Acoustic Instabilities in Aerospace Propulsion
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Module - 01
Lecture - 14
Multidimensional Acoustic Field-1

Good afternoon. Vikram, you have a question.

Student: Derivation of the non-dimensional impedance you took the common point f c equal to 0 for both the left and right running waves left running will travels a distance beating the material which impedance you have to determine. So, we should put x equal to l for the left running wave travelling.

So, the question is when we did this problem on impedance and related it to this growth rate and so on. The question is our sample if it is on the left side should we call that as x equal to l or if our convention is consistent with the travel times and so on.

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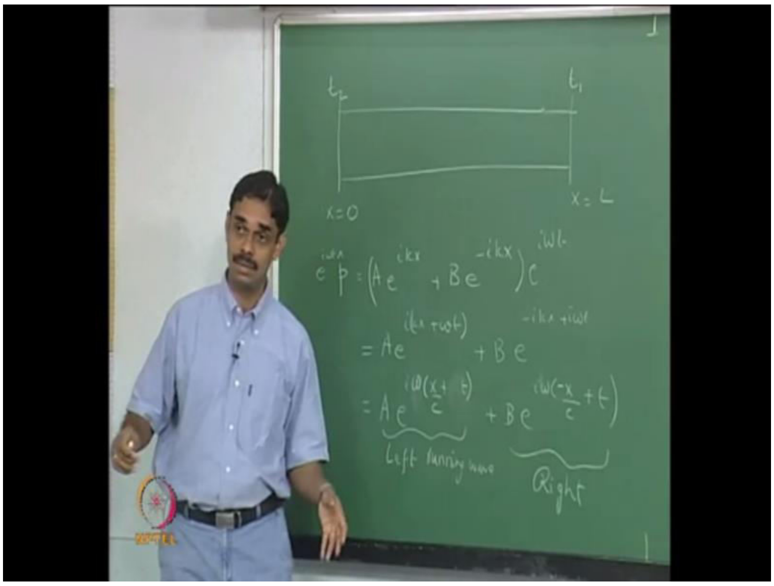

So, let us just take a quick look at this. So this was a tube and I had x equal to 0 here and x equal to L here and I will call this as time t 1, when the wave reached there and time t 2 when you're at some other time t two and our solution is we had p hat equal to a e power i k x plus b e power minus i k x. So, this is the left running wave and this is the want that your questioning and this is the right running wave and if I multiply by e power i omega t on both sides, then this will get e power i omega t. so, this would be A e power i k x plus omega t please correct me there is a algebraic mistake, because it will be crucial the signs and so on. B power minus i k x plus i omega t.

So, this can be written as A e power i k into x plus omega can be written as o k is omega over c. So, this should be x plus c t right. So, plus B e power I will take i k out minus x. Let me take omega out. So, this will be x over c right plus t and if I right omega here is this. So, far did i make a mistake did you get the same. Yes. So, we will look at left running wave and which is the left running wave this one right, this is the left running wave and this is the bright why do you call this left running wave, because it is t plus x over c and we showed that would corresponding left running wave. Now the let us call this t plus x over c or the other quantity t minus x over c they are called a retarded time.

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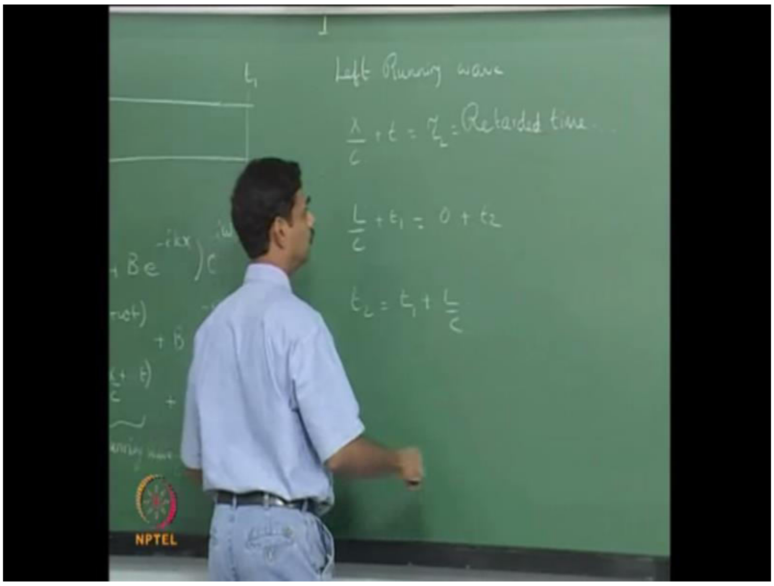

So, let us first take a look at the left running wave, left running wave and we say x over c plus t we will call this tow it is called retarded time. So, we can call it tow L corresponding left running wave. So, let us say you were at t time t 1 when the wave started from here and reached here at time t 2. So, at x equal to L over c plus t 1 equal to 0 plus t 2 that means t 2 equal to t 1 plus l over c. So, this is consistent the wave was first at t 1 and then after some extra time l over c reached t 2 now. So, this is consistent with our understanding. So, although we have 0 less than l but, that is what he out.

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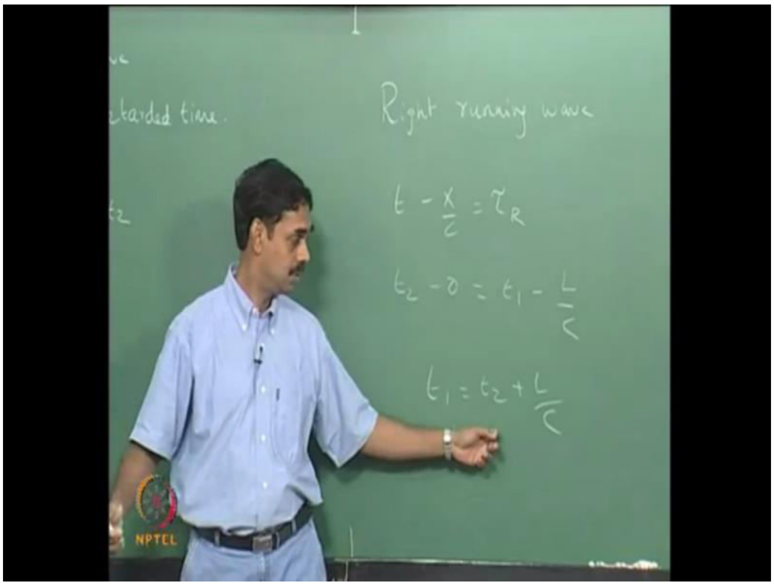

Now, if you look at the right running wave. So, there it will it will be x over c t minus x over c equal to tow r. So, at x equal to 0 you will have t 2 minus 0 equal to t 1 minus l over c. So, t 1 equal to t 2 plus l over c. So, the wave was at x equal to 0 at time t 2 and then after some time period extra t l over c, it reaches t 1 which is consistent with our understanding that wave is moving from 0 to l. So, both are consistent. So, there is no need to modify with the co-ordinate system. Is that any other questions?

