Acoustic Instabilities in Aerospace Propulsion
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Lecture - 20
Modal Analysis of Thermoacoustic Instability - 2

Good morning everybody. We were looking at this simple example; where we had a heat source in a pipe which is located at some location, and we look that the heat source having unsteady fluctuation. And, we module unsteady fluctuation in terms of the velocity fluctuation using the so called crocus entered model. This is a very famous classical model.

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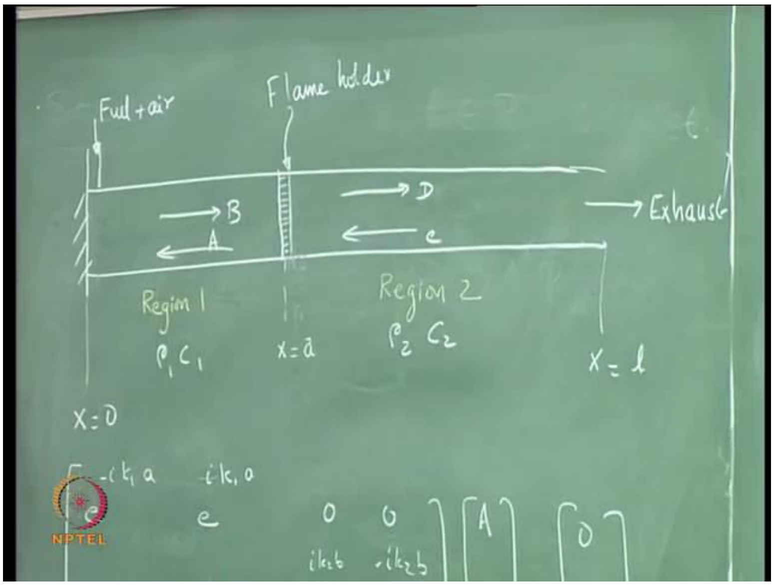

And, then we try to find the stability of the system so, just to recap this is the pipe with length l, and yes ask me this is the pipe with length l and somewhere in the pipe we have in the duck we kept heat source so this is the flame hold as a flame is there. And, we have a region 1 and region 2; region 1 is cold, region 2 is hot. And, we they can have different temperatures and it is a premix flame and coming here on the burning right here and coming out. So, we have a closed end here, and open end here. And, we wrote the solution in terms of this A and B, the left turning and right turning there on the left side in the left turning right left turning there and right turning there on the right side. And, what we did was to apply boundary conditions here that velocity zero, and apply boundary condition that pressure is 0 here.

These is the same as same manner in which we did carter way tube, but then we did something extra, at this flame we apply the jump condition that pressure is continues. But velocity jumps and then, we got 4 equations and 4 unknowns no sorry, in 5 unknowns we would 4 equations and 5 unknowns. And, we discuss why this is ok, why was there anybody remember?

Student: ((Refer Time: 01:59))

So, 1 is the omega itself we are solving for. So, we have Eigen value the problem and get the dispersion relation to solve for Eigen value that is fine. And, then we get the amplitudes we have then, you left with 4 equations and 3 equations so, why is that ok?

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We are having linear problems so, we cannot really solve for the amplitude or all we can get is your 4 wave amplitude we can get 3 in terms of another 1 or something other.

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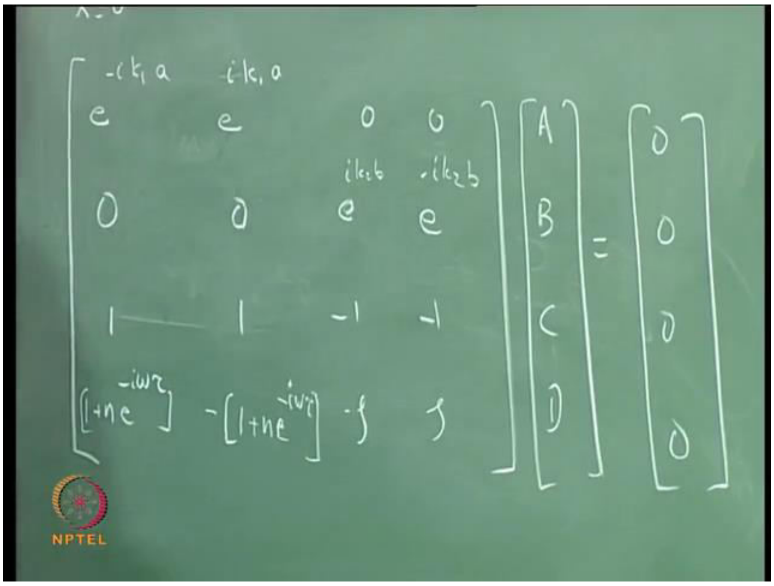

And, then if you put the determinant of these matrices to be zero, we can actually get the relation for the omega which is complex and the real part indicates the periodic quantity and the imaginary part indicates the growth rate or dk. And, we had a relationship of the form.

