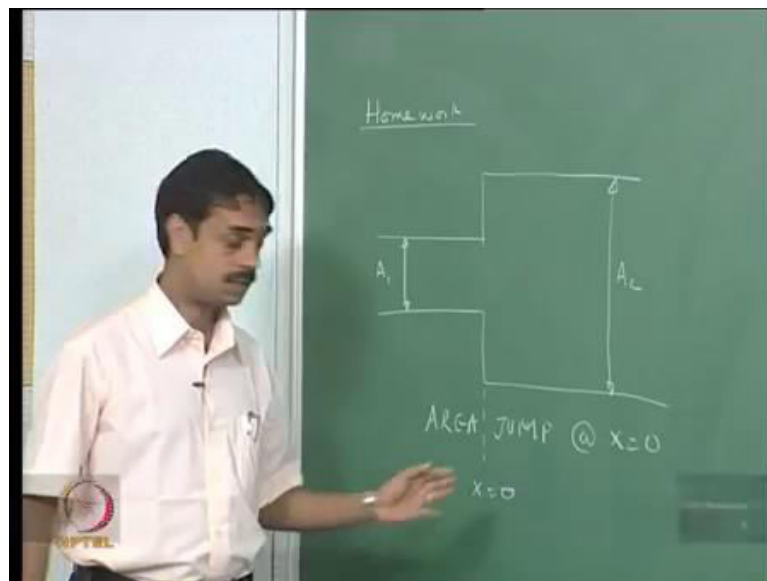


Acoustic Instabilities in Aerospace Propulsion
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Lecture - 22
Toy model for a Rijke tube in Time Domine

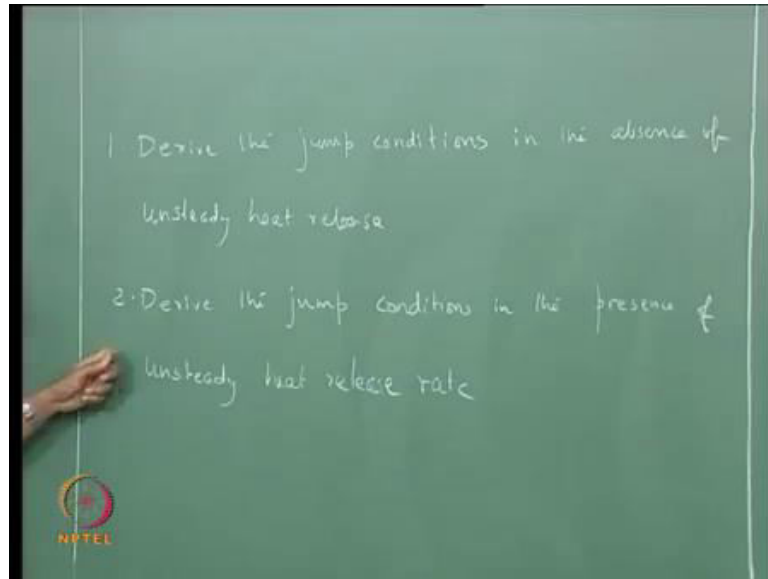
Good morning everybody, we will start with the home work problem, we derive the gem conditions for acoustic and acoustic velocity in the presence of a unsteady heat release.

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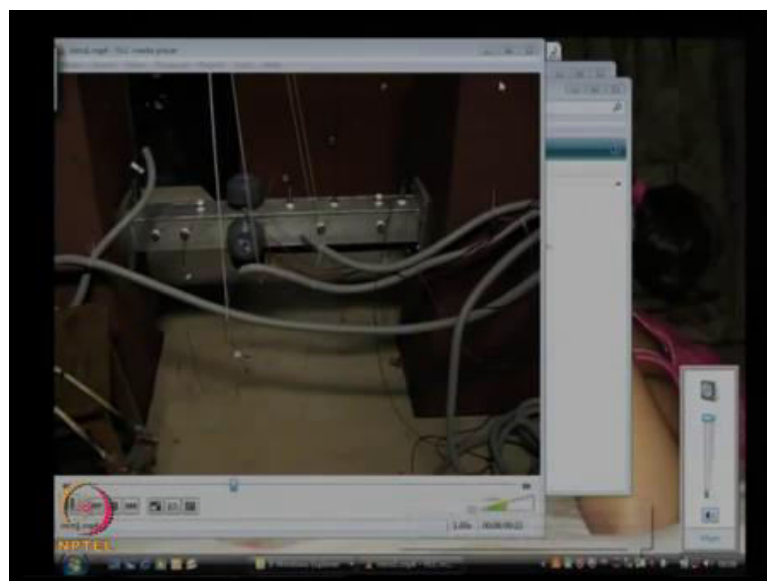
Now, I want to find the conditions if this area jump, if you do not have unsteady heat release rate, what are the condition for pressure and what are the condition for velocity. So, that is the first problem derive the gem conditions in the absence of any unsteady heat release.