Student: ((Refer Time: 05:47))

Speak loudly.

Student: Sir, when the attenuation problem. We took omega n by k was complex making that make c. So, how what is complex c?

The thing stands for growth or decay.

Student: Complex because if we take the first or the solution that we finally get that do not satisfy initial wave equation that would be.

It will not because let me work on this and get back to you. I am not having a of that answer and I think it should satisfy but, you mean to interprets complex speed as let me come back to you I do not have a clear I mean I think in terms of complex wave number and complex frequency but, if you fix on to be real and other to be complex, yes you should have complex wave get back to you sorry I am not having a good answer. Any other questions? Sorry, about this. So, I want to first talk about special functions. So, I will give some references and also nice NPTEL lecture exists.

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So, special functions are solutions to ordinary differential equations which are studied by some people a lot and then their properties are well-known. So, then they are called special functions. So, sin x cos x are also special functions. So, it is very special because we use it all the time. So, they are all special cases of what we call hyper-geometric functions and so on.

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So in particular we are concerned with Bessel functions.

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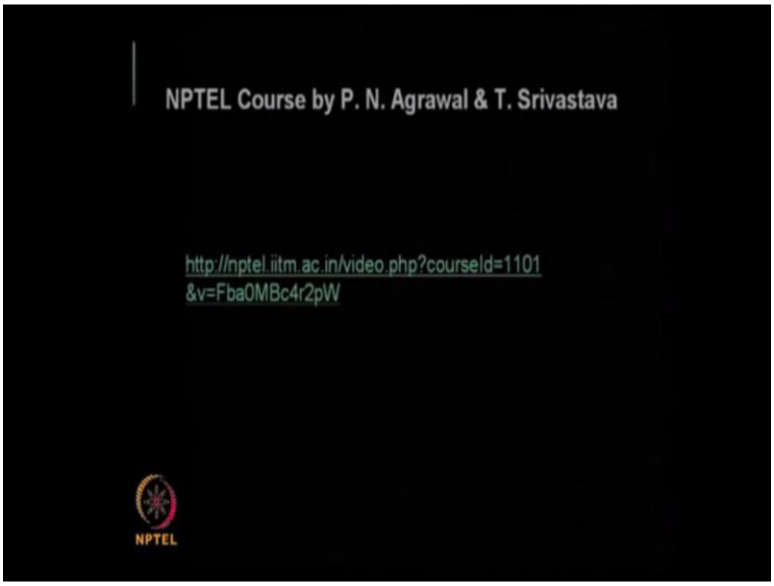

And there is a very nice NPTEL course by Agrawal and Srivastava. This reference is here I can mail you the link I think I checked in place very well on the computer. So, when such a lecture is there does not make any sense in redoing this whole thing. So, I am not going to do it.

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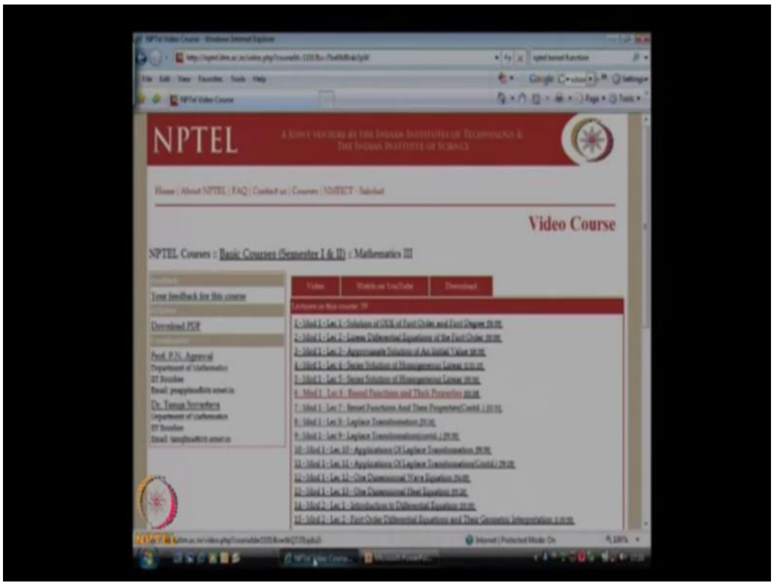

And this is the website of this class and you can see lecture six deals directly with Bessel function and properties and the earlier lectures on series solutions and on that deals with how to get a general series solution and so on. So, forth I recommend that if you do not have the background on special functions or getting solutions of ordinary differential equation the ordinary differential equations in terms of series and so on. Please go through it is worth it as a engineer one should know Bessel function I think it is quite critical.

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So, this is the course website and you can do Google and it will come out.

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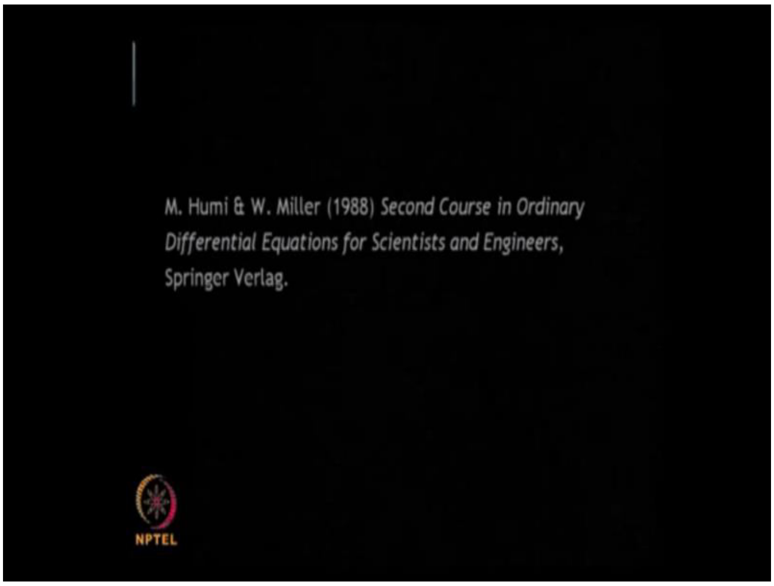

And the book that I used to learn is Humi and Miller. It is title second course in ordinary differential equations for scientists and engineers Springer Verlag. It is a really nice book and it is math book without you know sometimes we get worked on a math's book because they are. So, worried about existent and uniqueness and theorems and lamas and so on. He does not deal with any of that he explains to you how to solve the equations and what there will special functions of what are the properties? How to apply the boundary conditions? When it will work? When will it would not work? When it will converge and. so, on where the worry about theorems and lamas and. so on and it's really about for scientists and engineers not for mathematicians. So, I is ideal book I really enjoyed reading this book and I have read this book several times I think a Bessel function is chapter two. It is there in our library actually this book.

