

Acoustic Instabilities in Aerospace Propulsion
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Module - 01
Lecture - 13
Sound Propagation Through Inhomogeneous Media – 3

Good morning everybody. We were looking at sound propagation through regions of uniform, non-uniform temperature of course, we know that when you have constant temperature we have solution. Let us say for light wave of the form f of t minus x over c but, if you see we saw that we our pressure when there is non-uniform temperature we are non-uniform speed of sound. So, you have f of t minus integral $d x$ over c .

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Background : High freq. solution

- For a perfect gas $(\rho(x)c(x))^{1/2} = \left(\frac{\gamma P}{RT}\right)^{1/4}$
- Assuming no mean flow \rightarrow mean pressure is constant.
Also γ, R are constants

$$p'(x,t) \propto \frac{f\left(t - \int \frac{dx}{c(x)}\right)}{A(x)^{1/2} T^{1/4}(x)}; \quad u'(x,t) \propto T^{1/4}(x) \frac{f\left(t - \int \frac{dx}{c(x)}\right)}{A^{1/2}(x)}$$


And it is scale by a power half and t power one-fourth and similarly, velocity was going as T power one-fourth on the numerator but, a power half still under denominator times f of t minus integral $d x$ over c . We started with some kind of huge restrict arguments and assumptions.

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Acoustic pressure and velocity

$$p'(x,t) = \frac{f\left(t - \int \frac{dx}{c(\xi)}\right) + g\left(t + \int \frac{dx}{c(\xi)}\right)}{A^{1/2}(x)\bar{\rho}^{1/4}(x)}$$

$$u'(x,t) = \frac{1}{\bar{\rho}(x)\bar{c}(x)} \left[\frac{f(\zeta) - g(\eta)}{A^{1/2}(x)\bar{\rho}^{1/4}(x)} - \bar{c}(x) \frac{d}{dx} \left(\frac{1}{A^{1/2}(x)\bar{\rho}^{1/4}(x)} \right) [(f(\zeta)d\zeta + g(\eta)d\eta)] \right]$$

Note: $\zeta = t - \int \frac{dx}{c(\xi)}, \eta = t + \int \frac{dx}{c(\xi)}$ 

But, later on we derived first term special temperature profiles that the pressure can indeed we expressed in this form and the velocity was also in a similar form. But, there were extra term where we were and the characteristic t minus x over c . Now replaced by t minus integral s psi over c and other one is t plus integral d psi over c .

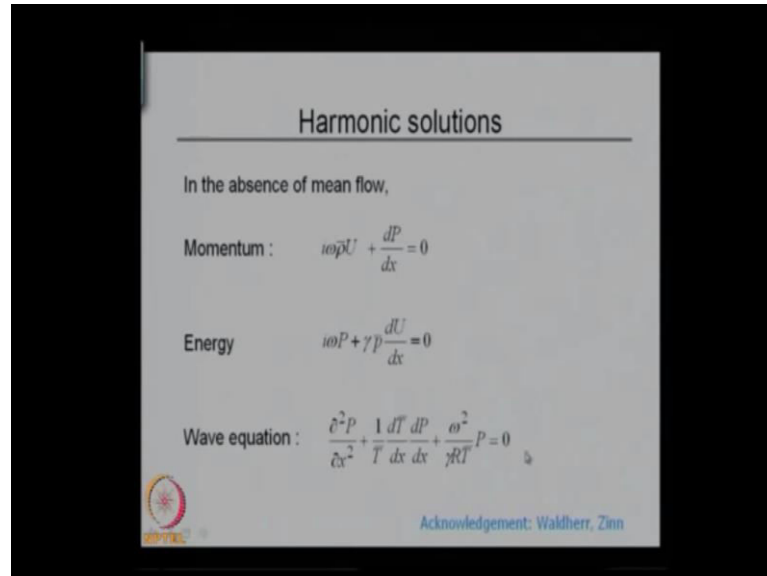
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- Questions
- What is the relationship between the pressure and density fluctuations?
 - Why am I solving only 2 equations?
 - When do I need to solve all three together?

Now I stopped last class with some questions, what the first question is. What is the relationship between pressure and density fluctuations? we worked it out and the answer was that p prime is not exactly ρ prime time t square but, there is a correction term

right we worked out do it a last step ha the second question was. Why am I solving only 2 equations?

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That is here I started with momentum energy. Even if you look at the time domain solution I was starting with momentum and energy we actually have three equations continuity of momentum and energy. So, why are we solving this two? What happened to the third one? Try this why do not we use continuity momentum and energy. And solve them together it should be ((Refer Time: 02:19)) yes I either use continuity of momentum or momentum and energy but, yes the answer is that the continuity equation gets decoupled from the momentum, and energy equations for mark number is zero or mark number 10 is 0 and if you had higher force like if you have mean velocity terms then if you have very small in velocity. You can still have \bar{U} appearing but, you can still decouple terms but, for moderate \bar{U} , you will not be able to decouple the terms and you will have to keep all three equations together but, when it gets decoupled, you can solve either momentum and energy or continuity in momentum and then solve. The third one plug it into the other equation and get the density.

So, in this case we use momentum. And energy equation solve for p' and u' then plug this into the continuity equation, and you can actually get the density fluctuations. So that comes out of it would not have to solve them in the couple manner but, you had higher velocity is you will have to solve them in a couple manner. So, clear.

