Acoustic Instabilities in Aerospace Propulsion
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Lecture - 21
Active Control of Thermoacoutic Instability

Good morning, we will looking at a ducted system where we had premix flame at the given location and then we solve for the stability of the problem.

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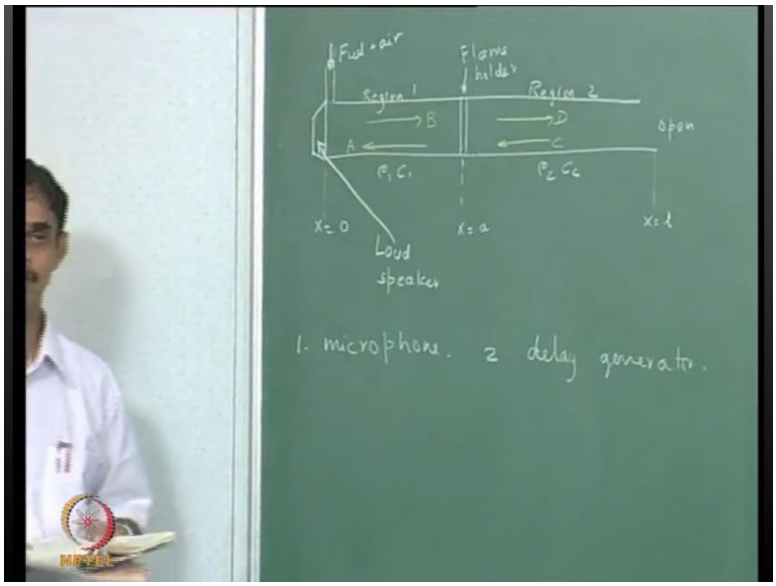

So, in this case, we had a flame here and we had a wave system on the left side and another wave system on the right side. And we actually try to set up the acoustic propagation equations on this side and also on this side which is our classical solutions. And we had a boundary condition which is closed here and this is opened here now. And we solve for the Eigen values and little bit of the sign of the Eigen values we had a Eigen values had a real part of the imaginary part.

And we showed that the imaginary part shows the growth rate on the d k rate and we could solve for that. And we could find out under what conditions you would have growth under what conditions we are d k. And all these assume a simple n tau model and of course, we plug in values often in the movement. But in reality n and in reality n and tau will come from some physical modeling based on the fluid mechanics and combatant

principles which we did not account for. But assuming that you know that then we can actually do this modeling. And now having d1 this come this for I will give you a home work problem on active control so very simple problem.

So, what we wish to do is to replace 1 of the boundaries in this case it easiest to replace that the close boundary with like a loudspeaker. So, we let us say we keep a loudspeaker here and we make a vibrate in some manner which is convenient to us or which is such that it will make the system stable. So, how do we do this? That is the question. So, if you if you look at our calculations, we had solve for the imaginary part Eigen value. And we had a suppose we had a imaginary part which is giving a positive growth rate. Then we have to make this system such that this loudspeaker will input something. And therefore, that would make the imaginary part change the sign flip the sign and a growth will come to a d k rate.

So, that is the idea now in the limit of things is there was no flame and if we put a loud speaker the oscillation will grow right. So, if we if we do not have flame and if you have a duct it is a naturalist able system there is no driving no dumping. So, you put a loudspeaker it is it can grow, but on to contrary if you put the loudspeaker. And if you play around with the delays then it is possible to make the system stable and that is what we are going to work on. So a delay is.

Student: Then the sound is release stable.

If you release at sometime later it will B the face.

Student: Face.

Means going up coming down many flats so, we can adjust the place.

Student: How do we know up?

That is what homework is our so we can So, the so this is like a very simple way of having a active control. So, what we mean by changing the sing of the imaginary part of Eigen value this is a formal name for this anybody know this pole placement pole placement control we have force and push it to the other side.

So, formally in control there is call pole placement control I am know expert in control theory. And this pole placement is most in rudimentary form control and practice the very advance controls such a control LPG control and so on so for. But we can we will still in the class room we can do simple small things and we try to illustrate things. So, the 4 key elements; 1 is the microphone which will sense, because if you have to do some kind of active control. W have to know what is going on to control anything if it is a feedback control that means it is like if you are studying well I need to I do not need to something But if dates are going down. Then I have to teach better or or yeah let you more loudly or something other. So, it depends on a performance what my intervention should be everything is fine then maybe it is to stand back and watch everything.

So, the same you have to put the microphone for example, everything is stable and then you turn on the loudspeaker and then the sound will come up right we do not want that. So, we have to know what is happening, so we definitely need a microphone. And we need a delay generator; yes Rajesh pointed out. We need to be able to adjust the time advice the sound comes in and so on or the phase of the sound it is not like we are not having pulses we are having continuous sound wave. So, it is more like a distinct the phase and we need a amplifier, audio amplifier 4 we need a actuators and so on. Actuator would mean something will which will accurate and make sound. So, in this case in this crude example I was saying we can use a loudspeaker; yes tell me the problems is I originally had problem where we had a close end here. And I was serving let say instability under some conditions we showed for when the instability can come for fundamental mode and when it can come for third mode.