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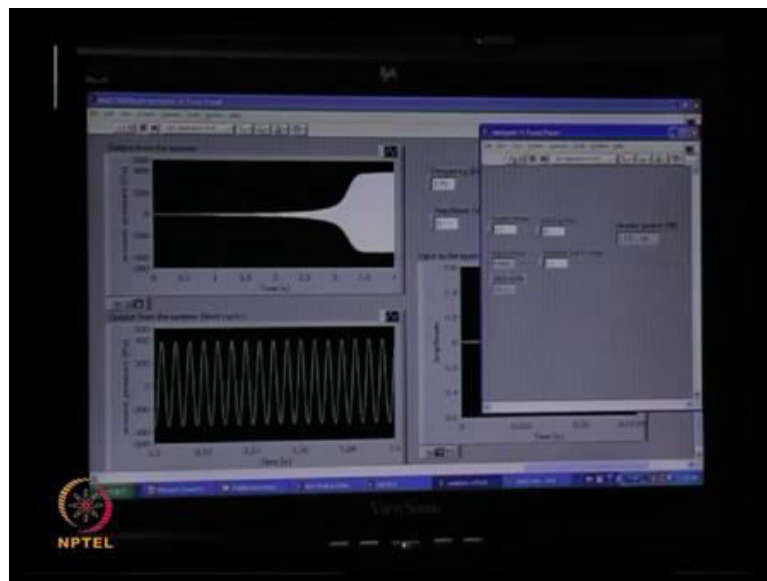
Second condition is let us say for a moment that there is a unsteady heat release here Q prime or Q dot prime. And what would be the gem conditions in the presence of the unsteady heat release rate, that is the second question, very simple straight ((Refer Time: 00:53)) you have any question from last time, no. So, we said that we will try to do to see the equivalent of model analysis or the frequency domain analysis, now we will do it in a time domain, and we will attempt to do it in the context of a recative. In a very general thing, it is very hard to do, that to particular in a class room, so I will proceed to do this in the context of a recative, so what we have I will show you a small video.

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So, the humming here is the sound of the blower, so it bend too fast, so this is the relative, on the left side there is a decoupler or a settling chamber, on the right side also there is a decoupler. This is the heater which is use to provide the heat release rate, so it is like a wire mesh, which is a lower resistance and we heated with a high amp current, and when you put this inside the tube at certain location, if you have the right amount of heat you will get thermo acoustic instability.

(Refer Slide Time: 02:24)



Now, let us look at the regular instability, so just came on it is, and you can see the tries on the oscilloscope, so this is like a it can spontaneously as not the heater power was turned off, it just came out. And you can set the heater power, we are having a program running with lab view, and frequency is 175 Hertz, and you can see the growth of the oscillations, and they are going to reach some kind of limit cycle in the end. And you are having very strong oscillations, and you can see the pattern of the oscillations, you can see very nice sign waves and sound.

So, this is the case of exponential growth, but it did not exponential grow for ever, eventually it reach some kind asymptotic state, where the amplitude stop growing, because is a perhaps growing, for ever eventually. I mean know something as to catch up like, I told you if you get money, eventually people come to take the money away from you, is this clear you want to see again or no, or yes I have no idea, no.

Volume will be too much, I will just, now I will show you show you what is triggering, so we said that triggering would be that, if there are disturbances. And disturbances are below certain threshold level, everything will died on, but they disturbances are above some threshold level will actually go to instability.

(Refer Slide Time: 04:17)



So, right now, we have the here the hum of the blower that is all, and we will give a small pulse and we see what happened, then will give a bigger pulse, we gave a pulse, but just eventually died on. We can see just the oscillations grew and then decay, and actually putting some sign wave for 7 cycles and then living the system at came blank. Now, we had a amplitude frequency is oscillations of 175 Hertz, and amplitudes got the signal given to loudspeaker was 0.2, now will back it up to let us 0.3 we can see that when to quite state.

Now, want to change it to, so it still then do anything, so it went to it is stability, and you can see the this was the initial condition that gave, this is the final state and this is the time revolution. So, this is the, so called triggering instability or you can say this is sub critical transition to instability, and the region which this happiness like a by stable region. Do you have any question about this.

Student: ((Refer Time: 06:31))

Yes, you can really look at one particular measurement and say anything, what have to do is to look at the net energy of the system, so will have to measure the pressure, and the velocity. And velocity may be indirect or direct measurement all over the tube, and then integrate all of this and get a measure the acoustic energy, and that would be growing we have check that for this case, so I think locally there will be differences during the transient, any other questions.

So, this is like a you know, let us consider just to give a some bad example situations acousticability, we are not use to thinking about stability although we see unstable situations all the time, but since I am worried about instability, I see everything in terms of instability. So, let us look at pros, the some pros whenever you come they would not say anything, you can come at 8, 9 they would not be saying to many things, now some other pros they are like linearly stable system also, all linear non-linear everything is stable.

The some other pros, if you do not come at 8 o or even may be will give 2 seconds delay or something anything after you will through you out. So, that the like a gets spontaneously upset, and he will be very mad at you if you are coming late, and he will be angry on grade also come down, but the some other professors 3 or 4 people are not coming, he would not worry, he would continue his thing.

And if you come 2 minutes late, it may be, 3 minutes it may be try to put it, 5 minutes it may be, the next day 5 minute and 1 second, he just through you out he got gets mad. So, that would be like a sub critical bifurcation, he till you cross the threshold he was not getting upset, but if you cross threshold and he got he got angry. It also depend on the context and which this happens if you look at life, so when people under pressure smaller disturbances can create lot of problems, so and let us consider a household.

