Acoustic Instabilities in Aerospace Propulsion
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Lecture No. - 23
Galerkin Technique for Thermoacoustics

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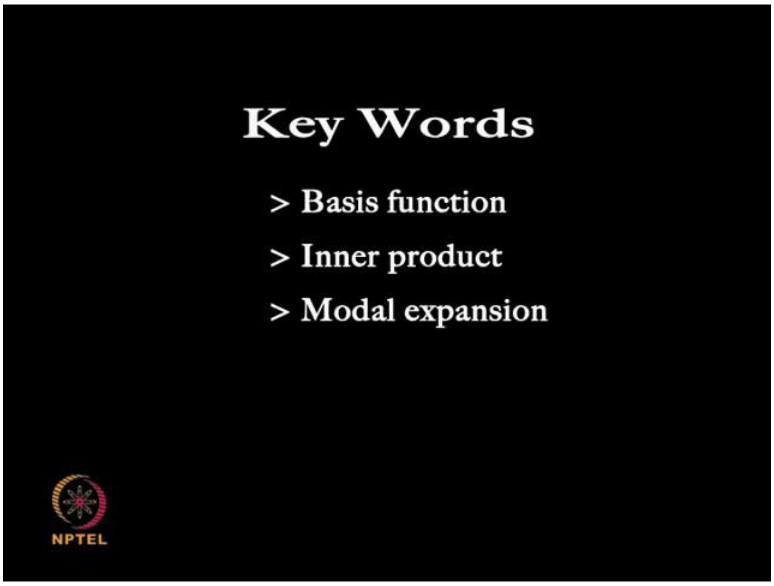

Good morning everybody we are looking here, we are trying to do thermo acoustic in the time domes. We have covered the frequency do mine up how which already and we also look that, active control in that frame word poor plus will control.

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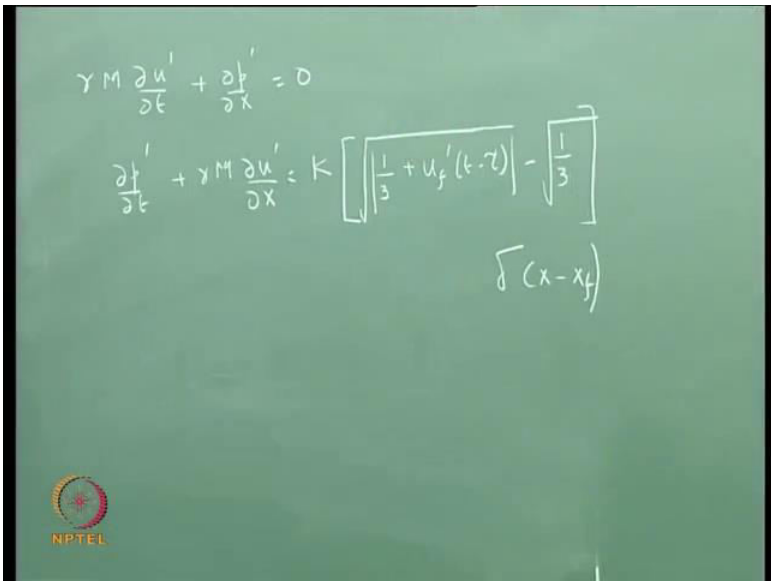

Now in the last class we drive the time do min equations, for acoustics and these to summarize this sequences they where, gamma m d u from to o t plus o p prim d x equal to 0. And this was the this is the moment do mice equations and this is the energy equations and we drive this at the last and this expirations and the right hand side, which the expiration for the heat released rate come from a correlation and I mention that, if I have to solve this problem a grossly you will have to live a outer scale for acoustics and in inner scale for the heat transfer and the hydro dynamics and then you will have to so, dos in requite tube if a locking at heat at via so, you have to be accuely solving for a flow around hard via.

So, we will have to solve the flow and energy equation, heat transfer equation and then calculate and net heat coming out the why and that would be coming out of this hydro dynamic flow analysis and that, would have to go and to the heat least street term. But, I have try to make a toy problem by passing that and using this correlation which we have was provided by my friend Marry a Heckle m a heckle

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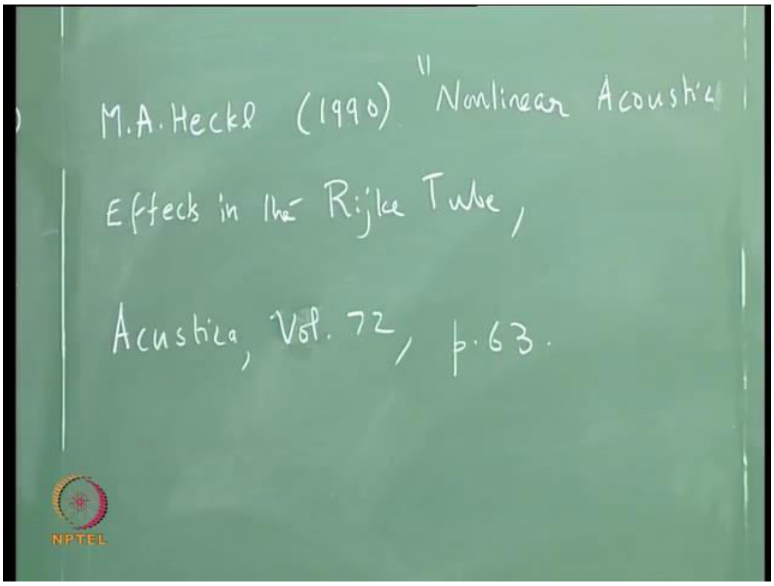

this is the reference non linear acustica effects in the weekly tube acustic volume 72 page 63 she said professor keen I think she had this paper as in cambridge and we use this correlation with actually (Refer Slide Time: 00:33) modified version of the keens law from the hardware anemometry the modification is in this factor of 3 because professor hicostick solve their experiments that, if you go to factors 3 for this matching experiment. So, it is in sometimes putting by hand or some other patient may put 1.5 somenody else keen put 1 so, it depend so, I am not going to debait as to where it is right and how to match experiment or you could something else either.

