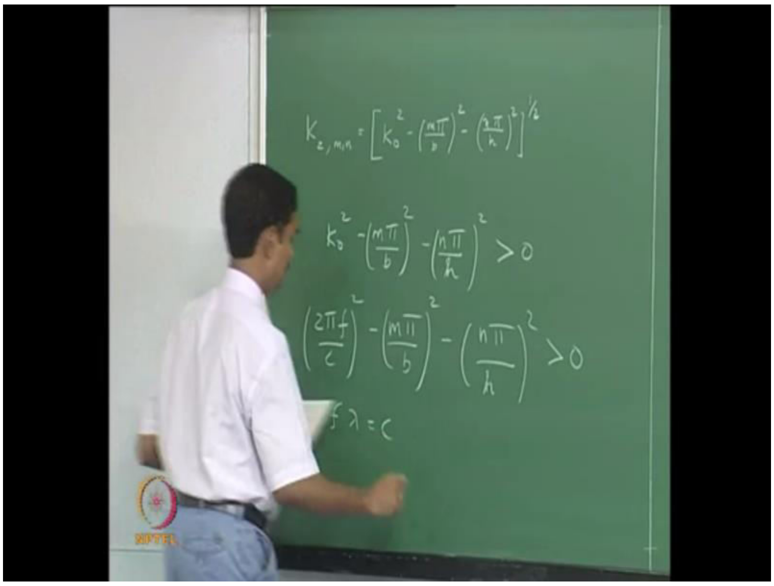
Acoustic Instabilities in Aerospace Propulsion

Prof. R I Sujith
Department of Aerospace Engineering
Indian Institute of Technology, Madras

Lecture - 15
Multidimensional Acoustic Fields-2

So, we are looking at sound propagation in a box or acoustics field in a box and we derived the expression for the solution for the acoustic field and now, we are looking at the relations between the wave numbers. So, we had the wave numbers K X K Y and K Z and these are not independent things they are expressed in terms of K naught as K X squared plus K Y squared plus K Z squared equal to k naught square.

(Refer Slide Time: 00:49)

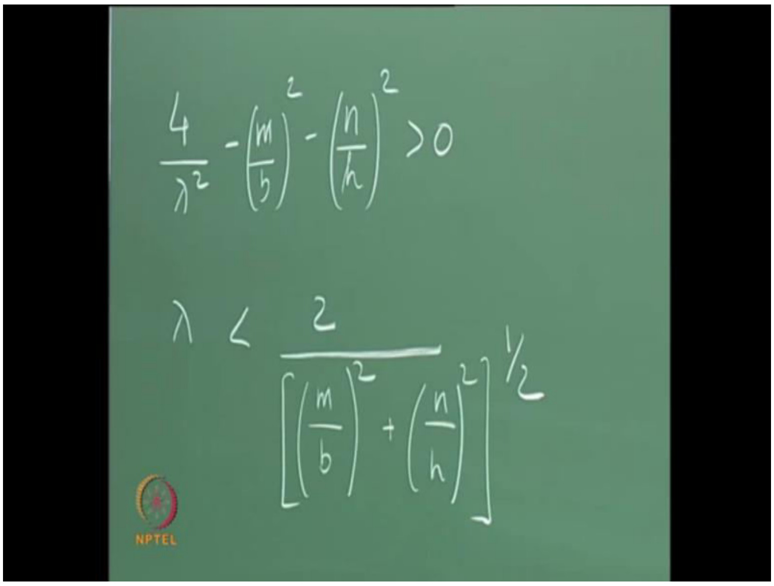


And so, if you write the solution terms of the modes and then the relation turns out that. You can express K Z in terms of the K Y and K X as follows it becomes square root of K naught squared minus m pi over b squared minus n pi over h squared per half and so, there is a square root over this quantity which is forward half but, this is some quantity minus some other quantity.

And if you notice this m pi over b the whole square and n pi over h the whole square the positive numbers. So, there is a a square is bit of possibility that the second and third term together can over the K naught. So, I that case you will get a real solution only when K naught squared is greater than these 2 quantity if not you will get a imaginary solution and then you will examined what is the consequences of that? So, this would give real roots for K Z comma m comma n and that would mean that you will have this propagating solution of the form e power i k is that e power and minus i k is that. When this quantity becomes imaginary then you would not get that kind of form because i times i. If you think of this quantity as imaginary let us say i alpha.

So, e power i times i alpha will become e power minus alpha. So, you do not get this propagating kind of solution but, you will get that alternated solutions. So, you get propagating solutions, in other words any module propagates only in this condition is true that is K naught squared minus m pi over b the whole square minus n pi over h squared is greater than 0. So, if you write rewrite this in terms of frequencies you will get 2 pi f over c square minus m pi over b squared minus n pi over h square great than 0 or we can alternately rewrite this as noting that f lambda equal to c.