When, you are small children, you can imagine, your dad and mom goes to work in it is morning, and 8 O'clock it is on, so normally you make a mess and no nothing happens everything life is stable you make a mess at 3 O'clock in the afternoon everything comes breaks, you broke lot of thing. Eventually through away of parents came and cleaned up, and everything become alright, but let say your mom and dad has to go at a 8 O'clock to work, and you broke something at 7:55 and you have to go to school.

Now, what happens dad's called you or mom called you, one of the which short temper, and then you say something, and in gets really under he starts breaking other things. And then the kid goes historical, that is of you yourself historical or your little sister or brother goes historical, and your mom was, so far, now get really angry, because a total mess has be made. And now she has got historical, now dad has two option whether to completely go crazy, and shut down and or he has to come every body down.

So, now maybe that is not even element sackers very keotic instabilities happen, whereas the same thing if it happen at 6 O'clock in the morning, things does has been peaceful, if it happened at a 8 O'clock at night also things should has been peaceful. But, someone 55 it gets small disturbances amplify a lot, because they have to go at 8 O'clock, so it depends on the contestor, which it happens. So, in this recative for example, that is the heater, and if I put the low level of power, whatever I do it will testable, I it wound goes spontaneously unstable.

And even if I break a bound there it wound go unstable, but as I increase the power first the sub critical instabilities will start happening. So, I have, now I have more power, and now above some threshold levels, it will go unstable below it will stay stable, and increase the power some more anything will become a stable. So, as you come close to this critical point, beyond that like at 7:59 and your dad has to catch 8 O'clock, as you come close 7:59 or something, anything can go and unstable.

So, it depends on where as you father away from this 8 O'clock it would be things will be quite stable, so life is also like this thermo acoustic systems, I mean in general I like to view instabilities in terms of life. So, we will show you more thermoacoustic systems in the coming classes, but we attempt to try to do this things in a time domain any other question, look at my glasses once again.

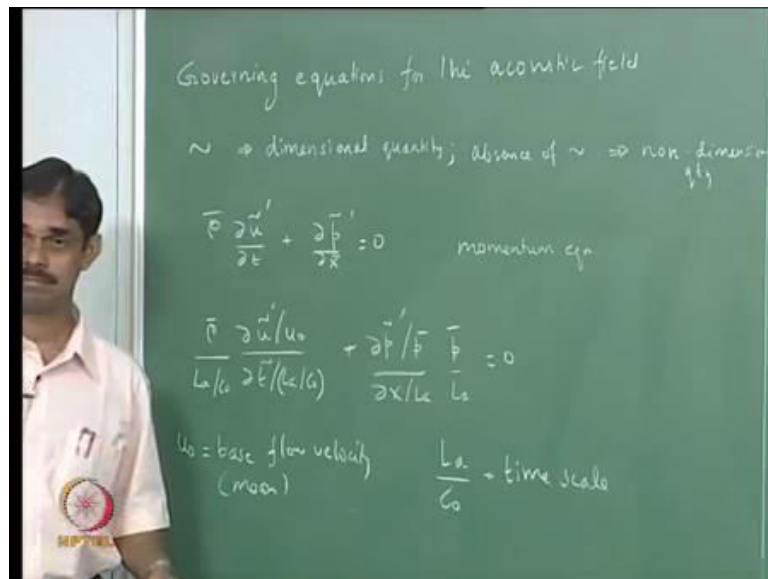
So, what we do is to derive the equations and the non-dimensional form, I think it is generally good to do things in the non-dimensional form, but then it takes effort to things in a non-dimensional form, that is why it did not introduce it. And a there will be more question has to be choice of non-dimensional variables, with I wanted to avoid and for postponed till now, but now we can face them.

And so, I will use this opportunity to teach you how to do non-dimensionalization, and will construct a simple toy model for a thermo((Refer Time: 12:12)) which is actually

what I showed here, a horizontal recative. Remember, that the classical recative is a vertical tube at the there is no blower, you just set up the heater somewhere in the tube and the heater creates a draft, and that draft main flow. And those devices become unstable, when the heater is at the lower half particularly at 1 by 4, quarter length it will be really unstable, but this horizontal tube.

The advantage of this is disadvantage, first disadvantage is that you have to have a blower to create the mean flow, but the advantage is that your blower power or the velocity mean flow or the flow rate which has driven by the blower. And the heater power can be independently varied, you have a one control ((Refer Time: 13:01)) for the blower another for the heater power, whereas in the vertical recative, if you increase heater power the velocity will also change. And you have no way to adjust the velocity separately, and this has, so more possibilities of getting unstable and more ways of, so more flexibility to operate.

(Refer Slide Time: 13:43)



So, let us look at the governing equations, and so the we will use variables with tilda for dimensional quantities, and without tilda for the non dimensional quantities, normally people do otherwise, but I wish to deal with things in a non dimensional way. So, most of the time we having non dimensional equations, so when I have most of the time something, I do not want tilda over it, that is only fraction. So, tilda indicates dimensional quantity of absence of tilda would indicate non dimensional quantity, so that

is the convention that we use, otherwise we use everything else the same. So, what is this equation, we have derive this except that does it tilda on top, otherwise we know this equation, what is this equation.

