

**Acoustic Instabilities in Aerospace Propulsion**  
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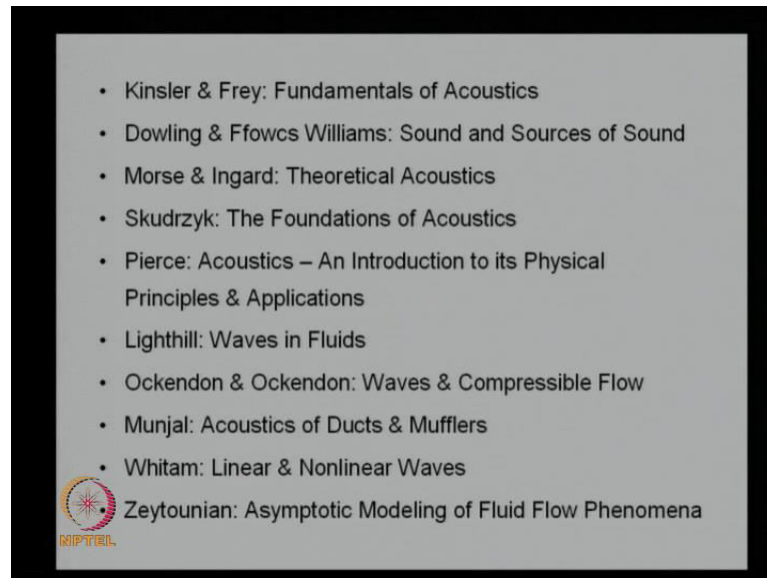
**Lecture - 17**  
**Reference Books**

Good morning everybody. I have been asked by many students to give you a list of reading material and so on. So, I have prepared a list, but before that I wish to give you two commentaries, I think one you know, elephant and blind men. So, if you, several different blind men write a story about the elephant, each of them will say different things. So, one would say, that it is like a rope; another would say, that it is like a pillar; some other want to say it is like something else, depending on what you touch.

So, I think, thermoacoustics itself is very difficult and there are lot of issues there and each person knows a fraction of the issues. And acoustic itself, even though it is somewhat well understood subject, I mean you have, I mean nobody knows everything and then, so what you speak depends on what you know, I think, very much. So, I think this issue is there.

The second thing I want to say is, the writing depends on the nature of the person also. Again, tell a story about an elephant. So, let us say, different people from different countries wrote stories about elephant or books about elephant. So, if Indians were writing, you would say how can elephants waste time in the evenings or something like that; a British person may write about how to kill elephants or hunt elephants; American will write about how to make fat elephants; Russian would say, which liquor the elephant likes and French would say love life of elephants and Germans, probably, elephant signs and 12 volumes and big books. So, it depends on what is the culture, origin of those guys. So, you have to keep all these on mind, but here is my must read list on acoustics, which will probably make you faint.

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So, this is the must read list and then, this another list, which is not so must read. So, I will give a commentary on what is there in each book. So, you can pay attention to the commentary. I think it is really important from my experience of reading books. I have read all these books in bits and pieces. Some of the books I have read fully, some here and there.

I think the simplest book is Kinsler and Frey. This book is found in different editions with more co-authors or less co-authors. This is a really nice book and you can get the full reference by searching in Google or something. So, this is more, I have put the names for, to give my version of the commentary. So, this is very simple. If you want to get started with acoustic this is the book to read.

Now, of course, we are really worried about thermoacoustic. So, we are not just concerned about acoustics in the sense of propagation of sound, but we are really concerned about how sound is generated also and from that sense a very simple book would be by Dowling and Ffowcs Williams, title, Sound and Sources of Sound. It is a really simple and nice and elegant book. Again, these are my views and it is need not be somebody else's views, but as the instructor here it is my views. Next in line, these are not in any particular order, but for each I will give you my commentary. So, Morse and Ingard, title, Theoretical Acoustics. This is more like a physicist talking about acoustics.

I mean, you have engineering stuff in it very much, but the flavor of it is like a physicist's view of things.

And the Foundations of Acoustics by Skudrzyk, this is a very nice and comprehensive book and it explains not just acoustic, it gives the necessary math ground, math background and this is the best book I found for multidimensional acoustics, like you know, waves and ducts ((Refer Time: 03:54)) and so on. Now, Pierce, title, Acoustics, an introduction, its physical principles and application, this is what I had to study as a, when I took classes in acoustics. This is a nice book, but it is a difficult book, but I mean graduate students are supposed to do difficult things. So, I think this is a good book, but it is not so easy reading, but you have to put effort to it, but it is worth reading this book.