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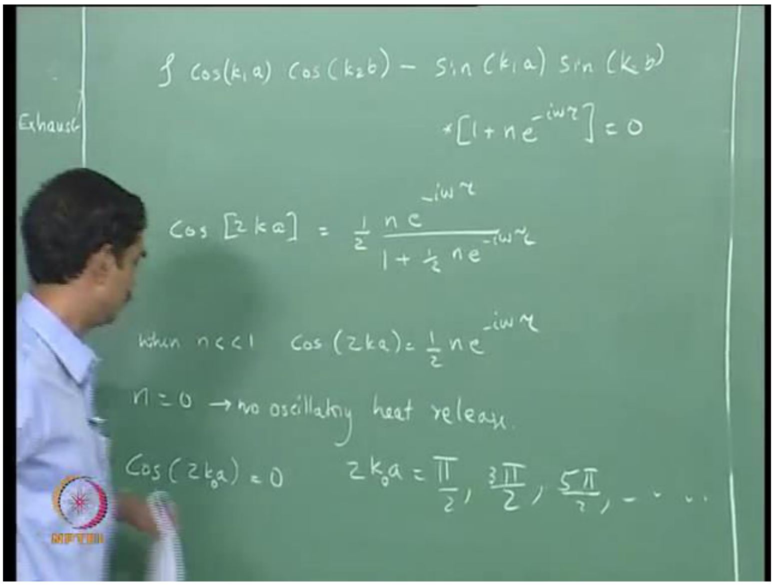

I hope you check this at home, check my calculation again it seems right. Now, in MacManus paper there is a slide differences anybody notice the difference? There he uses e power minus i omega T, I am using e power I omega t I generally like it that way for no reason you can do which ever way you like. And, therefore; some are the in between some different. So, after this we made a bunch of approximations these equations can be solved a numerically, but we want to get some relationships annual degrees so that we can see and fills pleased about relationships and then we can look at equations relationship an understands something. But then, we have to go lot of simplifications so, we made the simplifications that row 1 equal to row 2, and c 1 is equal to c 2 and so on.

That means; we are can to assume me average temperature in the duct which is constant, and then we get a solution of the form. So, k 1 is equal to k 2 equal to k, and we are also assuming that the duct is the flame is at the middle of the duct then, things really becomes simple. So, a will be equal to l over 2 if the flame is at the middle of the duct, it is naught like the flame as to recap the middle of the duct you can keep it anywhere we can get simple answer, because trigonometric functions can be simplified easily.

And, so we there are this relation, I hope we checked it I think this is ok. And, when n is much less than 1, we can get cos 2 k a so, this is quite symbol that we can now play with it. So, the idea is assuming that we know n and tau, what should we calculate? Omega, and in that we have 2 parts; 1 is periodic part the frequency and, 1 is the growth rate that is all objective. And, if you are having growth rate we say it is unstable if you having decay let we say it is stable. So, we can do this in 2 steps; this again similar to all the problems we did earlier where I look that a small admittance and then, I compare it with a open tube process some will have those problems we did so, it similar to this.

So, first let us study the case of without combust ring so, if you have a general result then we should be able to derive our earlier known results from it right. So, let us say n equal to zero, this would mean no heat release no oscillating heat release. Now, I must say that when the temperature jump is 4, you can also get a close a solution and I would give it as homework to work it out ok. Now, n equal to 0 would mean the oscillating heat release so, what happen to this relationship? Cos of 2 k a equal to 0, what would that mean? 2 k a is equal to cos of what phi by 2 is 0 then 3 phi by 2 is 0 and so on. Phi by 2, 3 phi by 2, 5 phi over 2 etcetera. We look at the fundamental and let us see k is so, let us call this is k naught because we corresponds n equals to 0.