So, here I am (Refer Slide Time: 00:33) constructing a model problem, model problem is a problem I thing critisise all the term model problem are recital problem. So, in model problem so, model problem which is construct it as a features we has te feature and such eature is seen in the experiment, in reality this should be in model problem rectile problem, rectail problem is mean that playing with problem. So, if I if you solve with in a fully couple way with the acustic calculation and the harling calculation and it takes to is to run playing with it to play this is not to be take any derivative reasons myself create this problem. So, I am not seeing bad about myself.

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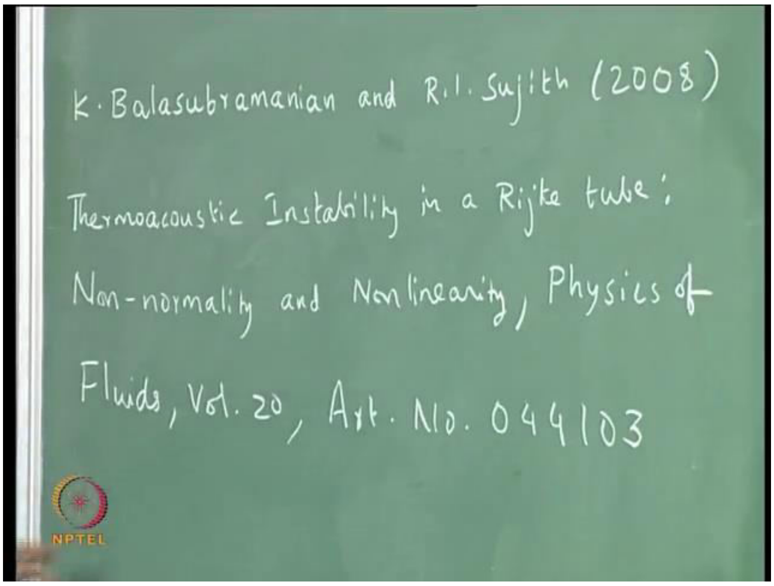

So, I following there reference in this paper K Balasubramanya and Sujith my own paper Balasubramanya was a student here, he was study in my apostolic class and my good friend also the moment here finish this theramacaustic theory theramacaustic equitive non monality and nonlinearity physics and fluds v o t 20 article number 44103 and we look normal mode analysis a problem, given by Metmonizda and I promise to speak a old non linearity, because of student are very interested non dimensional way day one, I will postponing but, I want to get some equation before I will deal with the greater than, speak in every generic sense because we speak generic sense we should attend in dynamic class in linier gupta.

And I will concern non linearity which my friend Koushic Balasubramanya this covered in oppostic and it was there hydro dymax it written in 50 years back but, this is a simple paper nice paper written with objective making things clear and so on. So, I will email this privative today and I can follow it hopeful it would clear but, I will go through this step and some paper we may find out and it easily shown but, I will got deal between case you have difficulty are they any equation from last class? Ok.

So, if the question is proceed with that so, the next taking the case we have a p d e actually oh partial different equation, which is in general p d s are hard to analyze some special p d s like so, moment will bring right hand side everything might say there we would peacefully solve with left hand side we would say here. So, the lot of techniques to solve p d c in problem is technique is specific to also the cause.

So, I will use a technique call model expansion some retention real exist method so, the lot of technique use the one of that we will use mod d thermostatic in technique but, structure the some case call model expansion again what exactly going to divide for the because of that, debate of this technique can be call technique ok, if you do not like that, if you think not like other technique to but, lot of people in term of static cam uniting offer to a technique basis model expansion of linier technique are you user then, thinking it very simple and deal with the in a very simple and these are very easy to words the only resend to the several methods which you can see basis methods.

So, you can use the oppostic equation and oppostic you can solve the saw few problem solving each and like a good scheme and so on but, I will not get that term and this term what we have to here causes. Now what we do is to dividing a summary what we do is to expand the variables? So, here is to variables are velocity and pressure in terms of some basis function and then, we project the p d and non basis functions and then, we are having get individual equation for the proportion now, if you think of you can imagine this that is the I have little bit imagine again but, I do not know it is peaceful what I said that is, you take the p d and project for the so, you write the variables as like a summation of expansion of the mode or any basis function. Basis function on function and put into the different equation and project the different equation on to the basis function does it make statement any sense? Everybody anybody can difficult with the statement ok so, what I would mean it.

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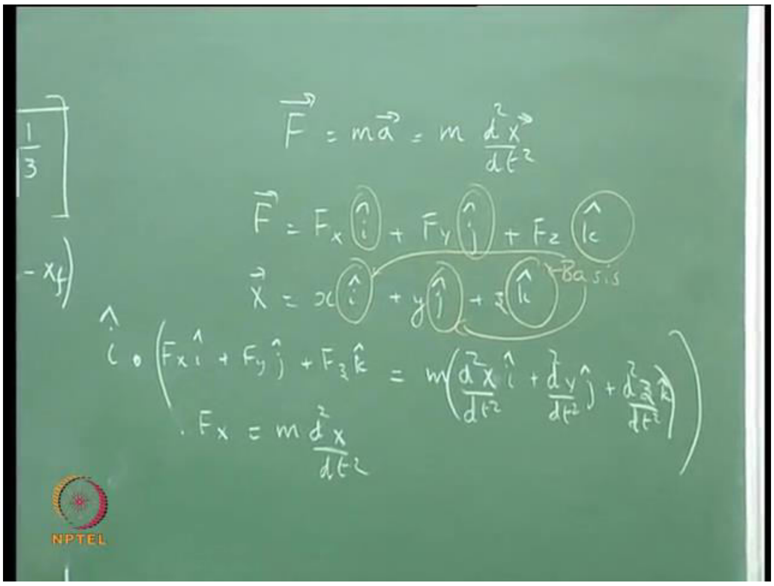

Let us look like a very simple equation vector equation so, f equal to m a so, this would be m actual ratio d square x by d t square right. So, this is strictly sticking on like vector equation. Now what we do with our x and f? We write it in terms of all vectors need base function basis for the vectors. So, f is say is f x times i plus f y times j plus f z times k help me write it go x we say is x i plus y j plus z k ok. So, we can write this has f x i plus f y j plus f z k we have to premier let us like let see, what happen equal to m d square x over d t square i plus d square y over d t j plus d square z over d t square and k.