(Refer Slide Time: 03:24)

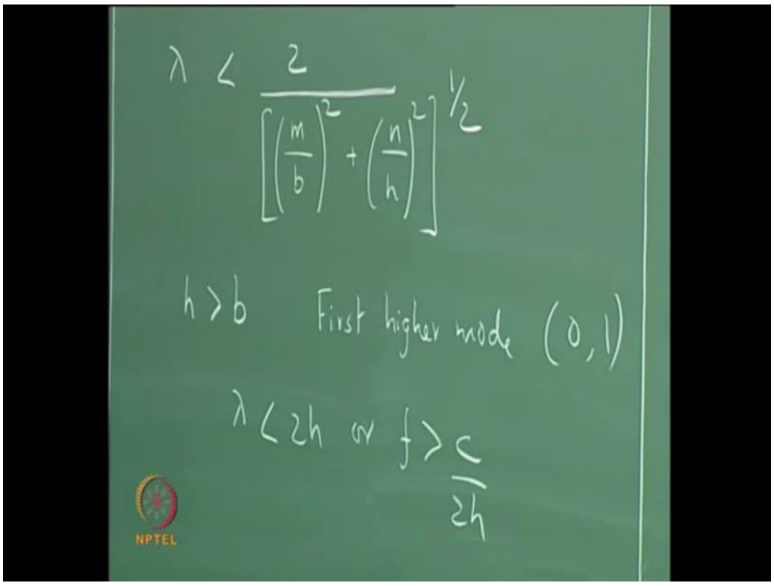


So, you will get 4 over lambda square minus m over b squared minus n over h squared is greater than 0. This can recast as this will happen if lambda should be less than 2 over m over b square plus n over h square. So, your wavelength has to be shorter than certain value for a mode to be established and that mode to propagate if not the mode will attend it rapidly and we cannot establish that mode in the duck it will only be person locally where you set up the sounds.

((Refer Time: 04:16))

Oh yeah, thank you. So, if your lambda is only lambda is small and the frequency is high. Then the cut off value you would be able to establish the wave in the duck if not the this would not get established, it will only be personed locally. So, if a otherwise plane waves should have no problem because we m and n as 0. If you take m and n as 0 this quality will always be satisfied. So, there will be no problem with plane wave but, if you have non plane waves with this mode number m and n you have to have this conditions satisfied.

(Refer Slide Time: 05:06)



Now, if you think of h is greater than b. I mean if I just pick the b the first higher mode will be 0 comma 1 that is 1. So, you should get lambda less than 2 h or f is greater than c by 2 h. So, if you do not satisfy this relation you will not be able to set up this higher modes in the duck. I mean, you can those modes will be very local and the amplitude will be k exponentially is this any questions ok.

(( ))

Yeah.

(( ))

Yeah.

Student: (( )) that is plus i something or minus i.

You can choose I mean both are possible but, it is ah I mean if you are. So, you have to pick which one is there. So, for the left running wave you will have to pick the solution such that the solution decays to the left. For the right running way you have to pick the solution such that the solution decays to the right so that, we have physical way of choosing it. In many case, I mean if you have a amplitude at some places set of it will decay rather I mean that is not physical to (( )) any other questions yes.

Student: (( ))

Yeah. So, if you have a if you look at this expression. You have to have frequencies higher than certain value. When replaces f over f over c with lambda and rewrote this way. So, either we have frequency has to be beyond some value or wavelength has to be below certain value. I mean they have inverse relationship for this relation to b satisfied and only if the relation is satisfied you will be able to get a real value of K Z which would only the make it possible to have propagating waves. Otherwise this K Z would be imaginary and then you will have the alternated solution. So, you will have only a locale locale against the established higher modes but, it would not stay in the dock along for higher editions is it clear anything else pass for the minus wherever you want right in this things here.

Student: Sir, when that K is that imaginary you said decay would happen.

Yeah.

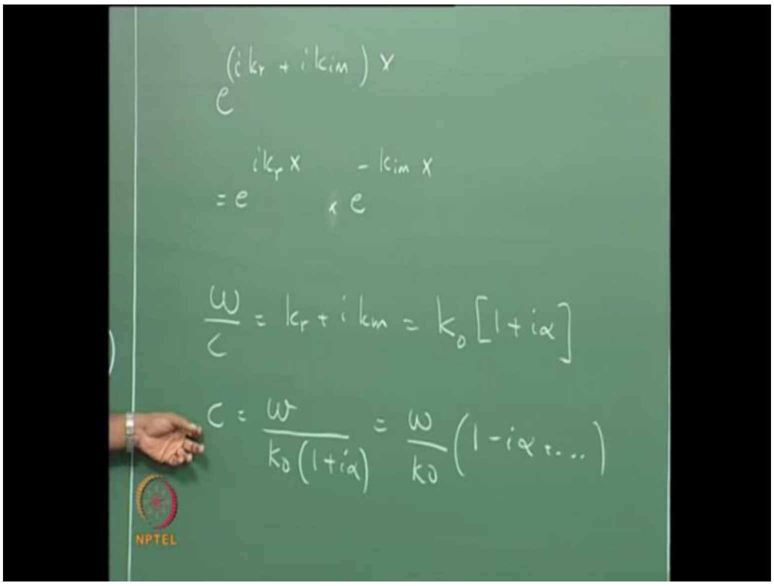
Is that decay for the amplitude or all the frequencies.

For the for the amplitude I mean that is that is what is the complex wave number means complex wave number means it will the amplitude will decay.

(( ))

The frequency has a periodic path and non periodic path. So, the wave number ex illustrates. How the amplitude will decays? e power i k x.