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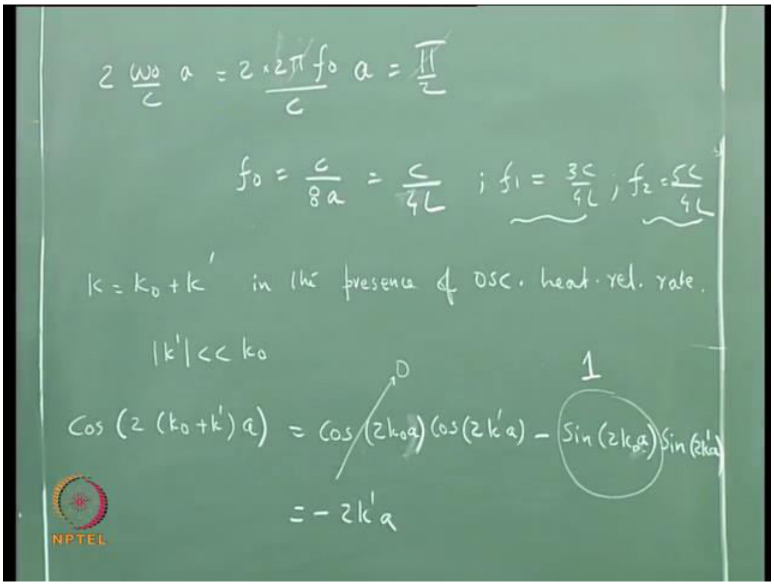

So, 2 times omega naught over c a which is 2 f naught over c and a equal to phi over 2, which it mean f naught equal to let us cancel this phi, this is 4 is this ok. So, do you have

4 or 8 here? 8 it is a, because 2 a is l so, we can rewrite it as c divided by 4 L. So, the other ones you can get it as corresponding to 3 phi by 2, 5 phi by 2, you will get 3 c y 4 L 5 cy 4 L and so on. So, just check to ensure so, if you have this real omega or real frequency, what happens to the growth rate? There is no growth rate would be accurate, but let say in the presence of combustion we have a departure from solution, and we express the due solution in terms of this solution then, may be things may be easier.

So, say k equal to k naught plus k prime in the presence of oscillatory heat release rate. And, we say that n is very small and k prime is very small. So, now we can expand this solution, cos of this can be written as cos 2 k naught cos 2 k prime a minus sin. Now, can you simplified any further? So, cos 2 k naught a would be 0, right. That would be the case when there is no heat release, and we are expanding around that so, we can drop this term. And, what will happen this term on perfect.

So, we would have this thing as just minus 1, yah I am doing first 1 for k naught I mean phi by like if you have look for 3 phi by 2 you will get f 1, f 2 will be 5 c by 4 L. And, we can work out. I will give the 3 quarter mode as a homework well, workout quarter mode and will give the 3 quarter mode as homework. But what about the sin you said minus this is ok so, sin if k prime is very small we can even call it sin theta as theta so, this would be minus 2 k prime a. You can actually do without all this approximations no problem, but with the approximation you can get some simple realer vessels which you may appreciate that is the idea at this I appreciate.

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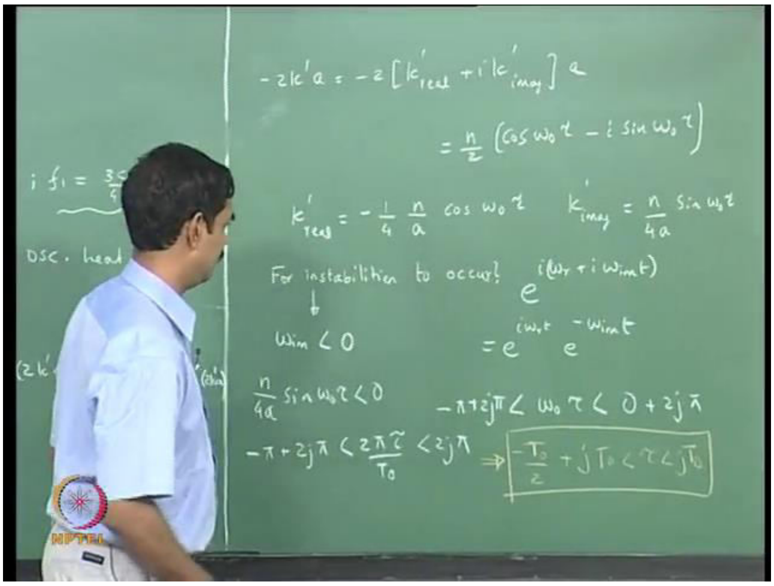