So, this is the vector equation but, I want to get equation for f x i and f y j so, you can just look and say that instrumentally obviously f x is m d square x over d t square but, what is the process behind distrubuly obviously so, what did you have is to we have the equation and I can take a inner product not product with i and what did have get is so, i times i is 1 and i times j is 0 i times k also 0. So, I will get and f x equal to m d square x over d t square actually summation but, I have this property that, it is i dot i is 1 i dot j 0 and k is 0. So similarly, if I take dot product with j i would get the equation f x equal to m d square x by d t square if dot product k I will get f e z equal to m d z d t square.

So, this is actually you have a equation and it projected on a basis function you see the same equation here and get of the what would you of other things we are conveniently use thing orthogonal property of the basis functions ok so, i dot j equals means that, i is orthogonal to k so i dot with to j orthogonal to k so, we get of those and keep only this.

So, the same idea of this vector algebra the same thing apply function also so, if you write expansion of the functions you take any product in product may be defined in this way may be defined in other way so, there is a inner product keep what you want to the whatever, you want and then you can take another product in keep what you want through ever what you want.

And so, here we actually having through these are get some color so, these are actually basis and the x y z are the co-efficient that multiple basis function so, I hope this is clear and we attempt you get x y and a z similarly, these also basis function i j k these are basis function and we are attempting to attempting to get the equations f x f y and f z. So, that is the idea what we did to state it again if you take the partial differently here is the ordinary equation we took the ordinary equation, which are vector equation projected on to the introduce a basis function and we got the equation corresponding to those solutions so, we will do this procedure is that clear? Any question of this p d e class giving a minute for this thing.

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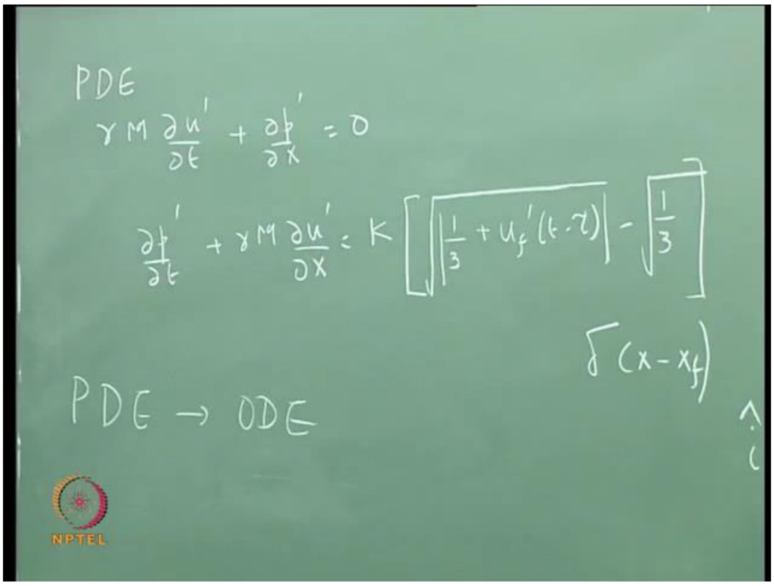

So, here I have a partial definite equation so, p d e so why I am eagle to convert p d e o d e, I want to convert p d e is to o d e so what is my big motivation behind concluding this? Yeah o d can be solve in principle numerically at least so, p d does not guarantee that can be solve once you can get it only guarantee you can get the solution that is the idea. So, we will look at a we solve rebate in the last class so that, is the duct open at and in the picture movie that I will show they were duct at the both side they big chambers with actually maintain the and hence open it ok.

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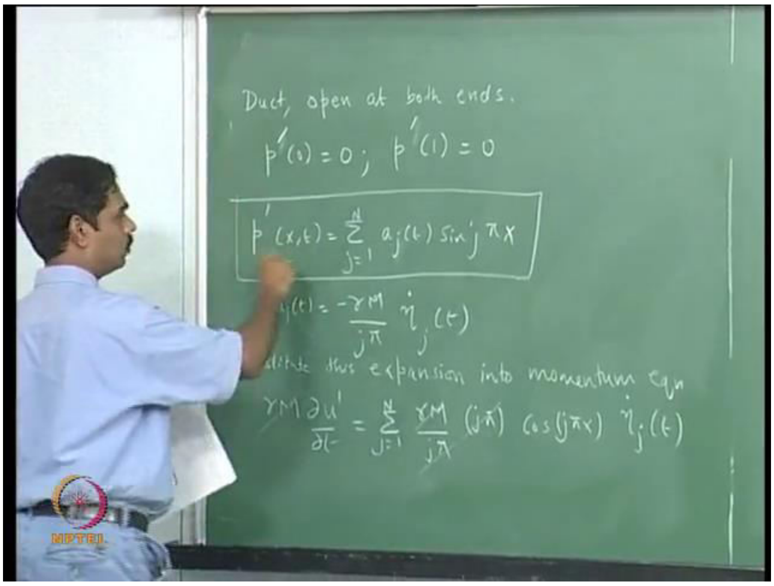

So, I have no lameness equation and the distances. So, the length with you excess non dimensional by the length derivative with you so, end of the tube is l divided by which is l a divided by l a is 1. So, I want to you say that the choice of basis function is not unique so, if we go back to this problem there was no reason (Refer Slide Time: 08:30) I have to choose i j and k you could a chosen e r e theta e I mean you can use some cylindrical corner you use other co-ordinates, circular co-ordinate or you can use pericular co-ordinate or other some co-ordinate what determines the choice co-ordinate?

