Acoustic Instabilities in Aerospace Propulsion

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Lecture No. - 24
Evalution Equation for Thermoacoustics

Good morning everybody we will looking at the solving the partial differential equation for the acoustic field. And now, classical equation for a we have store them, which is a heat related which really depend by some getting what did have a heat source and we at least partial differential and did the mode expression for the introgalic technique. And then we derived the alternate for equation for this.

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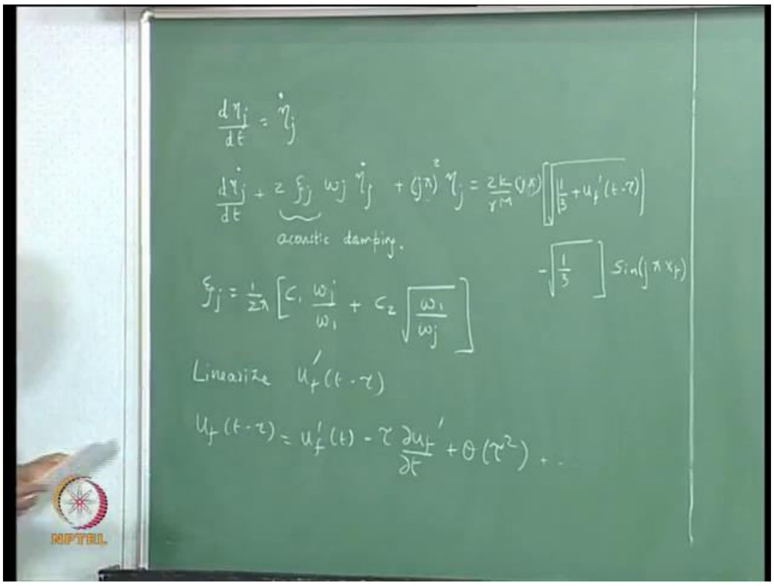

So, what we have this two equations d theta j o d equal to theta j data and this second equation this coming a g equation a for is not for d theta j dot and just a move for this is some this so, does the question as to what is the damping term that is this term that a used. So, which presents this will be dumping now, in speak of little bit of dumping modeling dumping very complicated thing and but, I just want to right issue the some of you may this one industry appreciated. So, in reality dumping is a very complicated thing; see acoustic waves actually, we have get converted to see where want you get to distributed this will be boundary layer

So, it just like acoustic distribution that by the, it is the mote that can be dissipate. So it just be boundary layer and in the boundary layer this and some are in valuate calculate and this also we have losses. So, we have radiation losses from in the open pipe heated for the you can have radiation losses going on other wise here any sound and if you have a in reality.

Now, more how thing values you get some amount you will get what the wall it is a vibrate and take energy so, this is the volume losses and depending what happens you there this is a suffix what you have all lot of damage paper dump must more than draw if you here must more than dump and so on. Now, that is also one more complication if you having a combustion experiment.

So, you have a steady same shape with alter in and you have machine damping it with same shape temperature with on everything would be comparison would different instrument will happens. Because same system would I change and so on happening would be value itself changing during it be during the comparison would and it is not it not this only way. How to find out the for they know moment radiation losses more order condition will be well so, at the moment and not going to getting this although reality is moment for further.

So, in a moment not going to this although reality is very important for the; because this step come before not depends on how much drive have some machine. it is like another you have money to spend weather you field depends not just on how much money you get but, how much money is spend; how was a like a said armature declared bank up here ninety three crores within him having you think own like me on very rich.

So, it depends on what are coming this is an going out so, the amount on dumping this system is critical because you have a instability the small drive dumping to very small. But, you have say amount of driving, the dumping is very use in just have instability so, we are this is a big topic and I get a large of amount need to done but, does not I am going to get into this but, I am have a simple model.

This model come from fluid a metric write the reference also and so, what they model is? So, omega j is wave number of the frequency of the j mode. Now, will not demonstrate sense all the way and j correspond to the same j-th mode, yesterday written m but, we replace n with j just sequence that the notation which have here and c 1 and c 2 are just constant which you get experiment of experiment c 1 and c 2 based as lot of people experiment and then we put value this in here.

So, this is kind of a adverb way of treating dampening and but, what to be derivation it is felly type. This term we have some extent putting in although have basis put in hope gradual with this the first mode ya fundamental with n gain values are for the natural mode in which we first order again the question has to at like a very natural different mode it is own. But, this is like we know the mode and based put on it in a dumping. So, dumping depends critically on the boundary layer dumping depends on discuss heat and condition on the so on here. But, every empathically obtained and put into the c 1 and c 2 and this you can see that this term this is you have high frequencies, this term it conclusion will be high.