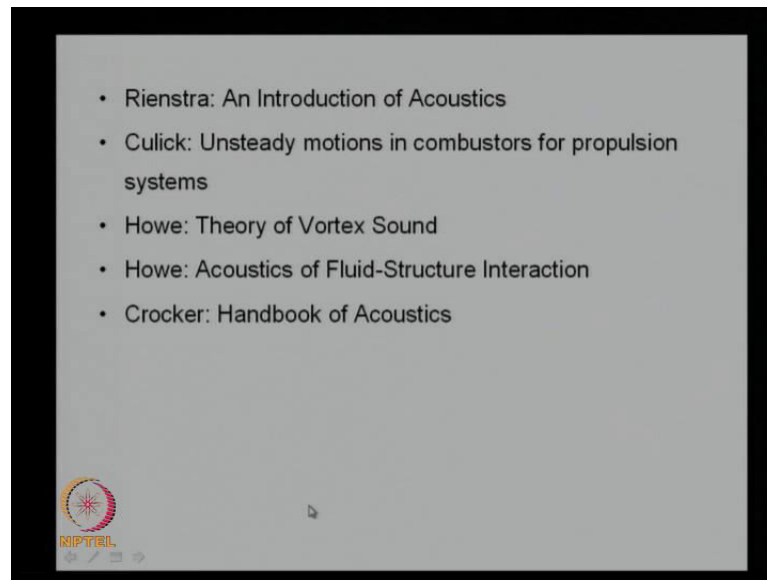
This is Lighthill book, it is a classic Lighthill, is a famous person in acoustics and even more famous in aeroacoustics. So, he has written a book on waves and fluids. It is fairly simple, I think you will follow it. And Ockendon and Ockendon, I do not know my pronunciation is right, Waves and Compressible Flow, this is again more like a physicist view of things. It is more general view, but it is very nice book. Now, Prof Munjal, famous Indian professor in acoustics, sorry, he has written a book, title, Acoustics of Ducts and Mufflers. This is very good book for duct acoustics. Here, duct acoustics is a big branch of subject as well as acoustics of middle of nowhere and so on, that also can be studied. So, this is a very good book for duct acoustics and yes given applications to silencers and other things as well.

Now, we term linear and non-linear waves. This is a really lovely book. It is math book, math kind of book, but it is really classic and each time I read it, I mean, I learnt new things and I think it is a must read book, I think the last one, Zeytounian, Asymptotic Modeling of Fluid flow Phenomena, there you can see how the acoustic equations are obtained rigorously. As I mentioned you, what is flow, what is acoustics and there is always flow when there is sound and how to separate hydrodynamic fluctuations from acoustic fluctuations, those things are the framework to derive those equations are given in this book by Zeytounian.

It is an advanced book, but it is really worth reading it because I, as I mentioned to some acoustics, you always have a basic flow. I mean, because there is an even under study circumstance you have a combusting flow and which is non-uniform and so on, over that

riding on that is the acoustic. So, how to separate fluctuations in hydrodynamics because hydrodynamics about ((Refer time: 06:27)) and so on. The fluctuations, how to separate that with acoustics, that is nontrivial task and the asymptotic modeling asymptotic analysis can be used to do this and that is given in this book.

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Now, the book by Rienstra, An Introduction of Acoustics, I think you can download it freely from the internet if you Google search and anything, free as good, and it is probably a bit mathematical, but it is coming. Some of you who are mathematically inclined what would really like it, there is no, not quite at a book on thermacoustic instability in itself.

But Culick has written a book titled, it is an e-book and in the sense it is a CD and if you write to him he will give you a CD freely. I think it can also be downloaded from the web, Unsteady Motions in Combustors for Propulsion Systems, but he is specifically speaking about, most of the time he speaking about solid rocket motors. But it is, Culick is a very famous figure in the subject and he has written a comprehensive monograph on this topic and it is very interesting. And if you are studying instabilities in solid rocket motors, this is a must read book I think.

And Howe is a very famous figure in aeroacoustics. He is the one who is credited, one of the person credited with the theory of vortex sound and he has written some very famous papers. So, he has written one book on Theory of Vortex Sound and another one

Acoustics of Fluid-Structure Interaction. They are very nice books, very comprehensive book.

And the last one, Handbook of Acoustics, it is a fat book, you can use it for weightlifting, but it has everything in acoustics. It is I think 10000 rupees or something, but really a nice book. It covers, if you are introduced to acoustics and you want to start off, every topic has articles on it by experts. So, this Crocker is, he did not write the handbook, he is like the editor of the handbook. So, this is my reference list with my specific commentary as to what you can get out of it.

So, I hope up, are there any questions? I can happily answer about the books itself, but there is no precise crisp text book on introduction to thermoacoustic or dummies gets to thermoacoustic, it does not exist at all. You have to read papers that is what it is. Any questions? So, there are no questions, we will get back to what we are talking about, that is, adding energy in phase.

Oh no, before that I have to talk about the assignment submission. So, I told you like in 2 or 3 weeks you can do the assignments. One week is over, so on March 14 you can submit it. I think you can get your computer program is running, if you know MatLab, it is few minutes I think and then, maybe report writing would take a few hours and analyzing the results, which you should do first over few hours. So, it is not a very difficult assignment. Now, if you do not know programming language or MatLab or anything of that sort, maybe 2 or 3 days to get to learn, to do stuff and then a few hours to finish it. I expect to meet, submit on March 14 a small report. I hope you know what was the assignment.

Student: ((Refer Time: 09:52)) temperature...

Temperature (Refer time: 09:59)) only temperature gradient no need of...

Student: Temperature gradient, we are ((Refer time: 10:04)) something...

Yeah, I mean you are, assume some, whatever is based on your experience and assume or see what was the values given in the papers.

Student: Solve for  $\phi$

1 d phi with close-close end or open-open end some classic ((Refer time: 10:18)). Yeah, you can ask any question.

Student: Yeah, in a closed-open tube if we connect a speaker one end and then, what will be the quality conditions at the speaker? Should we take p dash?