And I think it was a convenient certain geometry certain convenient certain think other geometry some other thing become convenient and there is no route you have to chooses what about wants what ok yeah thank you, here is the mistake the pressure is 0 for a whole now p prime is fluctuating it is not derivative not a mistake yeah what is the problem? You can ask me d t prime for a fluctuating pressure so, if I transform possible to keep this prime it was forget it apologies let me know I will put it back. So, when the pressure is to 0 both signing is a very natural choice. So, what we are going to choose are we solve the their analysis minus is also get shifted right frequency get shifted what we have going to choose has the basis natural function as the mode.

So, what did the natural mode in natural mode will happen there is no driving ok (Refer Slide Time: 00:33) so, if you remove this right hand side and solve for it you will get the natural mode or non other very saying of it is we take the leniarize the and self act join for the more solving use it that, is the very fancy say it just say natural mode that means the assonance whatever, the need for usefully basic function which means open tube pressure will go sin in velocity go like cos n fantastic. So, we will say this is purely terms to canceled the and to make the equation look pretty you do not need put this gamma and n over j phi all them this is make things look likely some people like that do not so, I leave with you.

Now what we do? We will substitute into the gamma than equation. So, let substitute this expansion moment in equation. Before a proceed I am say that, technique was introduce by introducing solve in what are give original reference in the next class and we are just using for the primitive and particularly because 1917's and so on now, may be possible to a solve with p d e and so on but, definitely not 60's and 70's so, at the time they will move this technique to a proper solution with word. So, if you substitute the expansion of the moment equation you will get this is gamma m d u prime by d t will to see a take and now, we need to different shade this different shade this is a j phi. Now you can see now a gamma m and j phi. So, this can cancel again this is cancel go here.

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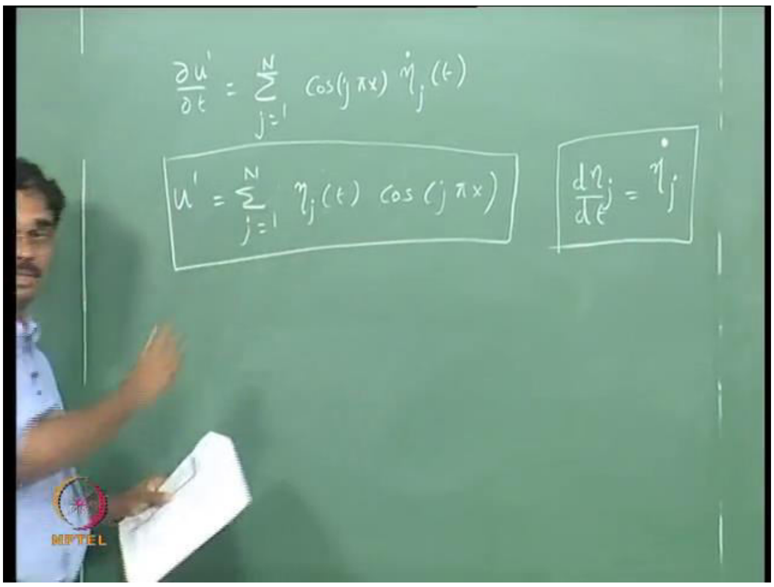

So I will get d u prime by d t equal to sigma j equal to 1 to n cos j phi x and I transuate so, if we inject this will again u prime equal to sigma j equal 1 to n so, if the answer if you put here and then, we integrated the dot enter you work out wants me make a less then you clean up you get things look by match velocity is this and this etc here both the function are x and time this is the x for fluctuate so, this is the expression for the fluctuate velocity this is clear?

So, we also so also we have interested in equation d theta over d t equal to theta j dot so this is and equation that is have you only this is valid you get this solution here right. So, it built into you can also write pressure has some coefficient, the velocity having into the solution have got into this side column is have same I thought it looks nice. So,now we need to get we have a pressure, we have d u by d t, we have a pressure we have a d p by d x and d p by d x and you use in the equation right.

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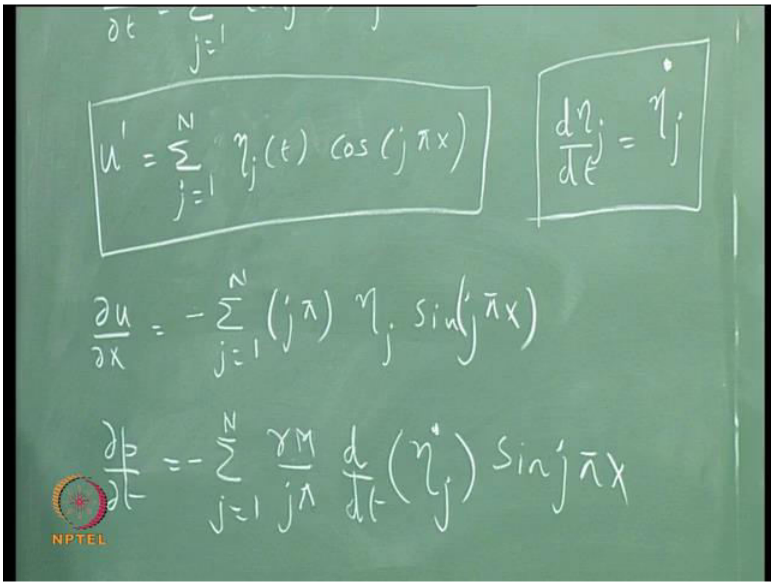

We need this term (Refer Slide Time: 00:33) d u divided by d x equal to minus sigma j equal 1 to n j phi theta j sin j phi x and d p by d t equal to minus sigma j equal 1 to n r m over j phi d by d equal to theta j dot sin j phi x just pause for a minute for you to check this equation differentiate this differentiating this (Refer Slide Time: 20:22) dimension for the next it comes to it through and your differentiate cost you have minus sin and j phi differentiate this term similarly, to get the pressure we have this is expression for the pressure and we substitute (Refer Slide Time: 14:37) a j here.