So, if you look at this so, if you expand this; this will be we had this relation for k as cos 2 k a equal to half n e per minus i omega tau again, this is for the case of n really n being really small. This is half cos i omega tau minus i sin omega tau. So, when you have a complex equation you actually have 2 equations; 1 for the real part, 1 for the imaginary part. So, we can say k prime real is minus 1 over 4 n over a cos omega naught tau and hope I have the sins correctly please, said me if any mistakes sins are quite critical above as if there is sin change. What you calculate as growth rate will be actually decay so this k prime it actually departure of the wave number from this natural modes, the wave number corresponding natural mode, ok.

And, we are saying that is the departure is small now, the natural mode has no growth rate would be decayed it is 0. So, if you look at the k prime imaginary from that we should be able to tell if there is growth or not, ok. So, what is the condition for the instabilities to occur that is what we are interested in? If I write this equation which I wrote so many times, i omega real and i omega imaginary t this is equal to e power i omega real t times e for minus omega imaginary t. So, if omega imaginary is negative we will have growth right so, instabilities to occur we need omega imaginary less than 0. So, this is the condition for instabilities to occur so as good so that is of main reason. We should mean n over 4 a sin omega naught tau should be less than 0 of sin omega naught tau itself should be less than 0.

That would mean that minus phi less than omega naught, this would be the quarter end then which the sin is positive from 0 to 180 degree from 180 degree to 360 degree sin is negative so, that is what I have written. But this always the ambiguity of some 2 j phi or something other of 2 phi or 4 phi or 6 phi I can rewrite omega in terms of time period. So, that you can get a ratio of tau over t so, you can multiply by tau naught and remove 2 phi you get this would be the conditional you get for instability. So, for certain time delay with in this which satisfaction this relation, if this occurs you can have the fundamental mode to be that the carter way to mode to be driven. So, it does not have to be delay as to be small, delay can be more than the period and then, it can coming in range, where it drive that is also possible.

Delay is need not be possible so, this is the conditional for in stability. And, if the delay is outside this range, if you does not satisfy this inequality then you actually have decay or it would not be driving now, the actual see we have not include the damping in our model as pointed is last class. So, the actual region will be smaller where you will get instability, because damping always to make thing instable this system stable. So, we have the instability region will be the small and damping is really lot, there are knowing instability a lot.

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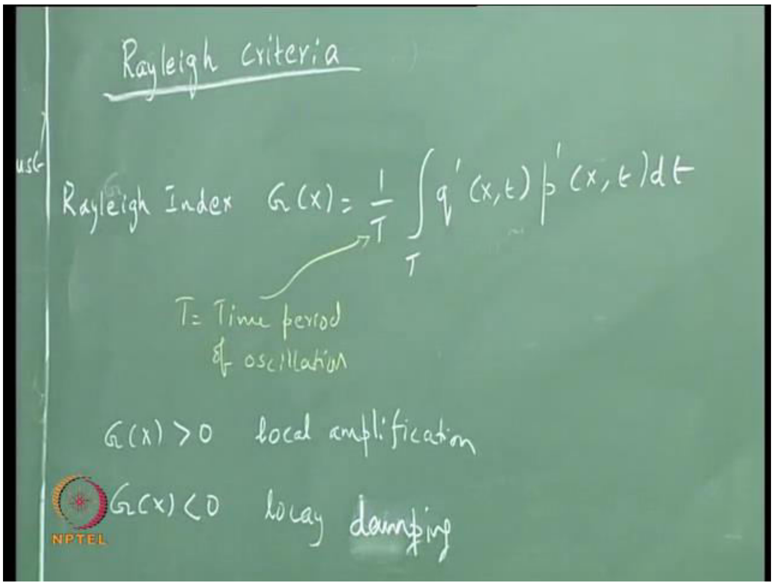

So, the next question I would have is so, we derived conditional for stability, but we also early a mention derived something on energy and like it new energy corny, which have a term which can be driving and damping depending on the phase between the heat release pressure and, so on. And, so this was called Rayleigh criteria according that if the heat release it was in phase in the acoustics pressure. We said I want to have driving out of phase with the pressure; we were going to have damping. So, really critics often use to relocated in often use to quantify it look at combustion instability studies can be actually use this to quantify the coupling between heat release and the acoustics pressure fields or unsteady heat release and the acoustics pressure fields.