Student: Momentum

Momentum, it is we have the belief that, first we have to continuity the we have to look before you see, so let us non dimensionalize this, so all variables will be non dimensionalize with some reference quantity. And you will see what are the references quantities, as I am doing, I will write the equation then explain, so divide it here the velocity by reference velocity u_{naught} , time by a reference time. So, you notice the base flow velocity of the mean flow velocity.

So, let it the base flow velocity of that mean, the base flow is the proper term because the perturbations are happening over an underlying base flow that is way we have to think about it . And so, in non dimensionalize pressure with that mosaic pressure, and l_a by c_{naught} is a time scale, and what time scale l_a is the length of the acoustic ((Refer Time: 17:20)) let say. And c_{naught} is the we written that the speed of sound is the constant, we have a constant c_{naught} , that is what constant value which is why subscript c_{naught} , and what does the times period denote.

Student: ((Refer Time: 17:35))

Actually, travelled time will be sound wave of that, so now I was expecting few question, now since in this I will ask you why did a used u_{naught} , and why not p_{naught} , so non dimensionalizing ((Refer Time: 17:55))

Student: ((Refer Time: 17:57))

So, what; that means, here I use a in pressure I used p_{bar} with this a small quantity.

Student: ((Refer Time: 18:23))

It is some answer somewhere there actually can you, so some non-linearity's are important, and you want to display that non something to be coming close to one, when non-linearity's are important. That is absolutely right why is a pointed out, that you are implying that as c_{naught} come closer to you bar, you will have non-linearity, what kind

of non-linearity. No, actually if you are really what it about that term you should non dimensional with speed of sound actually, so that is one form of non-linearity, but even earlier another non-linearity would come.

Yes, Anvisha, I cannot hear you, can you speak loudly, no, that is what he is also saying, but I am saying this another form of non-linearity in this recative. What do you said this perfectly right, there is non-linear acoustics, but let us think what else is there, so what are the things in the recative, there is acoustics and.

Student: heat.

heat.

So, there is a flow and heydro dynamics and so on.

Student: ((Refer Time:19:39))

Heat is always taken by the flow, now what happens when the velocity fluctuations increase, let say u' becomes of the order of u , then what happens to the flow, what is the physical thing that happens when u' is greater than u . You have a flow or cylinder let us say in the recative, and then what happened then u' is greater than u physically, we are having hated cylinder, so there is no blow, but.

Student: Direction of velocity

Flow reverse is that, so what would that affect the how would there affect the heat transfer with Vishnu said something happens to heat transfer, then flow reverse, no would the heat transfer depend on the direction of velocity or would it matter. If I blow with a blow this kind of way that is it, I feel cooling, now does it matter for blowing this way or does it matter blowing this way, speak loudly Anwy, if I blow left to right, will I get more cooling, then or less cooling then right left.

Student: No

No. So, it depends on the just the instantaneous my internal velocity, but then when we have flow reversal one time, you have what is the let us say we think of u and u' . So, when the mean flow and the flections if they are aliened in the same direction,

let us say that co co-flow what is the instantaneous velocity, when their opposite direction.

Now, we have \bar{u} , so there is some unsymmetry involved and, so on, now and the heat transfer is proportional to the it does not depend on the direction, so there is like a some modules sin sitting element, which is actually a non-linearity. So, in physical sense this would be some way to look at it, so we want to ensure that or non-linearity's become important, let say when u prime, so the order of \bar{u} .

So, then if you non-dimensional with respect to \bar{u} or u naught it helps, but if you want to make the other term look important, then I think you can non-dimensionalised with c naught or something. So, is that, so when you do non-dimensionalisation, you do whatever you want, so that you get what you want here.

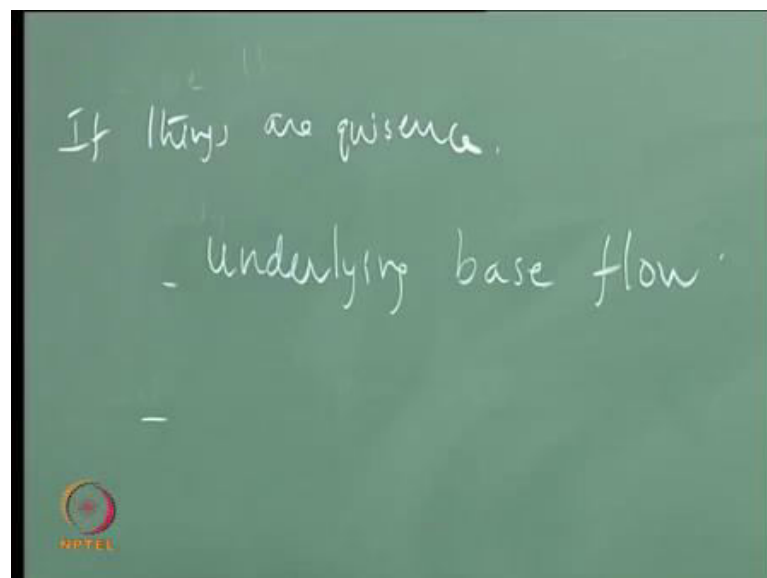
Student: ((Refer Time: 22:22))

Meter per second, 1 meter per second, half meter per second, 1 and half meter per second at least in that particular recreative.