(Refer Slide Time: 09:01)



Let us say e power i k real plus i times k imaginary times X. So, e power i k real times X plus e power minus. So, this is the propagating path sorry this multiplied by and this is the growth or decay. So, this is the decay part alternation path this is probably good time t answer the question that you asked yesterday you asked two questions one was what happens when you have a a what is the question exactly about this a complex.

Student: (( ))

Yeah. So, if you have a real omega. So, let me refresh the question. So, that it comes in the video. So, you be when we looked at the attenuation of the sound in a duck. We said that a loud speaker and its producing sound and we have said we said the experiment is running in kind of steady state.

So, we have a real omega and then we had a complex wave number and the question was whether omega over the omega over c is k omega is real c is c has to turn out to the complex for getting K to be complex. So, this is indeed right but, what we have to remember? Is that the wave is solved at it’s not a Eigen value problem we imposed the omega that means the impedance of the loud speaker has to match with the duck and so on. Which we dint take into account. So, whatever energy is up sought by the duck is being given that is (( )) is made implicitly.

So, suppose the Eigen value problem where we actually described conditions of the boundary and then we solved for it solved for the frequency and the wave number. So, the relation between omega and k that is the dispersion relation. So, the Eigen value problem you will get dispersion relationship and there you can say omega over c equal to K or something like that.

And omega can be complex K will also be complex but, c will have to be a real propagating filed that because that is what it is? Now, here if you say which I was said let me re caste the K naught into 1 plus i alpha right. So, I will get c equal to omega by K naught into 1 plus i alpha which can be perhaps written as. So, I mean this c is a complex number but, this is just a definition.

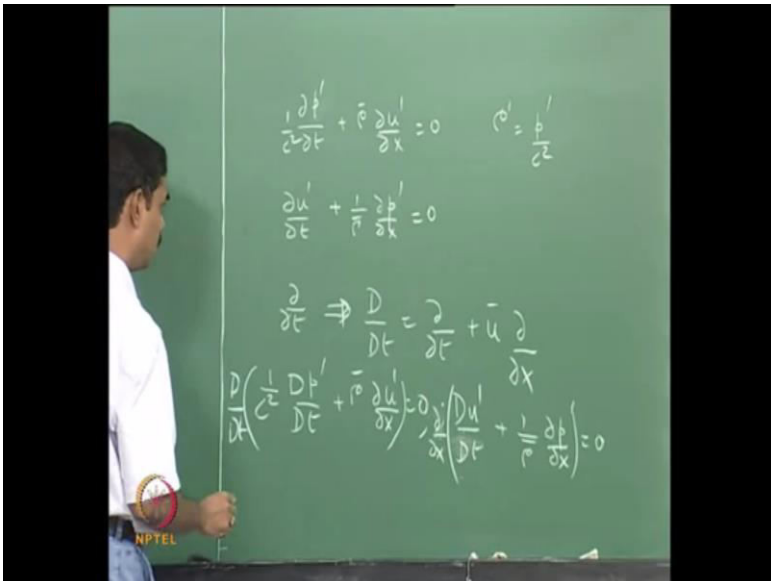
I mean it is not really the dispersion relationship. We would not get this as the dispersion relationship there is no dispersion relationship because it is not a Eigen value problem. So, this can be called a complex field and people many times call this complex field of sound but, we have to understand that it is just definition. So, omega over I mean this c which is gamma that is indeed the speed of the wave which is happening which is going which comes out as omega or K naught but, the balance is just a way of representing a (( )) but, if you were solving a Eigen value problem we can actually we will solve in the coming classes. When there is a flame and the flame drives or dance and then you will get the relationship between omega K and c and those would be like a property of dispersion relationship is that clear.

And the other question was.

(( ))

When you have mean velocity would you get same relation? So, I worked out the example quickly just 2 minutes before coming here and I dint get any answer which is fancy.

(Refer Slide Time: 13:24)



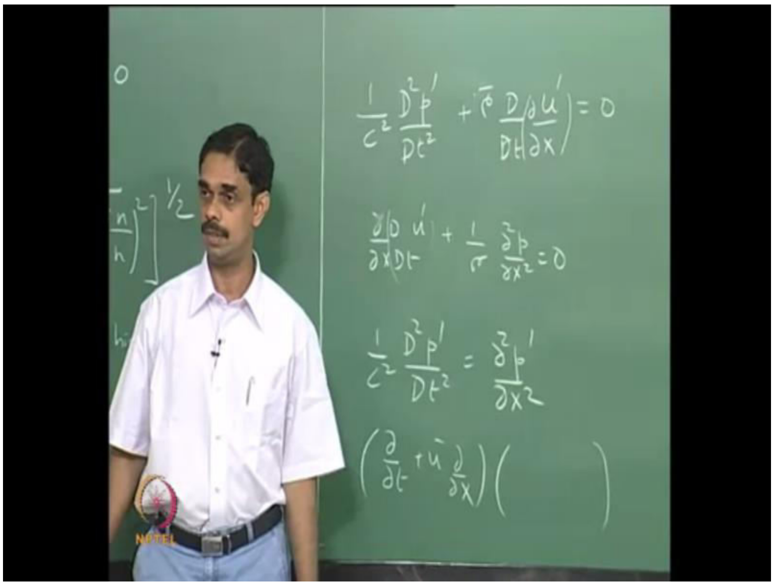
So, if you look at. So, if you let us say we the are. So, let us say there is a mean flow but, if you travel we are considering a constant mean flow. So, there is no variation in u bar or this no non uniform base flow it's just a constant base flow. And a question is can you translate the coordinate system and move with the base flow and then you should have the regular wave equation? We should be able to translate back and get the equation with the u bar and so on. So, what one would do? Is dou rho prime over dou t plus rho bar dou u prime over dou x is equal to 0. So, this is the continuity equation and I will replace rho prime equal to p prime over c square. I will say now.