So, I have a unstable system now I want to make it stable actively. Actively means I interfere with it middle with it passively would mean how would I do it passively maybe that is a good question ask. I have some have some dampers or 1 another thing would to do would be like where there is a special maximum put a hole. And then so at moss here tries to bring the pressure that atmosphere. But you want to maximize there is a conflict or yeah I have put a put a damping mechanism I think or maybe I think those are the move different location that would be another possibility I mean so you are not. So, that is passive control you do something, we are not interfering say everything is active you have to act to move it or to put to a damper, but we are not acting in the time scale of oscillations But active control mean that we are oscillation of the order of let say if you

are 1 100 Hertz oscillations the period is 10 mile seconds within the period we are middling with it as it is going on actively messing around with things so that is more likely.

So, I want to actively mess around and the any elementary control things should say that we need a plant. And if I speak the control jar again as a plant is our plant does not means some big cement factory or something it just is the device, which are trying to control which is our thermo acoustic system. We need a sensor is to find out what is going on if do something without knowing what is going on I think. Then you are doing wrong thing for example, you all came late the 2 possibilities. 1 is you are slept or may be the previous professor left you late. So, if I assuming that you are let say you are studious students. And the previous class you are aggressively taking notes and this everything professor saying. And you come here and I start telling at you how dare, you sleep to 9'O clock

Then actually you will be very upset and I am really spoiling whatever motivation you have, but on the other hand if you are actually sleeping. And if a assume that we are really working hard and all that. And then I say wonderful how wonderful guys you are that is also bad because I am appreciating your sloppy behavior. So, I have to sense what is going on so without sensing any intervention would be I think it is like a going to sometimes a going to hospital. And I want to tell the doctor the problems, but even as 15 percent of the problems they are writing prescriptions I mean you must had this experience they do not want to listen your. And they know everything about you these are not sensing.

So, we have to have accurate sensing to be able to intervening right that is a correct thing. And then of course, our model should be right we need a model. And then in the model we put a controller and or in a if it is the hardware you put a actual speaker and so on. So, we need a controller and the controller strictly means the algorithm base control. So, you have you get a equations you workout in this case, because we are doing the problem on the board. And then you say we have to have the actuator vibrator this phase delay and so on. So, that that is we call controller, controller does not really physically means some box which controls, but it is that control action which are the control which is what is generally referred as controller. And then there is a actuators so actuator is like

in my case they previous example of you coming late. And I am screaming at you the actuator is my vocal codes and so on so that is the actuator.

So, you have to have sensor actuator and the plant and that is basically it and you should actuate correctly. So, if you for example, you came late today, but tomorrow you come on time but instead of yelling today if I yell at tomorrow you will think that I came in time. And this guys yelling at me so there is something wrong and then you loss motivation where as if I yell at you may be oh I am slack in searched workout. So, the timing is very important for example, everybody goes up and down. So, when you are messing up I should get aggressively yelling at you and you are doing well I should say oh wonderful keep it up and so on. So, that is a modern management strategy.

So it is a same kind of thing in the timing is quite important, because if you give a another example I go to the park with my daughter I use to go now she is will grown up she does not want to come with her any more. So, she is swinging and I sometimes ask me to push and I have to push it the right time if I can also push at the wrong time. And I am sometime warning do away and I push at the wrong time and the swing will come to stop. But same pushing action at a another time within the cycle will actually keep the swing going even further. So, everything is put a timing I think in life also so, we have. So, microphone is sensor you can use a condenser microphone or piezoelectric actuator any such thing. So, this delay generator and amplifier together make the controller with and it is do whatever we asking it to do. And this is the actuator which is the in this example which is the loudspeaker we can also have piston oscillating.

And in of course, in a combustor it would be very occur it put a loudspeaker in a combustor system of such thing. The sounds levels are very high the loudspeakers cannot take it and this second thing is loudspeakers do not have the author. I mean it d1 produce that much sound the level that is required to stop this oscillation because they are very loud. So perhaps it is better to have like a mentioned earlier secondary fill injection you injectual such that the heat least coming out of that is out phase of the pressure. And then that creates acoustic damping; yes Anveeksha you had a question? So, this is a very, the question is in terms of active or passive control which is a lesser costlier option of course, costlier option and not costlier option depend upon what are the consequences. So, it is a very deep question, what you asking.

So, if you where to put a actuator first of all you have to make a control law. And you have to use analysis and then you have to put the hardware mount the hardware it should be tested. And it should work for so long it is a combustor it should work for like several years without this power there that is sitting a power plant which makes power. Let us a or in a airplane it should work for several years without having to do anything at all. So, it would be very reliable and any time ensure reliability it is expensive and so on. So, in fact actually in if you look at simple case of land based gas turbo engines I think there have been some cases where they install the active controller. But as have know to my knowledge it does not exist, because reliability is a issue and to ensure reliability is a very expensive. And then I think the engineers and the managers would go for reliability simplicity and so on. Now, if you speak to control guy he was say that yeah I mean passive control is not of active it will be effective only for 1 kind of thing 1 kind of frequency and so on.

