Introduction to Acoustics
Prof. R. I. Sujith
Department of Aerospace Engineering, IIT Madras
Indian Institute of Technology, Madras

Lecture - 2
Acoustic Instabilities in Aerospace Propulsion

As we saw in the last class, we saw that were happening because of the feedback between the acoustic field and the process and combustors and power plants and we also look at the consequences of the aerospace propulsion system. Then, we looked at different mechanism, which are responsible for crossing this feedback and how we could control this oscillations either passively or actively.

So, the subject has is basically because of coupling between acoustic and combust ants, so it is important that we build up a good background acoustics. So, the first part of the class as I mentioned lass time deals with a acoustics. So, acoustics deals with generation propagation and effects of sound we will be speaking generally about the first to last text.

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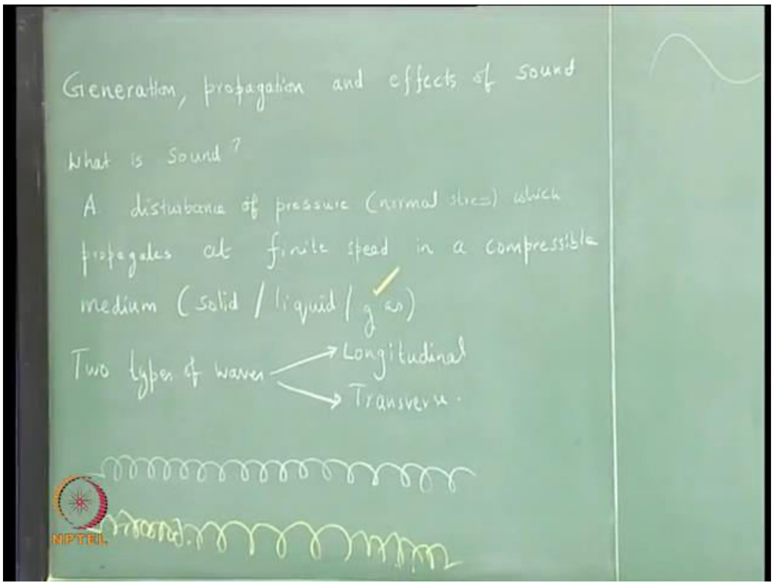

Generation and propagation although a lots of research work on the effects of sound on various things like the subjects like psychological acoustics and so on. Lot of study on how sound effects new born babies, how sound affects the moods of people, how sound how the environmental sound effect the mood of the community and so on. We did not look at that, we will restrict ourselves to generation and propagation, in fact it is much easier to study the propagation of sound rather than generation sound because generation sound is a much more complicated topic. So, it is easier to do easy things first, so we will start out with how sound propagates, then we will look at how sound is generated.

So, I must also emphasize that sound should not be a miss normal consequences in the range of human perception that is we can hear some sound. We should not restrict the definition of sound, it can fall into definition of sound can be a lot more general than that sound could be defined as the disturbance of pressure.

You could say even more general way disturbance of normal stress which propagates a finite speed in a compressible medium and compressible medium could be solid or liquid or gas. We will be mainly dealing with gas in this course let me write down this definition. So, we say that sound is like a disturbance or pressure or even more general stress normal stress, which propagates at finite speed in a compressible medium. It can be solid or liquid or gas, so in in this class we are going to primarily deal with this, so we can also have study propagation of sound and clause mass and also things like photon gas and so on. Sound is definition of much more than general, what we think that is in terms of what we can perceive on us.

So, in this class we are studying how sound propagates in gases because in combustors we have gases, we have fuel and burning and producing product of combustors carbon dioxide and water vapor. So, all this is gas we are looking to study propagation of sound in gases medium, so there are basically you must be knowing you must studied in school that two types of wave they are longitudinal waves as well as transverse wave two types of waves. We can say longitudinal and transverse, so a longitudinal waves is more like series of compression and refraction propagation.

So, you can have a mechanical energy in which you think of some kind of spring which you compress and then the compression propagates then verification propagation. So, if I draw a picture, so let us say I have a spring this way and compress it and I could have reigns, where this spring is very compressed followed by regions, where it is verified. So, you can have compression regions and verification regions followed by compression region. So, this is like the mechanical analogy of sound you can all also have transverse waves electromagnetic waves of transverse wave, but in general a gasses do not support that.

Although, strictly speaking that that segment is true only for inverse force, but in general we have compression verification waves which is the longitudinal waves happening in gasses and what we are going to study about. So, we are primarily dealing with longitudinal wave propagation, we can have another example of transverse waves would have a vibrating string. You take a guitar and plot the string and you take a cloth line and plot the string you can get waves like this.

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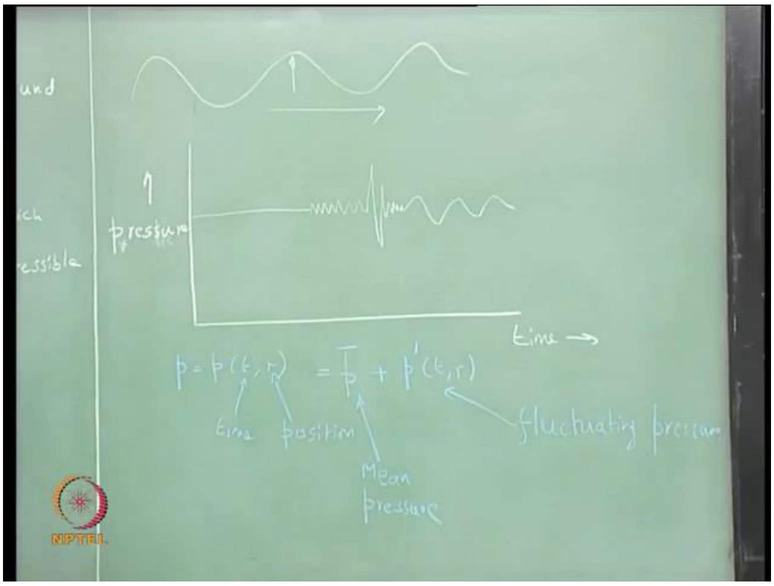

So, here the direction of motion the direction of the propagation of the wave is in this direction, but the particle is moving perpendicular to it. So, that is what happens in transverse wave, in transverse wave you have this particle motion perpendicular to the direction of motion wave in a compression or ratification wave. It is a longitudinal wave, you have the wave propagating and this motion of the gas particles also happen along the direction of the propagating. So, that is the main difference and we in sound is longitudinal waves that is the particle most along the direction of propagation to it. So, that is what we are going to study, so let us look at the air as the medium.

So, we have some kind of iamb impression the pressure here is one atmosphere and we are looking at disturbances about this ambient median. So, if you draw a graph, let us say this is time and pressure, so if you have squares and media that is if you have a silence, you will have constant pressure silence is just constant atmospheric pressure. Then, you could have sound then the pressure will fluctuate and it may fluctuate like this like this or like this.

So, you would have a variety of different frequencies, so frequency is the rate at which the pressure fluctuates and you can also have a amplitude is the how much pressure fluctuate. So, we typically think of silence that is sound is above the silence, the silence is there is a study base process pressure above which the pressure fluctuate. So, what are the different kinds of sounds that we experience in our daily life what is the decimal levels and so on. So, let us look at that, so we basically think of pressure the total pressure in it pressure equal to it is a function of time as well as position time t r is position.