So, this is the modified continuity equation and momentum equation must dou u prime by dou t plus 1 over rho bar dou p prime by dou 0. Now, if you want to see the now, you are if there is a mean flow or not decays and does not distinguish and you accept that. If it there is a mean flow then you have to be travelling at the speed of the mean flow that that is the way the equation is written. Now, if you now, get off the frame of reference and sit in the lab and way too my laboratory frame of reference imagine there is a mean flow. The all you have to do is to replace this term dou by dou t we will have to replace with D by D t which is dou by dou t plus u bar dou by dou x.

So, I will get 2 equations which are 1 over c square D p prime by D t plus rho dou u prime by dou x equal to 0. And the other equation would be D u prime over D t plus 1 over rho bar dou p prime by dou x equal to 0. Now, we can dou by dou x here and D by

D t here and you will get equation which is of the form. Let us do it p d minus b t and this is dou by dou x. So, we will be needed some space.

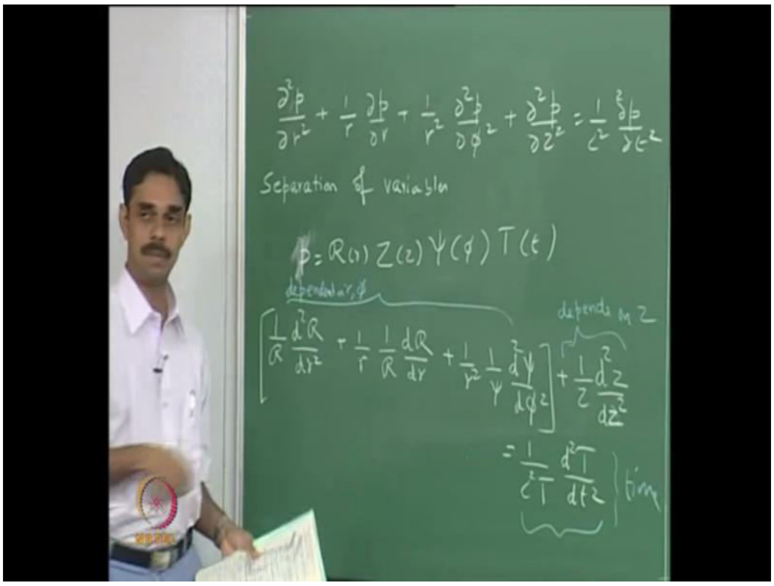
(Refer Slide time: 16:15)



One over c square D square p prime by D t square plus rho bar by and the other equation will give. So, this means that you have to operate d by D t first and then dou by dou x. I do not know how to write it? I hope this simple is ok or should I write D here. Yeah I think this is problem more appropriate. And you can actually commute the derivatives you can check it and therefore, you can get the equation as you multiply this. So, you get one over c square D square p prime D t square equal to and now, if you evaluate this expression as dou by dou t plus u bar dou by dou x acting again on the same thing and check you should get the wave equation in terms of partial derivatives. And I did not get any fancy things as you are saying. So, will stop here you can check it and get back to me right. Just check the full algebra m quite keen on clarifying this. Any other questions on the old topics? So, there are no any questions will move on to acoustic fields and cylinders. So, having got an idea about acoustic field in a box.

And we saw what solutions can propagate and not propagate. Now, we can go back to our Bessel functions and so on. And see what kind of modes can be set up in a circular duct. Usually in engineering applications one finds circular pipes more often if you go to a lab or any construction you always find a in general in mechanical engineering kind of places you find circular pipes often of course, in rooms are always like a box you do not have circular rooms yet. So, I guess both are important. So, we looked at the solution of will write the wave equation cylindrical formats and now, we can also do this spherical co-ordinates but, I am not going to spend too much time on this topic I will just wrap up this. So, we will look a solution to wave equation in general cylindrical co-ordinates. And this time will keep all this angular dependency and so on.

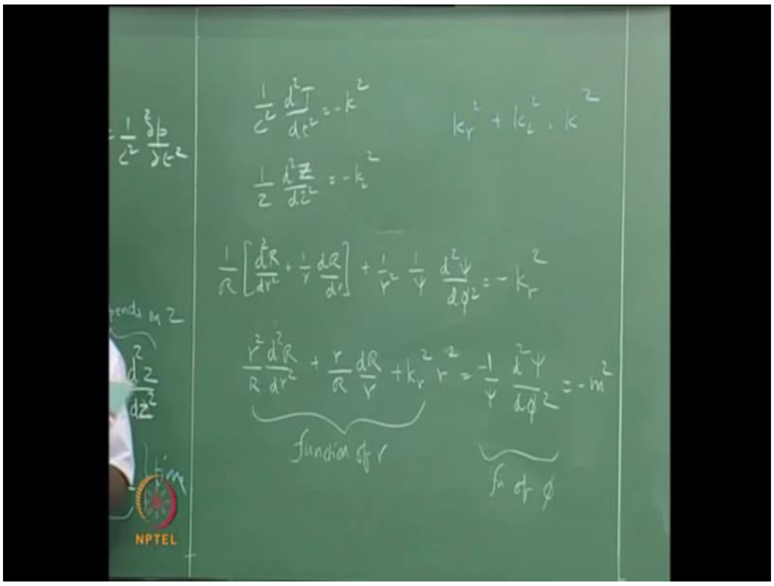
(Refer Slide Time: 19:48)