So, we go back to this equation. That is what rho prime rho prime can be solved by if you look at the continuity equation is zero.

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The image shows a green chalkboard with handwritten mathematical equations. At the top, the equation $\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} = 0$ is written. Below it, the text "Continuity eqn" is written and underlined. The equation $\frac{\partial \rho}{\partial t} + \bar{\rho} \frac{\partial u}{\partial x} + u \frac{\partial \bar{\rho}}{\partial x} = 0$ is written below, with checkmarks above the terms $\bar{\rho} \frac{\partial u}{\partial x}$ and $u \frac{\partial \bar{\rho}}{\partial x}$. An NPTEL logo is visible in the bottom left corner.

So, this is the Continuity equation. So, when we solve the wave equation we can get u prime right, I mean you get p prime from there you get u prime so on, also u prime and if you know the mean density and density gradient. Then we can plug it into this and get the. And get the density or in the harmonic domain will be $i\omega \hat{\rho}$ plus. So, if you know this and then you can solve for this clear.

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The image shows a green chalkboard with handwritten mathematical equations. The text "Continuity eqn" is written and underlined. The equation $\frac{\partial \rho}{\partial t} + \bar{\rho} \frac{\partial u}{\partial x} + u \frac{\partial \bar{\rho}}{\partial x} = 0$ is written, with checkmarks above the terms $\bar{\rho} \frac{\partial u}{\partial x}$ and $u \frac{\partial \bar{\rho}}{\partial x}$. Below this, the equation $i\omega \hat{\rho} + \bar{\rho} \frac{\partial \hat{u}}{\partial x} + \hat{u} \frac{\partial \bar{\rho}}{\partial x} = 0$ is written. An NPTEL logo is visible in the bottom left corner.

So, back to the equations. So, Any other questions? feel free to ask as if some is still bothering. ((Refer Time: 05:32)) What's that?

Student: I wanted to ask ((Refer Time: 05:40)) problem. We want the wave equation just T or T power. ((Refer Time: 05:44)) on that form. There will be bring the constants a and b.

Because there I said I wanted to only concentrate on the effect of fluctuations. So, I said a equal to b ((Refer Time: 05:55)).

That was imposed in that was imposed ((Refer Time: 05:58)).

Student: Earlier ((Refer Time: 06:06)) either closed event, or opened event how the sound is generated inside the ((Refer Time: 06:12))?

Sure that was mentioned in the first class, if you remember we I mean sound has to be generated somehow but, we are now solving for only the propagation. So, when we are solving for that complex I can value and then we talked about a traditional, and all that we assumed that sound was somehow there and how it is decaying? And how it is growing? But, it is simply because is much easier to do than to calculate for the sound generation which is still I have open problem with lot of controversies.

So, propagation is the kind of well worked out over the centuries but, we will deal with how sound is created after few classes. But, so for that we have to actually separate the unsteady base flow from the Acoustic equations and then the unsteady base flow. Actually drive the Acoustic equation and the Acoustic equation gets compared to the answers to the based flow. So, it is like a feedback and kind of thing.

Student: Today we are discussing about problem, some general admittance.

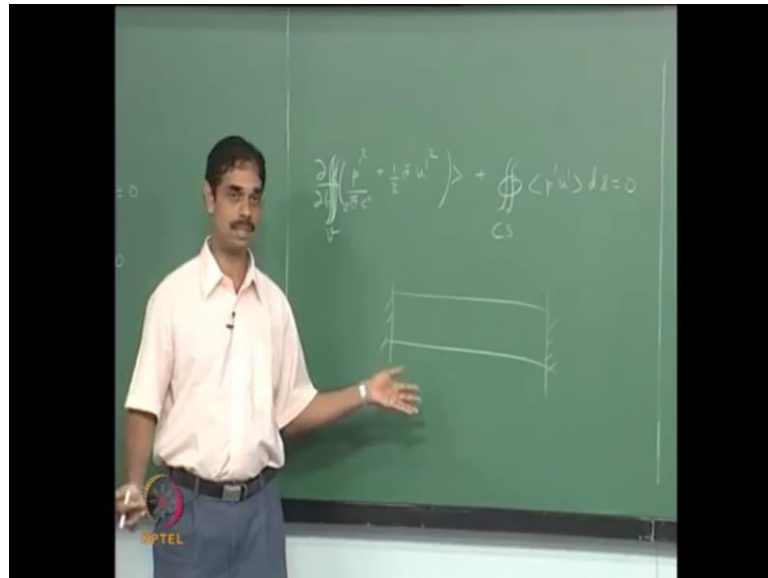
Can you speak loudly so that the camera will also record.

Student: So, for general admittance equations, one side any apply on the other side. Both sides should apply conditions.

I assume that the other side was closed, because I wanted to have a simple thing which I can derive in two lines. So, if I can actually have admittance on both sides, and that is

why in the if I look at the acoustic energy corollary I have sorry what is the factors to rho bar? c square plus half rho bar u prime square and the i time have it is plus.