So, they would advocate active control, but so active control is very good, but can you ensure 100 percent, 100.00 percentage reliability and marginal economic cost only, So, if you can and if it is necessary for example, you know that we are plying unstable aircraft without any problem. But you would not put that kind of you want to make the aircraft unstable, because you want to be able to control it very well. So that why, do want to control it very well, because you want to out many our enemy. So, it is circumstance everything is in the context. So, there it is a pressing need to out the other guy and then you have to have proper control lose. And so, system should be well understood and it should be very reliably be control so if that is not possible or in a commercial airplane you auto pilot that and so on. So, there this system is understood well enough and it is repeatedly performing without any problem and the cost does not seem to be going up and so on.

So, managers have confidence in it so if you have that then so, it depend on cost means if you can get it to work some work a 50 percent. Then cost may not be work much if you can make it work 80 percent cost will go up. But if it is 90 percent reliability it should work 90 percent time even go up if it work should work 99 percent time. It will even go up, it will work 100 percent of the time then the price will be very high So, then the passive control of course, it also as surprise because you put into surplus 50 Hertz

oscillation should want to anything with the 200 Hertz oscillation. And it want to any with a 5000 Hertz oscillation and you have to put something else in.

And then it is also psychological, because I mean whether you if you told the passengers at the airplane may be running at fly something then they may not get into the airplane It so, I think the managers think that it want to reliable, because if the things starts down if you are replays it takes a like several hours to work. And that creates lot of problem because power station there operating at capacity. And then you have to studdy down for several hours and they do not want to get in to this. So, as of now it seems like active controls perceived as a expensive option and reliable but, reliability again how much reliable. So, if I if you would come to class reliably that means whether you coming exactly 9 o o or some people come at 9 o 5. And some come at 9 o 9 o 8 some other people come a 859. And of course, you think about the effort there putting the fellow was coming at 859 always or 858 there actually concerned right from the morning some other get up the thing I should been here on time.

So, there paying a price for their reliability where as there was a class now I have to just go there. So, it by the time compare is 9 or 8. So, you are not having to pay the patience in terms of paying attention to this effort to come here in time so anything involves a effort now. So, this is 1 basic thing, but you want beyond that the other question like is the system reliable understood. And then again any 1 make a terrible say that this theory is wonderful and everything is understood. And everybody else may not believe it believe in it and can you actually reliably implement it.

So, I was spoken to many engineers running these things thermoacoustic instabilities are rarely problem in certain rocket motor. But it is just not amenable to active control, because I mean motor fire for 8 seconds and hits the target or something like that I mean you. And there is no possibility to put any active control there liquid rockets perhaps, but is not d1 yet. And the there may be possibility. But it is all passive in land based gas turbines they have d1 it some of the big companies, but all taken out I think it is not the animal

And so I think in airplane engines it could be may be some years later may coming, because I think if it comes will come first in aero plane engines. Because see you push the combustor performance beyond some limit. I mean there is a envelop is push by is is

created by various factors 1 of the factors is the instability if you push too much into it there is a increase a loading too much it may go unstable. So, there is limitation which is a provided by this and then if you can overcome that and give the piloter extra advantage. Then I think a people go for it But you have prove that we will bring the aero plane and the pilot back safely because you spend billions on both these person and the machine.

So, it is a very complex factor rewind by technology theory practice and economics and so on. That the land based people say that the actuators just not reliable enough that they cannot work several years without having to be able to replace it and they do not translate. So, these are the conditions now, but suddenly it can change it is very hard to predict long term technology changes and the short term only we can say. But these are some of the issues there is no easy answer for this and the answer depends on who gives the answer any supplementary question no. So, we will draw a block diagram for the controller bay with these components.

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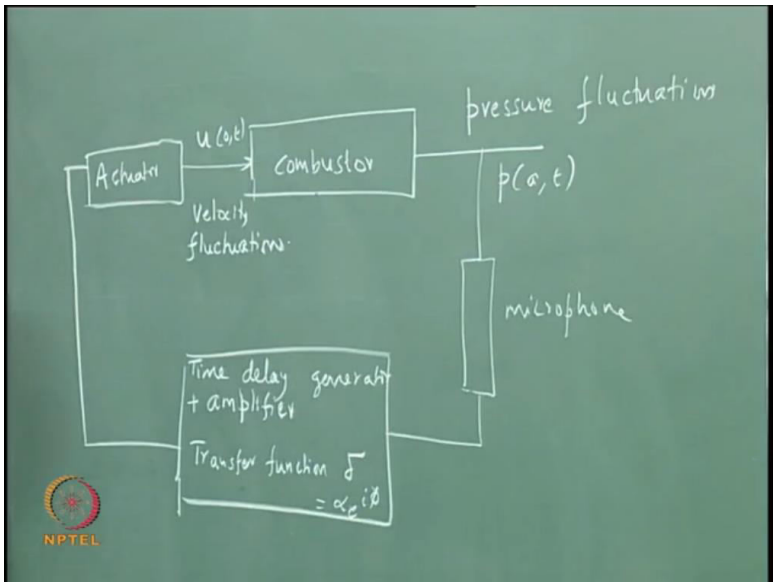