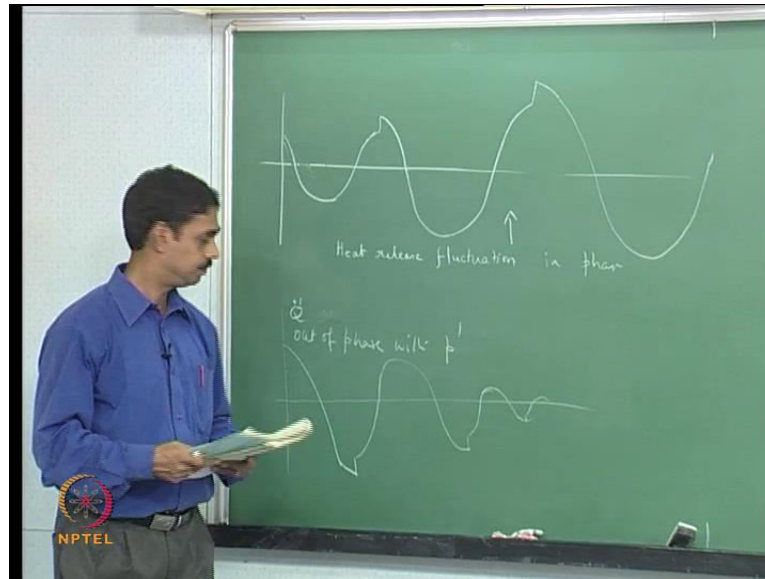
So, if you have a loudspeaker, so strictly speaking, close-close means there is nothing else in it. But if you have a loudspeaker, you are actually sending in a U prime, there you are imposing velocity fluctuations there. So, either you can impose admittance or impedance of the speaker or you can write some source strength based on that or you can, I think, I think best to write some equations for the admittance of this speaker or the impedance of the speaker. I think that is probably the best ((Refer time: 11:05)). So, if you put a speaker at the closed end, it stops being a close end.

So, if you are having, in practical terms if you have a speaker, you can put it and you can actually measure the admittance that is probably the, from the, with the two microphone technique or the impedance tube technique or something like that. But the moment you put anything else, because it stops being a closed end and you are actually putting energy into it, so energy is flowing in the linear framework, you have admittance.

Student: But can we say whether it is near to ((Refer time: 11:38))

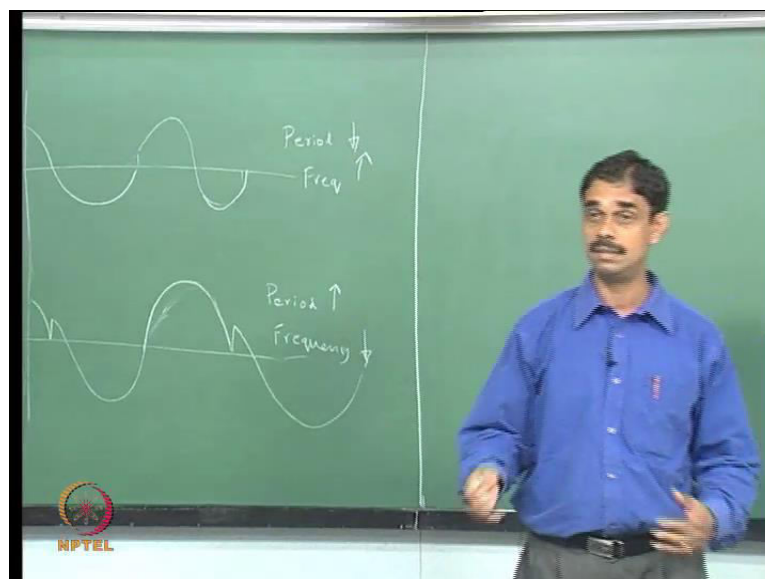
Usually it would be, you can look at the way, but if the, in it is very possible, that it is the wave structure looks close to that of a close-closed end. If you are having a close-close pipe and you put a speaker near close end and speaker drives very well at closed end and at the, if you put it in open end, it would not drive at all. So, yeah, I think it should be close to that, but there is energy coming in and energy comes in, there must be admittance. But then, there is also energy loss, which is why the amplitude eventually does not grow infinitely, but it will reach some value. So, there are losses and that have, has to be ((Refer time: 12:23)) also, either as volume losses or as boundary losses. Any other questions?

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So, we were looking at Rayleigh criteria and I asked you to think about what happens when you add energy at different phases. So, I will draw the diagrams, see if you have these kind of vessels. So, this is what would have happen if you had heat in-phase. So, here it is more like an impulse. Actually, in real combustors it would not be impulse, it will be distributed, but here it is in-phase that is what it is. So, next we will draw out-of-phase. So, this is  $\dot{Q}$  prime  $\dot{Q}$  dot prime out-of-phase  $p'$  prime. Now, if you have, so here it is at the, as the pressure is at the highest point here, I am adding heat as the pressure is at the lowest point.

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But if you can have here or here and let me draw those. So, if I am adding it here, so it is almost as if the, you know, the period shrunk or the frequency increased and could this hand waving explanation, we will try to work this out mathematically and if I am adding at the other end. So, here the period kind of elongates because you are kind of stretching the wave. So, you have frequency coming down. Here, period comes down, frequency increases in this scenario. So, here pressure is 0, but you are adding quarter period before and quarter period after, so here period increases frequency decreases.