And, the criteria say that if the heat release rate is in phase with the pressure, the acoustics field would be amplifier. Now, we can imagine that the heat release rate may that happen over finite zone; we did have lot of discussion in this earlier. So, in that cases we look at, we do not look at does a integral thing in each point you will have a different pressure. You can have different heat release rate, but the heat release rate is locally in phase then, that point wave is being amplified. At some other point it may be out of phase, it may be decaying so, net effect is what drive the system or dam the system but locally it can vary.

So, people define something really index which use to quantify the coupling wave and acoustics wave and the combustion process and it is a local thing. T here is the time period of oscillation, if G of x I mean having x difference says that is the local think it is acknowledgement this is the fact that some place is the less than some other place can be outer phase. So, if G of x is greater than 0, we say local amplification and G of x is less than 0, we say local author of delay, local damp let us say. So, we saw if you look at this the frequency we have to free in between phi 2 minus I hope it is clear have you got again, could I do it again?

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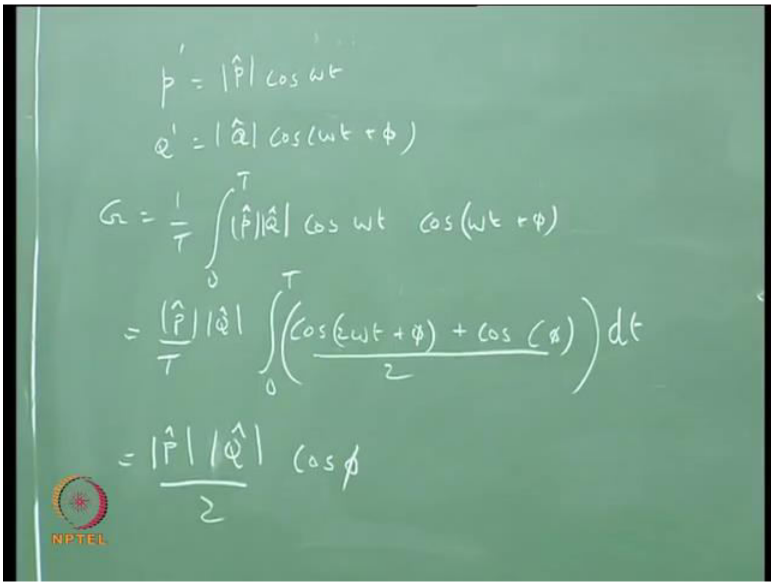

Cannot read why so, we have P prime has p cos omega t, Q has cos omega t minus phi. So, our G would be 1 over T integral 0 to T P prime Q prime cos omega t and cos omega t plus phi, this we can use the trigonometric identity and say; cos a plus b would be cos 2 omega t plus phi, plus cos a minus b would be cos phi divided by 2 d t. And, this is periodic quantity so, that will drop and you have only this so, this is say P hat and Q hat and t will cancel so, over 2 cos phi.

So, if cos phi is positive you will have driving, acoustic drive. If cos phi is negative will have dumping, so, opposite phi will come when this is depending sign on this. So, cos phi is greater than 0 implies phi is like minus 90, pi by 2 less than phi by 2 plus or minus uncertainty of 2 phi or n times of 2 pi. So, like wise n, n already in the interaction j time 2 pi or j times 360 degree. So, we have to see this criteria consisting with whatever we got growth rate and decade that is the idea. So, we can check our consistency or correctness of the criteria that will have is that ok. So, we cannot do straight away because what we have is the relationship of heat release in terms of velocity. But that is not phase radiant says that this is the phase heat release and velocity that is the maximum heat release between, it is phase between heat release and the acoustic pressure.

So, that means you have to find the phase between pressure and the velocity it is depends on the local acoustic. What is the quantity which indicates the phase pressure, what are factors effects?

Student: ((Refer Time: 25:33))

Yes go back to; it is a very good question.