So, there is a combustor which in the control jar and would planned you are sends the pressure fluctuations that is a so we have let us a we sends p at a comma t. Then we have a time delay generator and amplifier. And we will have a transfer function for it then speeds into a actuator it impose the velocity fluctuations somewhere in this case. We can keep the loud speaker wherever you want in this case have kept it an end impose

velocity. If we have combustor we sends the pressure in the combustor somewhere So, let us a around the plane with the microphone.

And then we look at the signal and delay the signal and also amplified adequately and what is the adequate we will see. And then we have to have relationship for the how much you amplifier and how much you pressure. And then you get the signal and force a actuator which will impose and velocity fluctuations. Because any loud speaker will actually here impose a velocity fluctuation, but I say told you there can be other kind of actuator also. And then you feedback the combustor here in this case we do it through the loudspeaker at the end. So this is the what is call the block diagram it is clear? So, if I go back here.

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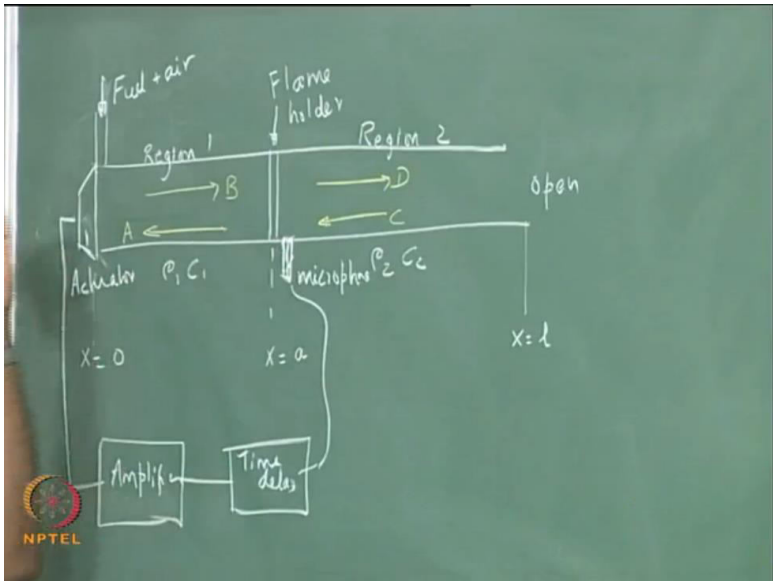

So, I locate my microphone somewhere here and then we have to have a time delay and then we have to have a amplifier and this is our actuator with it is the loud speaker. So, this is the system so we will proceed to make a simple mathematical model that is our ambition. So, we need to find a relationship between the velocity here and a pressure that we send here. Because we are sensing this pressure and inputting a velocity based on that by a some kind of delay and some kind of amplification. So, those things can be so, we we define a control transfer function.

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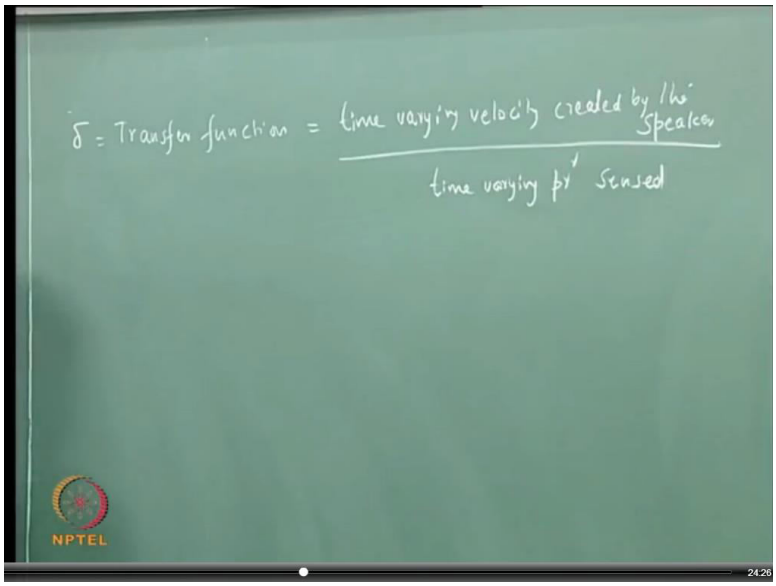

Delta which is time varying velocity created by in the speaker divided by time varying pressure that is sensed by a sensor. Of course, you have to be careful to put the microphone at place where you can actually measure pressure because if you put it at a pressure minimum the pressure note. Then you may not sends anything we have standing wave and some places a large amplitude some other places you have small amplitude. So, you have to locate the sensor wisely yeah you had a question yes yeah that is what I am doing. This is the example I mean you like anti sound you have like. So, I have sound in the system and now I put a loudspeaker in there and try to shutdown the sound. So, this is active control I mean I did not quite understand loudspeaker example of not of active control it is a example of a activator. But you are wanting; you can state your question.