So, which is that they model it, which has correct because higher modes have then dumping typically moves a high gradient in term comes to heat condition discuss; which has omega has the wave number is also have so, I just leave they want to go into greater detail but, this topic wearing into p h d teaches on. But, this only I want to speak about not lot of there are, people are measured dumping with in pipe and so on. And again they losses can be non linier for example, if you have radiation from that end.

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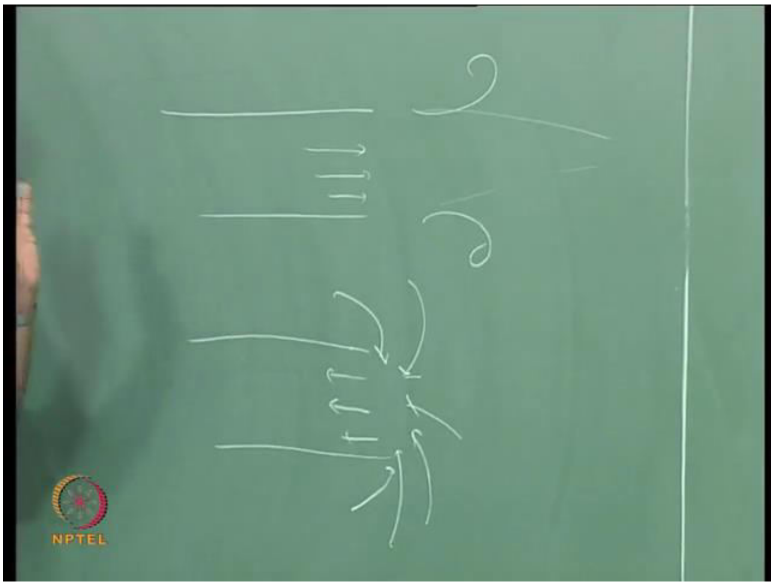

So, when you look at a pipe and mend flow is coming this way, what happened? How will be the flow out side; will be like a z mixture it is parties and when you go. We are cycling this way so, you have a unsymmetrical so, this is introduce non linearity boundary, boundary condition and. and so on. So, you can have a losses can have non linearity people are study musical instrument, have model this losses due to kind of over shedding and so on so forth. So the, I mean a lot a complication of associate this term (Refer Slide Time: 00:41) and this not linier but, we will just leave here any other question?

So, this like a breath in we are alive; because on this symmetric all border alive in the 3, 4 minutes. Because we will be if this was symmetric we will breath out, what you breathe there. And breather what will be the, which is eventually received on block will right so, thank you have on un symmetric here alive; when you heat it would when we close a dealing with equation that is, could they have you will complication then have a simple equation did have a bidder what are you talking about? You having a then solver which have a component d n with the everything then, you have a p prime will include everything; with converting to the monastic dimension.

If you having a n source you have a complex, complex you run so, the moment is a compressible you have a acoustic you have as well in built way but, then acoustic then a next that the issue and it is have a separate mode. If you do some analysis and so, in the waste separator going to but, the other thing is to do a full compressible analysis of real compositor is to separable do with a not so easy. People on do that actually represented have comparison in like to couple there some way to the other. And develop issues associated with this because what do the right equation use and so on. So, it is a form so, it is here is do that make something because more difficult to them, if you can solve everything have numerically it will have ready. But, if you remember I think but, 50 years later 20 years later things the way that will be the done.

At the moment you can do that for hydrodynamics you have a compositor but, the moves the instability. Because the acousting length care are border of meter and have length care of where the flame show the order of centimeter and reaction zone may happen over millimeter. So, the several lines care and therefore, it makes calculation way difficult. Next discuss time skill so, it is not to real just take a solve and we will get the answer for instability frequent, you do not have it. So, why are having wave equation? Because we do not have a anything else and we deal with simple way.

So, that solve we are trying to make think simple through away nothing else exactly; we are saying that is the say, we have do not in that is the say we have do not. In this particular example, this place to have dynamic thin way it sustain can be isn't to the, heat release right and then you go on to that place. But, I am not solving measure but, strictly in fall for the dynamic portion and after equation heat release and then you have to during the salvation and coupling hope it is some are clear.