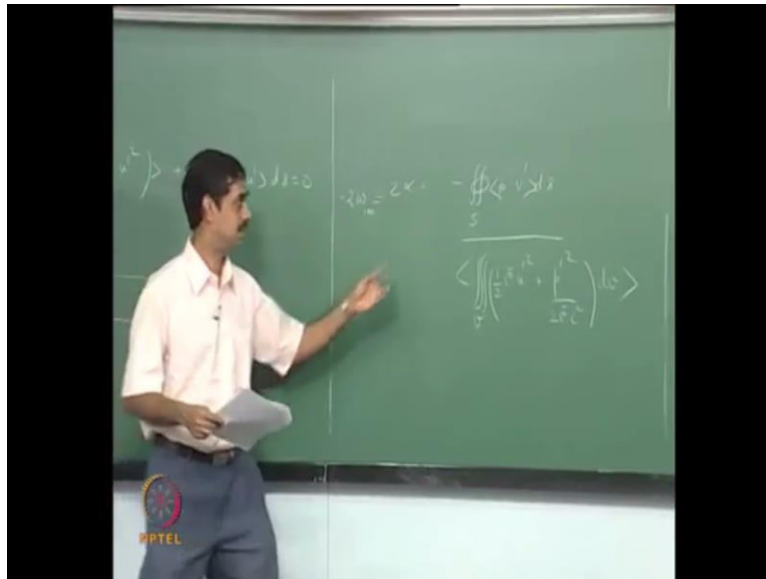
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This is volume integral plus control surface p prime, u prime, d x equal to 0. So, in principle it is over all boundaries. So, even if you one dimensional as a problem still the two boundaries on left and right plus. So, we have a box you can have intensity go out this way or that way, or you can actually have this walls vibrate and take out the energy also, which also happen in reality.

So, all that has to be accounted for. So, I wanted is simple very simple practical problem whose answer I can write in two lines to show the connection. But, in reality you can solve these complex problems and you can actually solve the problem. And that is why the final result I wrote ((Refer Time: 08:59)) this is somewhat general as long as you do not have mean flow.

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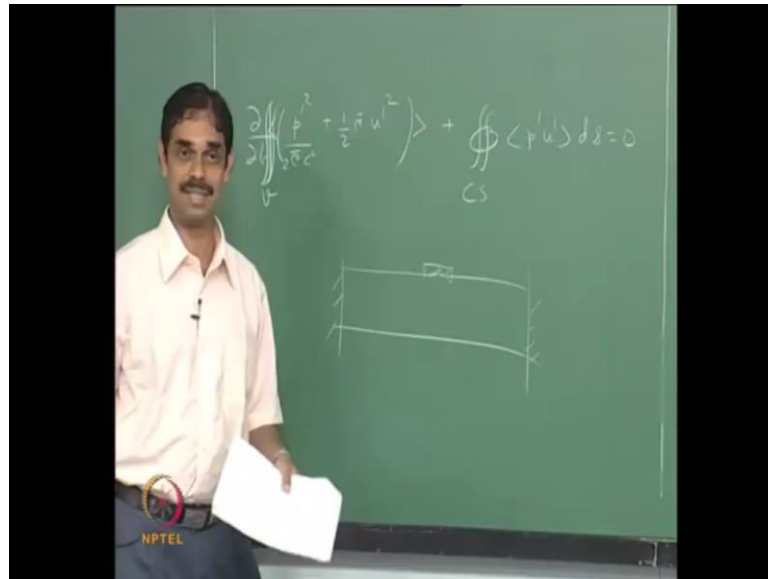


Once you have mean flow and then even a constant mean flow will make this expression as max and modified. I mean the non-uniform in flow which is what the result makes that a bigger mess and then the lot of controversy but, in the simplest case when there is no mean flow but, there is lot of surfaces that, this should be the general result.

Student: Sir that ((Refer Time: 10:10))

Just minus omega ((Refer Time: 10:25)). So, this can be contributing from any side I mean and need not be one dimensional problem and you can have the ((Refer Time: 10:36)) here or you can have a little hole here, or you can have some bonze kind of thing here, or various possibilities all sees the acoustic energy will grow decay.

(Refer Slide Time: 10:33)



That happens on how much of risk coming in and how much of risk lost. So that is.

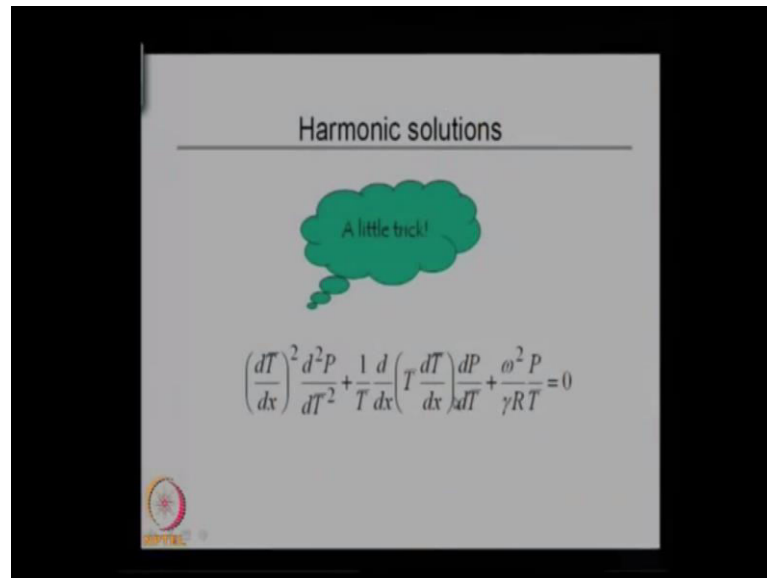
So that accounts for any possibility of coming in from anywhere clear anything else yeah nice to see you smile after long.