Student: What is the exact difference in these and these.

There is a no difference, it just a terminology that under, there is a study base book; that means, when there is no instability, there is some difference actually.

(Refer Slide Time: 22:57)



So, there is a base flow, so let say if things were quiescent, and we have underlying base flow on top of a build instability, now once instability happens and you are reach some state. Now, it take the mean of that, it may be different from this cohesion base closed state, because if you have non-linear terms, then the average of a fluctuating may not be the average quantity before the fluctuation started.

So, the mean flow need not be equal to, the flow which was present man everything was cohesion, which I would call as base flow, on top of this base flow needs to be derived, and now wants you instability is on, and the mean quantity themselves can change. For example, if I looking at flow around cylinder, you have a pulse sitting flow around a cylinder, will actually set up a secondary unsteady mean flow, which is called streaming, which is different from the that some extra thing over the original base flow that is there.

So, I mean you have a, if you have in solve rocket for example, when the instability comes on the everything changes, and the mean pressure itself will change, because now the burn rate has to changed and more things are coming out and so on. So, base flow means instability not there, and just came on and before it came on what was there, so it just if you are talking about a equation with cohesion medium, then we can think about mean variables, but I would preferred use the base flow term. I hope you understood the difference.

Student: ((Refer Time: 24:48)) 0.

yes, that is the base, we can have flow, but no.

Student: Smart flow.

We can have last flow also, it is no institute.

Student: Without risk

Yeah,

Student: flow without any instability

Without any perturbation, everything is steady base wise steady, but mean flow can be the average of unsteady quantity, and this average need not be equal to the value of the flow base before the instability or constant is a big difference.

Student: actually calculative ((Refer Time: 25:24))

See base flow is a thing in our mind, is it possible yes, but is it possible to stop its feasibility and see the base flow as it is, and I mean only to some extent, because they'll always be some fluctuations, but base flow is always there, because it something which was says that. So, that is a certain difference, but this base flow where is a mean flow sometime it can lead to some differences, and some particular cases I can show you later on.

Any other question very nice, can have steady base flow, it will be twice cohesion would mean nothing is fluctuating, that is my definition of cohesion not nothing is happen.

Student: Sir how steady is there are always some connections

Student: When you take cohesion how steady is that any till what can

There is a trick question, see again you have to this is a very trick question, I do not know if I am answering correctly, but I am really impressed by you and the question. So, let say you have a recative with cylinders know all that, so the flow is there, but there is no instability let say, so I have put the heating value low or I really want turn the heating. Let say my cylinders are big enough, so that I can gets or the flow velocity sign of, so that I can get the right or numbers 40, and I get 4 exiting on cylinder.

So, the base flow is now actually unsteady, but there is no instability, so I can have a unsteady base flow, but the and that is acoustic instability is not happen, but there is some hydrodynamic instability that is happening. So, the acoustics rights over a unsteady hydrodynamic flow, now if you all studying about if you are study hydro dynamic instability, you would call something else as base flow.

If you are looking at the instability of the flow over a cylinder, you would probably calls something else a cylinder, but I am looking at the stability of the acoustic waves in conjunction with everything else. So, hydrogen stability is there, but my eventual outlook is to do can acoustic instability, so that is what in its options what is there, so there is a unsteady base flow.

So, base flow, but if you view just zoom out and we only in terms of acoustics, then I mean you can think of it as the quite thing that exist in terms of their acoustic, now all

pressure is all unsteady things would not really translate to sound. I mean the hydrodynamic fluctuations, which do not necessarily have to be converted to acoustic fluctuations. And is a little difficult topic, but I am hoping that I will cover it, I think those guys they are not here, who gave me hard time in the first class that are all what are Piyush and Akshay.

So, all oscillations are not acoustics, what is the definition, what is acoustic oscillation, so if you take your equations and you look at the characteristic velocities or Eigen values you will have disturbance of propagating its speed of sound. And it is and that is sound, that is acoustics, now you have disturbances, which can propagate at speed of the flow, which are give example, vorticity hydrodynamic instability and entropy fluctuations like if your hotspot it can be carried away.

So, those are non-acoustic fluctuations, so that there is a distinction and it is a very sometimes it gets very difficult to deal with this distinction, thank you any other question I am please with this questions anything else. So, now, will try to simplify this try to go slow, so that let me collect things around, I think I made an I should multiply by u naught, and that is just alternately correct, any mistake I should use rho naught.

(Refer Slide Time: 29:53)

The image shows a green chalkboard with handwritten mathematical equations. The equations are as follows:

$$\frac{\rho_0 u_0 c_0}{Lx} \frac{\partial u'}{\partial t} + \frac{\partial p'}{\partial x} \frac{\rho_0}{Lx} = 0$$

$$\left(\frac{\gamma \rho_0}{\rho_0} \right) u_0 c_0 \frac{\partial u'}{\partial t} + \frac{\partial p'}{\partial x} = 0$$

$$\gamma \frac{u_0}{c_0} \frac{\partial u'}{\partial t} + \frac{\partial p'}{\partial x} = 0 \Rightarrow \gamma M \frac{\partial u'}{\partial t} + \frac{\partial p'}{\partial x} = 0$$

The final equation is enclosed in a hand-drawn box. In the bottom left corner of the chalkboard, there is a small circular logo with the text 'NPTEL' below it.