But, you are asking difficult way this even if you can again principle what do is practice at this moment way difficult I do not know how many years more take that is it. We have understand where to and this not one solution; get it is time own to you first it is do it why range parameters. Because you have a instable somewhere, do not have to be very expensive calculation here nothing has proceed. So, I will male you this frequencies few all things next is linear to time delay so, u is of course, based on here more chances are expression to for this. So, it is written to be right but, it is straight on to do this kind of algebra. Now, you do not do this, actually it does not linier equation and sub equation for solve d n equation. But, I will not of does not ask.

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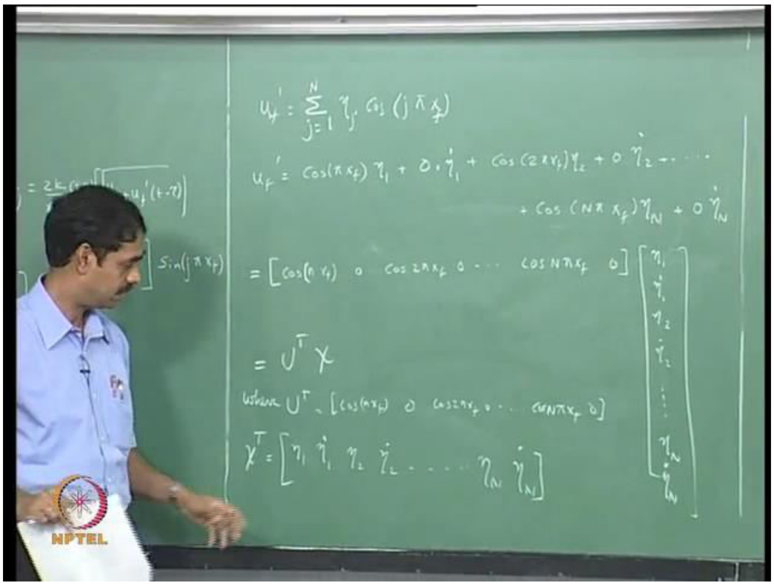

So, we will go back u t equal to sigma j n theta j cos phi x, you can use j or n or k, we were taken one time. I will try to I think wrong it to, you may one going to around about thing but, you will thing shift of a just we paste it on just follows. So, I will expand write to this cos phi x f and theta i plus does not know theta 3 row times cos 2 phi x f theta 2 plus 3 row times theta 2 dot plus 2 theta cos n phi x f times theta n plus 3 row times theta n dot. So, I have written long length.

So, this I can write metric form as cos phi x although it quite invert to write this thing on the board. Become do mat lab program thinking are quite simple, because all this can be done by metric as simple do groups and so on. Mat lab base very suited for quitting vector and metrics find the matrixes way vectors term multiply and very optimize thereby I will write everything long length for that clear so, it is clear pasperiment. So, this will write as j transpose x where j transpose cos n phi x y and x t so, it quite transpose this is theta 1 dot theta 2 dot and kind.

This would be, column theta and u transpose there u would be if u have transpose and u can everything write in a same row I think that the reason why they him wonted this rotation. They should multiply you transpose sky and call this is you and call this is sky you can see this will give this and so, this is a sleep notation that is all and so, we need one more thing for the second term.

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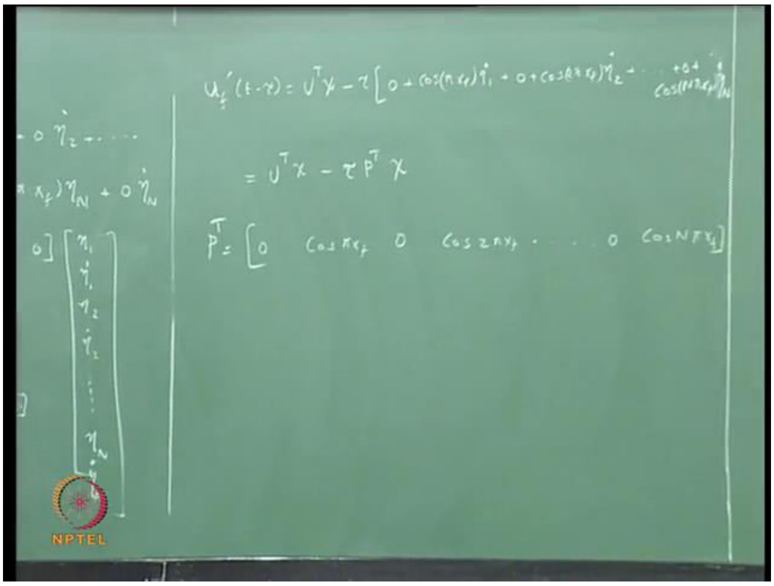