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I think many of you have studied the book by N. N. Lebedev. So, he has written a book on special functions and their applications and this is Dover book. So, it must be there in Tata book house. So, reasonable price and this actually Russian book but, translated. So, this is a good book also and again you can understand this book without any problem. It is worth getting on of this books of course, the other books is quite expensive, first once when I brought as a student twenty years back it was seventy and all over something.

So, I do not think there is a Indian edition and there are large number of books I went to the library sometime back and checked and whole stack of books on special functions and if you search on the internet or any book store you will find a lots of books on special functions and Bessel's functions to be specific and so on. So, there is no dearth of this material on studying this plus of as always there is Wikipedia which gives amount of materials and plus is lot of p d f's and also other sites like site of mathematic Wolfram's site on special functions and so on. So, I think there is no excuse to really not knowing special functions plus this wonderful NPTEL lecture is also there by this people from rookie. So, I think I am not going to deal with I am not going to teach this special functions.

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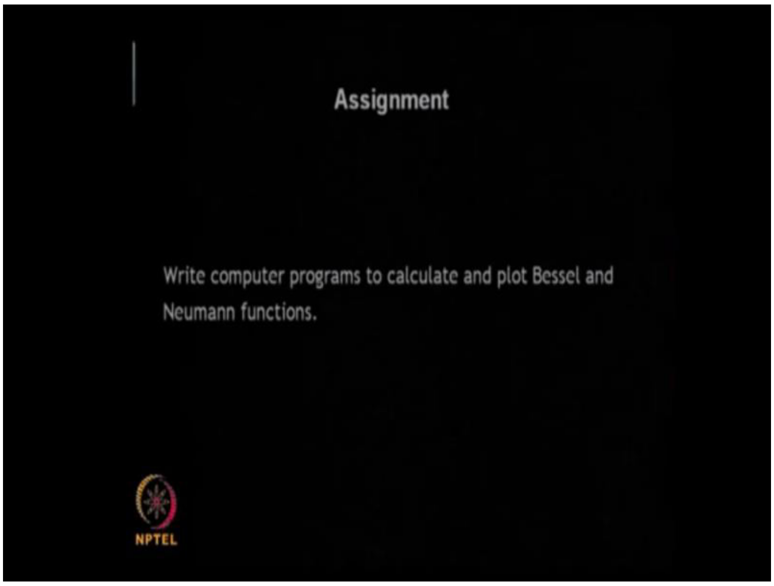

I used to give a few classes on special functions but, now that all this NPTEL is there already I will not teach I instead I will give assignment of do not worry I will teach assignment. So, you can write a computer program to calculate and plot Bessel and Neumann functions you can write first order and second order 0'th order and so on. And play with it and see how the functions look like I think it is quite important to play with this plots and you do whichever language or you can do it in C or Fortran and draw a graph or you can write the online in math lab or mathematic lab or excel I mean that is my favorite it works very fine.

And of course, if you have integer order you use Bessel non-even and if you have fractional order you use j nu and j minus nu that means ah is the same Bessel but, you have to use order and it is negative it would not work when the order is integer, because j of one and j of minus one are linearly dependent. So, you need two linearly in dependent solutions for second order differential equations. So, that is why we look for a linearly dependent one given j and that is what we get y but, I would not bother to teach you this but, like this plenty of places where you can learn this term and I will harp a little bit on multi-dimensional acoustics but, not a whole lot because I half the semester is end up.

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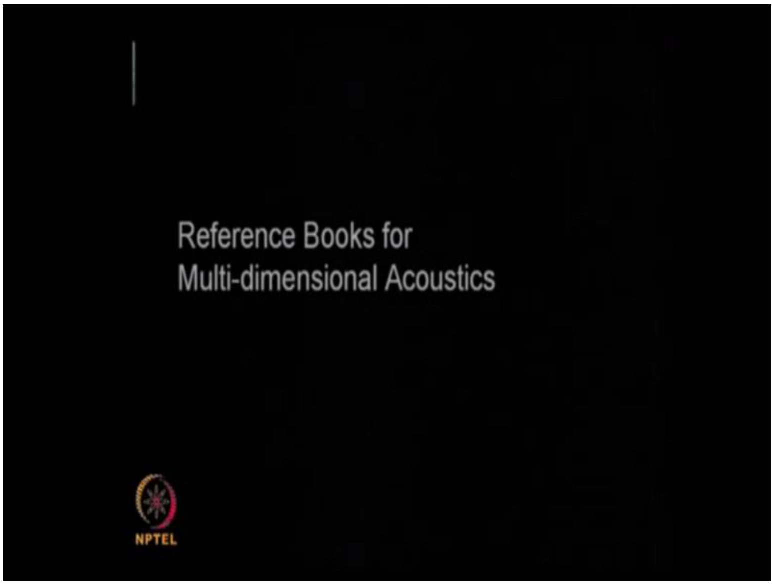

So, I want to combustor stability of thermo thermo-acosutics as soon as possible. So, but, I will very briefly cover this.

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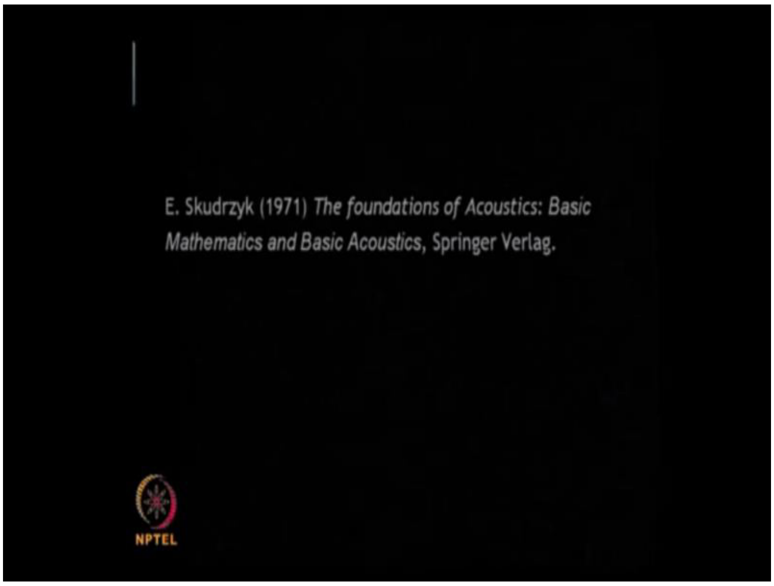

And the foundations of Acoustics Basic Mathematics and Basic Acoustics by a Skudrzyk this is very nice book, really lovely book it is in our library I really enjoyed reading this book and I think this is the best on this multi-dimensional Acoustics with best book I have seen, well at this in my opinion it is not a universal statement you may not like this book but, it surely there in the library.