Student: Sum for a active example of loudspeaker.

So, this is like a time delay based controller. So, this the whole system together is active control.

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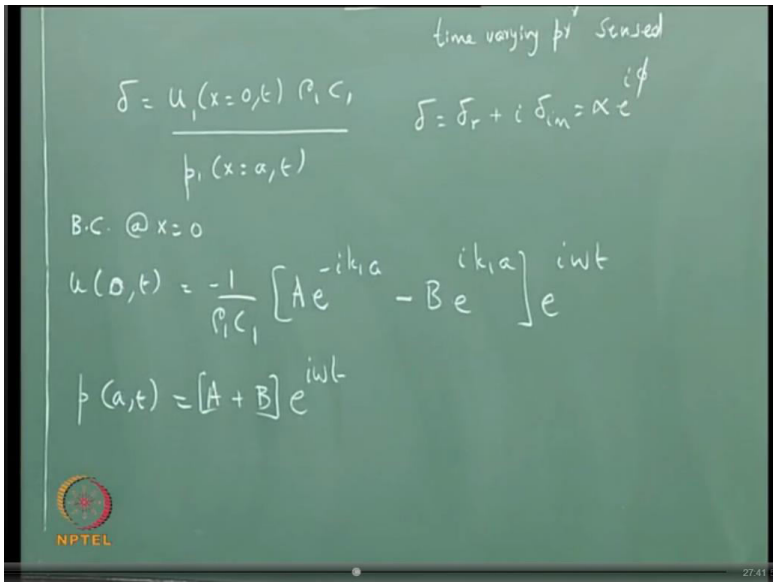

So, this is delta and it is a complex quantity so, the crux in the matrix, the boundary condition at x is equal to 0 is now different. So, this boundary condition is not closed than any more it is relation between it is some other velocity. So, we look at B C x equal to 0 so u of 0 comma t is equal to minus 1e over rho 1 c 1 A e power minus i k 1 a minus B e power i k 1 a times e power I omega t and p you have to remember that the sensor. And activator need not be at the same location we can choose to keep your sensor wherever you want you can choose to keep it a activator where you want that is your choice. So, here I am deliberately choosing a location which is different p a comma t what is p a comma t it is a plus B time e power I omega t so and I have a relationship between delta.

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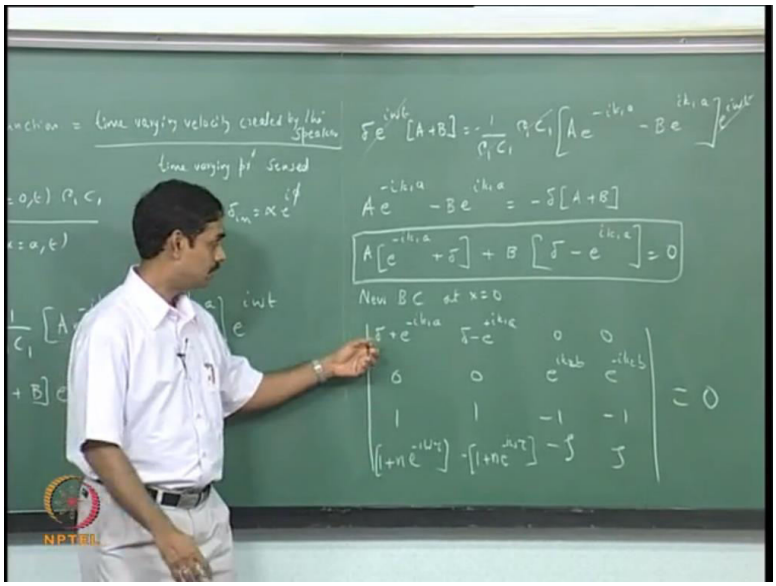

So, p 1 delta is u 1 right. So, that is the relationship that I should calculate so p 1 delta would be delta times e power I omega t time a plus B equal to minus 1 over rho 1 c 1 times rho 1 c 1. So, this is to make sure the delta is non-dimensional. So, the rho i c 1 factor will go away times A e power minus i k 1 a minus B e power plus i k 1 a times e power I omega t which you can cancel and this will also go. So, you will get A e power minus i k 1 a minus B power i k 1 a equal to minus delta into A plus B. So, you will have a times e power minus i k 1 a plus delta plus B times delta minus e power plus i k 1 a equal to 0. So, this is the new relation so we have 4 equations and those equations 1 there which was A e power minus i k 1 a plus minus B e power plus i k 1 a equal to 0, we will have to replace that with this 1. So, this is the new boundary condition at x equal to 0.