So, we can think of it as base pressure p bar the over bar stands for the mean time average quantity plus t prime stand for the fluctuating quantity. So, this is the mean pressure or the base pressure this is the fluctuating pressure strictly speaking all fluctuating need not be sound. You can also have fluctuating water city waves and entropy waves, but now we are going to study about sound and this is the fluctuating pressure. So, we are only looking at the crystal component at this point, so the minimum detectable pressure typically by human beings by the human ear is of the order of the order of 10 power minus 5 Pascal.

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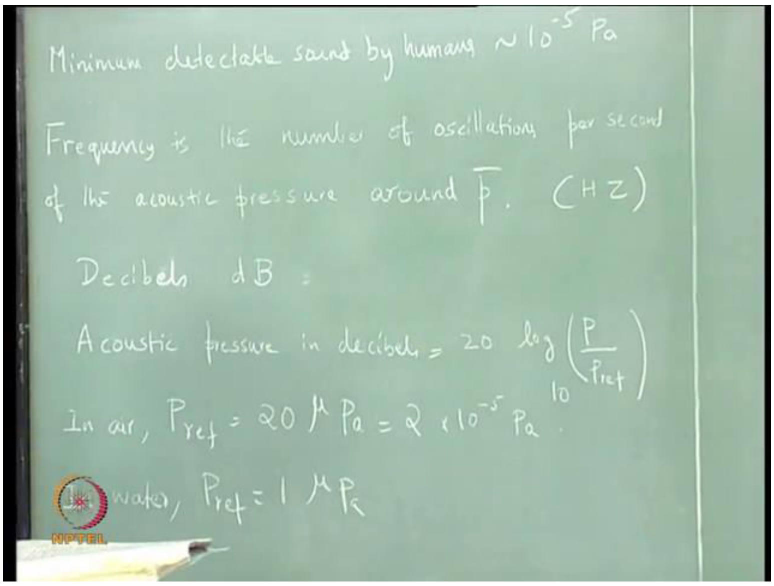

Of course, it varies from different people to peoples, some people can detect more sound then others some people cannot, but typical sounds that human can detect is minimum detectable sound by human, this is the order of 10 to the power minus 5 Pascals. So, let us look at we will look at various types of sounds and so on, so before we do that, we have to look at two quantities, one is amplitude the other one is frequency. So, amplitude is how much the fluctuation above the base pressure so that the amplitude though amplitude and frequency are the key characteristics that define the sound wave amplitude and frequency.

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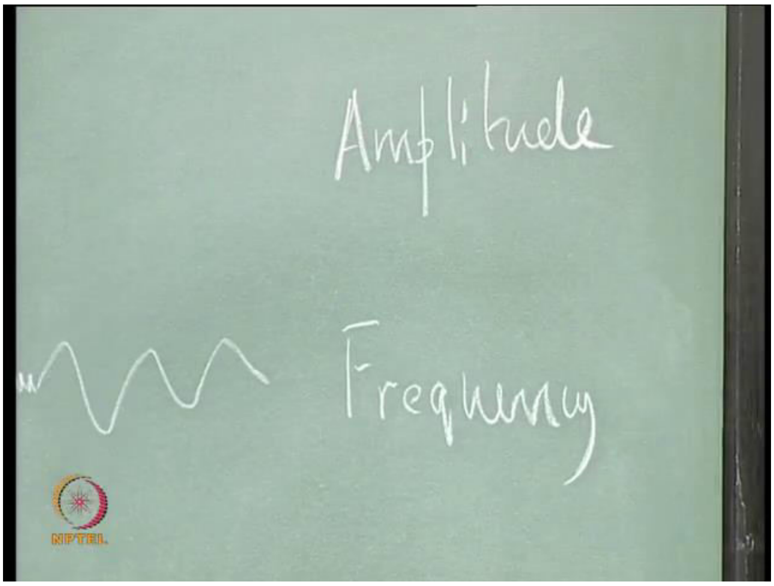

So, amplitude is again how much the pressure fluctuate frequency is the number of oscillations per second of the acoustic wave that happens around the speed bar frequency. So, here we live the term acoustic pressure and fluctuating pressure in the changeably and the acoustic pressure is often defined in terms of log scale. Now, why do we include log scale because our human hearing is kind.

So, if you double the volume in your loud speaker in your steroid phone, you would not perceive it as a volume doubled, so it is more than a scale because of that we have a definition of quality called decimals. So, acoustic pressure is measured in decimals, it is decimal written as dB small d capital B, so acoustic pressure and decimals is defined as 20 log p over p reference to the base 10. So, we take logarithm of pressure divided by difference pressure with respect to the base 10 and multiplied by 20, what is the reference pressure in air the reference pressure is 20 micro Pascals, but is 2 into 10 power minus 5 Pascals.

So, we are dealing with air, but in water we mention that reference pressure is one micro Pascal, I mean it is just a convention. So, this is this reference is a typically epic that the lower sound the human that is the order of a lower sound the human ear can hear. So, here human ear can hear, so next we speak about, so typically the amplitude is expressed in decibels and frequency is denoted in herds, so amplitude decibels and frequency herds.

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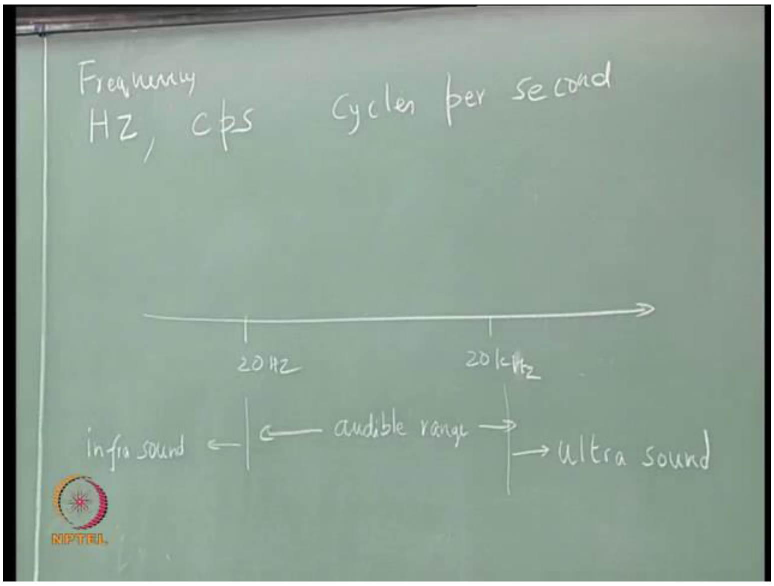

So, herds is same as cycles per second the number of cycles per second frequency is expressed as herds which is also called sometimes cps cycles per second. So, we have different type of sound we are audible sound which are not audible, but are high frequency or frequencies which are very low which we cannot see here. So, if you look at the spectrum, so typical human beings can hear from 20 hertz to 20 kilo hertz, so this is the audible range and below this there is infra sonic sound or infra sound and above this you have ultra sound. So, ultra sound is very important, infra sound and both have application in various fields infra sound is an application in geo physics in terms of when you talk about earth quakes in its measurements.