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And professor from Munjal from I I S E is a very famous professor from India. He has written a book on Acoustics of Ducts and Mufflers with applications to exhaust and ventilation systems design. This book is not in print I think. So, you would in I could not find any book stored but, it is there in the library. So, it is a very nice book and it is written that is the people read it would understand some books you would not get the feeling that, the author dint wanted to understand but, this is a really nice book I really enjoyed it and is a very famous Indian professor.

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And Morse Ingard is very nice book on Theoretical Acoustics and this is written more like a fesses view of things but, those of you are inclined to studying physics would really enjoy reading this book. Also, in our library of course, there are plenty of other books but, this is my view of the word. So, we were looking at model solutions and I wrote a differential equation for wave equation and cylindrical co-ordinates and we said we will drop the angular dependence. So, the equation was something like this.

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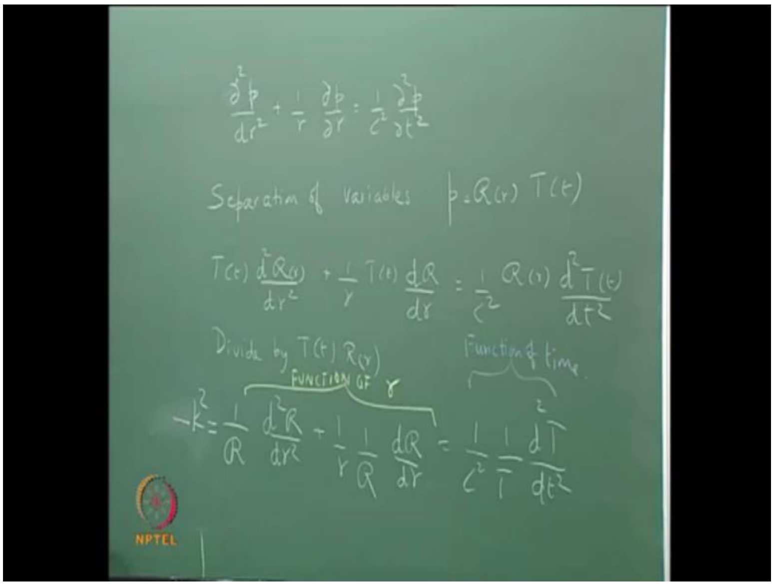

So ,I asked you where to obtain a solution in terms of separation of variables right when I said we cannot find the solution of the form f of x plus c t and g of x minus c t. So, we use. So, there is no way to know by the Separation of variables will work or not you have to assume a separable form put it in and see if you can separate it. If it works fantastic if it does not work nice try. So, p equal to R of r times T of t.

So, you will substitute in here. So, I will get a T of t times 1 over r. Now what we did, is to substitute this expression into this p d e and then differentiate it. So, if I Divide by T of t, R of r I would get 1 over R, d square R by d r square and I am writing the dependency in bracket now plus 1 over r this. So, I will write I will erase the board. So, what do you observed from this? The left hand side is actually a function only of r. So, this is a function of R and the right hand side is function of time.

Student: There will be ((Refer Time: 18:51))

Any mistake?

Student: R of r and T of t should be.

Should be?

Student: Non-0 term and all r small t's.

Why? Because where I mean only thing is these thing should exist and.

Student: Divided by R of r and T of t.

Student: There will be non-0 term.

Why I mean if R is 0 this cannot be go to 0 and there limit can exist.

Student: So, you can tell only limit is 1.

So, you have a function of r which is equal to a function of time. So, if a function of one variable equal to function of another variable Left hand side is purely a function of r, right hand side is purely a function of time and then what should be there should be both should be a constants this will be equal to constant. So, we will call this minus k square there is no sintity about the minus sign and it just to make things look pretty we can relook at this issue later. So, we know how to solve the both the parts.

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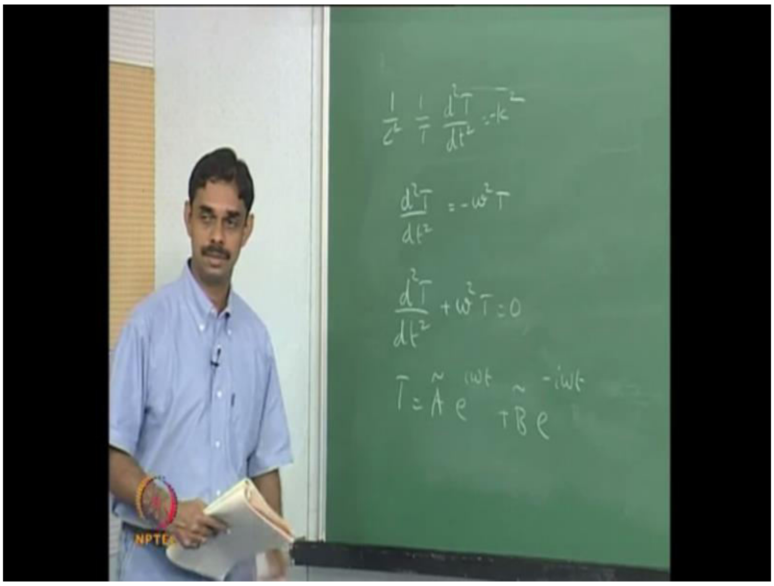

So, first part would be 1 over c squared times, 1 over T, d square T b y d t square equal to minus k squared. So, this will be d square T by d t squared equal to minus omega square T or you have get the solution of this form. In fact you can show that when you take the total product in right pressure you will get the same relation whether you keep I omega t or e power minus I omega T or you keep both. So, it is enough to keep just one of them and just be consistent.