So, we are writing the wave equation there are no mean quantities separately dealing we are dealing with and so on. So, I will take the liberty to drop the primes if that is with you p actually means p prime here. Now, we can do. So, this is the wave equation in cylindrical co-ordinates or what is call cylindrical polar? We can do separation of variables. So, we can say phi equal to R of r times Z of z times. So, if you do this and work out the algebra I will write the final result. So, if you work out the algebra that is you take your pressure as function of R and function of e z times function of phi times function of time and substitute into this relation.

And then we divide throughout by R Z psi t, then you get something of this form. So, this term is dependent on r and phi and you have this term this dependent on e z and this is on time. So, if you have a function of time b equal to equal to function of e z which is space co-ordinate this should be equal to both of them should be equal to constant and if these 2 things are constant the function which is depend on r and phi should also be a constant. So, such the logic here.

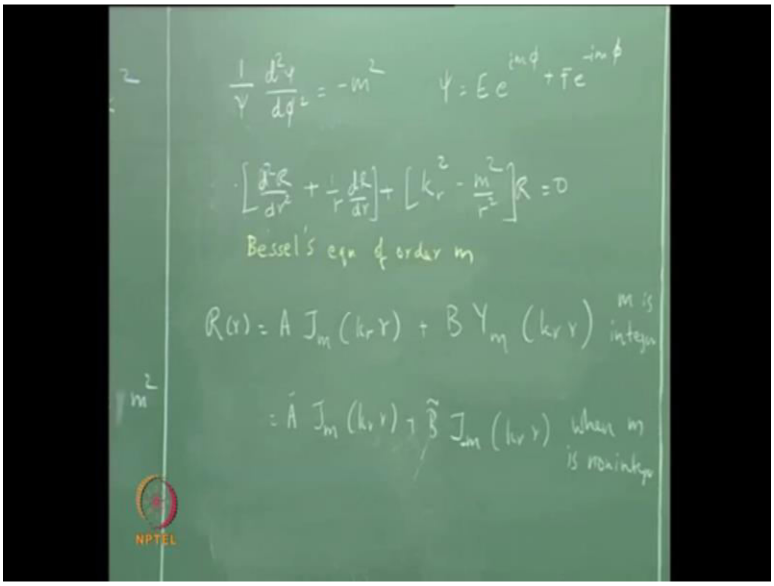
(Refer Slide Time: 23:08)



So, we can then write this as 1 over c square d square T over d t square equal to minus K square and the minus sign is just to make things appear pretty and you have the first term 1 over R. So, I should have yeah. So, will have this constant and they are independent we can write a relation between them. So, K r squared plus K Z square equal to K square. So, this is like a constraint compatible relation. So, if you expand this thing out we can write r square d square R by d r square plus r over. So, you multiply this through by r square and then.

So, r square comes here this goes away from the denominator r squared comes here then you brings this term on the right hand side to the left side take this term to the right side. Now, you have a function of r being equal to function of phi and therefore, both should be constants. So, we will call this constant as minus m square. Now, we have separated out p d e's to order different equation for each of this R these are psi and T. Hope this is straightforward.

(Refer Slide Time: 26:16)



So, we can write this as 2 equations actually. So, 1 and the other 1 is. So, we have 2 equations which are actually coupled. So, you have the psi which is the angular dependence you have differential equations for that and you have to note that the same m that is appearing here and what is this equation you seen it?

(( ))

Student: Minus m.

Where? Yeah.

Minus.

This is. So, here there is the minus right no I want to call it minus.

M

Yeah, no; this is this should be thank you. Yeah again you can write plus m or minus m it is just to make the things look nice and so on. So, what is this differential equation? Bessel equation of order m that means we if you put m equal to 0. This will become psi is independent of c you can get that kind of relation. So, we looked at a last class. So, this is a more general version where you can have angular dependence another many problems in which there could be angular dependence. So, you just leave it. There I will only look at the radial variation and so that.

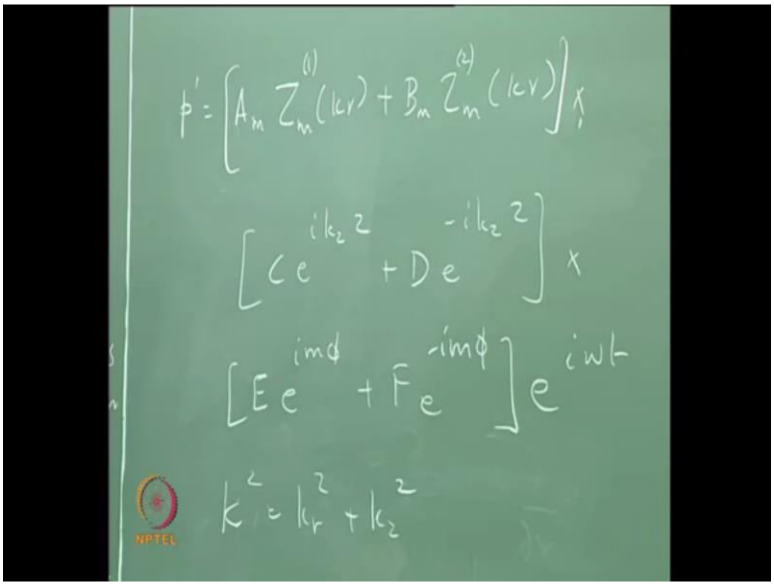
So, you will get solution of this form when m is a integer and if m is non-integer what happens? So, this is for m is integer and but, if m is non-integer. What is the solution?