So, there is an infra sound to detect if in the ecological application were if some cities are buried inside, you create sound wave low frequency sound waves and you try to repair what is reflected of and try to say if there is something like a arch logical site. You have city hidden or something many animals can here have a different hearing range for example, elephants can here low frequency sound some other animal. Here, high frequency sound and humans young people can hear from 20 hertz to 20 kilo hertz, you grow older hearing deter ate.

So, your range can get narrow, ultra sound can also have lot application in doing things like mammogram were people are trying to detect cancer or ultra sound scanning, where you try to look at whether the new born baby is healthy so on. So, we try to do a scan and you since sound see how the sound is scattered and reflected image and look at the picture to get the idea of health of the new born baby. So, typically acoustics deals with low amplitude sound the linear acoustics that mean the amplitudes are very low, but when the amplitude is very high. The acoustic wave would become steeper and steeper and you tend to found short wave.

So, that is the subject of non-linear acoustics in our class, mostly we deal with linear acoustics low amplitude acoustics and not really with short waves. So, just to get a idea of various levels of sound involved, so if you are talking about a calm breathing where somebody is sitting calmly and breathing you breathing itself is very fine. That would be order of 10 dB the normal conversation would be the order of 55 dB, perhaps 55 to 60 dB. We have animated loud conversation that could be tending to 60 dB city traffic could be around 80 dB again. It depends on which city and the loud Indian cities, it many be slightly higher that complain that I would make around eighty dB and lawnmower would make 90 dB.

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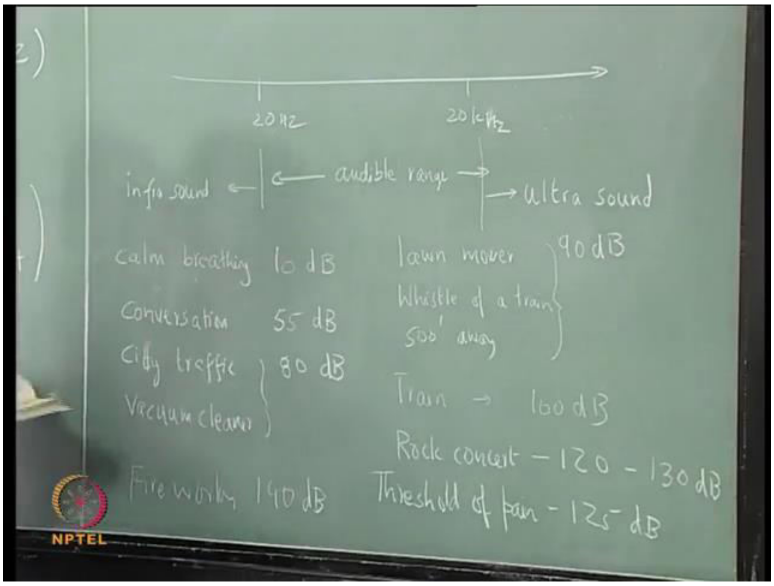

The whistle of the train or the horn of the train if you are about 500 feet away would also 90 dB a train itself when it is moving its probably making about 100 dB and would be around 120 or 150 to 130 dB and your thrush hold of pain. It is when the ear start hearting support 125 dB and fire work would be like one 140 dB. So, this is rough figure, they are given just to give you a idea of the typical sound pressure seen in various day to day situations. I think if you have anything louder than 140 dB, if you hear that kind of sound, even you are in this ear, there is serious danger of losing the hearing.

If you are even listening to rock concert regularly, you are hearing will calm down, in fact it study has shown that musician have low hearing because every day listening to very loud sound in their own concepts and sound. So, there is just a rough idea, this is not our basic interest, our interest is just study not to study about hearing and so on really to look at how sound propagates inside combustors. For that, we are trying to build up a background in acoustics theoretical acoustics, so having said this we need to be able to quantitatively describe the propagation of sound we need.

Of course, we need to quantitatively describe generation also first we are going to talk about quantitatively the propagation of sound, so we need equations for that and that is the that is next half of this term. So, as I mentioned that we have to we are attempting to describe quantitative propagation of sound, so when you want to describe something quantity, we need a equation, so the equation that describe quantity is the propagation of sound is the wave equation.

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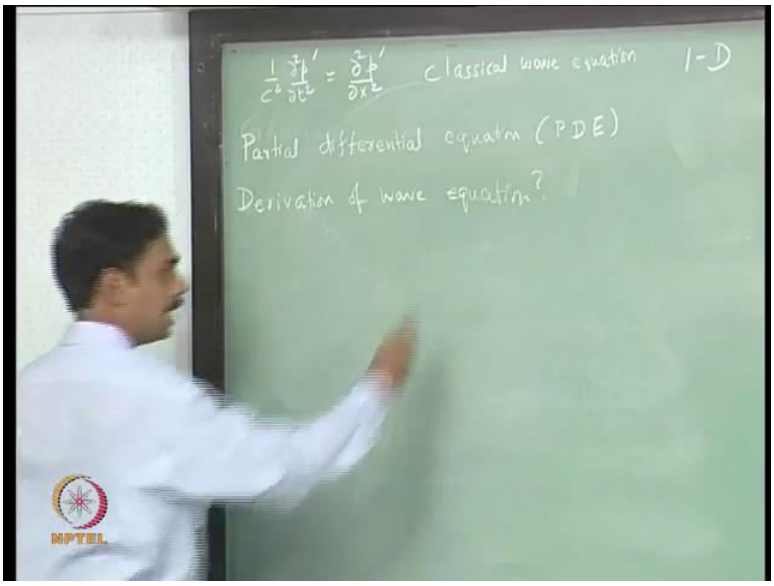

I just want to first of all write it so that you have a idea how it looks like, most of you have seen it. So, this is the classical wave equation, so this is an equation that is being studied by mathematicians for a long time and in very great detail. So, we are going to derive this from a fluid mechanics prospective, this is how the subject actually acoustics started in the first derivation of wave equation from fluid mechanics. Later on, the other process to acoustics came, but in this class we are going to describe it based on fluid mechanics.

In fact, in the 80s, this approach came back strongly because of the success of aero acoustics, since acoustics were you have to deal with the generation sound and to the generation sound you have to study fluid mechanics. So, I think in this class we are eventually going to deal with generations of sound, so we will stick with the fluid mechanics approach. So, to simplify matters I will be deriving this in one dimension, in fact this equation that have written in this one space dimension a x, but we can very easily do it for three dimensional. In fact, I will write down the results for three dimension, so this is a partial differential equation popularly called PDE, partial difference equation.

We are writing a partial differential equation for the acoustic pressure, you can also write wave equation which will be another partial equation for acoustic velocity or the density and so on. Although, writing in terms of the partial is the most popular form, now how do we derive this wave equation that is the next step to derive the wave equation. We will use the basic equation of fluid mechanics, what are the basic equation of fluid mechanics, they are continuity equation momentum equation and energy equation.