So, this gamma sin j over by so, when we d o by d t the this sin s not the function of time. So, this is the function of time and when you get d o d t we get d t of d transpose of this is clear? So, now we substitute all this into this d t and (Refer Slide Time: 00:33) we can get the equation ok.

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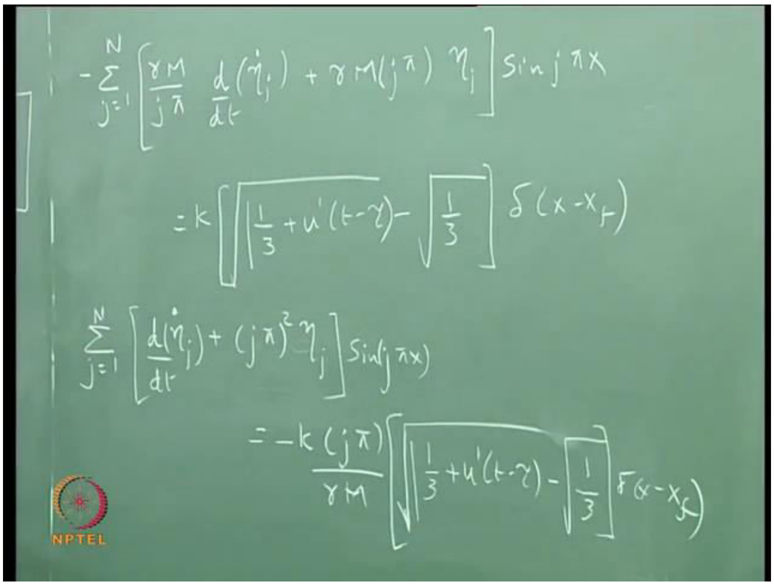

So, we substitute this energy this term d p by d t has the sin j phi x (Refer Slide Time: 22:59) d u by d x sin j phi x it is a taken the thing out so, d o by d p this terms right (Refer Slide Time: 22:59) and I have a gamma n to d u by d x so, gamma n multiplied by this term so that what is here that is equal to the source remind a right here is that clear? So, we will clean up to make up this equation look it will good so, we will put minus sin to the right side and multiplied by j phi and cancel the gamma n then little clean l l p so, interest the remember the we have the delta function because y r is the heat y r is very small of the order of mili meter some mili meter that length reduction of a meter.

So, this is the equation we have to solve of one more equation and other equation d theta equal to theta j if you want you can substitute this here and write this as d square theta j o d t square and so on that is the way it look like a you have the term which are theta 1 for cos j phi r j phi x theta call 2 phi so, each of the terminate differentiate call cos phi coming in front 2nd one has 2 phi so you have to put sigma outside each term have a different conclusion depending on to j is that ok?

These are so in the traditional way of doing things if you see the whole 760 data they actually do this write this d square theta j whole d t square and solve but, I will not do that, I will keep it this way in this form d theta by d t equal to dot theta j and then so, this is my one equation and this is the other equation and this is not do that I can stack of d j dot.

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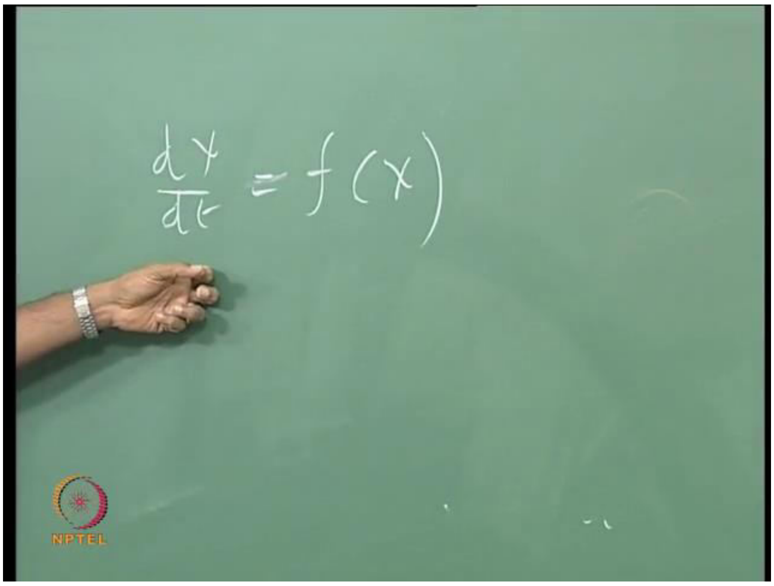

And then I can get equation of this form I want to express the entering equation of this form, it will unable to took use the tools and machinery available from the dynamic system theory. So, I am not going to convert (Refer Slide Time: 24:36) into a this 2nd order differential just I will keep it to way is this clear?

