, man and woman going to work and children are going to school and you can think about this as 7:30 and this is 8:00.

If you plot time versus disturbance and beyond 8:00 you are unstable, because they are staying together at home. And in this region, so let us have we are far away, that 7' O clock if the kid spills coffee on father shirt or mom shirt, father will go and take another shirt and there will be no serious problems. And as you cross 7:30 AM and let us say that, some critical time then, if the kid make some break something big then, there will be disturbance.

But, if it breaks some small thing, through away eraser or cannot find something, also spills little bit coffee, it will die down, but this will make a big disturbance like tare away dads paper or something. And then, you go to instability, dad gets upset and mom checks everything and go. But, as you come closer towards 8' O clock, when everybody suppose to leave and even a small disturbances can actually make ((Refer Time: 33:08)). At 7:58, you spill coffee on dad shirt and he will get angry and kid will start crying.

And once the kid starts crying, mom will get upset and school bus is there, the school bus is waiting, the school bus horns, but the kid is ((Refer Time: 33:22)) by then, you pacify the kid and bring out to the school bus, the school bus was left. Now, we have to drive to the school bus and drop them, so this is the hysteresis zone, where all these things can happen. So, this is like a real life analogy of that, it is not I want to go analogy, but I hope you get the spirit of it. That is, I mean, the same action under some other circumstances like 7' O clock or 6'O clock would not create any problem, but close to 8'O clock when everybody has to go, can create a problem.

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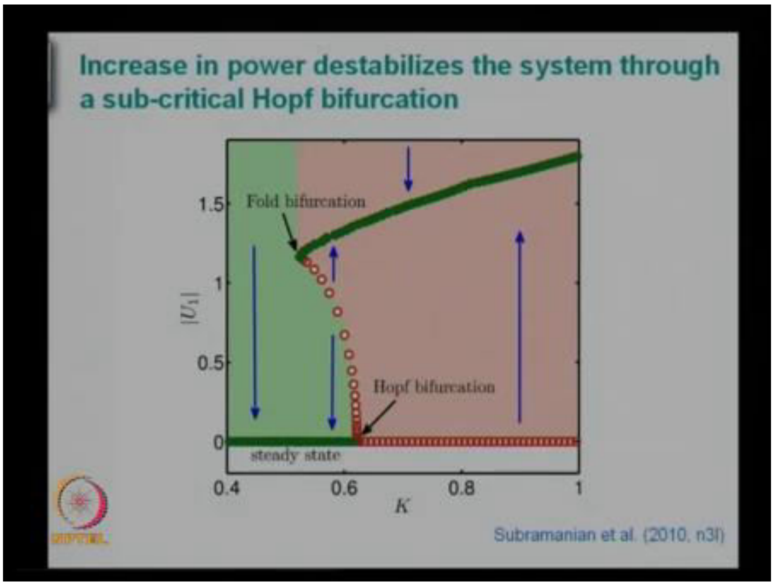

So, if this is calculated one, the earlier one was the experimental one, so if you increase the heat about this case, some kind of non dimensionalized. So, let us not worry about this, but it represents the heat released, the heat of power, if you increase the heat of power, you come till here and then, you will go to instability here. And of course, theoretically you can increase the power as much as you want, does not worry about heat of burning and so on.

Whereas, once when you do the experiment, you have to see if the heat will burn off and so on. When you come back and you come here and only then, it will come back here, you can come back where you took off, we have to actually undo some more damage ((Refer Time: 34:34)). Let it come back some more and only then, you reach a steady state. So, in the language of dynamical system people, this point where, so this is the loss of linear stability

So, our linear stability will predict this, because any solution is, so there are active solutions here, but they are unstable. You can have steady solution, but it does exist, because beyond this heat of power, any small disturbance will become unstable. So, this half point is called the, is also can be thought of the loss of linear stability and this point is called fold point. I will teach a little bit specifically about bifurcation later, but at the moment, we can identify two points.

So, this is the linear stability, loss of linear stability and this is the place, where you below this is always stable, here there is a region where in between this two powers, it can be stable, unstable. So that means, there is some kind of base and boundary, which separates these two solution, that is a limit cycle solution and this fix point, which is fix point means, everything is quite or equilibrium point solution and we have some other solution, which is high amplitude limit cycle.

So, there is a base and boundary which separates this solution from that solution and of course, basic boundaries are surface, because many surface in the direction of this, we have several dimensions for the system. So, a limit cycle will lie on this, you should be able to find the limit cycle on this base and boundary, but typically it will be unstable limit cycle. That means, in principle solution exists, but you cannot find it, the solution wants the chaser, because any small disturbance will perturb that, so there was actually unstable limit cycle lying there.