So, they have to expand this so, what I will do is to follow the same step see this bit term. Now, became the step the small symbol so, we can big mantises but, they symbolically way small and even constructing big mantises us the involved in writing a paper. But, on computer on it is via personal computers see. When I differentiates this u (Refer Slide Time: 15:27) if I differentiates this, the first term will time the first time will be it a 1 dot time scotch by axis. The next time will be it a 2 dot time pi axis, there is no it a 1 time or it a 2 time. Because when we differentiate this term, is stay not coming here and this term will go away. So, there is 1 it a 1 or it a 1 time there is only dot terms.

So, if you say that we can call this in a same manner here. We would a 0 0 any dynamical system. We do in time done you will take the support so, done after even after study some complicated vibration problems structured problems or some problems in mage into head and make in term. I thing you will follow the same, if you want to converted into definable solution. Reform d k b d d d f f sky and so, it just looks like several pages and in the ended when the dark struggles down.

We are, really neat and trim that you are wonder have write so, many peace its good thing. Now, our next term is to deal with this big thing. We have dean with the transpose. Now, we are after assemble this thing as a matrices and I have must keep n number of times. It just the assembling the procedure with looks difficult but, actual assembling in the computer is trillion.

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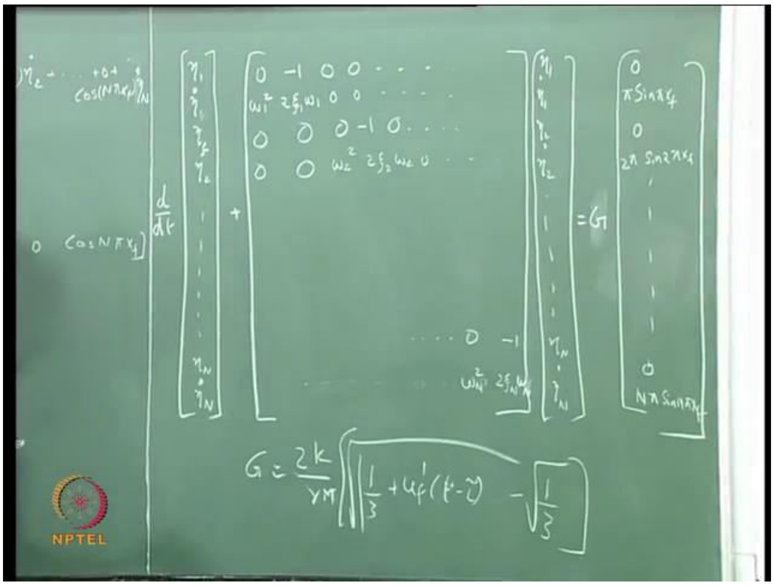

So, let me write the, because you do not know with hand in the computer; you can assemble in this way or you can put all the it an first or all the dot. Next that is all up to you some lie assemble. So, if you go back and look at this equations (Refer Slide Time: 00:41) de it a dot by do it equal dot or minus it a dot is equal 0. So, that equation will assemble so, what we need is some kind of matrices are times it a 1 it a 1 dot it a 2 it a 2 dot it the and it a and dot.

Now, it is clear what is the if that equation wish going to do fast it a one dot equal to d it a 1 d it equal to 1 dot so, what happened the questions? What would be the first one? Let me write that equation here we write d it a 1 d t minus it a 1 oh equal to 0 so, this s the first equation. Yeah, yeah will be 0 minus 1 and everything as will be 0 excellent can you look at the next equation and say what would be the equations? So, you will have this j 5 which i will call mega j.

So, that would be multiplied it a 1 and this term will be multiply it a 1 dot and so, you will have the mega one square mega one 0 0 and then comes 0 0 0 minus one and here would be 0 0 mega 2 square 2 do it a mega 2 0 and here last one will have lot of 0 0 minus 1 and this one would have again call 0 and m square and 2 square n mega n.