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So, back to harmonic solutions and we had the momentum and energy equation and we just now discussed why it is enough to deal with only two of them, and because this third one can be used to derive density fluctuations, and we do not have to solve them in a couple manner. And then we get this nice little wave equation the harmonic domain and this is quite consistent with whatever we had derived earlier as the harmonic equation.

So, if you look at a middle term we have you have a one over \bar{T} $d\bar{T}$ over dx which is actually zero when there is no temperature gradient and the last term is ω^2 over $\gamma_r \bar{T}$ $\gamma_r \bar{T}$ is nothing but, square of the speed of sound. So, ω^2 over c^2 is k^2 which is a constant for the case of constant temperature. So that is why we had these $d^2 p$ or $d^2 x$ plus $k^2 p$ equal to zero. Now this in general I do not know how to solve it we have to use some trick, and if you have a general solution fantastic write about it and you can immediately get the new result.

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So, I did a little trick out of the blue. So, you can do any tricks and the idea is to get the differential equation transformed into some other equation, whose solution is not that is the idea you know something and we have a unknown thing and is there somewhere is the unknown thing can be converted to a known thing. And then you can look at the tables or a way of integrals and write down the answer of course, you can also if this equation has not been solved.

And then you can solve it with a new technique like if it is a ordinary differential equation how do you solve it in general

Student: ((Refer Time: 13:00))

No that is to separate if there is some other driving function and all that but, it is a basic homogeneous part ((Refer Time: 13:12))

We used this for linear expansion what I did was. So, if you have a differential equation which is not solved you look at the books and it is not solved at all. And there is no transmission existing to compare it to the to convert it into a existing solution. What we can do is you can do a serious solution and get a new solution and the of course, you have to study the properties of this new solution and with hell of it and then tabulate, and draw curves, and all that and then you can call it your own function it can be direct function or Vishnu's function. So, Anveksha function, and Akshay's function whatever

and you can get your own name famous and so on. I resorted to the is this thing I dint try for the Suchit function but, I tried to pick it back on somebody's solution or exists. So, out of the blue I did this transformation.

The trick was that I tried to solve this equation quite hard, dint get anywhere. So, I abandoned this problem and wrote a numerical solution for it and it is squared simple an we will do that as a home work problem or assignment.

For this it is quite trivial to solve these two things these are two o d e's and how we solve a o d e's numerically this you must be knowing.

Student:((Refer Time: 14:40))

So, what is simplest o d e?

Student: Runge-Kutta

Runge-Kutta, you can use a Runge-Kutta second order or fourth order and get a solution peacefully but, I will I want to do some analytical things. So that we can get some properties coming out with very nicely and. So, on but, we will do this Runge-Kutta as some kind of solution, which will work out for assignment and some ((Refer Time: 15:08)).

So, what I thought was that normally we say x as a function of temperature pressure as temperature, velocity as function of temperature. So, I got in a inspired moment I thought that we will think about no sorry we think about temperature as a function of x , pressure as a function of x , velocity as a function of x . In an inspired Momentum I thought that x is function of temperature, pressure is a function of temperature, velocity is a function of temperature, and then suddenly I solve the problem and in high sight.

So, I dint do all this big writing I just wrote down the answer directly but, So, what this is the general way of doing it. So, if you rewrite everything as a function of temperature and write the different equation in terms of temperature, no one said that temperature is a function of x I could also think of distance is a function of temperature. No one ever said I means it is not written in a Veda's or Bible's, that temperature function of x not the other way. So, when I wrote this somehow it worked whatever works in getting solution

right. So, I got this solution and then you plug in various different temperature profiles, and for lot of temperature profiles we can get a solution peacefully with this approach.

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Linear Temperature Profile

$$\bar{T} = T_0 + mx$$

Eq. reduces to $\frac{d^2 P}{dT^2} + \frac{1}{T} \frac{dP}{dT} + \frac{\omega^2/m^2}{\gamma R T} = 0$

$$P = c_1 J_0\left(\frac{\omega \sqrt{\bar{T}}}{a}\right) + c_2 Y_0\left(\frac{\omega \sqrt{\bar{T}}}{a}\right) \quad \text{where } a = \frac{m}{2} \sqrt{\gamma R}$$

$$U = -\frac{m}{\rho} \frac{1}{\sqrt{\gamma R T}} \left[c_1 J_1\left(\frac{\omega \sqrt{\bar{T}}}{a}\right) + c_2 Y_1\left(\frac{\omega \sqrt{\bar{T}}}{a}\right) \right]$$

So, will look at the simplest temperature profile is the linear temperature profile. So, we say \bar{T} equal to T_0 plus $m x$. So, m is a constant, and T_0 is another constant and x is the distance. So, I rewrote the equation in this form. So, we are replacing x by T . Now this will work if your monotonic function but, if you say that your temperature goes up and fall down then you have to do it in two segments but, if it is wiggling out of this place and it is not possible to do this but, usually it is a monotonic function it decreasing or increasing.