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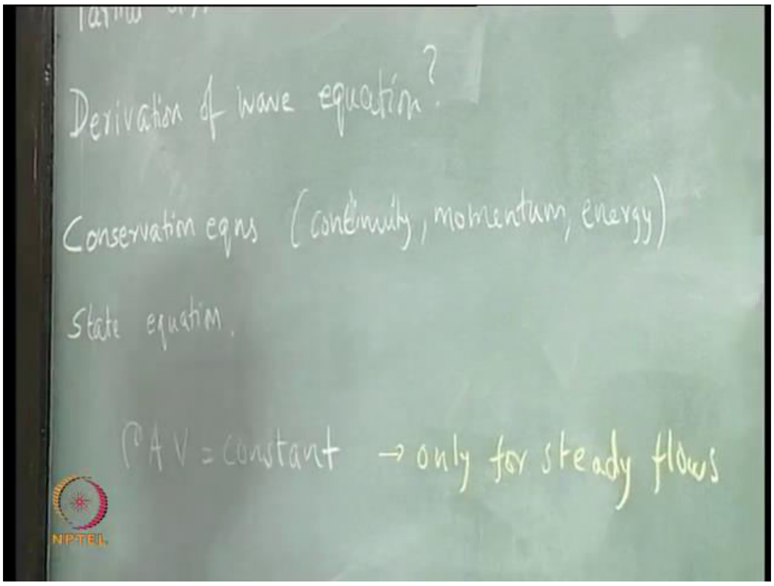

So, we use they are called conservation equations, so for a gas we will also need a stat equation. So, we will use the moment we speak about continuity equation what comes to your mind is suddenly you think row A V is equal to constant. So, I am quite worried about this, because we are condition to be thinking this is the conservation equation, but you are dealing with. This is true only for study flows, only for study flows and acoustics is primarily an unsteady phenomena, so we would be in quite bit of trouble.

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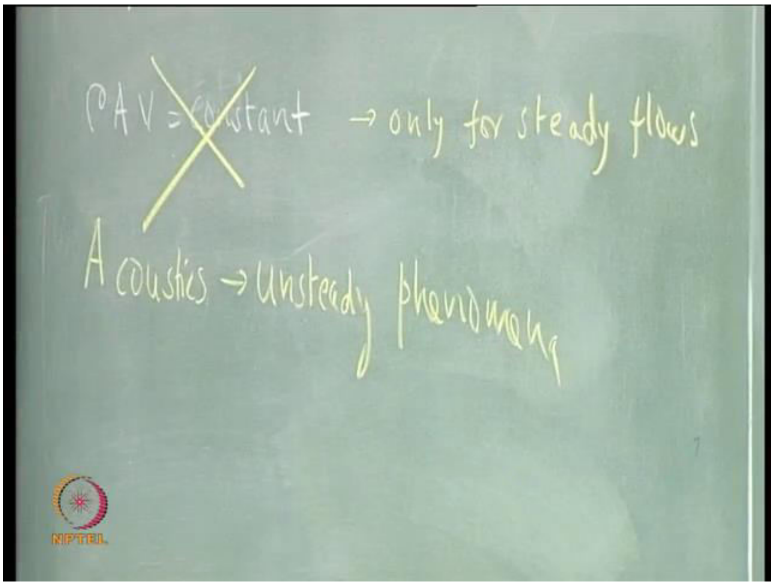

If we use the equation row A V equal to constant we have to un condition ourselves to not to be using that. We have use some other form of equation for continuity, we should actually be able to deal with and that is the differential form or kinetic equation, when you do not make the assumption that the flow is study.

So, similarly we have to use the un study form of the momentum equation, so I am not assuming any back ground on fluid mechanics. So, I will derive the continuity and momentum equations from scratch, but I will not do the most general derivation I will do a simple one dimensional equation derivation of the one dimensional equation. Now, if I am interested, you can go ahead and look at fluid mechanic text book to see how a general three dimensional equation for all kinds of situation could be derived, but here I am going to restrict myself to just to one dimensional equation.

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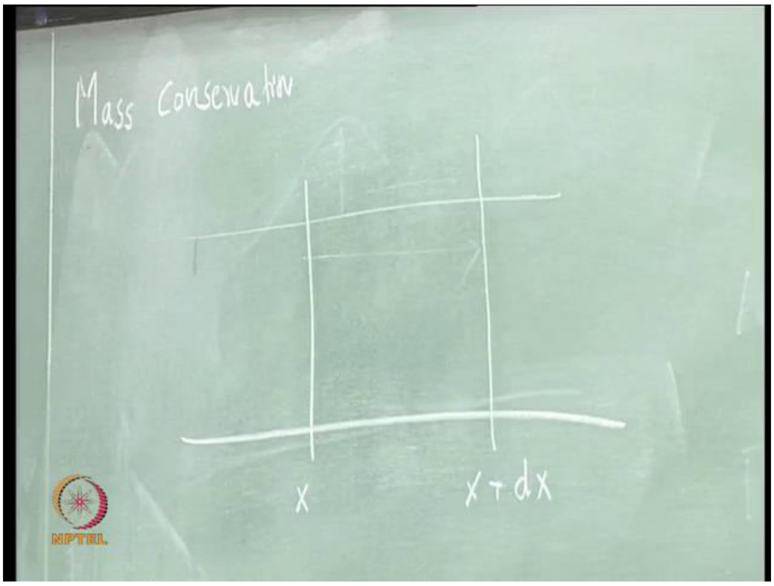

So, we are looking at the mass conservation or the continuity equation, so let us look at the one dimensional control volume and look at a station x and a station x plus d x and we see what happen to the flow that happens across these stations. So, we want to if you think about what should be the consideration law, you can say that if suddenly mass product is coming in and mass product is going out. The rate of change of mass florid in the control volume will be equal to the head of mass accumulation. So, mass florid in minus mass florid out equal to d by d t of mass in the control volume that is the very simple statement of conservation of mass. So, we are following a control volume approach you can also have particle approach.

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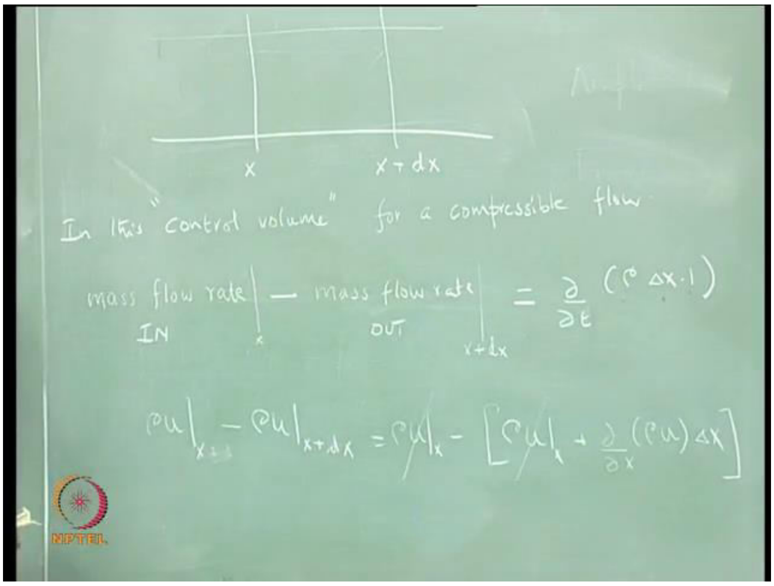

Here, we are going to follow a control volume approach or a oil arena approach, so in this control volume which is bounded by x plus x delta x. So, we must emphasize that we are dealing with compressible flow, we will say mass flow rate so that would be like mass flow per second in that is happening at x minus mass flow ray out which it will be at x plus dx delta x this should be equal. So, we take the cross areas one we can also take it as a or some kind of thing will simply keep it as 1 as to keep things simple.