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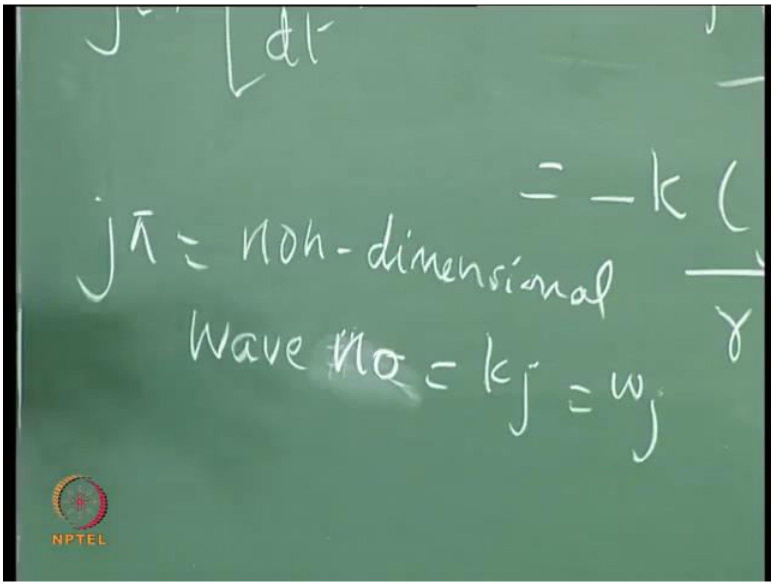

So, we can the j phi is like is non dimension wave number. We call k j or omega j all would be same because this are non dimension so, this k g or omega j it would be just not because the dimension I am going to slow so, here on the corridor private opinion some say going to fast some say going to slow but, do not have a only random sample I have a average opinion front of camera I want to say so, we have the equation and again maths here we check on the j phi and sigma what is that, if you want introduce the sigma and get individual equation we will adopt this methodology with a breeze in the beginning that is in the case of vector equation I multiplied by j k which said is this like a i j k not will be multiplication highly special multiplication inner product.

So, p will take a inner product in the functional space how would take inner product in function space. So, multiply with this same base function we take a whole the support integrate the 0 to 1 and 0 to but, these are non dimensional 0 to 1 that is we take a inner product we multiply the whole equation by individual base function so. one time you can multiply the sin phi x and 2 phi x and 3 phi x what about and then, we integrated over to the enter domain that is will affect to the inner following I am not doing right those but, multiply definition just small explain it some easy way to understand

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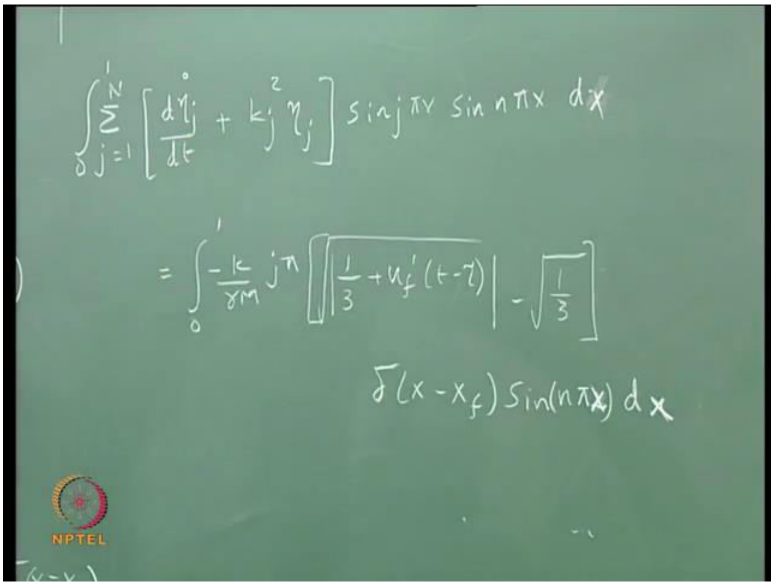

So, when you do that this small x neutral in this case so, what is the distinction between j and this you can choose is like a double some so, j and sin j so it so it is like you have this term velocity which is having (Refer Slide Time: 22:59) all the term have enter it and under you put different equation. So this is the sum of everything but, then we are taking one of those numbers so just we take either sin phi x or 2 phi x and 3 phi x and then you multiply and then, you inject period.

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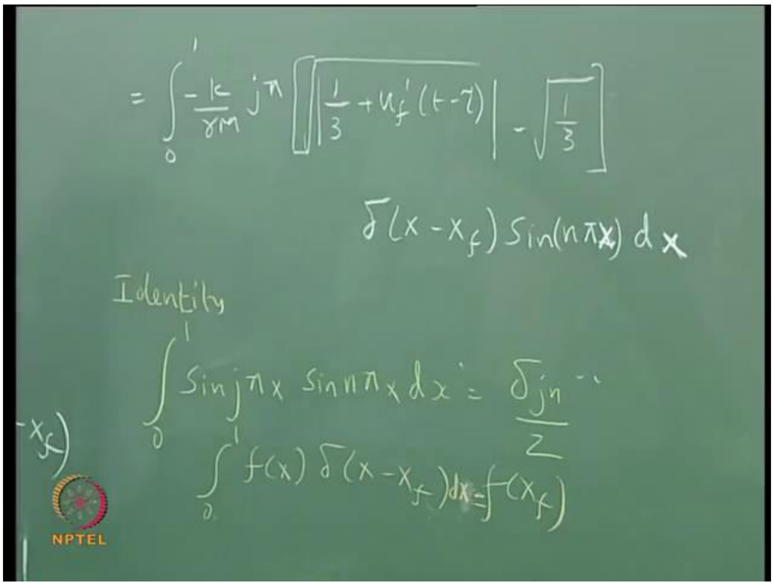

Now that is the magic happens because the signs have some very nice property identity in diagonal that is got 1 sin j phi x and n phi x d x equal to delta j n by 2. So, we can certainly use the and we can use one more identity so, if you integrate multiple delta to the d x so if you multiplied a function and delta function integrate over to the you will get on integration value of the function at that location so at this x f.