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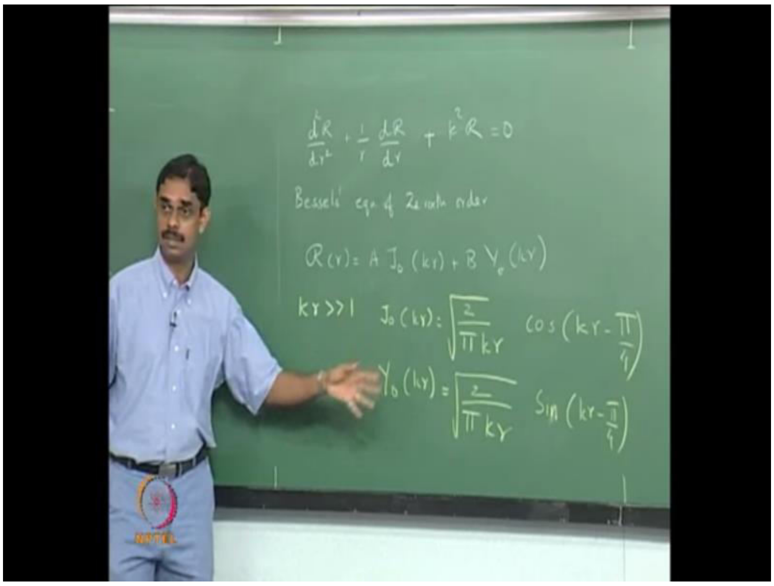

And the other equation is d squared R over d r square plus one over r, d R by d r equal to minus k square R. So, I will take this rather. So, this is the Bessel's equation of 0'th order and the solution is and I emphasis that if you are not familiar with Bessel equations study about it. So, just to understand how the Bessel functions behave we can take a look at it is the asymptotic relations of Bessel functions which will give you a hint of how things work.

So, Bessel functions have asymptotic nice asymptotic relations. So, asymptotic relation means when k r is much larger than 1 you can show that j naught of k r equal to root of 2 over pi k r, cos k r minus pi over 4 and y naught of k r equal to root of 2 over pi k r, sin of k r minus pi over 4. I must emphasis this formulas are valid only for k r is much greater than 1 and if they are low lower than one near 0 these formulas are not valid any questions.

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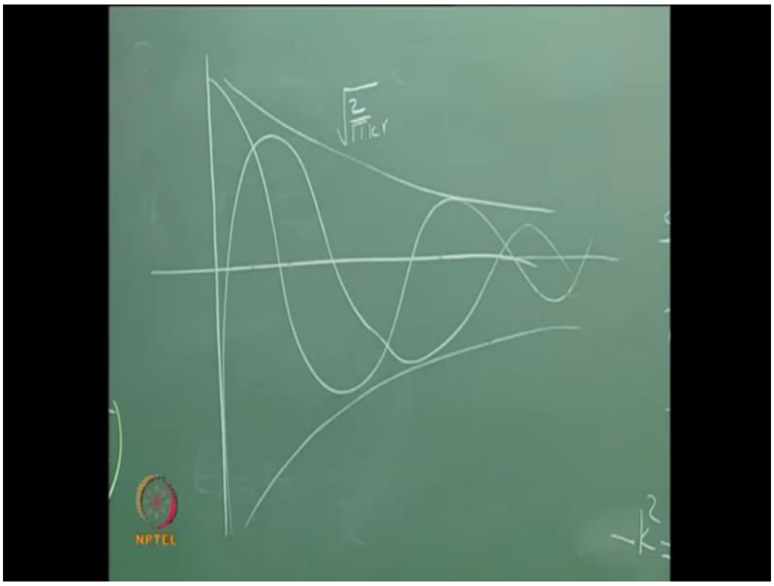

So, if you look at the Bessel function j naught and y naught plot. So, j naught would look like this. So, this envelope will be going like two over like here. So, you can see that some kind of periodicity but, then there is a kind of enviousness decay. So, if you rewrite with this asymptotic formulas again. It is valid when k r is much larger than 1.

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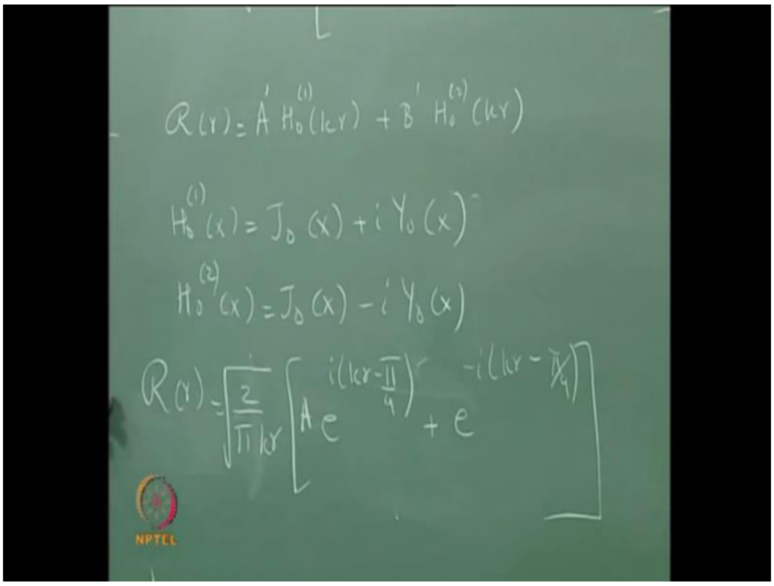

So, let it write nicely. So, this is kind of un-analogous to our expressions. We had an sin and cos. So, this is something like that. So, j naught is kind of analogous to the plane wave cos k x minus pi over 4 and y naught can be thought of yes the other one.

And we can also write in terms of Henkel's functions and maybe some of you are inclined towards math's they can see physically maybe. So, I should put A's prime here just to denote that. This is different when you are writing for Henkel's. So, the 2 Henkel's function of first kind and second kind and they are like J naught plus I naught and J naught minus i Y naught you know that e power i theta is cos theta plus i sin theta.

So, this is some kind of generalization of that form. So, if you again go by the asymptotic formula then you can get of course, this formula is valid only in the far field. When k r is much larger one but, it'll give you a hint that you have a progressive wave e power something an e power minus something e power i k r there is a phase shift by pi by 4 similarly, e power minus i k r but, divided by a root r dependence and this is quite easy to imagine because when you have a spherical spreading.