So, whereas there is no change in the amplitude of the oscillation, so we are not really driving or damping, but we are just changing the period of the oscillation or the frequency oscillation. So, we said, that there are some, this is, this is like a graphic illustration or a crude hand waving illustration of Rayleigh criteria. What the Rayleigh criteria says is, that if heat is added in phase with the pressure, again I am speaking only about fluctuations, then you produce acoustic energy and if this acoustic energy is more than the losses, then you will exhibit instability. So, this is the Rayleigh criteria, we will write this in an equation form.

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Rayleigh Criteria

$$\iiint_V \int_0^T p'(\vec{x}, t) \dot{q}'(\vec{x}, t) dt dV >$$

Driving

$$\iiint_V \int_0^T \sum L_i(\vec{x}, t) dt dV$$

Losses / Damping

NPTEL

So, this will be the acoustic driving and if this is more than, more than the losses, write this below, so if this is greater than... So, this equation would basically mean, that this term here is driving and this is losses or damping. So, if driving is more than damping, instability will set in. So, this term indicates acoustic power is, acoustic energy is input



by the combustion process, but you may, you can have acoustic energy being created by the combustion process, but you may still not have combustion stability or thermoacoustic instability because your losses maybe more than what you are putting in and then a disturbance will decay.

So, you have to have that much driving, which is more than losses, then only you will have the onset of instability. I hope this is clear. So, this is the interpretation of Rayleigh criteria. Any questions? So, L means loss, write, yeah, it is written. So, there could be a several loss mechanism, that is why have the subscript L i.

Student: Sir, in practical scenario how do we control where we had heat ((Refer Time: 18:48))

A very good question; I am surprised. In fact, that is what is written here. I wanted to have an example. So, let me repeat the question. In practical scenario how do we control where we had heat, how we had heat and how the losses are?

So, it is easy to address the losses. So, I mean losses can be through nozzle or of different things at the boundary and so on. So, if you have access to that you can have access to that, but generally you may or may not. If it is a ground based combustor, perhaps you may have some access like a burn or something or a furnace. But if it is a rocket or something, I mean everything is designed based on performance and then the instability sets in.

I was not here last Friday, right. So, I went to see some instability and then the rocket, I told you, is designed to give incredible performance. Actually, I am really impressed with the design, but it is designed only for that, it is not designed for stability. So, and then the guy says do not touch anything. The program manager and I want a solution today and it is such a complicated problem, that even to we do not know what equation to write, but they say by end of the meeting tell me want to do and this is the greatest. But in fact, if you read some of the historical accounts by Professor Price in some of the papers or books he says, that this has always been the case. So, they call some black magicians and then, you do something to fix it.

So, I will give you example of how you can change the way where the distribution is affected. For example, if you have a gas burning thing and like a premix burner, let us

say, and you want to adjust the location of the heating place. So, what you can do is to you can send the same flow rate that cannot be touched because of the performance, but then you can adjust the inlet diameter and change the velocities so that you can find out how much the vortex moves, you can alter that. Perhaps you can alter the shedding time of the vortex, so you can have some leeway over the time scale. You can, once the vortex velocity changes, I mean, it may burn at some other place that is one thing. If you have like a, like a airblast atomizer or something like that what you can do is, you want to change the timing, let us say, or the heat release. So, basically timing goes like droplet size rate, I mean, the evaporation depends on droplet size.

You studied D squared law and all that and then there is a modified D squared line in the presence of velocity fluctuations and so on. So, you could actually change the injector, which will inject same amount of mass flow rate, but a slightly different droplet distribution or something and, so that the, you have more droplet with smaller diameter or less droplet with larger diameter and then that will change the time scales. So, that is one thing you can physically alter the location of the injector, then it will change differently. You can alter the air velocity in the injector and, so on.

So, there are few things you can play with, but there is and you can, if you are talking about fuel supply, if it is not choked and if the flow rate is pulsing, then you can choke the fuel flow rate. I mean, you can choke the fuel supply line, so that fuel flow rate does not choke, so, or you can play with the lengths of the tailpipe if that is allowed to. It is hard to give a general answer because generally you have to see what can be changed because you are not planning for this because management says get things done yesterday and this is the subject where you do not even know how to write equations and so on.

So, you and performance everybody knows, everybody studied propulsion, one knows how to design a vehicle for performance and you design for incredible performance and you are not accounted for anything, and suddenly instability happens. And then, your bosses are breathing down and telling you, that you have to fix it and their bosses are breathing down, the parliament is breathing down on annoying the hell out of this incompetent company or organization, that they cannot stop this. So, there is lot of pressure and then you have to fix something. So, it is never really planned for being this.

Although, I mean, if you listen to big shots they will say, that you have to design for instability performance and they say, that I mean, there are programs, which for whatever it is worth, calculate the Eigen values of the systems and whether they are stable or not, some company will bring in some sense. But it is really calculated apart from satisfactory and so on. But nevertheless, it is a secondary thing. I mean, the primary thing is performance. So, when the thing comes, then you have to do some fix or black magic and then or other passive controls like you can put baffles where there is a velocity maximum, so that you try to, the acoustic standing wave wants to go somewhere. So, it wants to be at a velocity maxima and you put baffle there, forcing the velocity to come down.