So, this approach is the change of rate of mass in the control volume, the volume is the control volume delta x times 1 multiplied by density row, you get the mass and rate of change of that is dou by dou t. So, we can ah write a tailor expansion, so the mass fluoride would be row u times 1 and the mass fluoride. So, this is what we need to look at so this row u of x plus d we can write it as dou of x minus dou of x plus dou by dou x dou u times. So, we can cancel this, so this should be equal to dou by dou t delta extends 1.

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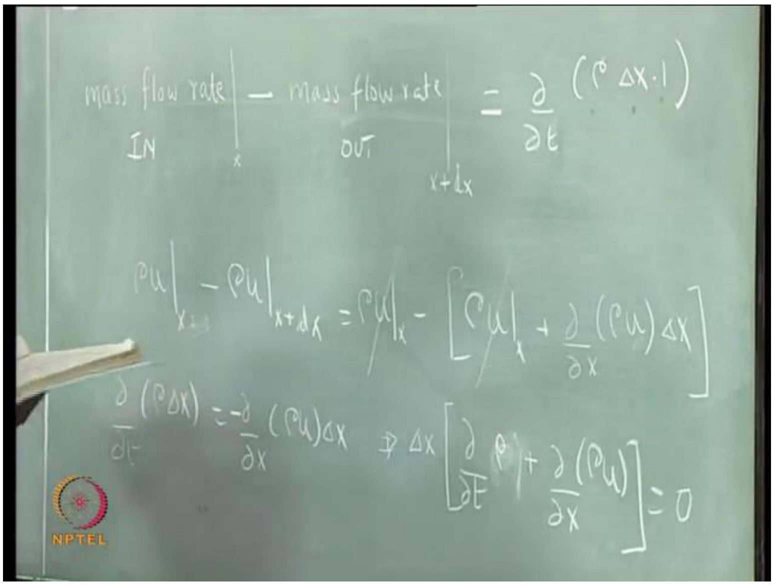

We can re write this as, sorry I missed a minus sign, so I can re write it as delta x times dou by dou t of row as this is a universal law. So, whatever be the delta x, we can choose we would satisfy this equation any choice of delta x. so, if that is the case, then what is inside this bracket dou row by dou t plus dou by dou u should be equal to 0. So, let me write that here, so for any delta x if this equality is valid, then what is given here should be equal to 0.

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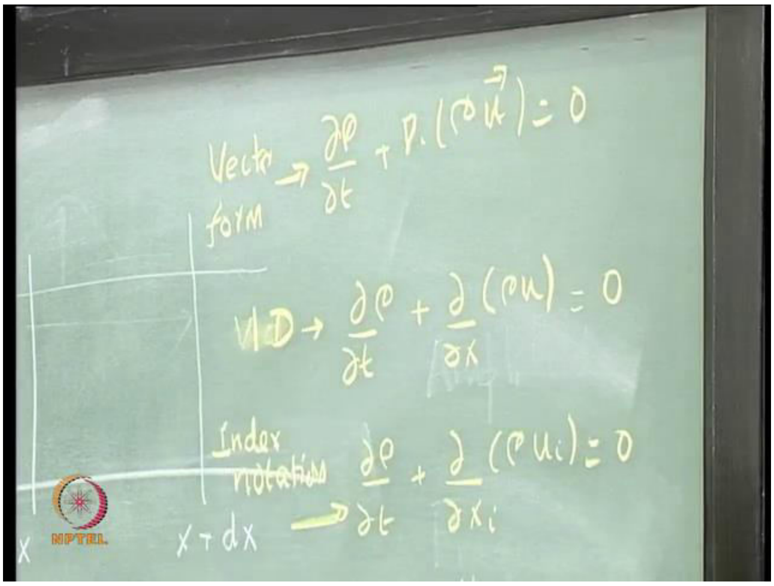

So, our continuity equation is dou e by dou t plus dou by dou x of row u equal to 0. So, this is our continuity equation in this, if you write it in a vector form in the general 3 d science it will be dou row by dou t plus del dot row u equal to 0, U is now a vector and I would leave that u to derive this and if you were writing in the Einstein notation or the index notation. Then, we can write this as dou row by dou t plus dou by dou xi of row U i equal to 0, sorry this is the vector form and this is the index notation, correct this is one dimensional.

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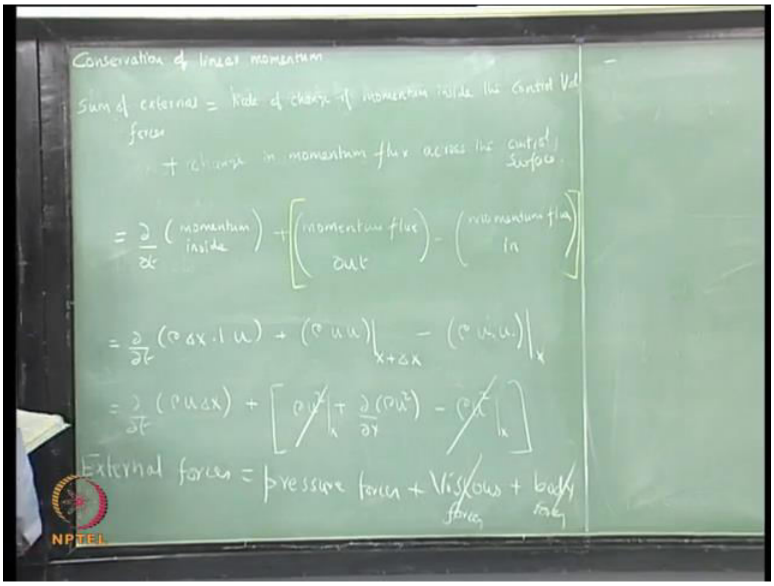

So, now we have derived the conservation mass equation and the next step is to derive the conservation moment equation rather the conservation of linear momentum conservation of linear momentum. So, we must have studied in school that this can be expressed as f equal to ma that is the nutrients low, but in a fluid we have written slightly differently. So, what we say is and we are going to do it in a Hilary frame work, so what we can say is some of the external forces is equal to rate of change of momentum inside the control volume plus change in momentum flux across the boundaries.

So, that is what the statement would be in the Hilary frame work and this can be this is equivalent to the usual statement in the frame work. The theorems which shows the equivalence of that, but I am not going to any of that, but a standard class in include mechanics would deal with these things. So, let me write the statement some of external forces is equal to rate of change of momentum inside the control volume plus change in momentum flux across the control surface.