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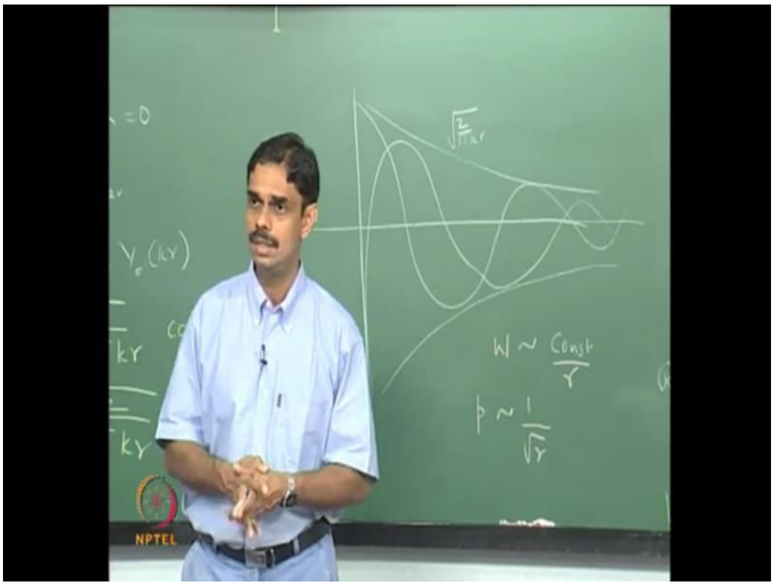

You have your power goes like. So, pressure will go like 1 over root r same with velocity but, this is valid only in the far field in the near field things will be quite complex. So, which this is why you get this 1 over root r here, because you are having spreading of the wave.

So, it really not although the amplitude comes down as you go away, you should not think of it as being some kind of damping your attenuation it just spreading out, it just like if you have a lamp turned on and it shines bright but, if you go further and further away the amount of light you that falls on some object will keep coming down that is because the same amount of light is spread into everywhere along. So, the same thing with sound I mean when you are far away the same amount of power is spread to more and more areas. So, the intensity goes like 1 over r and the pressure and velocity will go like 1 over square root of r, which is why you see this kind of dependency's here 1 over root r here and here also.

And which is why I mean the Bessel functions capture this dependence perfectly and that is why you got the solution as J naught and K naught instead of getting sin and cos and J naught and Y naught they capture cylindrical spreading. So, they are call cylinder functions this J naught and Y naught and they capture cylindrical spreading. we are having a cylindrical spreading that is the key thing.

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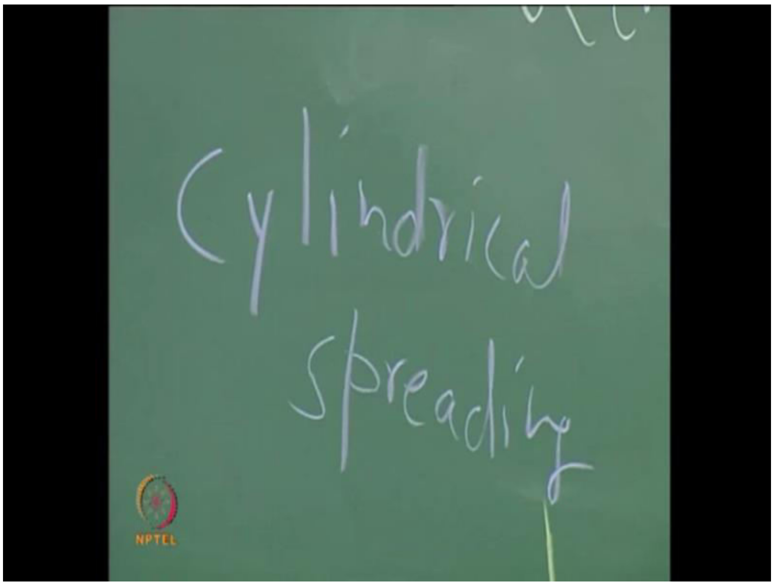

You could imagine what would happen? If you are having three d propagation what would happen? We would be constant over r square and pressure would be like 1 over r and velocity will also be like 1 over r, in fact you can actually get rigger's solutions that way in terms of travelling waves. Whereas here you cannot in the near field there is a problem which is you cannot get a solution but, for spherical you can actually get pressure as f of x minus or r minus c t over r plus g of r plus c t over r you can actually work this out to the homework and what would be the corresponding solutions in harmonic domain in spherical co-ordinates? I said we get solutions as f of let me write it down.

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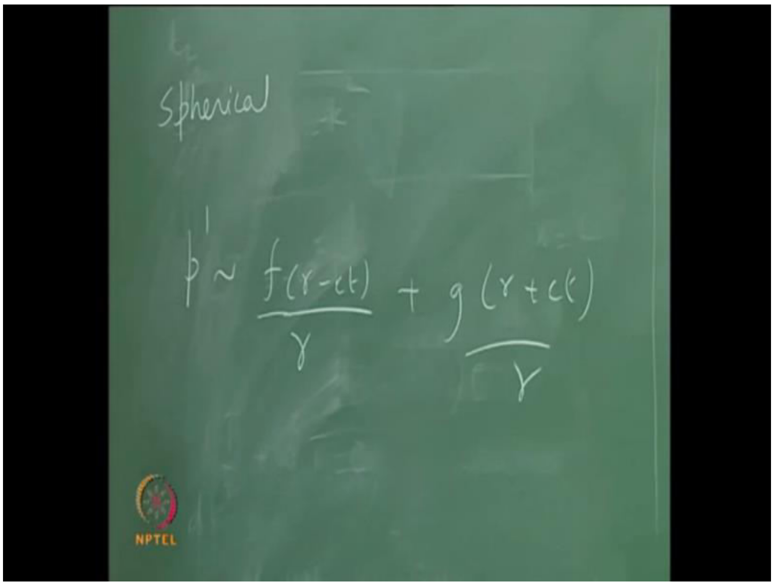

You can get p prime where are f of r minus c t over r plus g of. So, what would be the corresponding solutions in the harmonic domain. Here, we got J naught and Y naught what would you get for spherical propagation must studied in some other class and physics [phy/physics] physics minus here. So, you must answer.

Student: ((Refer Time: 32:27)) Question

What would be the answer can you guess?

Student: Spherical.

For spherical instead of J naught and Y naught. What would will be J half and J minus half and any questions on this? So, I wish to do more of multi-dimensional little bit more but, before we do [be/before] a general circular duct and look at the natural frequencies and so on. We will work out a simpler problem of sound propagation or acoustic field in a box that it is a simple problem and will do that we will look at natural frequencies and things like cut-off and cut-on and then will come to this cylindrical thing again. So, what would you call it box like that is word rectangular prism what must be some fancy name but, will call it box.

Student: Cuboids

Cuboids. So, we are looking at acoustic fields in cuboids which I call box and of course, we are we can be we thrill because we can leave with only sin and cos or e power i k x and e power minus i k x, no Henkel's, no Bessel through them away. So, the wave equation would be.