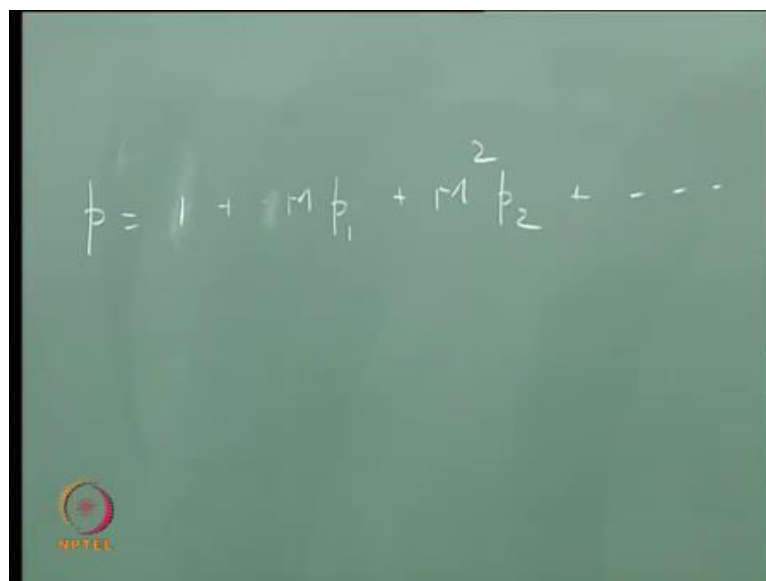
Actually, it is be consistent, so that this naught is ignore that I am just keeping a constant value and so on, although it could be changing, it will change right rho naught and u naught density will jump, velocity will jump temperature will jump, but we are ignoring

it just putting a mean value or something rather. So, if I divide throughout by \bar{p} and I can remove this \bar{p} , and I can bring this \bar{p} here, so I can multiply both top and bottom by γ , I think I should use \bar{p} also here, $\bar{\rho}$, \bar{u} , \bar{c} , this is if I make mistakes please let me know.

So, $\gamma \bar{p}$ by $\bar{\rho}$ what is this, \bar{c}^2 and there is a \bar{c} here, so that will go, so I get $\gamma \bar{u}$ over \bar{c} , did you get this same equation, thank you is a big mistake. Any other mistake Anveksha assignment, when you snap your fingers, how does it produce sound, so you can when you steady acoustics, when you snap your fingers why does it mean, so bubble acoustics is very big subject actually lot of people study, but we are not study.

Now, back to the a question with I think you ask long time back, how do we know this equation is right or how do we know this describes, so I have assumed this equation in some sense, and if how what did you do to get this equation. If you remember go back to the second class, third class, we said u equal to \bar{u} plus u' , and when we drop the \bar{u} , but how could we just do all this things, because \bar{u} is also fluctuating. I mean it is not really \bar{u} , in sense \bar{u} plus u' u' has a hydrodynamic component and acoustic component, and how did you packed hydrodynamic component.

(Refer Slide Time: 34:23)


$$p = 1 + M^2 p_1 + M^4 p_2 + \dots$$

Hydrodynamic component would defiantly be important hydrodynamic zone, so we really have to write some kind of expansion in terms of p equal to \bar{p} plus or in the

non-dimensional form. P equal to 1 plus mac number times p 1 plus mac number times square p 2 plus dot, dot, dot, dot. And in the end little turn out that, this is acoustics and this is hydrodynamics and so on, but now we are going to do this, if time permits we will do it to words end of the thing.

So, I mean there is there is some delegates issues that, pushing out carpet here because I am not dealing with hydrodynamics rho, instead I am going to model hydrodynamics, what do I need to model hydrodynamics or why do I need to model the heat transfer. Where does the rho, where does unsteady fluctuations and velocity and the hydrodynamic zone and so on, why is that important, because the heat transfer comes end.

And why is that important, that is what is actually driving the acoustic feed, but instead I am going to replace it with like a co-relation for the heat release rate, rather than solve the equation. Just to make thing simple, but if you have to release solve it you have to address all this issues, and if time permits I will teach you at the end, always privately it can be done. But, it is a very interesting an exciting topic, and it is a very open topic, I mean people are doing research on this even now, so I mean only now some minimal break close are being mate, so it is open to you after this class you can get to do very exciting start.

(Refer Slide Time: 36:06)

Energy $\frac{\partial \tilde{p}'}{\partial t} + \gamma \bar{p} \frac{\partial \tilde{u}'}{\partial x} = (\gamma - 1) \tilde{Q}'$

$p_0 \frac{\partial \tilde{p}'/p_0}{(L/c_0) \partial t/(L/c_0)} + \gamma \bar{p} u_0 \frac{\partial \tilde{u}'/u_0}{L_a \partial x/L_a} = (\gamma - 1) \tilde{Q}'$

NPTEL

So, now let us do energy equation, so I just forgot mention non thing, when I started doing this first I did this on own, and another one thing what to do with rho naught, but it turns out that. So, I divided by rho naught multiplied by rho naught and all that, but it turns out that, rho naught will just disappeared because you will get absorbed in to the c, and then that will come in to this over beautiful mark number and so on. So, you do not have to deal with it is this, is there any mistakes and I making plenty just let me now.