Yeah this so it must be peaceful assemble the left hand side. So, if you look at the right hand side 1st term 1st equation 0 on the right hand side. Then d it a j by d t minus into 1 dot equal to 0 so, we can put 0 the 2nd one we will have is 2 k j if I over times as fine p i p i axis. The 3rd equation will have again 0 the 4-th equation will have again is time fine 2 pi axis its clear right. So, I will call this posting as some kind of as for some. So, this would be like o let me write it off. No space as some a symbol can you tell me which we have not used. I want you will all for later season some symbol, I will say g times 0 five. and five x f 0 2 five sign 2 five x 0 n five x f let see. So, this g would be 2 k for gama m times this square root of 1 by 3 square of p minus tau minus 1 by 3.

So, we have put it in some metrics form. As some esterase thought this full thing in one day. So, I guess nobody understood the answer. So, they have not asking anything. So, we said that we want to do a linearized analysis. So, we want to linearization and how would you we will have a non-linear term yes square roots. So what would we do? Binomial expansion, we will do a binomial expansion but, we have to make sure that your prime is small. So that we do not know an individual form.

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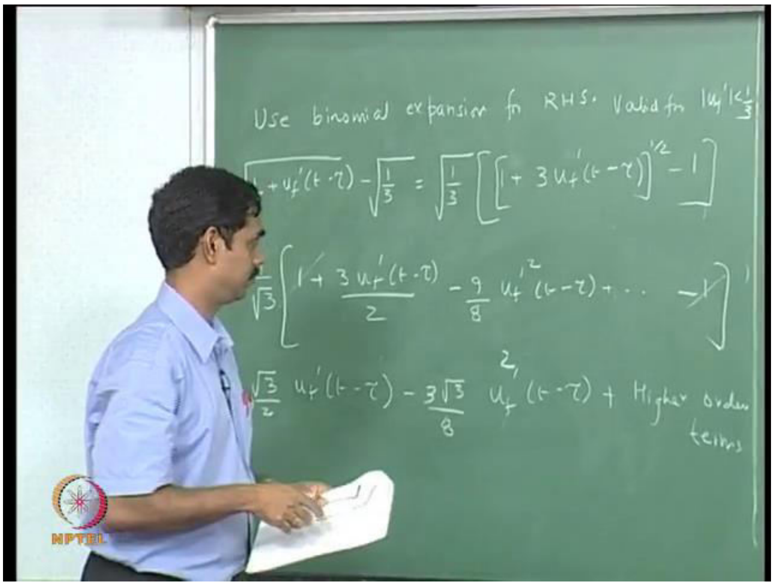

Let see, expansion for right hand side valid for you have a be little of animator. So, we expand the terms, we get 1 plus 3 u at 1 bar t minus tau divided by 2. And next I am would be what did expand? Now, 1 plus explorer. So we will be s next timely minus half minus 1 will give minus. So 9 divided by 8, cancel this 1, this 1 and now, we can simplified by taking it off. What they do not analysis, what we do not have to do this. But, I will do the non-linear outsource with this, when you can see some nice is it ok.

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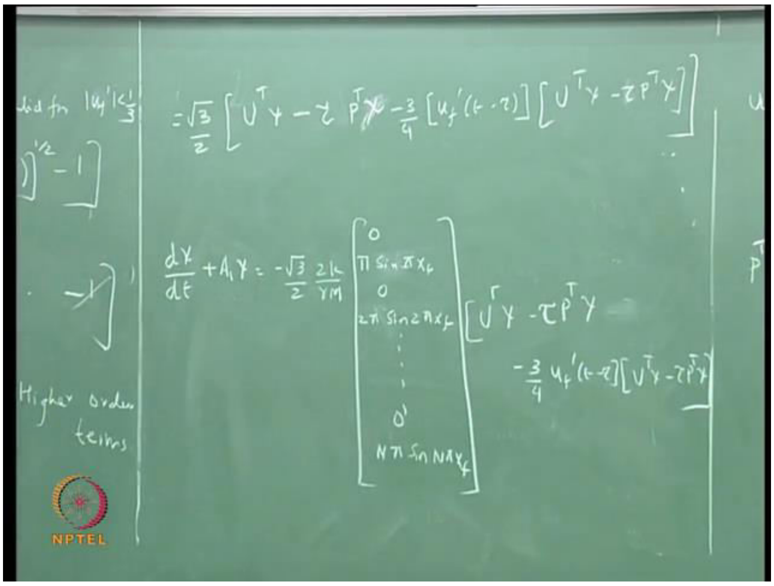

We know, how to expand this so, we use that root 3 go to this is ok. (Refer Slide Time: 29:47) The reason for this square term and spiting prime as u of prime and all of them I replace by this expansion. The other one I have keep in that way and I have bother the whole thing and I see which way it comes nice. So, that is the reason for this and first time on the other one had become a big moths.