So, we can use this to identity and then this equation, domestically simplify any question of this one? In like also be use basic function for example, if you see watching movie and you are watching hindi movies, telagu movies, tamil movie and english movie. So, the you can spilt what you are doing in terms of those bases function. So we are always characterized by day by day things terms of basis function so, if you do this right away you get simplify this term (Refer Slide Time: 31:50) and also we get this term.

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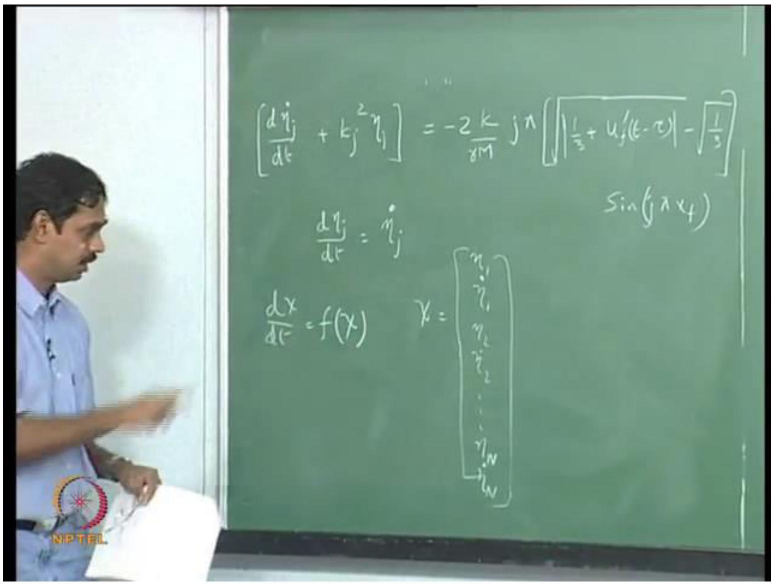

So, sigma vanishes and you will get and you have to put those square routings so, instead of u you have to kept u r. So let us go back here again so when we integrate this equation (Refer Slide Time: 31:50) you have the identity (Refer Slide Time: 33:52) you have plus delta x minus if you integrate from 0 to 1 so, answer is the phi x f what you have do is just put everywhere, the value of x f and you will get the answer.

So, we have just view x f here ok. So because of the delta function we do not have n element 0 in terms j you could do vise also you can by the n term not n of the j ends up to you just have a index is this clear? And we must not forget that, we have a one more equation so, this is the r i j equation. So, we can take this to the other side.

So what do this is except of arminatary equation now and we can actually write them in the form if phi equal to theta 1 theta 1 dot theta 2 theta 2 dot up to theta n dot and how many terms you have to take of conversions this is the lineage dimension term f of x by this linier. So, the next question is within have any dimension in this equation at within have any dimension and reality the more systems have dumping so, we take dumping in adverb given by in present call matrix the details given paper which has given you.

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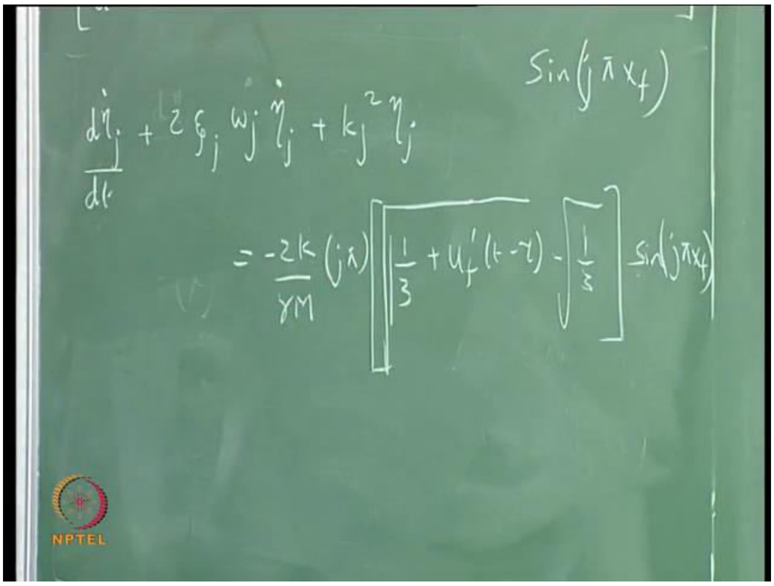

So more the detail as this so, if you want you can corporate model for dumping you can put simple experimentally and empering co-relation for this psi j s let us go back this.

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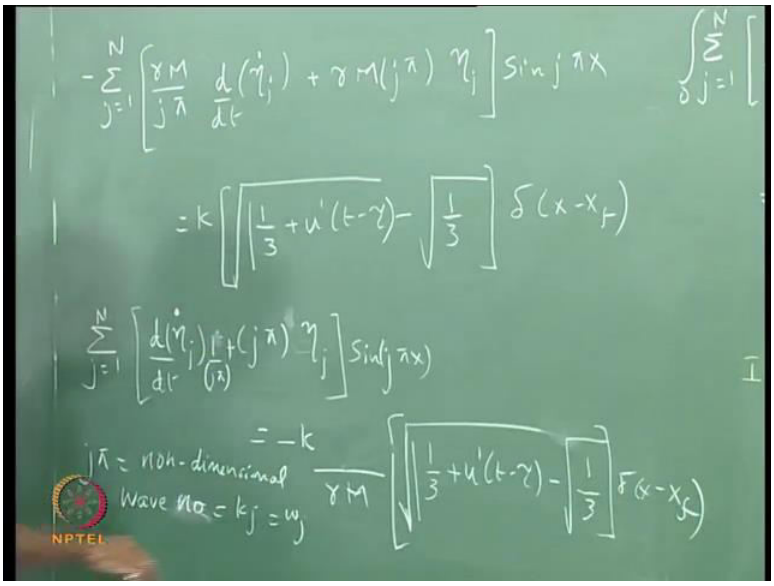