So, this is a very brief way of, brief introduction to the dynamical system way of representing it and I must emphasis that, these are non linear phenomena limit cycle and triggering instability and so on and so forth, other any questions for the things as far.

Student: ((Refer Time: 36:52)) can we have some active controls, which can use and which can avoid these instabilities.

That is a tricky question, so you have a system then, do anything say, you cannot do anything controlled, because control means, some measurements and then, only you can control. But then, they are controllers which come on and when some instability happens and if transient growth happens I mean, if this growth happens and so on, the controllers which can take care of it. Like I think, LQG controllers should able to work there.

Student: ((Refer Time: 37:35))

No, I have this is, this multiplying by W, but I am hoping that, the two norm of this vector will go like acoustic energy or like some kind of energy of that particular system, that is all and nothing more.

Student: But, mathematically what is the significance of that ((Refer Time: 38:10)).

I mean, physically there is no, because I am not altering anything.

Student: But, you are changing the, we can think of this is changing our bases for the x vector.

Exactly, so you are having not the bases, you changing coefficients of the bases, you are changing the units along this thing or something. So, you find the energy as like a length of the factor, we are not changing the bases, we still uses the same bases, it is like we are stretching some coordinates or shrinking some coordinates by way.

Student: ((Refer Time: 38:56))

Not necessary, you can have any W or anything I mean, any W with, I can use any W when I can change this equation to this equation, they then depend on it is diagonal.

Student: But, the only changing the length, that will happen only if W is diagonal. If it is not diagonal then, the whole vector base will change and we will get linear combination of those two ((Refer Time: 39:26)).

Now, you are in this, yes we can change the, you are having a linear combination, we have different direction for it. We still use in our original thing, sin and cos all that, but then it just like transforming the coordinates, you constructing new basis function as combination of those function. And then, you are still measuring the length with that and we are weighing those directions also differently, that is a possibility. So, we can amplify, you can use the same bases function and use which is what you doing here.

But, just multiplying or shrinking or expanding, but you can also use combination and find new directions, both are ok. So, that is mathematically physically we are trying to see if you can get, physically the objective is, in maths we are just changing the distances and perhaps the direction of the bases function. The objective is to make sure that, the two norms, two norms means, just squaring everything, would represent the physical energy.

Like we have the acoustic energy as a two norm when you write d k by d t of p prime u prime, but weight by some factors, you will get acoustic energy. So, some kind of it, because to measure growth in it, a measure kind of previous state, but it is a deep statement, you have to measure growth of energy that is what you are looking for. When we study any instability, we are looking at the, see we cannot track all the in vectors, there are may be in a real system 100 bases functions, 100 Eigen vectors or in a more realistic situation, 10000 or 10 million and so on.

We cannot really look at each of them, but we need a measure, why do we need a measure, to measure something, to measure the growth or decay, it is like you have salary money coming from stocks, here coming from rent, here money coming from lottery whatever, a lot of things. And the income tax department is caring about the total income and they will measure the total income and put you some tax, whichever the way you get it.

And so, it some lump, because they do not want to keep the track of individual things and then, collecting individually five cheques, they want one cheque from the ((Refer Time: 41:52)). So, we like to, it like why do you need GP, you can have a credit card then, it does grades, it has 50 subjects you have done over or will be more and has a great associate with each. Now, human being do not like this idea, he always want to tag somebody good, bad, ugly, something like that.

So, we say we will weigh, we have weighing functions there also, so we can just average or take the RMS value of all the grades, but four grade courses, three grade courses, lab courses, one has 4, another has 3, another has 2 and so on and then, you weigh. Because, they think that, if you weigh this way, physically it represent how good the student is, overall performance, something like that, that is it or it is weighing the efforts that is going into the courses, something like that.

So, there is some bases and how good your bases is and all that and many the measure will be wrong, because just as I cannot distill you into a number. You have lot of attributes and the moment I say, we have GP of 9 or 10 or 8, I am really losing lot of information, because I had other ways 50 grade look at and, but we would know, we will just have one number. It was more important thing that, many things we may know, which is none of this 50 grades will measure.

You may be brilliant in something, you may be very creative you may be able to do new things, you may be very ballet study whole things, but you may be discovering the new theorems and all, making new hardware and engines and so on. But, this GPA does not measure any of that, so then in that sense, if wanted to worry about those things also, this norm is really not a norm, it does not account for everything, so it is really a semi norm or it accounts only for partially.

So, it is important to construct, we speak about it later on, the context to thermoacoustic not in the conduct to GPS, but it does not measure your expertise in Tamil or English, which may be important for your job. But, your GP does not measured, GPS is measured how good you are memorizing and answering or something. So, the critically the norm, so the bi norm is actually distills everything into one numbers, that is because it is our philosophy.