So, whatever I have written words in the right hand side can be just written as in equation format dou by dou t of momentum inside plus momentum flux out minus momentum flux. So, this term dou by dou t term momentum insert plus momentum flux out minus momentum flux in that would be equal to the some of the external forces. So, let us try to evaluate what this term is in right hand side, so this would be equal to again we consider a one dimensional duck with a unit cross section area this for convenience, but you can have any cross section area and just find.

So, this would be equal to one is the cross section area, so row theta x that is the mass time times one that is the mass times u that is the momentum plus U square at x plus delta x minus row times U to the mass times velocity at x. Now, this should be equal to the external forces. So, the external forces are pressure forces plus discuss forces plus body forces and so on. So, we are not considering any body forces or visit forces, we are looking at in visit fluid with no body forces and pressure force actually in acoustic propagation the pressure forces will be what is dominant.

So, let us before that try to expand this term delta x plus cancel this term, so this is the simple simplified form of the term, now external forces equal to pressure forces plus forces plus body forces. So, we are not going to considered those two looking at in visit force with no body forces and so what is the net pressure force in the x direction, so pressure times area is the force.

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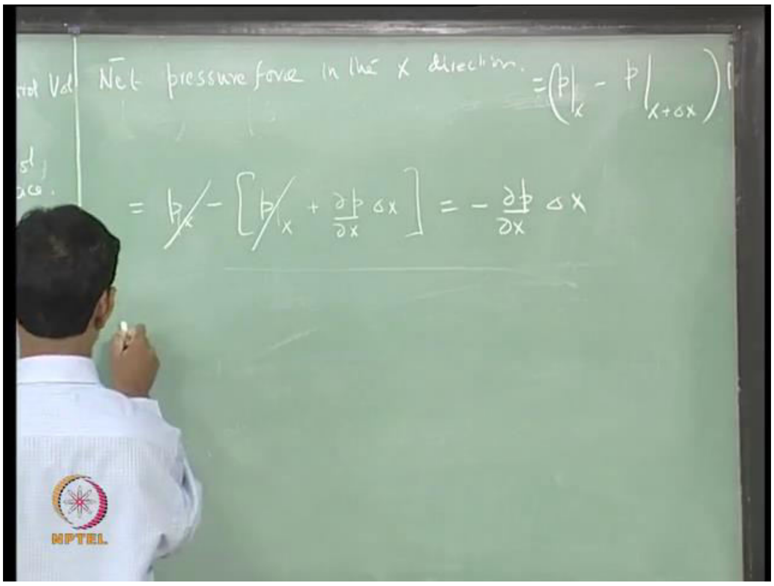

So, here we are talking about unit area so p times 1, so this would be equal to p at x minus p at x plus delta x whole thing multiplied by one because of unit area cross section. So, this can be written as p f x minus pressure at x plus dou p by dou x and dou delta x. So, this will cancel, now if you collect these terms to gather then this plus this, so you will get what you get, now we can simplify this.

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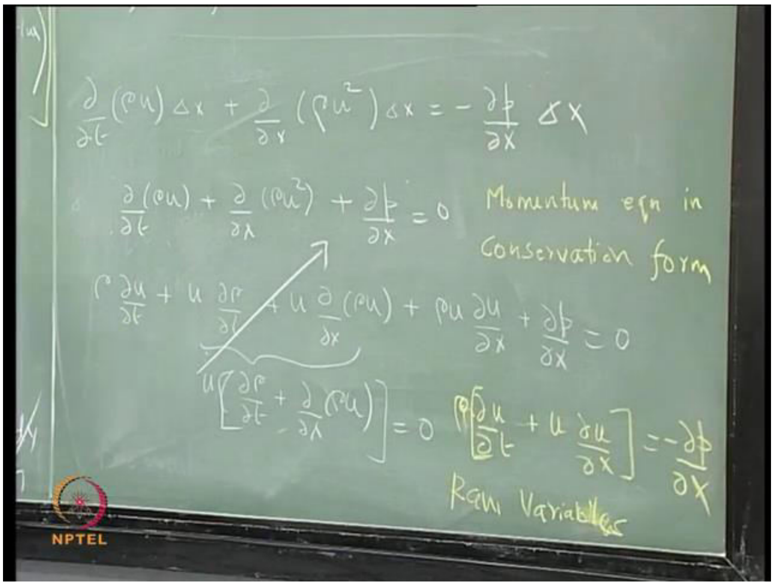

We write it nicely s p and again as I mentioned earlier this equation should be true for any arbitrary control volume space delta x. So, that means the term without delta x should be 0, so we can write this as delta x times dou by dou t of row u plus dou by dou x of row u square plus dou p by dou. So, we should have this equation work for any arbitrary delta x, so I will drop this and say that of the momentum equation this momentum equation is given and what is called the conservation form. This is very important for solving a things numerically and computation dynamics, this is the momentum equation in conservation form. Now, what we can do is to get this in terms of the raw variables and that is also quite useful.

So, that would be let us expand in this term we can say row times dou u by dou t plus u times dou row by dou t plus u times dou by dou x of row u plus row u times dou u by dou x plus dou p by dou x equal to 0. Now, if you see this term these two terms can be clubbed to gather as u times dou row by dou t plus dou by dou x of row u. In fact, this total thing you see from here that this some of these terms is equal to 0.

This should be equal to 0, so we can drop this term and you are left with just those terms. So, the momentum equation can now be written nicely in another form which is row dou u by dou t plus u dou u by dou x equal to minus dou p by dou x. This is the one dimensional momentum conservation equation in terms of the row variables, raw variables because these are the variables that you can measure directly.

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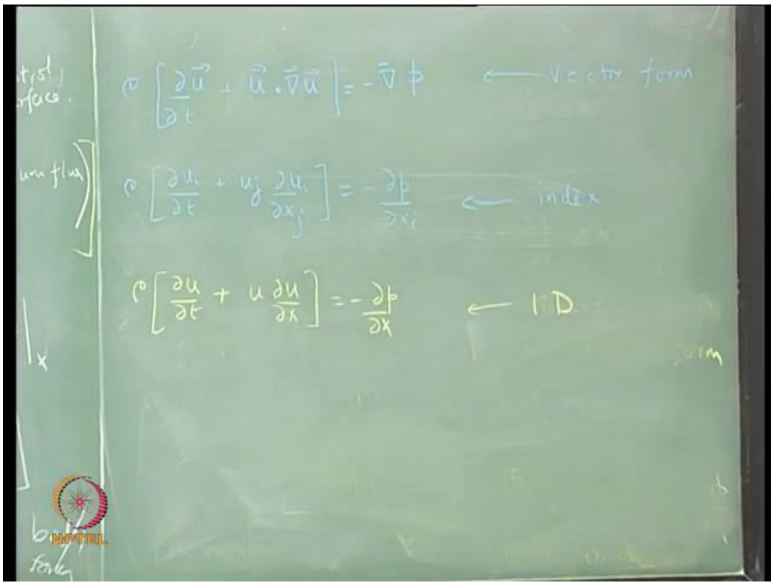

So, we can also write this in in the other forms, so I can write this in a vector form as dou u by dou t u dot lu times row equal to minus del p in the index notation. This would be row times dou U i by dou t plus u uj dou j of dou x this would be u dot del times U i equal to minus dou p by dou x i. So, this is the vector form this is the index notation or the Einstein convention and you have the one dimensional form here, so let me write that write below, so this will be our one dimensional form. So, now we have derived the equations of continuity momentum and next we need a relationship between pressure and density, we need a pressure density relationship.