(Refer Slide Time: 37:47)

The image shows a chalkboard with three equations written in white chalk. The equations are as follows:

$$\rho_0 \frac{\partial \tilde{p}' / \rho_0}{\left(\frac{L}{c_0}\right) \partial t / (L/c_0)} + \gamma \frac{\rho_0 u_0}{L_a} \frac{\partial \tilde{u}' / u_0}{\partial x / L_a} = (\gamma - 1) \tilde{Q}'$$

$$\frac{\partial p'}{\partial t} + \gamma \frac{u_0}{c_0} \frac{\partial u'}{\partial x} = \frac{\gamma(\gamma - 1) \tilde{Q}' L_a}{\gamma \rho_0 c_0}$$

$$\frac{\partial p'}{\partial t} + \gamma M \frac{\partial u'}{\partial x} = \frac{\gamma(\gamma - 1) \tilde{Q}' \left(\frac{L_a}{\rho_0 c_0^2}\right)}{\left(\frac{L_a}{c_0}\right)}$$

An NPTEL logo is visible in the bottom left corner of the chalkboard image.

So, let us recap this, so I can remove this prime, and I am bringing p naught here, I have a L here, and this L here, so that can come to the top, so L a can go, now there is a c naught here, which is coming under numerator. If I divide throughout by the c naught, that c naught will come under this u, so that will be under this gama here, for gama u naught by c naught and, so divided by c naught, I should get as c naught at the denominator.

What I have let me check this is, so this can be even re-written as tou p prime by tou t plus gama m tou u prime by tou x equal to, this let see we can make it pretty. So, if you multiply by gama, and divided by gama and then you if I can multiply by, so gama p naught is what, gama p naught is rho naught c naught squared. So, you can leave with that we also ((Refer Time: 39:45)) if you think this is pretty, of there any questions, anybody all are.

So, now we need to model this Q' that is the requires to the matter, and how can we do that and I say mentioned we have to set up a 2 scale problem, and then solve for the hydrodynamics in the inner zone. And that for you get it, but if I would not avoid that I could is a correlation, do you know any correlation for flow passing, heat transfer flow passing have you come across, I am sure you would come across.

How do you measure flow, turbulent flow fluctuations, what device we use some kind of anemometry, how to I have how do you start their work, what do you measure there.

Student: Consistency change is ((Refer Time: 41:05)).

So, that is deal with temperature and heat transfer all that, how does the relate to flow velocity.

Student: ((Refer Time: 41:13))

So, what how do there calculate calibrated using what, there is a correlation rate between heat release rate and the velocity fluctuations, what is it call, what law is ((Refer Time: 41:37)) does not doing a bell ((Refer Time: 41:42)), never mind it, does not matter, it is just a name.

(Refer Slide Time: 41:53)

The chalkboard contains the following equations:

$$Q' = \frac{2Lw(T_w - T_0)}{5\sqrt{3}} \sqrt{\pi \lambda c_p \rho \frac{dw}{2}}$$

Heck's Correlation:

$$\left[\frac{u_0 + u'_f(t-\tau)}{3} - \sqrt{\frac{u_0}{3}} \right] \delta(x - x_f)$$

$$= \frac{2Lw(T_w - \bar{T})}{5\sqrt{3}} \sqrt{\pi \lambda c_p \rho \frac{dw}{2}} \left[\frac{1 + u'_f(t-\tau)}{3} - \sqrt{\frac{1}{3}} \right]$$

$$\times \sqrt{L_a(x - x_f)}$$

If you discover your are a head your name on it, so basically I need relation for Q' prime in terms of u' prime why, but we saw from our earlier analysis that when things

that happen today are not affected by what your actions are now, but what are the actions yesterday. For example, you are sleepy, because you did not sleep last night, so it is, so there is a delay, and that is what is affecting.

So, we want get some correlation of this form, that is that cuts of the matter, so here come handy, I am not saying that going to have a correlation is the way around it, and just for convenience and tractability I am doing this. And you miss a lot of things any time you make a simplification, I mean that is the way life is a anytime you change something you will lose lot of things. So, it is a question about what do want gain versus what you are willing to lose, everything comes to that finally, I will explain the symbol.

So, this is the correlation that we have used L w is the length of the wire, T_w is the wire temperature, T_{bar} is the gas temperature, I should use T_{naught} , actually just to, and λ is the conductivity, c_v is the specific heater constant volume, ρ is the density. D_w is the wire diameter, and $2 u_{naught}$ is the base flow velocity, u_{prime} is the fluctuating velocity, subscript f stand for the fluctuating, in the fluctuating velocity changes across the tube, I mean it does a standing wave and so on.