I want to call you ugly or beautiful or pretty or smart, intelligent and clever and then, I do not have to worry about it, otherwise if you have a lot of attributes and lot of color to you, but I cannot keep track of it, that is not because of mathematics or physics, the human mind. So, with this, I call you beautiful or ugly or whatever, so I have a norm, GP of 9, 8, 3, whatever and the success of the norm or success of a choice of norm depends on, whether your measure or the norm can measure what you want to measure.

So, in some sense, what you have is what you get and if you have a wrong choice then, you are doomed. So, the bases of what is the significance of doing this is, what you want and what you want is, what you get. It sounds outwardly very shallow or simple statement, but actually very deep statement. I hope you can appreciate it if not think about it, now any other question, thanks it is a very beautiful question.

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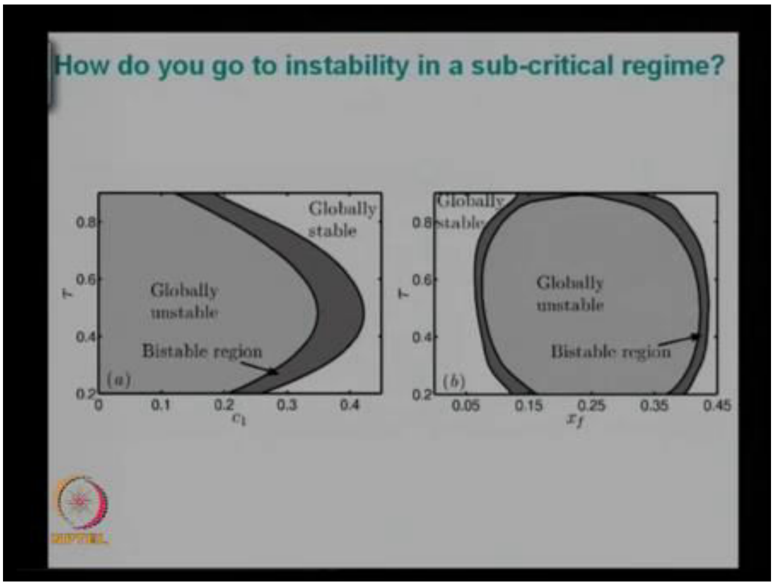

So, if you go back to the plot, so we saw those region in this figure, there was a region which the green is stable and the pink is unstable. And the green region below this point to left of here, you will always be stable, in this region it is like potential instability, everything can be quite on a nice day and another day everything will go wrong and you can be here. And here onwards, everything will wrong on anyway, you know some days are also like here, that day what would you go know, some other day whatever you talk will go good and some other day could be this way or could be that way.

So, it is same here, you have globally unstable that means, anything you touch you will break that day and globally stable, whatever you do, you can go and call the professor as idiot and he would smile at you, that is a deep statement you will say. And another day whatever you do, he will be annoyed and he gave ((Refer Time: 46:33)) and in between, there is a region of, it is called bistable region. I would call from a point of view of life as potentially unstable, things can go wrong and it may go wrong, but need not you can survive also.

So, this is actually variation with damping and time delay for Rijke tube system and here, I plotted with... So, that is not really life, that is Rijke tube calculation, this is time delay and location of the heater, but the some acceptance what I am saying is, you have a region which is globally stable. That means, whatever you do, you can burst a bomb and still be stable, as long as do not change the base flow, that is we have some constraint like that.

And the some region where, whatever any infinitesimal disturbance would block, that will be globally unstable and the bistable region, which you have, if you have right disturbance, you can mess it up, you can go to an instability. But, right or wrong depend on the if I want to be stable or unstable, if you are theorization you may like the instability, but if you are engineer, you do not like it. So, if you have some other disturbances, disturbance may died out, so that this three regions, I hope this clear. And if you have class after this, those of you can leave no problem, so the issue is, how do you go to instability in a subcritical region, that is a question.

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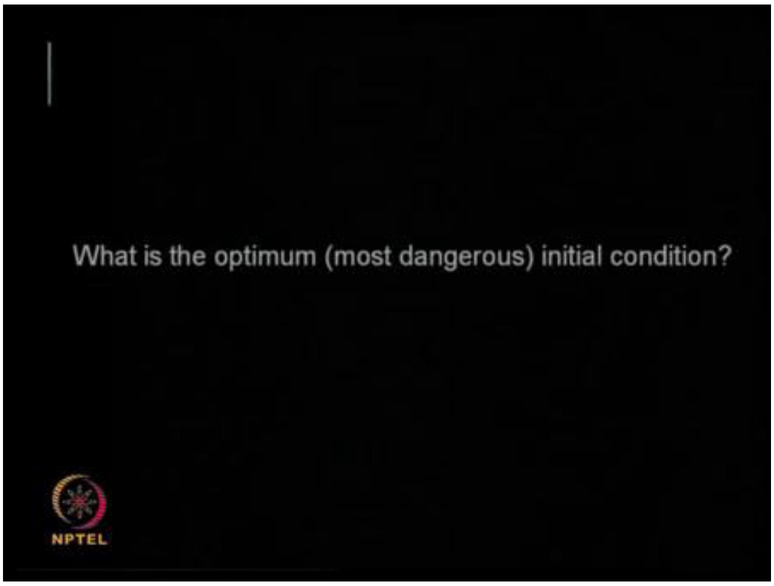