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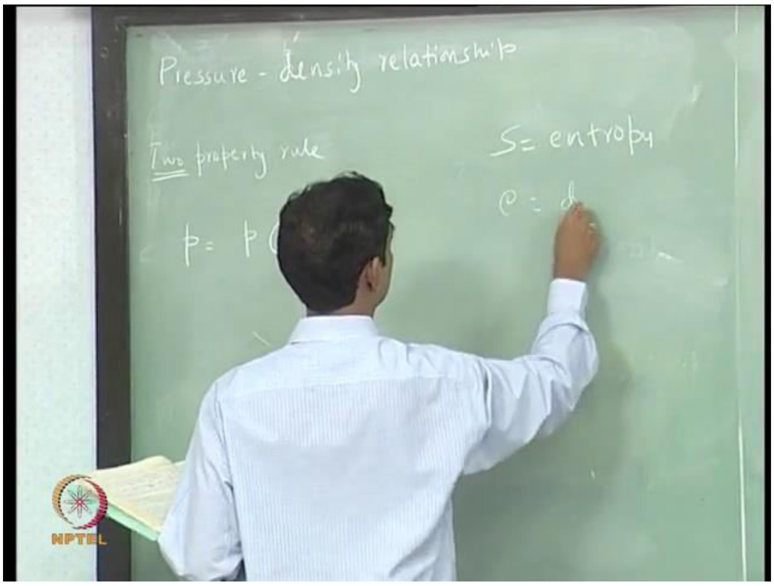

So, there is a rule called two property rule in thermo dynamics a thermo dynamic state is defined by two thermo dynamic variables at some instantaneous time in a uniform flow and a single flow. So, you need two thermo dynamic variables to that is called two property rule you need two thermo dynamic variable to describe the system. So, we can say that for example, p equal to is pressure is a equation of row comma entropy. So, these are let us say you specify density and entropy s equal to entropy row is density p equal to pressure.

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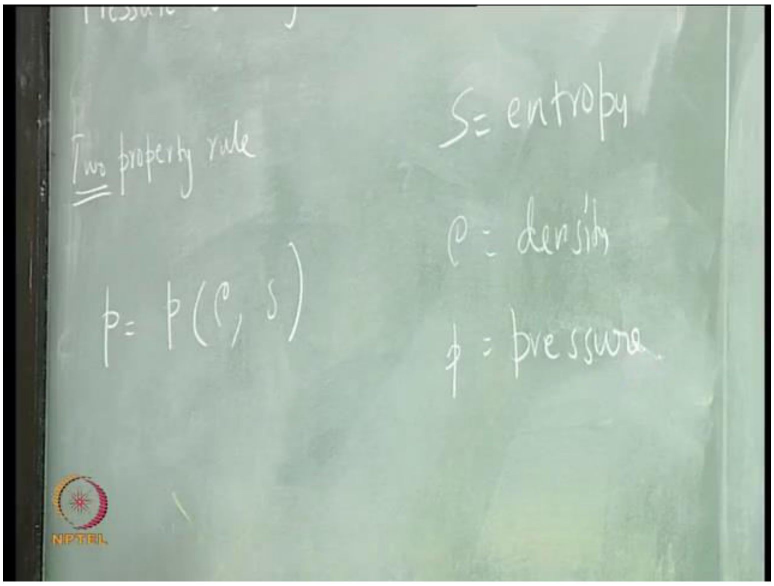

So, in sound propagation we assume that sound is propagating in an isentropic way, so there is no there is no heat flow in and out of the particle no heat flow in and out if a fluid particle.

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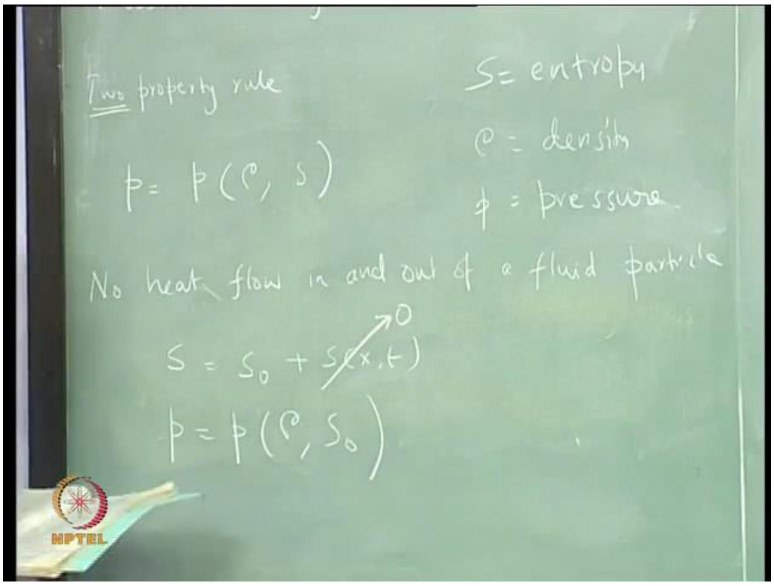

So, we can say s equal to some reference quantity s not that is the entropy is not changing, what we can say that is pressure equal to a function of row comma s not where s not is constant value to we say this is 0 for isentropic sound propagation.

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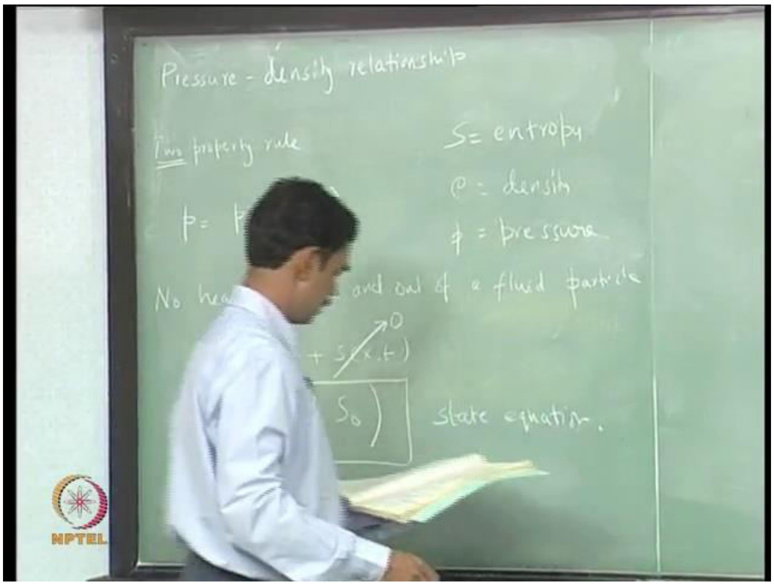

So, we now have a relationship let us say p equal to p of row comma s not there is a unique relationship between the unique relationship between pressure and density. So, this is the state equation.