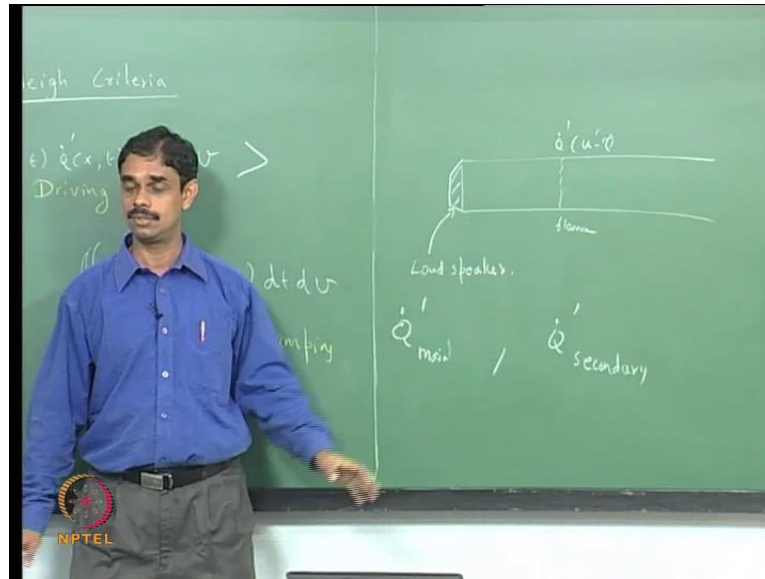
So, this will, this will make some kind of mismatch and so there are lot of ways we can stop this and you can even change the length to the combustor, change the acoustic line scale. But that in an aerospace situation may not be possible because weight will increase if you increase the length; if you decrease, there will be not enough resistance time and so on. So, you have to see there is almost nothing you can touch, but you have to stop it, so some magic has to be done. So, there is no possibility to give a general answer or even if I give, it would not be of any worth. But I think I have given you some ideas as to if you think to play with, so that depends specifically on the situation at hand, but eventually this equation is always holding.

But what causes this  $Q$  prime to come in face with  $p$  prime that depends on the specific mechanism of instability in that specific configuration. So, one way is to adjust the timings such that you can, so you can play with appropriate thing to adjust the timing of heat release with respect to the pressure, which will remove the instability or create some kind of damping mechanism and interfere with some increased, boost this term or decrease this term. So, that is, that is what it is.

Student: So we can send some of the acoustic instabilities so that this condition is satisfied.

Yeah, so let me give you an example. We will work out this problem soon.

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So, we can have a combustor, let us say, and that is a flame here and let us say, it has certain  $\dot{Q}'$  and now it is going, it, this is the function of  $u'$  or  $u'$  minus  $\tau$  after a time delay, and then it is growing unstable. Then, what you can do is, maybe, you can put a loudspeaker, which now, this without loudspeaker, if you ((Refer time: 26:09)), this system has certain Eigen values, but you can alter the minor part of Eigen value by putting a speaker and maybe make a system, which was originally unstable stable.

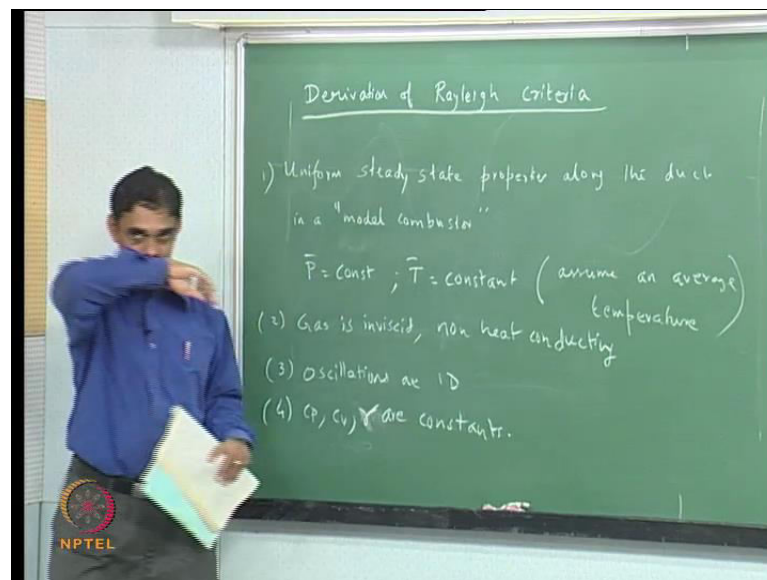
So, this is kind of like the anti-sound concept and this is the crude form of active control. They are very fancy and advanced forms of active control and in reality, you will not be able to put a loudspeaker and a combustor because I think nobody would want to do that. You would probably put a pulse fuel injection or something, so that you create another fuel oscillation.

So, you have a primary heat release rate, but you can have like a  $\dot{Q}'$  one or main, which is in phase creating all the problems, but you can have  $\dot{Q}'$  secondary and this can be designed such that the damping it produces, it interferes with pressure such that it creates damping and that can be more than what this main driving is. So, that is probably the actual strategy. So, you are right, in principle it is possible, but in practice can you do it reliably that is the question.