So, this differential equation as a solution which is $c_1 J_0(\omega \sqrt{\bar{T}}/a) + c_2 Y_0(\omega \sqrt{\bar{T}}/a)$. Now I perceive you know what is J_0 and Y_0 .

((Refer Time: 17:23))

How many of you know J_0 and Y_0 ? Raise your hands Thatser

Thatser everybody know everything about thatser? Only Rajesh knows. So, if you have any doubts you can ask Rajesh what is J_0 ? And Y_0 Rajesh I know you are expert in give some commentary on.

Student: The Bessel's function.

Speak loudly

Student: The Bessel's function for the first order J_0 is.

J_0

Student: J_0 and Y_0 is the.

((Refer Time: 17:52))

Why do you use this?

((Refer Time: 17:53))

They do use to see this kind of function.

((Refer Time: 17:58))

So, I guess I have to give a supplementary class on differential equation is that what it comes down to.

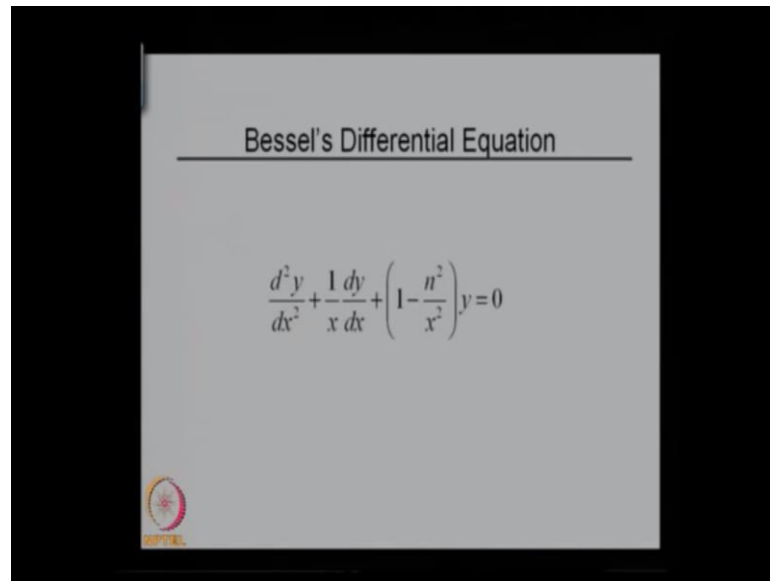
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Tell we know about ((18:10)) function.

You know about does everybody knows that Bessel's function everybody shaking their heads several different. How many do not know about Bessel function? In this only two, five, six, seven. So, I along with my ((Refer Time: 18:25)) thing I will have to hear about Bessel's. So, you have a solution for pressure and velocity. Now of course, solution is useful only in you know what is Bessel's function? So, at the moment I will give a two minus lecture on Bessel's function and the will have a elaborate lecture later no but, you can write it down at least you feel very happy there a cool again and tell your classmates I wrote Bessel function in notebook. So, J is called Bessel and y is called ((Refer Time: 18:58)).

And they are the names of people, and you can try to work out this transformations at home I have E-mailed the paper which has this solutions and or you can see.

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Bessel's Differential Equation

$$\frac{d^2 y}{dx^2} + \frac{1}{x} \frac{dy}{dx} + \left(1 - \frac{n^2}{x^2}\right) y = 0$$

So, a one minute introduction about, What is a Bessel Equation? This is the Bessel equation, the very simple equation. I am sure you are familiar with $d^2 y$ by $d x$ square plus y equal to zero what does that give?

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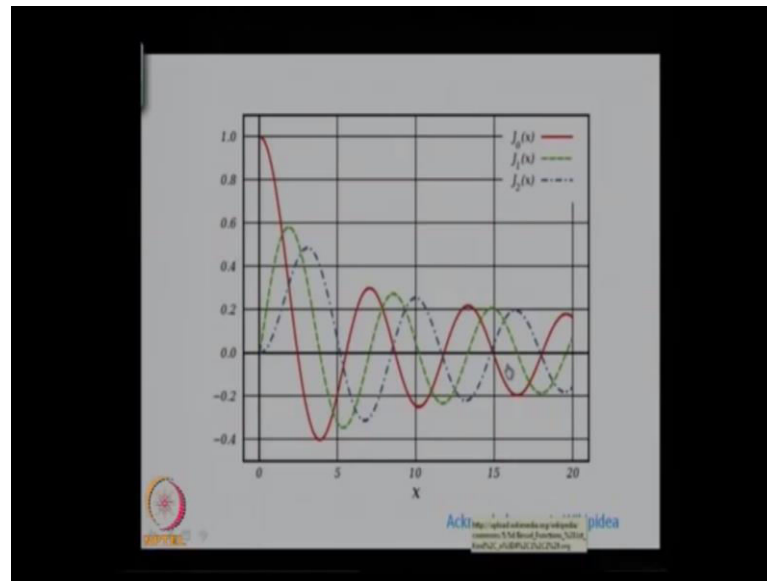
Like sin and cos and

((Refer Time: 19:45))

So, we have x term middle term and the y term is modified by one minus n square overed x square.