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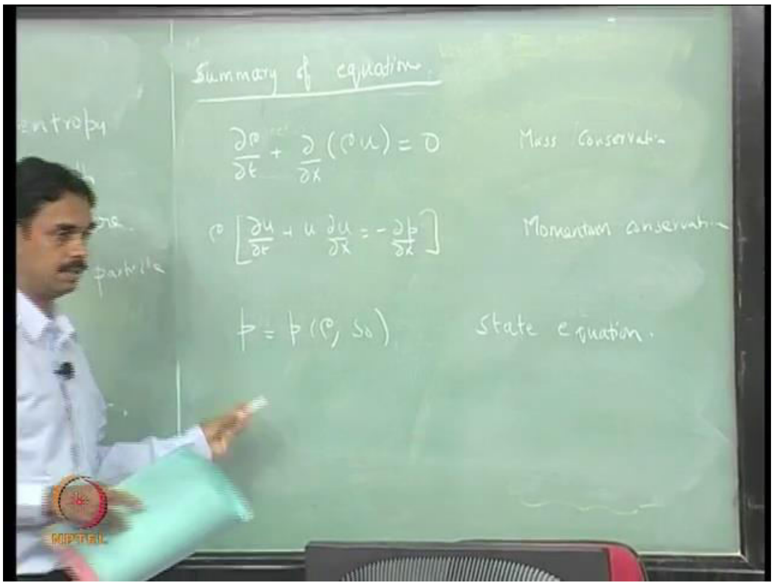

So, to summaries what we have the full set of equation that we derive, so we have the conservation of mass conservation and momentum conservation will be row times dou u by dou t plus u dou u by dou x equal to minus dou p by dou x. So, this is momentum conservation and we have p equal to p of row comma s not this is the state equation. So, we have these three equations, this summary of the equation and we have conservation of mass momentum conservation and state equation. So, we have derived the basic equation of fluid mechanics, now I did not derive the energy equation that is the reason for that.

In fact, for simple situation, we do not need it because we will see it later I will leave that equation for subsequent class, why we have not going use energy equation although when you have a combusting kind of a situation. Here, there is non uniform temptation we will need to use the equation. So, we will eventually derive it, but at the moment I try to keep thing simple energy deriving energy equation is little bit more complex.

So, we have three equations, now some of the things we want to see they are coupled equation because the variables appear in all the equation they are non linear, see the terms row du by dx. So, you have non linear coupled partial differential equation that is what we have we have non linear coupled partial difference equation. These equations have very limited number of equations and often we will have to solve the numerically, but in linear acoustics we if we can lean rise this equation and write a linear. Then, we can actually derive the linear wave equation and then we may be able to get exact analytical solutions.

So, that is what we will proceed to do in the next class what we do is we will leave lean arise the equation of mass and momentum and also lean arise the state equation that means we will write variable as a mean quantity plus fluctuating quantity. This fluctuating quantity will have value which is much smaller than the mean quantity and then we will neglect product of two fluctuating quantities. We will get this linear equation and couple them, we will some algebraic manipulation and derive the linear wave equation and that that is what we do in next class.

So, to summaries what we did is lecture we talked about what is acoustics what we talked little bit about what is there in the subject acoustics, let us talk about what a wave is talked about amplitude and frequency. Then, we spoke about various kind of infra sonic sound and so on, we spoke about practical situations where what the situation decibel levels that we see.

Then, we said we will attempt to derive conservation of mass momentum excreta and then we will and we derive this things in a attempt to be able to derive the wave equation which we will do in the next class. So, you should work out this exercises in the first exercise, I want you to do similar derivation for a variable area duct.

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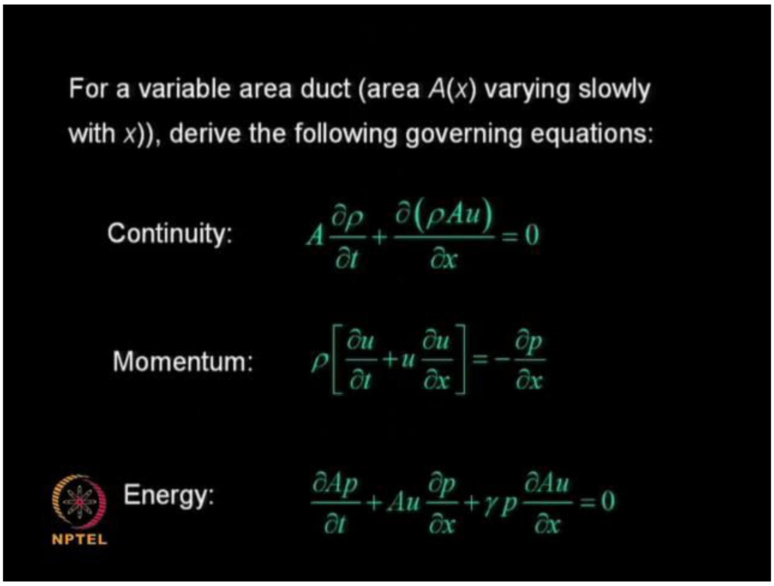

Here, you have a general function a of x wining slowly with x and derive the following gone equation that is this continuity equation, which is a first equation given in the slide. Then, the momentum equation the second equation given to slide and the last equation is the energy equation, please do this, it is quite a simple problem, just do it carefully the final equation of water given here try to get this answer.

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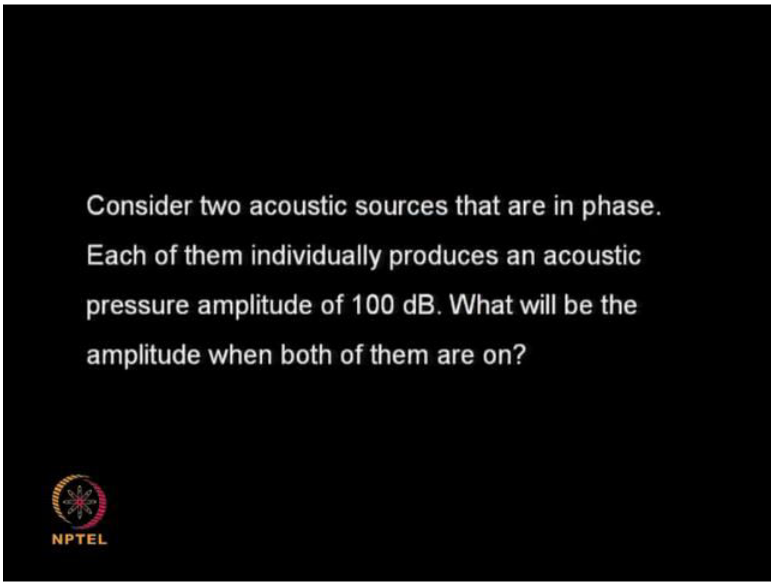

Then, the next question is let us consider two acoustic sources that are in phase each of them individually produces as acoustic pressure amplitude of 100 decibels, what will be the amplitude of both of them. If they are on simultaneous in the phase is a 200 dB or is it something else you will be tempted to say the answer 200 dB, but what is it? I will stop with this and have a good day, we will now in the next lecture, we will proceed to the wave equation first and then we get the solutions.

Thank you.