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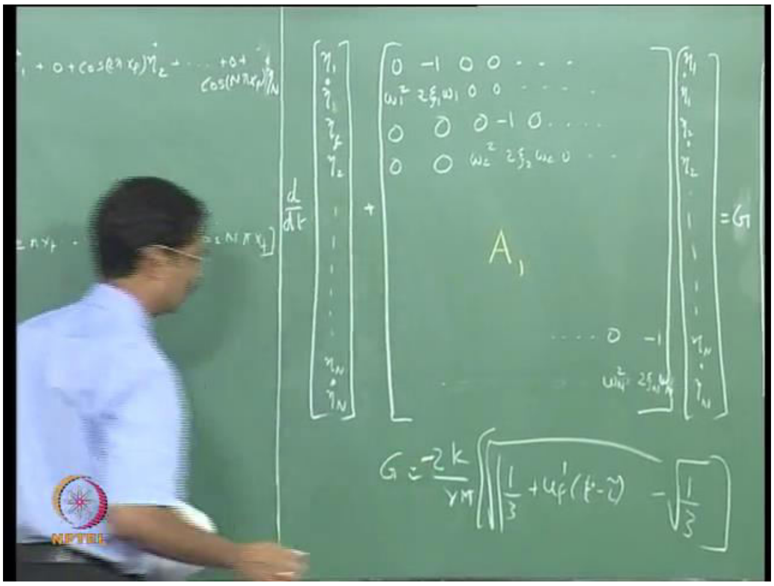

For now, we can assemble so, let me call this matrices A 1 (Refer Slide Time: 22:57) we call matrices. We call this matrices a 1 so, I one come to this form this is my hidden reason and doing all this equal to 1 minute check. There is my mistake times testing this times minus t times minus. We have half way there but, we want make whole think in this compound in this form. We make machinery your sitting in the maths you get it to some problem this all, that machinery solve here.

So, always reduce to slandered mathematician wanted working hundreds of years, getting this standard things. I must take a time out explain, how we did not things? So, I did not dynamic system unit I credit this camera neither I did know anything about matrices and edit linear algebra but, edit could to power of edit of the so, when we did this weight first in the standard way. The use second dollar solve, it is solve want we will see in interest of thing like transient growth and sub critical biographical. So, if continued that way full dosing are way but, suddenly the cut me that all of these things we done. So, then we took a time out put everything in to the frame work of things, that already will done and will person and really have and wise view.

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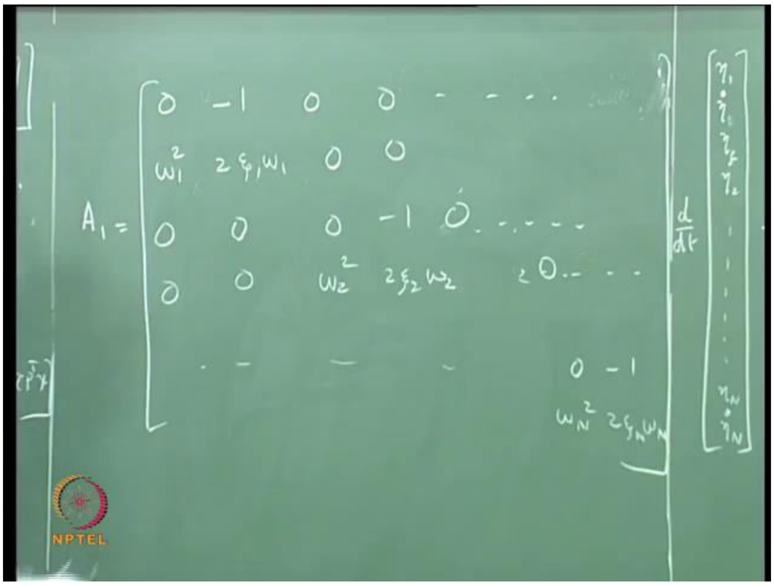

That is the wheel, take it put in your vehicle go, that is ideas. should I write out what is a want I will do right so, a want equal to 0 minus 1 0 dot dot dot 0 0 then next called we 0 0 0 minus 1 0. Then here 0 0 0 minus 2 square minus 2 sight 2 0 and come back to 0 minus 1 and mega 2 squire sorry this particular yaw thank you, a 4 component make a more their dots is 0. So, given that this is a 1 want my objectives here is to I am doing this step to step. Because I could do whole thing step but, then it may be confusing.