And what is the optimum or most dangerous initial condition, so if you have, there may be some directions, some particular disturbances, it may be low, but just get over to instability, and may be some other direction that means, as you mentioned here combination of basic function, whatever. So, some other direction along with, which you need a really amplitude disturbance to get to instability.

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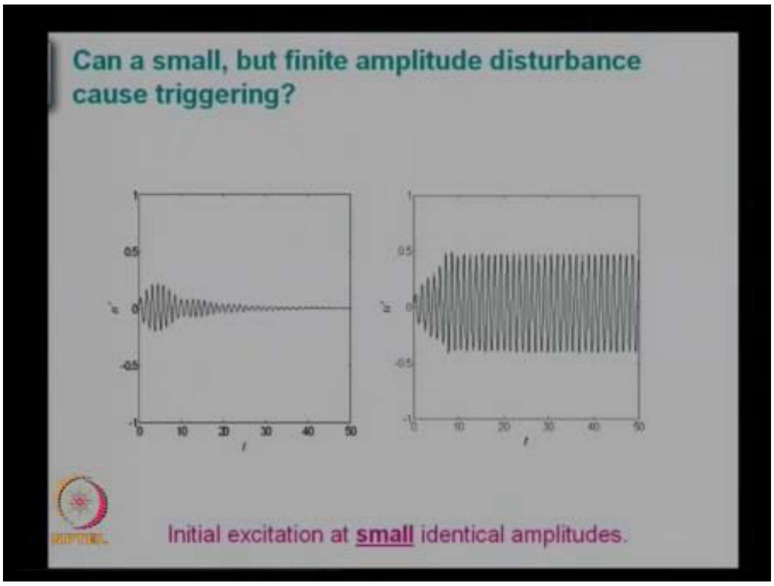

So, can a small, but finite amplitude disturbance cause triggering, so infinitesimally small amplitude, infinitesimally small means, you have small value, you can still find a small value even that should cause instability and even smaller, even smaller, even smaller, anything should want. But, now you are looking at the case, where can small but finite amplitude disturbance cause triggering, So, this simulation is one of the first calculation we did, so we start from small disturbance and I was thinking that account to linear theory, it should died out, it did not died out, it grow and then, and decayed.

Now, we increase the amplitude and disturbance little bit and then, it went to a limit cycle. So, we exciting at almost identical amplitudes, but one actually did some strains thing, it did the growth and decay, I was surprise that a linearly stable system can grow and then, decay. And then, another one which I can relevant to limit cycle, so the issue is, can a small but finite amplitude disturbance cause triggering.

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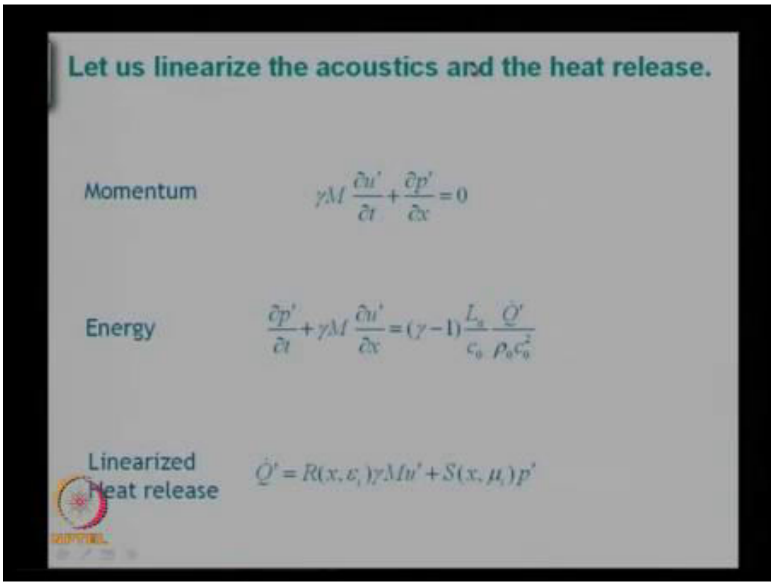

And what we will do in next class is, we will take a look at the equations and we look at the nature of the operator and more details, whether it is normal or non normal. And we will then, we will worry about, what is the consequence of the operator being normal and non normal and so on, so I will stop here.