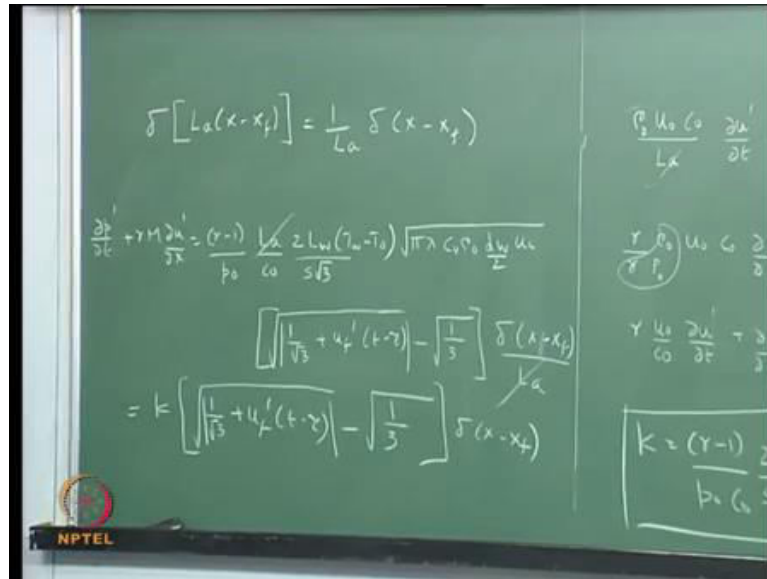
So, at the heater location should strictly stand for flame, but it is here we are heater now flame, so there subscript f , t is the time, τ is the characteristic time delay with this heat transfer happened, and δ is the delta function. So, we can rewrite this as s is the area of the tube, area of the duct, and now I forgot to mention one, so there is a factor 3 that is coming up, with this if you seen ((Refer Time: 45:45)) it does not have this factor 3.

So, the original kings law if you look at the correlation, Q_{prime} is a linear function of u_{prime} till almost about by you will want, and then the equation starts becoming non-linear. So, there is a lady called Maria Heckle, she works in Keele university, so she claims that the non-linearity is will come in much earlier, and now the non-linear terms are important when u_{prime} over u_{bar} is of the order one third. She saw in her experiment, so she change the correlation to this form, it was purely based on her experimental evidence and so we use this correlation.

So, this is we will call this hackles correlation, she is my friend you know in their, we if you have politicians is fathers, sons will be politicians, grandsons will be politician Karunanidhi, Kanimozhi and so on, so forth, Rajiv Gandhi came from Indira Gandhi now how is this Ruhul Gandhi. So, Jermany father is a scientist, daughter is a scientist,

so Maria Heckles father, professor Hackle was a famous, very famous acoustic professor, daughter also studied acoustics she is in Keele which is in England, but so here we do this only for politicians, there professors also seen do this. I will take this u naught out, so then I can write in terms of the non-dimensional fluctuation multiplied by delta of L a into x minus f x, so this is the correlation.

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So, I will write identity, you can check this yourself quite easy to prove, 1 second let me write this, yeah

Student: ((refer time: 48:33))

It is, so I divided by u naught, so u f prime tilda over u naught is u f prime, so this is a identity for the delta function, so if you have delta L a times x minus x f, you can take the L a out, and written down, so delta of x minus x f. So, if we use this, and then we can say time and I can cancel, this 2 L s, I can rewrite this entire right side as k times square root of more or less 1 over root 3 plus u f prime of t minus tau minus 1 over root upon 3 times delta of x minus x f. So, I call this whole thing is k, so this whole thing I can call k, and says k times, this terms can delta function, delta function is because we have a compact heat source. So, if you see the typical dimensions of a recative the size of the heater the order of half millimeter or something.

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$$\gamma \frac{u_0}{c_0} \frac{\partial u'}{\partial t} + \frac{\partial p'}{\partial x} = 0 \Rightarrow \gamma M \frac{\partial u'}{\partial t} + \frac{\partial p'}{\partial x} = 0$$

$$K = \frac{(\gamma-1) \rho_0 c_0 \sqrt{3}}{2 L u_0} \sqrt{\pi \lambda c_v p_0 \frac{d u_0}{2} u_0}$$

Whereas, the tube itself the like 1 meter long, so 1 divided by half into 10 power minus 3 is like 2000, 1000 or 2000 is a big factor, so we can say our heat source is compact and here is a delta function there. I hope this is clear, are there any questions, suggest you summarize, I would write the equations together, so I have the what is this equation again, momentum equation, so this is the momentum equation.

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$$\frac{\partial p'}{\partial t} + \gamma M \frac{\partial u'}{\partial x} = k \left[\frac{1}{\sqrt{3}} + u_0' (t-r) \right] - \sqrt{\frac{1}{3}} \delta(x-x_0) \quad (2)$$

$$\text{MOM} \rightarrow \gamma M \frac{\partial u'}{\partial t} + \frac{\partial p'}{\partial x} = 0 \quad (1)$$

$$K = \frac{(\gamma-1) \rho_0 c_0 \sqrt{3}}{2 L u_0} \sqrt{\pi \lambda c_v p_0 \frac{d u_0}{2} u_0}$$

And energy equation is $\frac{\partial p'}{\partial t} + \gamma M \frac{\partial u'}{\partial x} = k$ times, so this is the energy let us call this 1, let us call this 2, and the k is here. So, now,

the issue is what we are done, so for is to derive the non-dimensional equation for momentum and energy, and as I mentioned for low base flow values. You are continuity equation can be later solve for to get rho prime, you do not have to solve it in a couple manner, that is the first thing now the issue is how to get the solution in time domain will stop here.