K of m.

Yeah, k of m k r r plus r j of minus m k r r when m is non-integer. Now, the reason we can write a solution in terms of j of m and j of minus m is because j of m and j of minus m are independent in independent functions when m is non-integer but, when is a m is integer j m and j m minus m are linearly dependent. So, then we have to look for alternate solution which is what is the non m function which we get? And I give you references in last class about this things if you do not look at them study them this will appear as beautiful pictures and not as anything which is meaningful.

So, I argue to go and look at those that nice NPTEL video on Bessel functions and so on or any book or Wikipedia, whatever without it I think this is not make any sense. So, a general solution is product of this we know R of r. And we know solution to this psi would be e power and for yeah for e z we can get e power i k is a plus e power minus k is i i k z. And you can multiply all them and you get the general solution for p prime.

(Refer Slide Time: 32:25)



So, e z is the generalize cylinder function which can be written as j or y y or j j of m and j of minus m or you can also write in terms of angle which is j plus i y and j minus i y. All can be represented by this general cylinder functions that is e z and the superscript denotes the first kind second kind which always have 2 solutions right and m denotes the order. So, we multiply this by multiply by E e power i m phi plus e power a and we have to also remember that this m here is coupled with this m here.

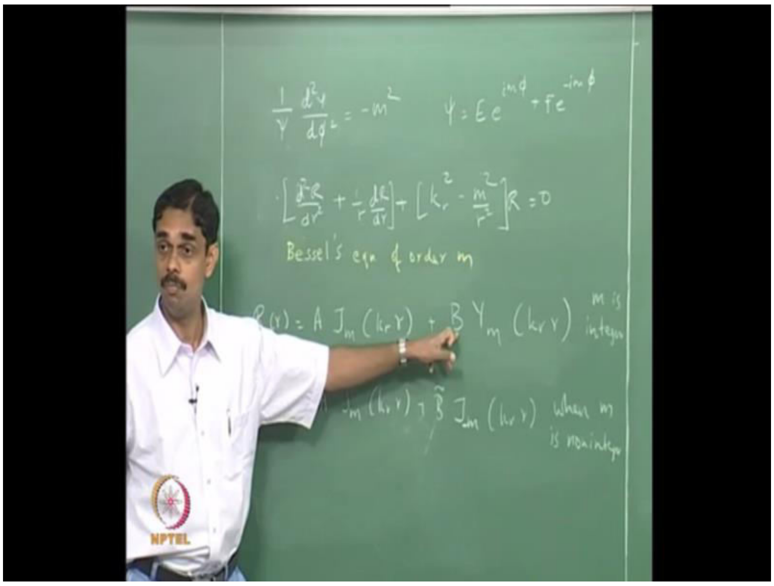
Student: Sir e i.

Yeah. So, times e power i over here thanks and enough is to keep or i omega t or e power minus i omega t just give the same thing thank you. Can I erase this term you have written.

(Refer Slide Time: 35:38)

So, next will study sound propagation and circular tubes oh acoustic fields and we look at rigid walled tube.

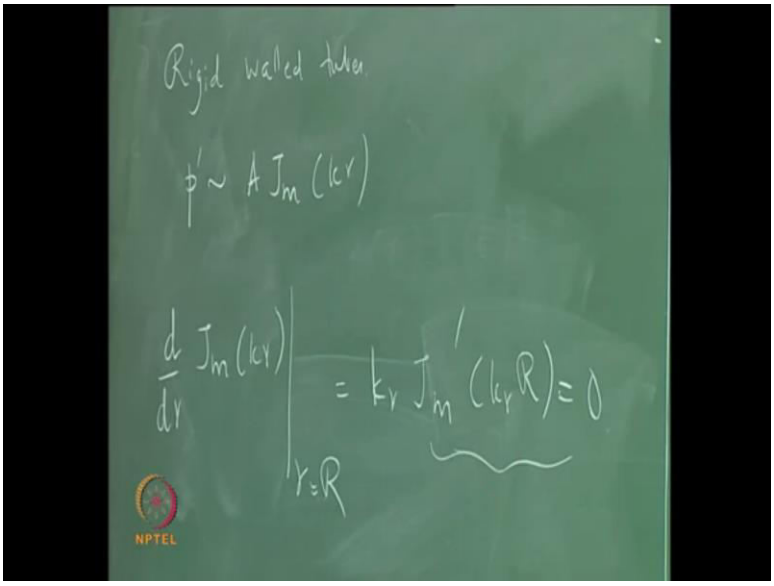
(Refer Slide Time: 36:05)



Now, in a tube if you look at the solution and we have this radial dependency here and what would be the value of the can we guess the value of B. 0 why yeah. So, at r equal to 0 the value of y is minus infinity. So, he has probably done the homework no I think he is expert in Bessel function, I learnt from him earlier. So, for those of you are not experts you should do the homework and figure out how the y and j likes. So, all y's go to minus infinity when r equal to 0. So, if you have if your domain includes r equal to 0 you can peacefully drop b will be 0. If you domain does not include r equal to 0 that is what we are talking about a annuals sound, annular combustion chamber which your probably studying then we have to keep b so that, is clear. Now, we can safely if circular tube we can just safely say that out expression is j m what is a value of j naught at r equal to 0 and what is j 1 j 2 j 3?