So, active controls are in principle suppose to ((Refer time: 27:25)), but the engineers and the managers do not have faith in it because they have to work for several thousand hours without failing mechanical components. That is the reason they give for not having these things, but I do not know some instant may, they may come. Passive controls are quite popular because they may not work everywhere wonderfully and all that, but if it works now, it will work tomorrow, it will work one year later. So, it is this simple. So, engineers like simple things, so they have passive controls. So, you are absolutely right. Any other question?

So, the next agenda here is to derive this with, in some sense. So, what I am planning to do is to get a derivation of Rayleigh criteria with some very simplified assumptions. You make get very annoyed with the assumptions, but that is what it is and I will explain why you have to make this assumptions. Then, I will work a problem out like this, a very simple model problem, where you can actually translate this Rayleigh criteria into Eigen values and Eigen vectors and whether you can calculate the growth rate and decay rate and so on, a very simple configuration. So, that will give you the idea of how to set up this calculation.

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So, we are trying to derive Rayleigh criteria and the assumptions are... So, uniform steady-state properties along the tube, so then you cannot really call it combustor. It is more like a model combustor or something. So, we will say  $\bar{p}$  is constant. That may

be ok for many combustors because you are having low Mach number flows and change, I mean,  $p + \gamma m^2$  is a constant. So,  $p$  goes like  $\gamma m^2$ . So, small values of  $m$ , you may not change things so much.

But the next assumption,  $\bar{t}$  is constant is really crazy because the moment you burn something, temperature has to go up, that is the present idea. We are burning things and so, I mean, this is really off the roof, but then if you do not keep  $\bar{t}$  constant, I mean, the equations are much more difficult to deal with and so on. And that is the reason we, so we will take the easy way out and say, that we will model the things for some kind of average temperature in the combustor. We can always justify about you are doing, but and every model is wrong anyway. So, if we can learn something from the answer, then the model is good or if you can calculate accurately, as an, if you can predict whatever is happening, then the model is good, so, and any model is wrong. The issue is, whether it is useful or can you do anything with it? Can you learn something from it? So, we will say, assume an average temperature. There is one configuration where this is ok, one aerospace propulsive system where the  $\bar{t}$  is constant.

Did not here you. Loudly, can you speak loudly, I mean?

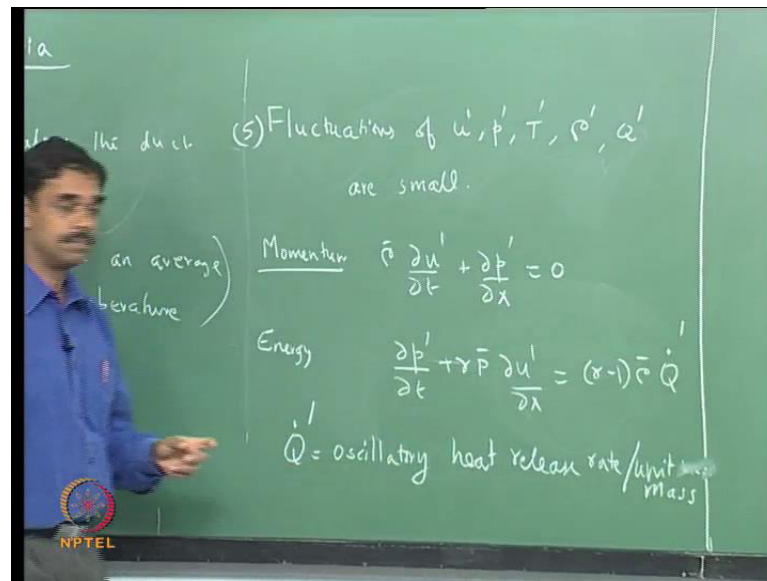
Student: Rockets

Yes, absolutely, in rockets temperature is almost constant in the few degrees. So, yeah, in rocket  $\bar{t}$  is constant,  $\bar{p}$  will change, but not so much. So, I think  $\bar{t}$  is constant, there you can get away with, but not in a, like a combustor with a flame, like a gas turbine low class flame. I think it is a very bad assumption. And in rockets, the flame is uniformly there everywhere and really, it is kind of well insulated from the surrounding and so on. So, it is a very good assumption for rockets, but not for furnaces or gas turbine engines and so on.

And second, gas is in viscous and non-heat conducting and I said, that we discussed this at low frequencies. This was, and we will do oscillations, which are 1D. I think we can do 3D and so on. I will even derive something in a generalized sense, but we will work the detail for 1D problems just to keep things simple. This is really not a serious problem. If you have ability to calculate then there is no problem.

And the fourth one is,  $C_p$ ,  $C_v$ ,  $\gamma$  are constants. This is ok for a gas turbine or a domestic burner and so on, but not ok for maybe a solid rocket motor and so on. But I think, compared to this assumption, these assumptions are really not that bad and we will say, that we are having small fluctuations, so that we can deal with a linearized framework. So, this is kind of assumptions for what is to follow for the entire semester except I will deal with some non-linear effects later on, but mostly I will deal with linear effects.

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So, having declared, that I am guilty about everything. Now, we proceed to, I get the questions. So, we, have the continuity momentum and energy equations, and it is quite convenient to use the momentum and energy equation to derive the equations. So, energy would be where  $\dot{Q}'$  is oscillatory heat release rate per unit mass. Sometimes you see it being expressed as just  $\bar{Q}'$  that would mean, that it is heat release rate per unit volume.