So, if you assemble the new matrix you will get i will just write the answer. So, we have to this the four equations we have 3 of them are same only the 1 corresponding to the boundary condition at x equal to 0 where are replace the hard and by the activator is changed and now you have to. So, this will be like this matrix times a B c d will be 0. So, determined of matrix which is what I have written here should be 0 and that will give a equation which relates the Eigen values. Now, we want the Eigen values such that your growth rate is negative. So, now so you will get a relation a dispersion relation for the Eigen value just like you had derived on earlier when this controller was not there right I mean earlier this delta was 0. And then we could relate derive a relation right everyone has very blank pages can you see your notebook.

And you such a relation was in did derived do you see you see this matrix and all that no at least the pg students look like they have seen. So, so similarly, we can get a relationship for the and now it will involved delta and n could n is given tau is given. So, you will have to tweak delta such that imaginary part of omega gives decrease rate not growth rate. So, that is the idea so leave it, you to do it as homework you can work out controlling the first mode and the third mode. And we can discuss the results some 2 classes later or something Wednesday, we can discuss the results, but please do it other ways all the discussions will be pointless actually.

Student: Sir.

We are adding u prime at pressure maximum. So, its high pressure measurement u prime is one more 0, but can you see that it is the minimum I do not know. So, we have to work it out so what you do is you can actually saw for a B and c in terms of d or something like that. And then you plot the standing wave and you have the jump conditions and all this. So, you plot the Eigen function and let me know what the answer may be on Wednesday you can show me. So, this is very important question so if you are if your n is very small and your delta is also equally small you would not be changing that. But if you are n and delta are big. You may actually we turning that changing that you can also workout this question without the flame which is probably what you are interested will if you put a driver will change the standing wave I think it depend on how much you drive.

So, can you volunteer to work the solution out and throw this if you have need help you can see me. I will explain how to do that is a very brilliant question. Thank you, any other questions? So, is it clear? What the homework is? It is clear Vishnu cool it is peaceful right. So, will take some time perhaps to figure out which is which but please do this and come back then will discuss. So, I will take a momentary break from this but we can discuss this on Wednesday.

So, this n tau model that can also be like your relating the velocity and the heat release rate. So, a control person would say that does it transfer function between heat release rate and velocity. So, when we call it n tau model but that is what basically it is now these are all this analysis was d1 in frequency domain right not in the time domain. We actually solve for modes and we are solving for 1 mode at a time. So, this would be call model analysis right in frequency domain that is what we did model analysis.

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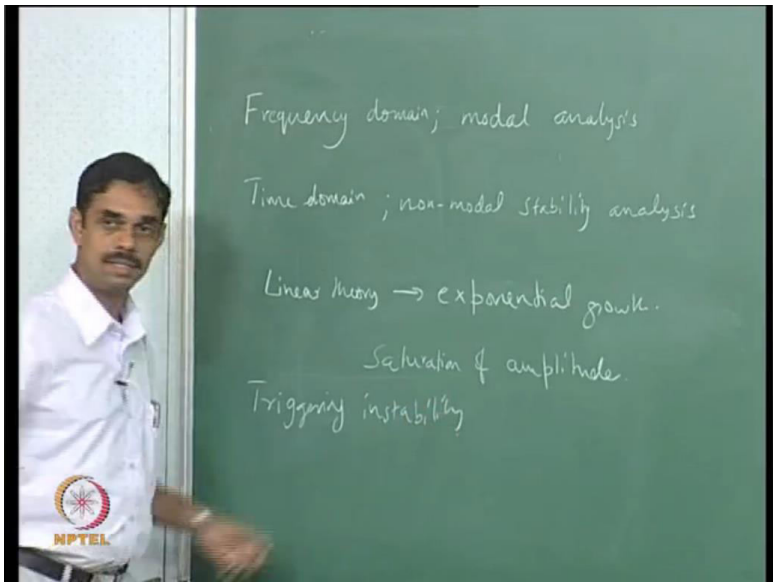

So, there is a alternate set of analysis in the time domain. So, the way things are setup and frequency domain kind of forces you to do stability of modes and in time domain. you are not constrain that way. So, you can solve for all the modes together and but essentially frequency domain analysis is much more simpler and easier to do then time domain analysis. So, which is why people preferred to do that, but when you go to time domain you can actually afford to do a known model analysis will this B f any use will have to analyze. And see now when you so all that our theory did we did basically linear stability analysis in the model frame work. So, we can get a frequency the periodic component and the growth rate that means oscillations are growing indefinitely. But in reality it wound nothing will grow indefinitely.

I give you example suppose you somehow got a new job or something like or found way to get new income in the begging the income will go I mean you are getting lot of money and you are in figured out waste spend it. But soon you will know a new things to buy a i-pod and I do not know what are the ipad, iphone. And all this things will drain a various phone an in case you are a humble fellow you do not spend all this you know need all these things your friends will find way to spend your money. They will pil on you or your family would take the money and blow it. So, eventually what happens it you are your growth this initial exponential growth. It will take some time for your family members or friends to figure out that your money.