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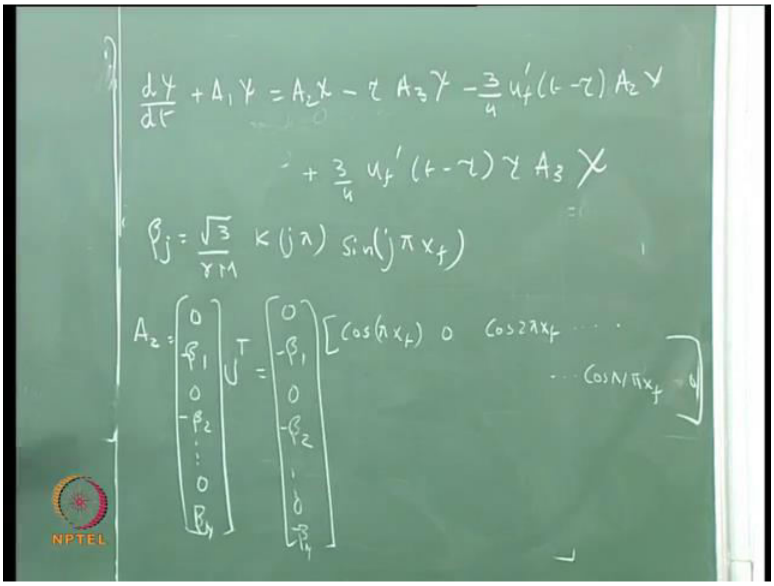

So, once a do this next time onwards do this much fast, purposely going slow. This is the plan so, get this so, what we do this. We need something to all this (Refer Slide Time: 32:34) term.so, we define a new constant, not constant variable be tally parameters equal route 3 are m k j times sin j etc this is subscript peter j. So, that a constant but, its biter 1 biter 2, biter 3. So, we will look this term, we know that this as to; we take this multiplied 1st term a 2 multiplied a 2 constructed 0 beta 1 0 by the beta 2 and 0 beta n times u transport that is would be 0 minus beta 1 0, beta 2 0 minus beta n and cos the x 2 0 cos x 5.

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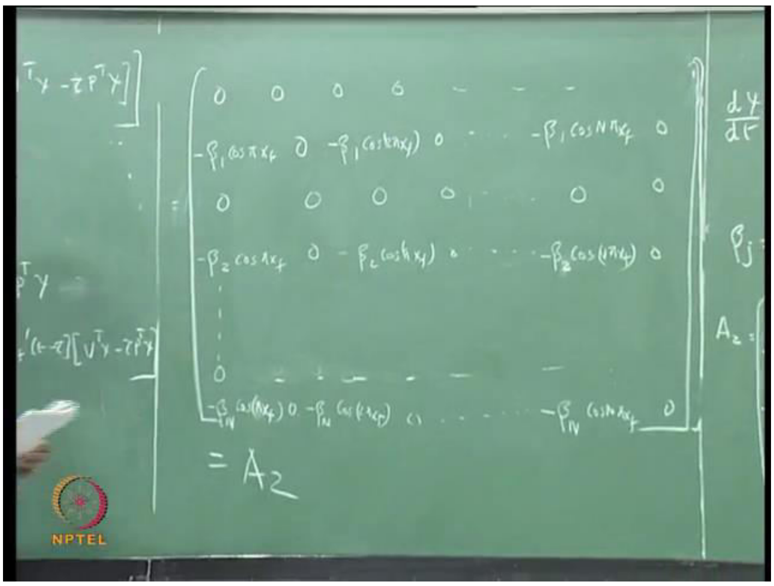

This multiplied by what kind of yeah this would be equal 0 0 0 0 and what will be the next multiplied minus b this kind this right minus beta five x next angle the 0. What will be the next term right? 0 minus beta 1 cos and f x 0 minus beta 1 cos x y 0, next what will be the 0 0 0 0 0 next I would be minus beta 2 cos f x minus beta 2 cos x f 0 minus beta n sorry beta 2 n 5 0 come dot 0 dot. Then here minus beta m this multiplied (Refer Slide Time: 39:45) this is multiplied this to multiplied. pi x m 0 minus beta n cos by x f 0 to this is our a 2 this clear any problem in this. So, try to do A 3 we in this term (Refer Slide Time: 39:45) here this comes from here.