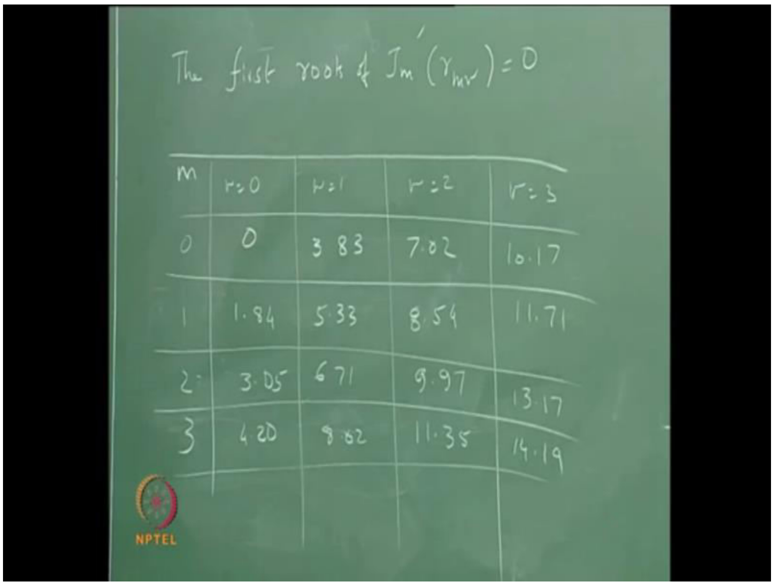
Student: 0. 0 j 2 ha j naught is at the 1. Fine yeah all the j's are 0 except yeah. So, for. So, if you have a circular tube this enough to keep the solution. And in summary we can drop this because Y of 0 goes to minus infinity which should imply B is not 0.

(Refer Slide Time: 38:18)



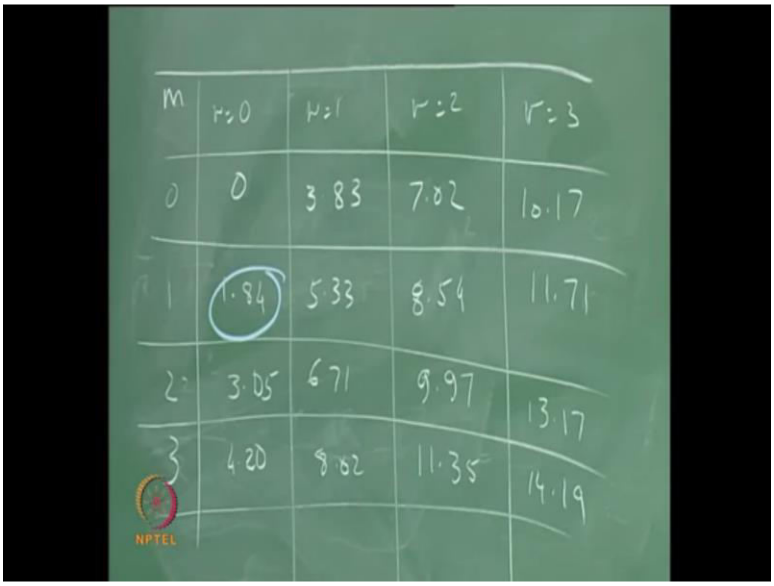
So, our p prime will go like a j m k r and. So, we need the velocity 0 if you have rigid wall. So, velocity will go like d by d r. So, d by d r of J m of k r at r equal to capital R where the tube is boundary is k r R equal to 0. So, the roots of this equation will give the values of k r R and in Sin and Cos have simple relation like if Sin x 0 then x can be 0 pi 2 pi 3 pi and so on. But, we learnt that Bessel function has a very important characteristics with about the roots what does yeah the roots of the Bessel function are not evenly spaced. So, it is a big subject by itself to lot of people do p h d's on roots of Bessel functions and so on. We will not do a p h d but, look at first root. So, this quantity.