So, it depends, you have to read the, if you are reading a paper read it carefully to find out what exactly is the notation. Also, many people do not use this dot, which indicates, that it is the heat release rate. They just write  $Q$ , but it is implicit that it is a rate. Now, what we can do is we can multiply this by  $u'$  and multiply this by  $p'$  over  $\rho$  bar, let us see what you get. So, and I have assumed  $\rho$  bar to be constant.

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$$\frac{\partial}{\partial t} \left[ \frac{1}{2} \bar{\rho} u'^2 \right] + u' \frac{\partial p'}{\partial x} = 0 \quad (1)$$

$$\frac{\partial}{\partial t} \left[ \frac{p'^2}{2\bar{\rho}} \right] + \left[ \frac{\gamma \bar{p}}{\bar{\rho}} \right] p' \frac{\partial u'}{\partial x} = (\gamma - 1) p' \dot{q}'$$

$$\frac{\partial}{\partial t} \left[ \frac{p'^2}{2\bar{\rho} c^2} \right] + p' \frac{\partial u'}{\partial x} = \frac{(\gamma - 1) p' \dot{q}'}{c^2} \quad (2)$$

So, rho bar ((Refer time: 36:14)) del t of half... Now, if I look at the energy equation and multiplying it by p prime over rho bar. So, you get... So, we can call this term, let us say we call it c square and like the speed of sound. So, we can divide throughout by this term and you will get dou by dou t of p prime squared by 2 rho bar c square plus p prime by dou x equal to rho minus 1 p prime Q dot prime over c square. Now, we call this equation 1 and 2.

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$$= 0 \quad (1) \quad \frac{\partial}{\partial t} \int_V \left[ \frac{1}{2} \bar{\rho} u'^2 + \frac{p'^2}{2\bar{\rho} c^2} \right] dx + \frac{\partial}{\partial x} (p' u') = \int_V \frac{(\gamma - 1) p' \dot{q}'}{c^2} dx$$

$$\frac{\partial}{\partial t} \int_V \left[ \frac{1}{2} \bar{\rho} u'^2 + \frac{p'^2}{2\bar{\rho} c^2} \right] dx + \oint_{c.s} p' u' d\sigma = \int_V \frac{(\gamma - 1) p' \dot{q}'}{c^2} dx$$

Acoustic driving

Acoustic

$\oint_{c.s} p' u' d\sigma$

$\frac{p' u'}{c^2}$

= 0 for closed-closed duct  
open-open  
closed-open



We can add one and two and you would get  $u' \text{ squared} + p' \text{ squared} / 2 \rho_0 c^2 + \text{div} \text{ of } p' u'$ . If you are doing 3 D, this would be like  $\text{div} \text{ of } p' u'$ , this should be equal to  $(\gamma - 1) p' Q' / c^2$ . So, this is very similar to the derivation we did when we derived the classical acoustic energy corollary. So, if you average this over a control volume, so and then, this can also be average. So, in 1 D it is like area times  $dx$ , right, and I drop the area. So, I will get...

This is, this would be what you get in a general sense, but here it would be  $p' u'$ . I mean, you will get  $p' u'$  across the boundaries, the difference. If you have a close duct we can drop this term, this will be equal 0 for, let us say, a close-close duct or open-open or close-open. So, any of these thing, this term would go away. So, then you can see, that if heat release rate is in phase with pressure fluctuations, then the acoustic energy and the system will grow, right. I mean, that is what the message comes, coming out of this.

So, this is like the losses unless energy is input from the surface and in the case of a solid propellant. For example, the propellant can add energy from the surface, but in a furnace kind of situation or a gas turbine kind of situation this term would represent the losses and this would be the acoustics driving. So, if there are losses the acoustic energy will grow. If the driving, which is the correlation between  $p'$  and  $Q'$  acoustics represent the heat release rate is more than damping, but if the damping is more than that of the driving, the net contribution will be negative. So, acoustic energy will decay. This is clear? Any questions?

(Refer Slide Time: 42:24)

The chalkboard shows the following derivation:

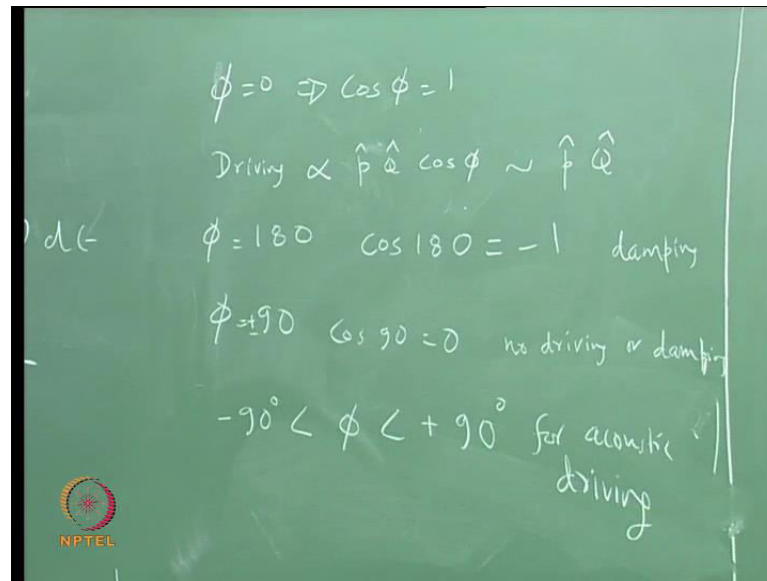
$$\begin{aligned}
 p' &= \hat{P} \cos \omega t \\
 q' &= \hat{Q} \cos(\omega t + \phi) \\
 \frac{1}{T} \int_0^T p' q' dt &= \frac{\hat{P} \hat{Q}}{T} \int_0^T \cos \omega t \cos(\omega t + \phi) dt \\
 &= \frac{\hat{P} \hat{Q}}{T} \int_0^T \frac{\cos(2\omega t + \phi) + \cos \phi}{2} dt \\
 &= \frac{\hat{P} \hat{Q}}{T} \frac{T}{2} \cos \phi = \frac{\hat{P} \hat{Q} \cos \phi}{2}
 \end{aligned}$$