So, at that time you will have this exponential growth your wife may not know that use full as so much money or your children may not know or your friends may not know. But after some time delay tau everybody is going to or some people know in time lay tau 1 for close to you some other fellows will know to sometime delay tau 2. And now suddenly after some another time delay tau 3 this people are not close to you. Now will start becoming close to you in another time delta tau they will suddenly become very close. And adding this tau 1 plus tau 2 plus tau 3 plus this delta tau they would starts quisling you taking out the money.

So, eventually what happens to your exponential growth? It will stop growing exponentially growth little come down eventually what happens you can have 2 possibilities. One is you may saturate out and you may have earlier; you are having no money at all or every time you had like 5 rupee 10 rupee kind of thing. From that you are now 100 rupee of tau 1000 rupee or 10 thousand or billion like I heard that Amitabh Bachchan had 93 crore. And still he was banger I cannot imagine how he can be banged up having 93 crores. But whereas, I mean I knows I do not even know much crore is I am feel very rich.

So, it is all perception it all depends on what you non-dimensionalized with if you non-dimensionalized with the amount of money you had yesterday you may look good, but if you non-dimensionalized with Amitabh Bachchan that was 90 crore. Now, we are talking about 200 thousand crore and think 20 thousand crore and all those things which certain people have. So, compare to that I mean what is 1 lakh or 1000 or 10000 nothing. So, it depends on what is non-dimension less you will see that for example, you have u prime; you can non-dimensionalize with u bar it look very good non-dimensionalize with c it look very small. So, depends on what you non-dimensionalized and is a point behind what non-dimensionalized with. So, so we talked about this growth now there are 2 possibilities the one is hopefully you will saturate out something somewhere other possibility is you can go back after some time with the exponential d k and asymptotically become very poor.

So, this is like a transient growth a both possibilities exist and these our linear theory would not predict. Because linear theory deals with linear things and these are non-linear affect for example, this front was not close to you. So, you acquaintance now he saw that your money and always suddenly became a friend. No linear theory will not account for

it, because there are non-linear effect, because if you have to your amount of money in your pocket our bank as to grow beyond some threshold for that guy to get that guy to get interested in your money. So, it is like let say amount of money is m and he will active he will be active only if m minus m threshold should be greater than 0. So, such things are non-linear right I mean then so this is this function. So, there is some function which is acting on this kind of thing those are non-linearity's and our linear theory would not predict any of this kind of term.

So, you can have these things plus now let say you are a quite fellow poor fellow nothing happen. And yours having your day to day existence as a fixed point no money and some of barely leaving without making much noise. And suddenly you get a lottery bang so that is like a suddenly some initial condition happen and everything change. And when you have rich then all this hope same mechanics you suddenly have lot of money; you are reached high aptitude then lot of people are going to come. And take the money out and eventually you can reach a stable place hopefully or can go to kenotic situations. So, you can go to reach different attractors. So, this would be call like triggering.

So, in a combustion situation so what I first mentioned about saturation. So, we have linear theory giving exponential growth. But this was stop being that and some kind of saturation of amplitudes happen a possibly the oscillation I go to a elements perhaps now the other thing is triggering instability. So, you have a system which is completely quite and, but suddenly it become unstable or I will give another example the first it was this this kind of behavior was notice and rockets solid rocket motor. And you have a same rocket motor it will be fire 10 times twenty times there will be no problem, but on twenty first time let say instability comes on. And then twenty second to 38 may be it will be quite and then suddenly it may come on.

So, it depends on the initial conditions what is the specific circumstances that day and and some particular initial pulse was there somehow which might go. So, this is called triggering instability. So, this was discovered in the with the solid rocket motor and so this things like triggering and saturation we can study in the time domain very easily and frequency domain it may not be all that simple to look at it. So, these things see you you probably know that those are studying non-linear dynamics they are I mean this things are describe well in that. So, this triggering would be call sub critical transition instability. And but, this sort rocket people saw all this instabilities this kind of

instabilities even before this language of non-linear dynamics was by well established which happen sometime in let 70s 80s and so on.

So, had their own terminology. So, I will try to connect them. So, what a rocket person would say is that system is stable to amplitudes of certain below some threshold value. And above the some threshold value the system goes unstable if you speak to non-linear dynamics you will say that if you are within the base now attraction of one particular attractor. You will go to word that attractor, but you are if you are outside the base now that particular attractor than it would not go there will go to some other attractor. So, these are all same things, but it does describing this same phenomenon different language. So, if you we can actually use the dynamical system theory to advantage to.