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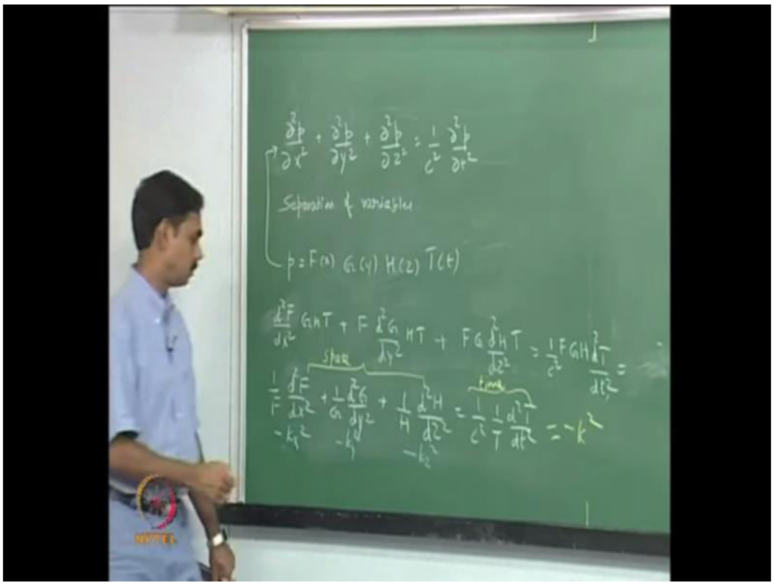

So, again we can do our Separation of variables. So, we say will use a same notation p equal to F of x, G of y the h t H of z and T of t. So, you can substitute this in here and now by we know the trick. It divide throughout by p and then you can separate out a terms as function of x function of y function of z function of time.

So, if you do this you will get f times d square no d square F by d x square times G H T plus F d square G by d y square H T plus F G d square H over d z square H T equal to 1 over c squared, F G H times d square T by d square this is. So, this is a function of function of time and this is a function of space. So, if a function of time is equal to function of space then both must be constants. So, we will again call this as minus k square and sorry i to divide. So, I jumped the gun sorry will have to edit and erase it out. So, I must divide throughout by F G H T. So, I will get 1 over F d square F by thanks plus.

So, now I can say this is a function of space and here is a function of time. So, both must be equal to a constant and. so, once this is a constant and then you can keep these two on one side and bring this the other side then you have function of x and y being equal to function of e z. So, those must be constants and then you can bring y to the other side and say function of x equal to function of y. So, that should also be constant. So, we can have each of these terms independently must be a constant. So, this can be called minus k x square minus k y square minus k z square yeah.

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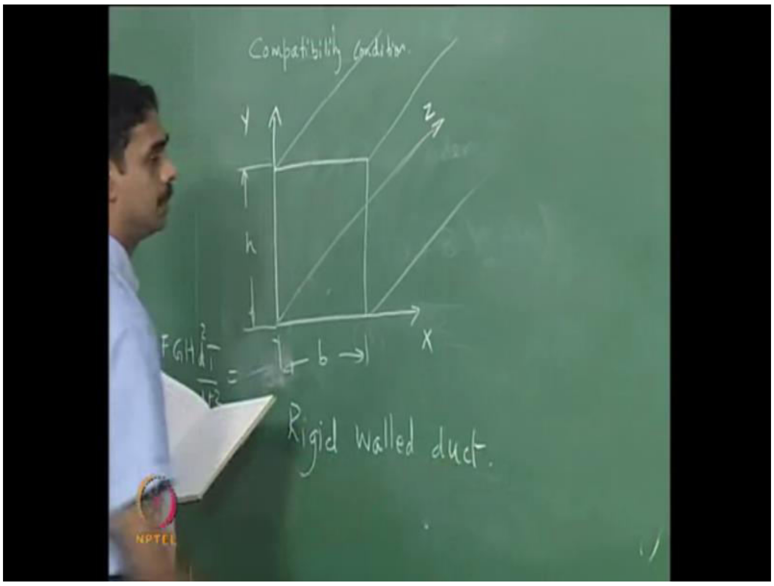

So we have this relation now or alternately I can remove this minus sign. So, this is like a constraint or a compatibility condition or something like that like a. So, now we need to solve it for a very specific power. So, let us look at a case now comes the box, this is x and this is y h. so, let believe that we have a wall which is rigid we have walls, which are rigid we can also have walls which are flexible or imprience walls and so on this is very simple. So, we can work out the problem algebratly. So, when we have rigid walled duct what is a boundary condition.

Student: Velocity.

Velocity should be 0. So, velocity will go like d p by d x.

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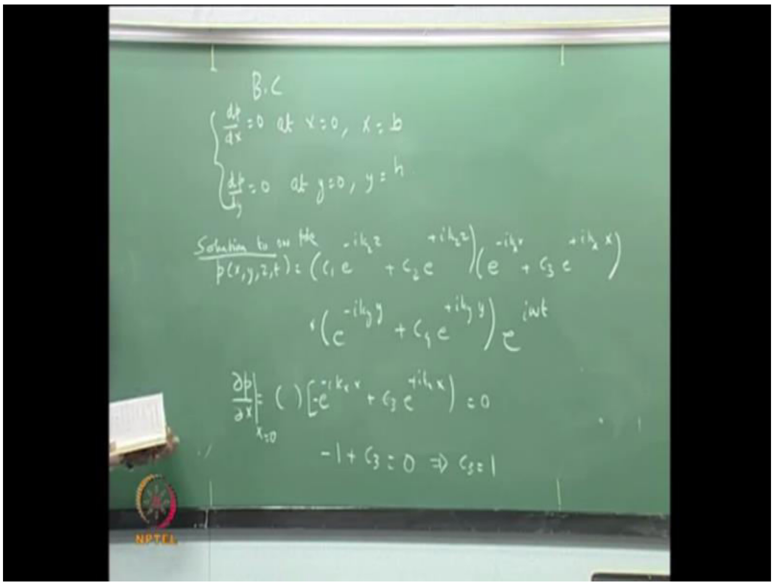

So, can say x equal to 0 and x equal to d this dimension is b and we can say d p by d y equal to 0 at y equal to 0, y equal to h. so, if you differentiate we can get d p by d x will be you will have some constants time minus first let me write the solution, dint write that. It is by know obvious to you the solution. So, p of x comma y comma z comma t would be c 1 e power minus i k, z z plus c 2 e power plus i k, z, z multiplied by e power minus i k x plus c 3 there should be a constant here but, it is a linear problem.

So, I can I will recover all the constants. So, I can divide throughout by if I had c 3 and c 4 I can divide 3 throughout by c 4 and rewrite this multiplied by. So, this should be the solution to our p d e and now. So, this is a solution what is given here and this is the boundary condition I will pass for a minute.