(Refer Slide Time: 39:41)



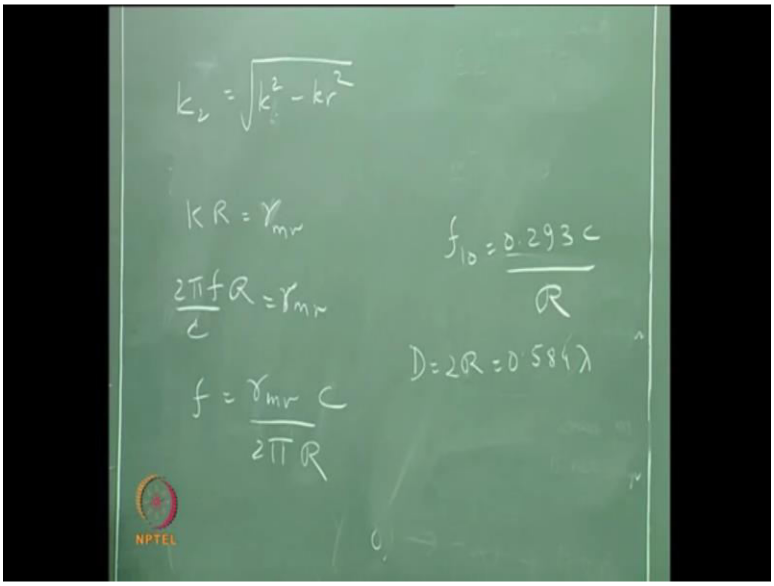
So, of course, the there are interesting relationship between J m and J m primes for example, if you have m is 0 J naught prime is it goes like J1 one and therefore, higher one's there are recurrence relationships. In terms of like the primes can return times of other J's but, it is not a simple a sin and cos except in the case of J naught but, hand books are available tables are available and the math lab and so on gives the roots will peacefully. So, let us look at a table m is the order here and 0, 1, 2, 3 we looking and we look at the first root which is m equal to 0, m equal to 1, m equal to 2, m equal to 3. So, you have 0 here but, here.

(Refer Slide Time: 41:46)



So, this is the very first 1 that is coming that is correspondingly m equal to 1 the 0 m equal to 0. So, of course, why the roots are this way and lot of stuff are available but, we can get this in hand book and the interesting thing is that this correspond to the lowest frequency that can be set up with radial mode and you can have frequencies are only higher than that anything lower than that would not propagate.

(Refer Slide Time: 42:35)



So, we have to have a k which is more than what the K R gives and what is the value of k r. So, sorry.

K z is equal to square root of.

Yeah. Sorry, thanks fantastic. So, K can be written as 2 pi R over c r equal to we calls root as gamma. So, you this is the cut-off frequency the frequency established has to be higher than this. For the K z to come real otherwise you are not be able to get a propagating solution and so, this would from what we from this 1.84 will correspond to over R over this will correspond to D which is 2 R equal to 0.584 lambda which is of the order of lambda over 2. So, you have to have frequency which are higher than what you get from this formula to be able to establish radial mode in a duct anything lower are only be the local.

Here you will not propagate or the wavelengths have to be here should be shorter than the D divided by 0.584 or you can think the other way for given wavelength the duct diameter has to be like half the wavelength approximately. And anything below you will not be able to set up these radial modes you only be able to have plane waves. So, the same way we saw in the case of the box except there it was simple m pi by b n and n pi over h. Where here we need to have this table or computer programs and so on. So, I am going to stop my discussion about multi-dimensional acoustic field with this but, in reality there is a lot more to study I think annular combustors have this modes which are annular modes and they are having this radial kind of stuff as well as they are rotating and so on.

But, we will not speak much about this in this class we will concern with purely axial waves but, just I wanted to give an introduction to this kind of topic. So, I will stop here with my teaching on acoustics and in next class we will. So, we are free first half of the class, first half of the course which I said was dealing with propagation of sound. So, just to recapture whatever went through half semester we derived equations of fluid mechanics well 1 dimensional and then you can 3 D if you want but, the important thing was that the continuity equation is not rho a equal to constant but, dou rho by dou t plus del dot dou rho v equal to 0. And it is really important that we keep the terms D by D t in both continuity and energy and moment. And then we talked about linearising and this was purely because we wanted to keep things simple because linear equations can be solved.

And we did manage to solve some of them not all for example, everything dint get dint give out solutions form f of x plus c t and so on. Only some with the temperature get in or with the cylindrical co-ordinates, we dint have a general solution right. So, we linearized we got travelling base solutions we got sterling base solutions we found this speed of the propagation which came out to be square root of dou p by dou rho at constant approved and entropy. And then we talked about different kind of boundary conditions closed and open ends and then imperious boundary conditions and then we will talked about how to determine the imperious in practical experiments? We said that it is easy to see the solution and then get the boundary condition that is what you do in reality and then we. So, there we set up a real frequency.

And looked at the standing base structure got the impedance but, then we are able to translate it to the Eigen value problem with those that n and you are able to get the real and imaginary particular frequency real parties periodic solution corresponds to periodic solution the imaginary parties corresponds to exponential growth over decay. So, we linked admittance to the energy flow and that we linked to the growth over decay. Because if energy is coming in from the boundaries your acoustic energy duct will grow. If there energy is going out you your energy will decay and then we looked at some propagation to non-uniform temperature regions or non-uniform temperature then attenuated waves and so on. And then lastly we looked at we also looked at combination means the temperature gradient and this damping.

And then we looked very quickly at a multi-dimensional acoustic field that is, you have like a acoustic field in the box and acoustic field in a tube. So, of course, this is big subject and the 3 semester classes on acoustics but, I have crashed everything into 2 months or something other that idea is that the second half of the class which will start from tomorrow will speak about the more acoustic instability of combustion in stability and this is more like a background towards that. So, we in summary we studied propagation of sound and we are going to look a generation of sound. And why it should I mean the other way but, propagation is much easier to study compared to the generation this propagation is well established something.

Thank you.