Student: ((Refer Time: 25: 37))

So, answer of this is based on try to look at recall what we did for admitted so on de feel pressure. So, when you have a perfect example; that is close close and open open end, we got all phase difference in percent lost has 90 degree pi by 2. But if you have left running wave and right running wave you always got it as 0 degree, 180 degree right, on case it is for the right running wave pressure was P prime over rouge c, for left running wave it was minus p prime by rouge c. So, if you have a travelling with then, you would not get 90 degree at all, because it is energy flow so, there will be admittance and admittance time's p prime square shows the power flow. Admittance time's p prime square shows the power. So, as long as the power is flowing then you would not get 90 degree and you have 1 or something in between and so on.

So, and the direction depends on which direction the power is flowing so, I mean it is not necessary that you have it at all 90 degree. But accept in the case of perfect terminations then, it is possible like completely open and completely closed kind of terminations. Is it clear? So, this is what local acoustic which decides the phase between heat release and the velocity. It is the acoustic feel in the duck and so it critically depends on so, that gives the idea for example, if you found system to be un stable for a given boundary condition I can alter the boundary condition, and the same with the same tau time delay and all that this is time may be becomes stable. Which is actually the principle for active control which we see some so.

If you can some way adjust the phase between pressure and velocity then I think we can solve the problem make in a system stable, which is the clues towards the active control, anti sound and sound. Very good question any other question? So, we need do look at the express pressure and velocity at execute a, which is the place for flame right.

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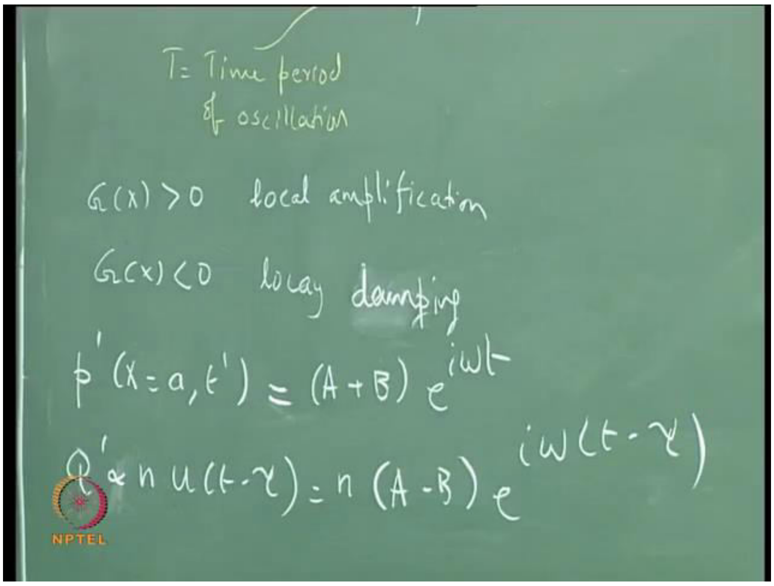

So, what is this, what is value from our last class? So, this is the pressure at this location now, what we need is heat release it which depends on the velocity at this location, but delayed by time tau. So, what will be n times will be right, where rough c an all will get cancel with a fractures and I will just say proportional to some constant. The gamma minus 1 factor is that from is proportional to, and u is this; we put the n and time delay tau is like e power minus of i omega tau it is ok right. We need a relation between a and b that is only thing that is stopping from calculating the phase from P prime and Q prime. So, we are saying at the solution is a small departure from the quarter wave tube. So, we can use the quarter wave frequency into the boundary condition and see we get a approximate relationship between A and B, and then we will plug that in here.

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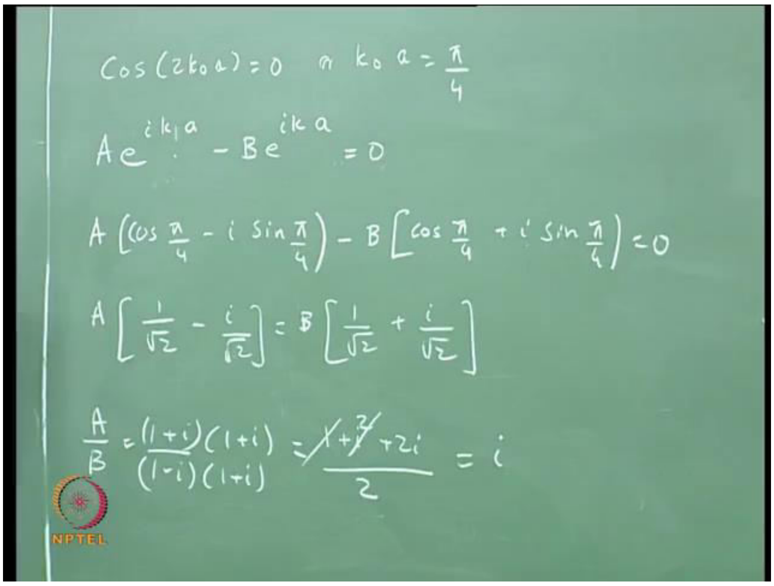