So, just keep their phi and just bring the gamma so just category the gamma so I will have like a so, this will be j phi and there will be a 1 over j phi here yeah

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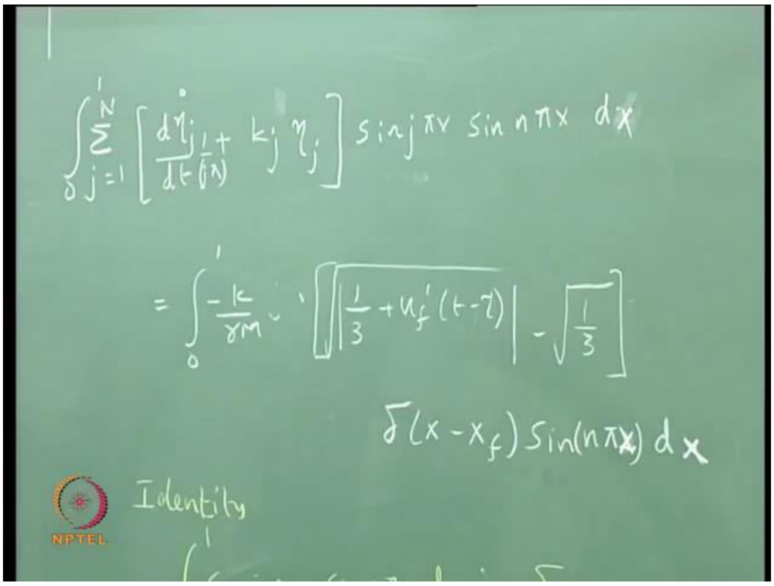

Now if I do this multiplication in this 1 over j phi here and that is will be there and yeah now, it is it to bring that is why because it is can actually

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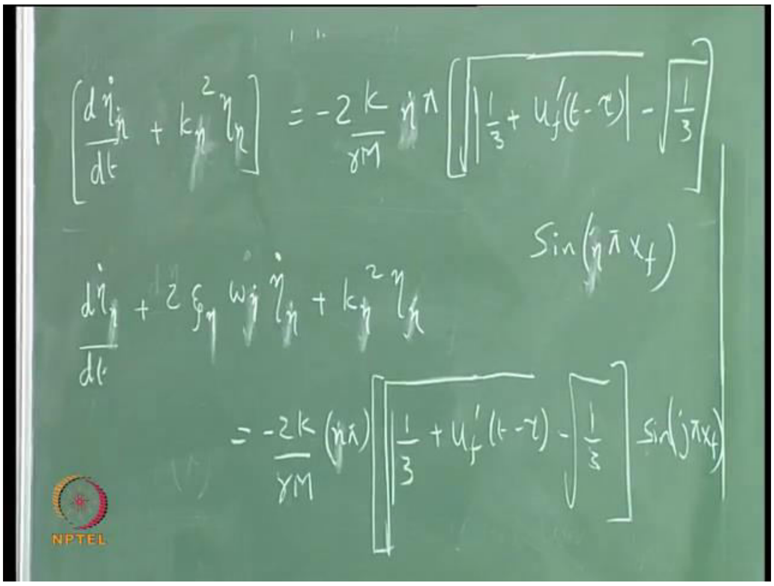

So, it is you are saying n should be here yeah now it is does not matter by change it to n r j but, i do not minding do here but, you can do any induction want you show it is keep it j now it keep over change to n so does not rooting and you can use any index here and but, here middle takes and it began sorry about it. So, now you have a yeah thank you. So, now we actually have a like d l different equation I will not deal try with to that, I will try to expand this term in a round v o for t and it will be find that, I want to keep thing simple and I introducing new complication.

So will linier time delay and we say that possible only a time expand and experiment for found time delay to small or other some system may be large. Let me expand large delay as but, here it is way small.

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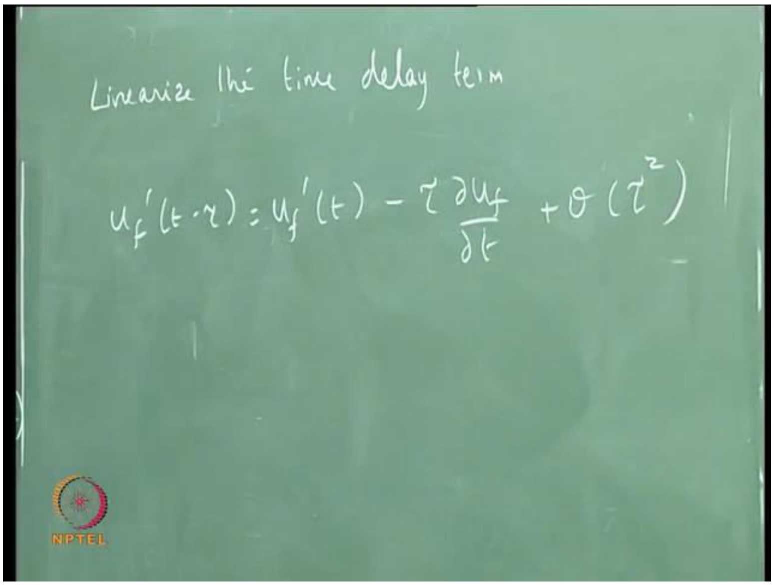

So, nothing I will not gave to finish this in 5 minute little take more that, is why stop it this point and continue in next class because it write this in matrix form it take some time if any questions? You can ask from this otherwise summarize, what we did? We took the partial equation and we substitute for the variables pressure and blasty in terms of the basis function, which are in actually more delta than we substitute the to descend to the p d and project d c on basis function then now, we got different for the coefficients. Now we are attempt it to solve for this, that is way here so to change the we will in next class change from a delay equation to ordinary different equation.

Thanks.