So, if you evaluate dough p by dough x it will go like some constant bla bla times e power I k x x plus and I'm writing the k term and putting clubbing everything in else into some bracket here. Now if this is 0 at the x equal to 0. So, I will take 0 equal to 0 that would mean that minus 1 plus c 3 equal to 0 which will mean c three equal to 1. So, this term would be. So, if I where to evaluate this at x equal to b next d p by d x 0 at x equal to b.

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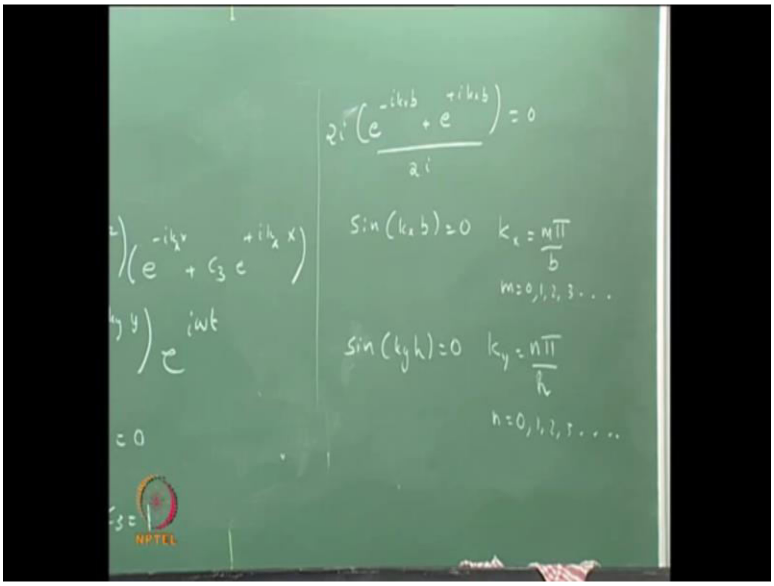

So, I will get e power minus i k x b plus e power plus i k x b, because I have got c 3 to be to be 1. So, it is 1 here. So, this should be equal to 0. So, divide and multiplied by 2 I and 2 I and. So, I will get sin k x b equal to 0. So, this should give k x equal to m pi over b where m is 0, 1, 2, 3. So, this should be fairly peaceful. So, similarly, if you apply the second boundary condition, if you apply this boundary condition you should get k y. You will get sin k y into h equal to 0 and you should get k y equal to n pi over h and n equal to 0, 1, 2, 3 you can work it out it should work out the same way exactly the same way yeah mistake. So, let us write the final solution this thing multiplying.

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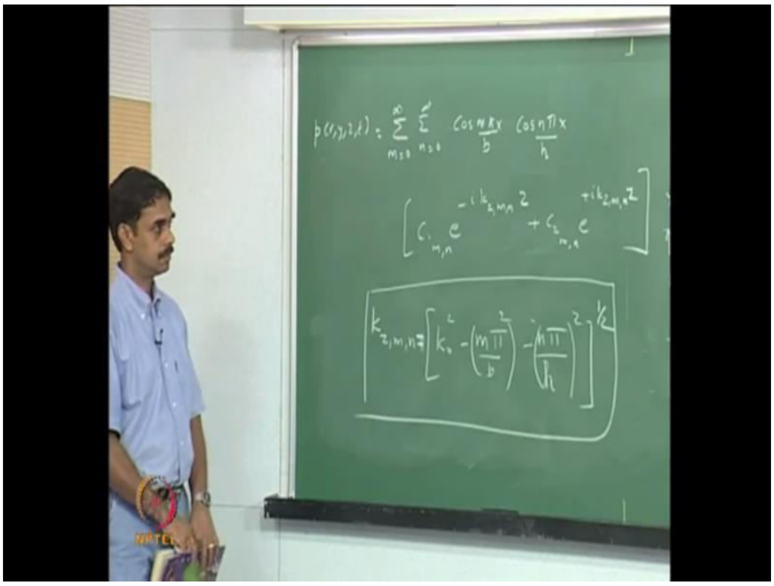

So, this would be our general solution and it will be a summation of all the modes of course, depending on what mode is set up only that mode will be there or you can have combination set up and so on. but, what is important this k x and is not a independent thing you will get this is what you get for a k, z the axial wave number depends on a frequency and also the m and n the modes which are set up across the done. Then pass for a moment for you to see this is a. So, this is straightforward algebra I hope there is no mistakes.

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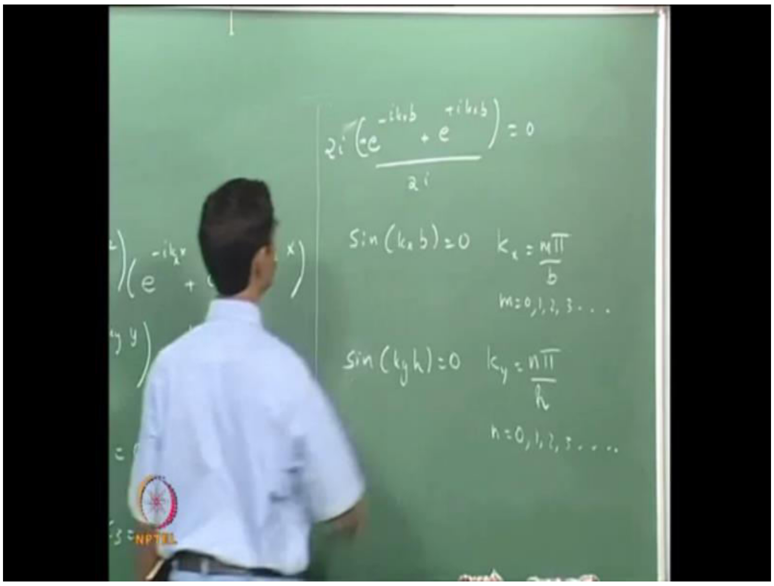

I made a mistake here. So, the minus here that is why I get sin.

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And so, when you have c 3 as 1 and then you will get e power minus i k z plus, e power i k z z multiply and divided by 2 and you will get cosine and the same thing for y term will get e power minus i k y plus e power i k y multiply and divide by 2 you will get cos k y. Now, you got the expression but, this has very interesting implications which will look at it in detail in next class but, I will give a hint of it.

What happens when the second and third term over once k naught? What happens when k naught squared is less than m pi over b squared plus n pi over h squared? This whole term becomes negative take a square root of negative number it becomes imaginary. So, what does k is a being imaginary mean. So, it will not be anymore periodic anymore e power i k z z will actually become work like e power alpha e power i k z times e z will work kind of like e power alpha e z. So, which will be like exponential decay and. So, we will see in next class under what conditions you will have a periodic kind of solution under what condition will have a decaying solution and therefore, this will relate to which modes can propagate and which modes cannot propagate. So, I will stop here.