An NPTEL logo is visible in the bottom left corner of the chalkboard image.

So, if you say  $p'$  equal to  $\hat{P} \cos \omega t$  and  $q'$  equal to  $\hat{Q} \cos \omega t + \phi$ . So,  $p' q'$  integral over 0 to  $t$  would be  $\hat{P} \hat{Q}$  integral 0 to  $t$   $\cos \omega t$  times  $\cos \omega t + \phi$ . So, this could be written as  $p' q'$  integral zero to  $t$ . This would be  $\cos \omega t + \phi$ . So,  $\hat{P} \hat{Q} \cos \phi$  over 2. I hope this formula is right. And the first term, what will it be? If we integrate the first term you will get 0, why? Because this is a periodic term and the second term will give  $\cos \omega t$  times  $t$  over 2.

So, this would be  $\hat{P} \hat{Q}$ . So, if I average over a cycle I put a 1 over  $t$ . So,  $t$  over  $2t$  times  $t$ . So, this is  $\hat{P} \hat{Q}$ . So, oh sorry, I miss the phase,  $\cos \phi$ , which is equal to... I hope this right. So, if ((Refer time: 44:21)), so if you have  $p'$ , which goes like  $\hat{P} \cos \omega t$  and  $q'$  going like  $\hat{Q} \cos \omega t$ , I should put dot here just to be consistent and then the correlation would be  $\hat{P} \hat{Q} \cos \phi$  over 2.

(Refer Slide Time: 45:20)



So, if cos phi is 0, then what happens? What is the value of cos 0? Cos 0 is 1, so then phi equal to 0, this implies cos phi equal to 1. So, the driving would be, is, there is also other factors like there is a gamma minus 1 over c square, I will not worry about it. So, driving, I will say, proportional to p hat q hat cos phi, which is going like p hat q hat. Now, if we had phi equal to 180 degree, your cos 180 would be minus 1. So, this is like damping and at phi equal to 90 plus or minus you will have cos 90 equal to 0, right. So, that way there is no driving no damping.

So, what is the phi? The phase between, phase where you get driving. So, you have to have minus 90 degrees less than phi for acoustic driving and... So, I hope this is clear. I do agree, that I did a very simplistic argument and keeping rho bar as constant and t bar as constant was very bad assumptions, but we get some result out of that. Any questions and what determine the phi? Where does the phi comes from?

Student: ((Refer time: 47:27))

Loudly

Student: time lag

Time lag between phi dash and Q dash and, so every combustor will have certain p prime and certain Q prime and the relation between p prime and Q prime depends on the acoustics, impedance geometry and all that. But in many combustors heat release rate is

proportional to velocity fluctuations. In some situations heat release rate maybe proportional to pressure fluctuation. In some other ones it may be having components from both. In any case, it is, it depends on the specific mechanisms.

So, there are pressure couple responses and velocity couple responses, like in solid rocket the pressure couple response would be really dominant because the burning rates depends very much on the pressure fluctuation. In a gas turbine situation, it is the velocity couple response, which is quite interesting and in a ((Refer time: 48:25)), which we saw in a demo yesterday, what was it? Was it pressure coupled response or velocity coupled response?

Student: ((Refer time: 48:32))

It was velocity coupled response. We are looking at heat transfers from the hot wire driving the sound in a leakage tube and the pressure fluctuation are almost nothing compared to that most depression. But the velocity fluctuations are comparable to the mean flow and they are driving the oscillations as we saw in the leakage tube.

So, in different situation it is different, but in whatever be it, if you have this phase coming between plus 90 and minus 90 and this phase depends on the various time delays involved in system, which depends on the physical mechanism, which can be different. But if you are driving you have to have this condition, that is, you have your phase between heat release and pressure will be between minus 90 and plus 90 degree or that heat release is correlated with pressure. So, then you have driving if it is uncorrelated or if you have contribution of  $\cos 180$  degree, then you actually have damping.

So, outside this regime you actually have the flame damping the acoustics; within the regime, minus 90 to plus 90 if you have the time delay or the phase, then you will have driving. So, that is, that is the message of this derivation here. So, Rayleigh criteria says, that if the heat release rate is in phase with the pressure oscillations, then energy is added to the acoustic field and if this addition is more than what is being lost, then what happens? Then, you have instability. There will be growth of the oscillations and if the addition is less than what is lost, then you have a decay of the oscillations. So, I will stop here and see you tomorrow.

Thank you. If you, any more question? Thank you.