So, the solutions are Bessel and n can be a integer or a non-integer can be a complex and so on. but, when n is a integer we have one kind of solution, when n is not a integer will have another kind of solution. So, let us for the time being say that we are worried about only when n is a integer. So, these kinds of equations are quite common, when you deal with cylindrical furnaces spherical hornets cornets so on. And then these solutions Bessel function they also call cylinder functions or spherical functions, and so on and the main if you have studies heat conduction and circular rod or a sphere or something you would see this functions might be you have done he transferred.

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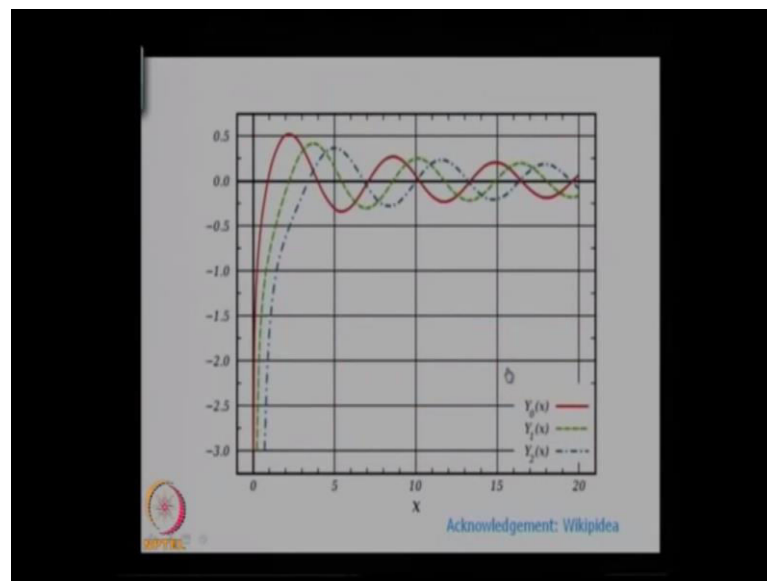
So, this is the red line here is a J_0 . And green is J_1 and blue is J_2 , the important thing is J_0 starts with 1 all other J 's start with 0 and the important thing to notice is that there is some kind of an envelope where the maxima's decayed and. So that is the first thing. So, if you had $\sin x$, it is a periodic function and between each of the roots of $\sin x$ or the technical name mathematician used to call zero's.

So, between each zero means, where $\sin x$ or $\cos x$ where they actually touch 0 function the distance between zero's is the same. So, \sin when the $\sin x$ is 0, it is for m 190 degree, 360 degree, 270 degree. So, they say zero's are equidistant some angular where as Bessel's. It would not be in fact you can see that as you go in terms of increasing x the roots will come closer and so on.

So, the two features one is this maxima's are kind of not constant the decay and the distance between the roots are not constant. So, very easy now world cup is going on, you can look at thus hockey and when somebody tries to hit the losted ball, the ball will if I am batting here then and then and so on. then of course, if there is a fielder here you would be out but, if some nobody catches it is bounces like this of course, in J_0 some \sin waves and all that but, in reality it is this and you can see that, you have like a decay pattern of the peaks and then the roots are of the zero's are non-equi distance and there is a class of curves which are used to fit this kind of stuff and they are the Bessel functions, there the solution a merges naturally.

And you can imagine that in a cylindrical geometry. For example, you are spreading out stuff right. So that way your amplitude of whatever thing is occurring. It can, it will keep coming around as you go away right. Let something intuitively you can imagine. So, here the temperature that is a same kind of spreading out, that is what we saw there right. Where this kind of scaling? That is why we get the messes. So, this is Bessel and I this is a good section Wikipedia on Bessel functions. So that is probably the easier thing to do and this is now m a.

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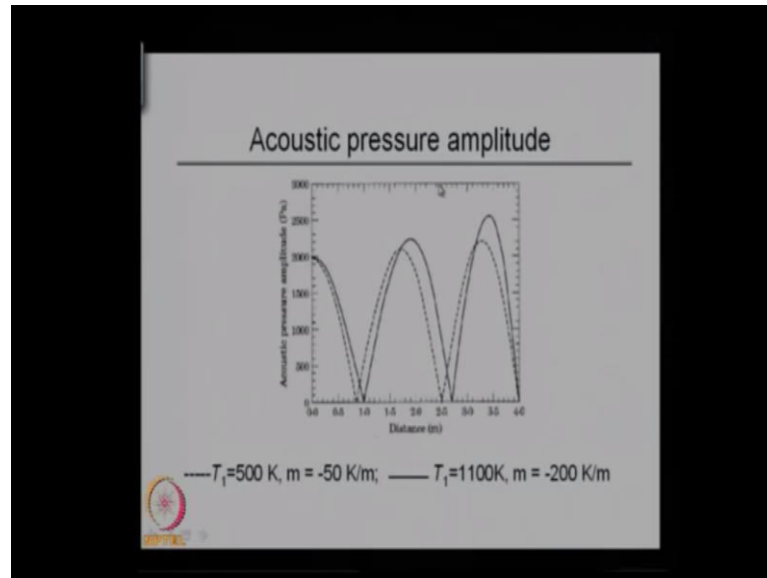


So, the nowman functions all of them start from minus infinity all of them. So, if you have x equal to 0 in your domain, you can happily through away the nowman function. Because if you want to solution to be bounded then this should not exist, if x equal to 0 domain but, in our solution we do not have T equal to 0 T is erasable. So, you have to keep it but, in a visual conduction problem in a cylindrical rod or something going to be 0 but, if you are having a (()) then you will have to. So, again you see the exponential decay or also, rry decay kind of things and then there is also this periodicity it is going up and down but, crossing zero many times but, the zero's are not equi distant and. So, this is the now man function.