So, our relationship was cos 2 k naught a equal to 0 or k naught a equal to pi over 4. So, this will be boundary condition execute 0 right, velocity is 0. So, what will do? We can use anything we need a relation between A and B so, it is goes too have relationship which you have just A and B. So, this can be written as A into cos pi by 4 minus i sign pi by 4 minus B into cos pi by 4 plus I sign pi by 4 equal to 0. A into 1 root 2 minus i over root 2 equal to B into 1 by root 2 plus i over root 2.A by B is equal to 1 plus i by 1 minus i. If I multiply top and bottom by 1 plus i equal to bottom will be, it would be 2 and this would be 1 plus i square plus 2 i this will go and I will get i. minus i?

Student: ((Refer Time: 31:28))

A e power minus i k sorry, and that for calculation is minus.

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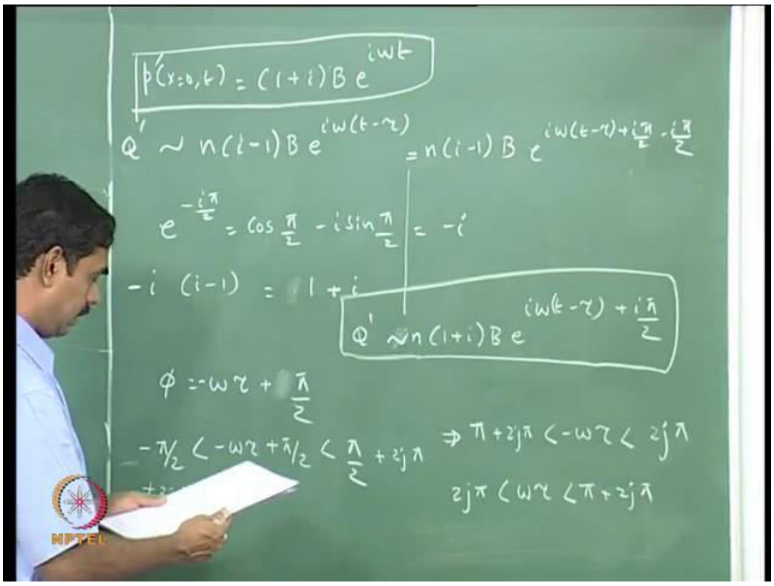

So, we can see p at x is equal to (0, t) is we can substitute for A as B i so, you will get 1 plus i times B e i omega t. Q prime goes like point slows like so will be some constant some factors of some other by the n times i minus 1 B e i omega t tau. So, A as i B so, let us we write this is n times i minus 1 we have to get same factor here only then, we can write the phase here as the direct value, e power minus i pi over 2 cos pi over 2 minus i sign pi by 2 equal to minus i. So, this we like multiply equaled into multiplied by minus i so i minus 1 time i would be 1 plus i. So, we can write this as 1 plus i B e i omega t minus tau plus i pi by 2. So, we have here is expression for pressure, and here is the expression for Q prime.

So, the phase is minus omega tau plus pi by 2. So, this would be between minus pi by 2 and plus pi by 2 or plus 2 j pi so, this would give, what would give phase pi plus i so, just if I multiply minus by signs square. So, this should give I can write omega is 2 pi over t so, 2 pi over t and if I divide throughout by 2 pi I will get so, in the elk either be multiple I stop this. So, in reality there will be multiple ok I will pass. In reality it is possible that that the multiple modes can be pressure we do not know be will come and we can also have the individually we can have, you did not get fundamental model. You can get 3 carter modes as somebody pointed out so, the next thing as we can actually analyze that also.