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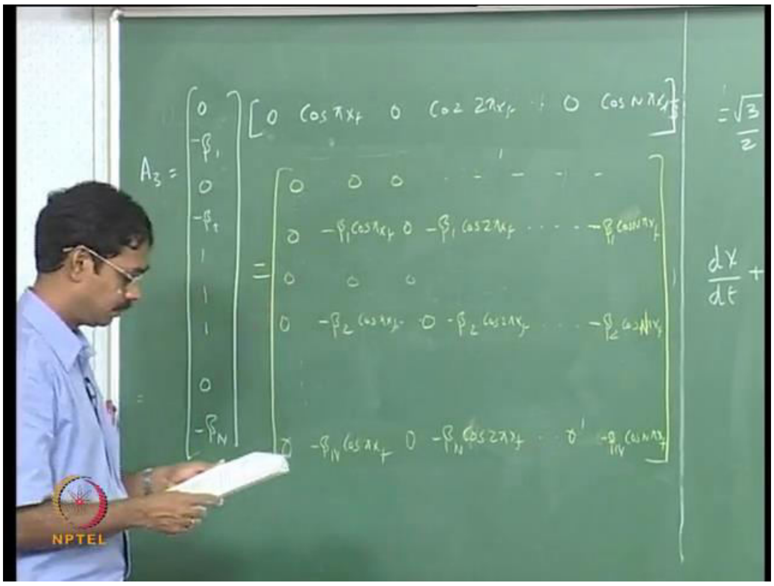

So, we have to right A 3 equal, this same for all term. This is nothing but, (Refer Slide Time: 32:34) this column this constant added. Times be transited which was 0 cos y x plus 0 cos 2 y x plus 0 cos m y x is this clear. So, assemble here and five to do here another that column this equal to so, first term will be 0 0 right and next term be the first will be the 0 (Refer Slide Time: 42:12) and next term have to will be the 0 that is different from here. The first term 0 have and next term 0 that is because your transpose and alternate the shift terms. So, you have the next process will be 0 and after that comes 0 minus beta 2 cos y f x 0 minus beta 2 cos to y x minus beta 2 cos x n y x and this right last 0 minus beta n cos y x n 0 minus beta n cos 2 y x n 0 beta 2 cos y x n.

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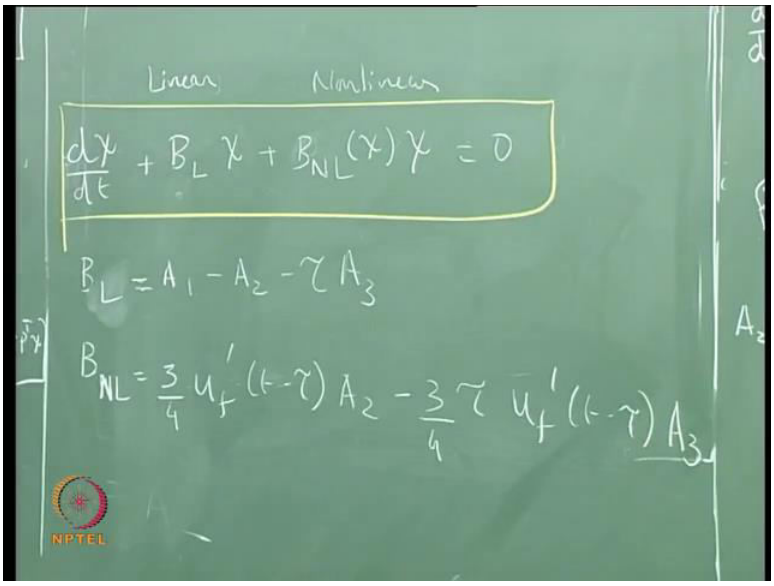

So, we have here we can read do this are set to be assemble the d k by d t plus. This is linear matrices and this is non-linear function here. Linear and non-linear and if this check this home. So, we have a one term (Refer Slide Time: 39:45) minus a 2 term and then this term and next we will going to b f sorry b l and b n l non-linear. So, all though this terms are kind of y c, I have to added this is asperity equation I can get out this thing, its liberal. So, if linear drop of this term of so, we have d k t p this full thing have of linier so, stop here.