So, I will give some supplement actually lectures on what this is but, I will not teach enough to compensate for a maths class but, they just give a more courage to go and read

a book something like that I will get to that stuff but, at the moment you can believe that Y naught looks like result curve here. And J naught looks like this curve here lets fair in a.

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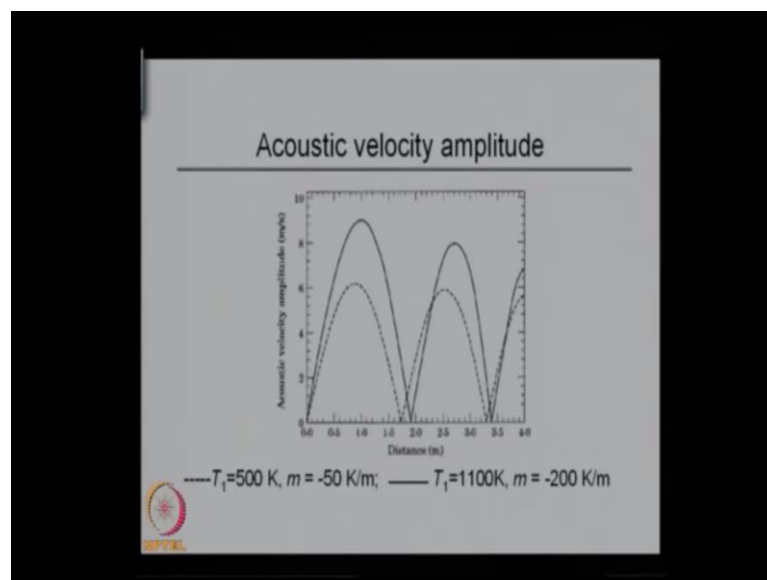
So, if you ready to plot that Acoustic Pressure amplitude distance for some closed pipe or something, this is what you would get I mean. So, you can see we saw that where here there is a decrease in temperature of profile. So, when pressure when the temperature mean temperature decreases what should the pressure should do it should go up because there is a one over T power one-fourth in the denominator right. So, you can see that the as the as you go for left to right temperature is decreasing but, the Pressure amplitudes are going up each of the maxima's have a different amplitudes and progressively increasing and you can also see that the wavelength is changing.

Because you are temperature is coming down. So, if you are cooling down the wavelength also comes down. This is the where the Acoustic Pressure amplitude is yes you should pass for a minute where you to look at it and ((Refer Time: 26:10)) linear We are whole course is about linear acoustic. So, the acoustics we do not have this differently equation we will have wave equation with terms like $d p$ by $d x$ squared and so on.

Student: Sir linear the

This is for linear variation sorry; I thought you are talking about linear ways of pressure linear variation of T . So, you do not have a general solution. So, you can get solution for linear or exponential or polynomial and so on. If you have to have a any general arbitrary profile with if you give me a table of numerical values temperature with x I do not have a functional form. Then I will have to do numerical integration but, this linear variation and temperature's nice to get a J naught and Y naught because I mean an immediately I know the solution what are the features I should look like. So, when you are having the pressure amplitude go up that does not necessarily mean that Acoustic intensity is created it just this kind of spreading by the temperature.

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So, here for example, if you look at the velocity it is going the other way the peaks are coming down, and we saw that the velocity scales like T power plus one fourth right. So, as temperature is decreasing velocity also decreasing, which is what we are seeing and again the distance between the routes are known they are not same. So, these are the two characteristics and the Bessel function beautifully capture these things, I must emphasis just because the pressure amplitude is between two peak's other does not mean energy is created velocity maybe the other way. So, I mean when you have when you deal with non-uniformity I have to be careful in studying all.

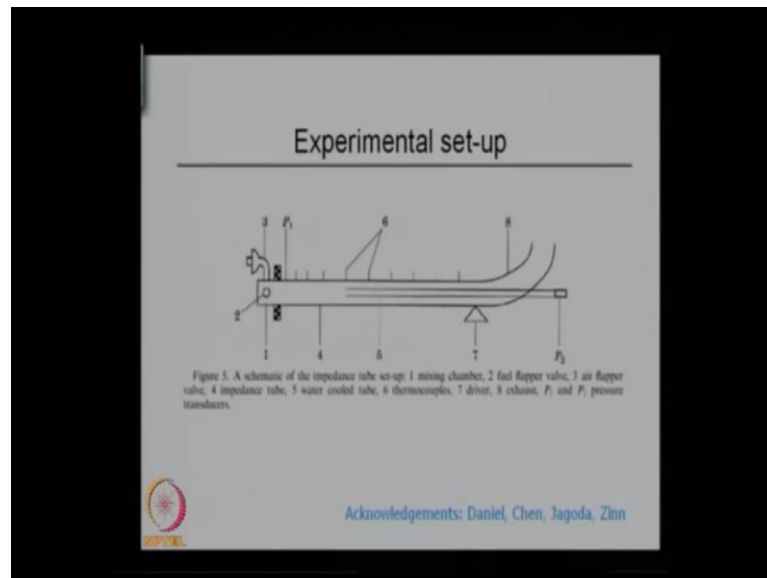
Student: Sir ((Refer Time: 28:45)).