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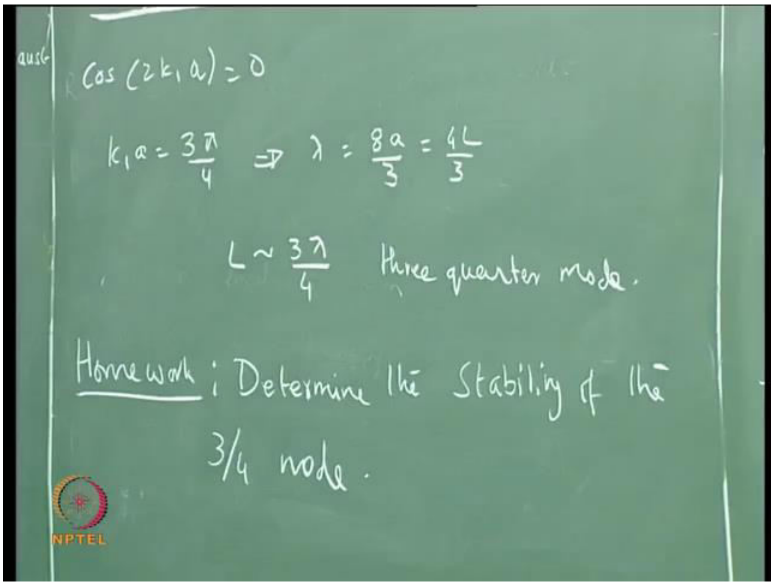

So, if you look at the next mode you will get 2 k a equal to 0, it will be k 1 a equal 3 pi over 4. And, this would be eventually you get lambda equal to 4 by 8 a over 3 or equal 4 L over 3. L is like 3 lambda pi over by 4 this is the 3 quarter mode, the home work is to determine the stability of the 3 quarter mode. So, if you do this problem and look at the regional stability and you find that the problem, 1 would wish is the time delay that creates instability, in suppose you have instability in the first mode and in the change in time delay to something else.

And, there is actually region when the first mode is stable and but second mode is unstable. So, you do something make the first mode stable and we alternate time delay. Now, the first mode is quite but the second mode might come up, this is very similar to the life when we solve 1 problem whatever we did to solve that problem we will create a new problem. So, I thing is like if have to do I have to living barrow money from the bank and business, but then the even the bank takes away all stuff because I cannot payback of the bank.

So, you do something to bad example; you do something to eliminate on problem then that anything create that another problem so, that is quite possible, there is a overlap of overlap region where you have this to possibility to first mode can the stable make the second mode unstable. And, then there is even more complicated things which is come to, but what is interested is now, if you go back to this and everything depend on the phase the disdaining because of the relationship between A and B and we got A was B i, and this critically affecting the phase.

So, can we do something to alter this n condition so, it should having a hard work can be put a loud speaker there and adjust the phase between the pressure and velocity such that you can actually make the system stable. The alter this stability conditions and whatever condition will be what apply so make it a stable that is the basis of active control we look at that in the next class. What will be the involves is, we have to put some loud speaker and what phase should you put you do not know, we have to measure acoustics feel from the using the micro phone whatever is they acoustics pressure some place. And, then you have to find the appropriate phase that you have to put for the upper the boundary condition report so, that the acoustics field dice down.

So, this is the principle for active control, we might have some of the may have both speakers would know how it censer knows the sound. So, what it involves the so combustor you have to put a piece of transducer piezoelectric transducer or a microphone measure the sound? Then, amplify and appropriate the phase system and use a actuator this can be a simple actuator loud speaker, or a piston complicated actuator can be the secondary fuel injection mass addition some delay. And, in the end you want to try system to stability, so we look at that system in next class.

So, in summary given a heat release model we can calculate whether you have stable system or unstable system. Then, how to derived this n and how to derive the tau and that is the cuts of the modeling of the heat release that we are not be such that it is a given thing. And, in reality your model may be much more complicated that there may be other anxiety process such a hide to dynamic instability under that you may not get the velocity frequency. So, the pipe as the, you may not get close to the rational frequency in the pipe, and the actual system may be highly non-linear and we will have to account for all that. But now or less is like the professional looking under the lamp fours for the range. So, the look somewhere and found solution tract solution that also objectives, and illustrated something that is some time delay under which you can get stability and so on, so stop here.