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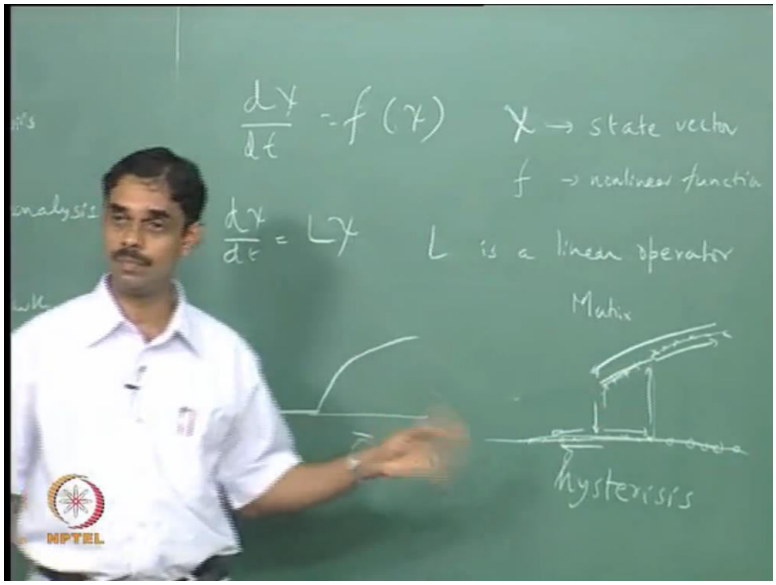

So, we write the differential equation it this form f of kai, here kai is the state vectors if you. So, we have all the variables that you need to describe system and your describe that so pressure at several different point velocity at several points may be temperature several different point heat release rate several different points. So, kai will be the state vector and f is a non-linear function now a linearized version of this can be one is a linear operator if you discretized it you can you can do it to a matrix. So, if you write in this form, we can use all the theories or the tools that are available in non-linear dynamics. And if you simplified by linearizing to this system then we can use the all the theory of linear algebra to our advantage and then study or instability with those things.

So, the earlier frame work was in the frame work of acoustics, but we can now use this framework of dynamical systems. And we can take advantage of those tools and see we can milk more things out of this analysis. And we will see if you can calculate triggering and saturated. So, that is the idea. So, the question is what is kai? The kai is the state vector. So, what it is depends on your view? So, you can think of it only as acoustic person velocity your as I think some of the discussions. We had earlier we can think of a combustor like a flame in acoustic field as a 2 scale problem or a 3 scale problem where you one scale for the acoustic field another scale for the hydrodynamic zone. And perhaps some other scale for the heat release zone and so on.

So, each zone will have to write equations and then mash the inner solution with the outer solution. And then all those variables together will all the variables together will form this state vector. So, now this would be quite intricate and so on, but that is for real. So, if you are talking about the thermo acoustic engine you will have the acoustic field, but there is also the complex hydrodynamics going on this stack which you have to model. So, that will also enter this enter this state vector.

Now, if you are going to use a transfer function approach what you do is we will write like n tau you say q prime is going like n times velocity t minus tau or pressure t minus tau. But here we are actually solving everything in a couple manner in the time domain. And so, what my objective here is to do a simple time domain model simple means you should be able to write the full model try to all this functions by hand and even write the linearized matrix by hand. And then we should be able to calculate this triggering and saturation and so on.

So, we will make up a model power. So, this a difference between a model and the model problem. A model problem is a problem which a lookup which has which is based on physics. But you are constructing you are saying it is that way or a toy problem again toy problem is not in any derogatory sense as some people would take it. But toy problem would mean that it will play with it is a toy you can play but, real model may be so expensive to run that if the run take six months to run. Then you cannot play with it a toy model would mean you heat enter and within a second you have all the results. So, I will make a toy model it construct a toy model or model problem.

And then we will learn to write things in the way the dynamical system people write. And we will try to use tools from dynamical system theory and we will use tools from linear stability theory from the linear algebra and try to unless the system. And so, this triggering would be called sub critical half bifurcation just you explain what it is. So, if you vary some parameter let say for example, if you have a flame and you send more fuel or something.

So, the more heating value more heat is release and let say so more heat release by changing the equivalence ratio or fuel flow rate or something. And let say have a quite condition and then the oscillations come up above this way. So, this would be a what kind of bifurcation super critical bifurcation. But I can also have a situation where everything stays quite and then suddenly jump of and then keep on doing like this. But, when I want to come back so, here I had a stable points all over plays and here the solution, but they are unstable. So, as soon as I come here I jump up here and continue on. And when I come back I if I come back here it would not come back I will have to come back much more and then only it will drop back here and go this way.

So, the hysteresis would be I come this way jump up go, but then I have to come back all the way here and only then I can works. So, there will be a hysteresis I do not know this spelling is right. So, this would be example of a sub subcritical half bifurcation where you go this way and you can jump up, but to come back will have to come back all the way. So, we will we will calculate these things and study triggering the modern dynamic system frame work that is the idea. So, next class, we will make a will write the equation time domain make a toy model and so on, but please do that homework. So, that next week we can discuss when this is over about the active control.

Thank you.