That profile was T naught plus $m x$. So, m is the temperature gradient $d T$ bar over $d x$.

Student: Like they those are two temperature.

I have drawn for two temperature profile very good.

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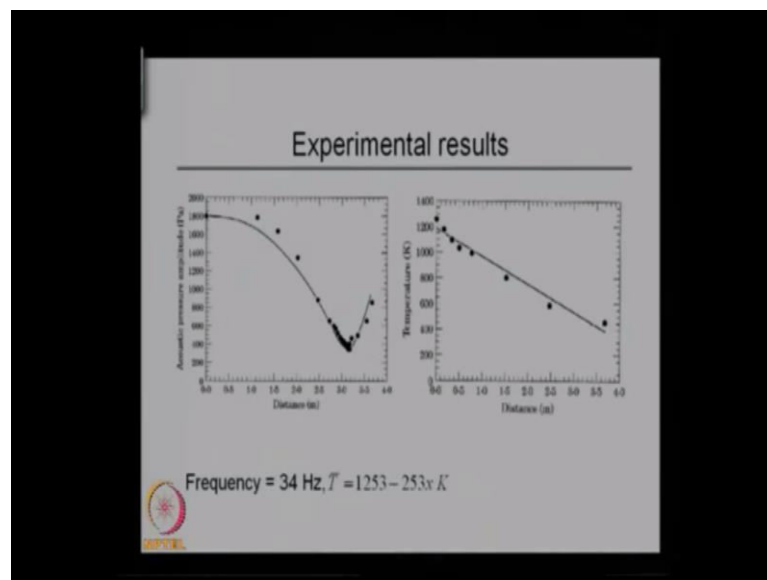
So, I you can do experimental. So, I did some experiment that is why I started my carrier with this non-uniform temperature, and then I wanted to understand the results and therefore, I did lot of theoretical analysis to understand the results and I told you in those days twenty-five years back or more it was really hard to do this experiments with there was no labs view, there was no modern computer, there was this data equation program in a paper tape which had holds much time and the then you have to bound on a spool and so on. It was really ridiculously difficult and. So, it was. So, difficult to do experiment which will now be trivially simple.

So, to understand you have to understand the subject to maximize or chance of getting some reasonable success. I think that is the reason why I even went to the I looked there was no solution. So, I derived a solution myself to understand the problem, and I think it was really worth. So, if you here I was trying to look at the admittance of the mixing chamber here. So, if you are having a non-uniform ah temperature ah experiment you do have to put thermo couples here for example, we add more lot of thermo couples all over the place in fact where the temperature is following rapidly at many thermo couples, and then you need to traverse the micro phone the acoustic filed and I had only one micro

phone with me actually two micro phones one two, one at the left and as a reference to get the face.

And the another one here and you traverse but, if you are having fifteen micro phones you can bound them all and get a reading in a snap and if you are looking at a admittance measurement in a rocket then there is no possibility but, to have fifteen micro phones, because the experiment will over in a second or So, I have mail this slides to you I hope you got it, if you dint get it I got your email address wrongly because few of them bounds. So, just in case any of you dint get this email with this slides then give me your email address.

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So, you can once the solution you can plot the solution but, not just that if you have the solution you can actually recover the admittance and so on. So, by curve fitting. Or a fancy navy's multi micro phone technique. So that is what I have done here I just showed some typical result to do this.

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Exponential temperature profile

$$T = be^{-cx}$$
$$T^2 \frac{d^2 P}{dT^2} + 2T \frac{dP}{dT} + \frac{\omega^2 P}{\gamma Rc^2 T} = 0$$
$$w = P\sqrt{T}; z^2 = \frac{4\omega^2}{\gamma Rc^2} \frac{1}{T} \quad \frac{d^2 w}{dz^2} + \frac{1}{z} \frac{dw}{dz} - \left(1 - \frac{1}{z^2}\right) w = 0$$
$$P = \frac{1}{\sqrt{T}} \left[c_1 J_1 \left(\frac{\omega \delta}{\sqrt{T}} \right) + c_2 Y_1 \left(\frac{\omega \delta}{\sqrt{T}} \right) \right]; \delta = \frac{2}{\sqrt{\gamma Rc^2}}$$




And you can get profiles solution for other temperature profiles for example, many times your temperature may drop when you rapidly near the flame and then might study off for example, many times you can have temperature profiles of this forms. So, exponential maybe a better function to fit them I manage to get solution with exponential temperature profiles, again in terms of Bessel functions and do not worry I will not ask you to derive this in the examination and so on. but, I wanted to show the solution to those who are interested because then it is really nice to be able to you can appreciate the solution and physics behind it very much.

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Particulate damping

- Solid rocket motors with aluminized propellant
- Acoustically excited incinerators
- resonant driving of packed beds and spray dryers
- drying of slurries in pulse combustor tail pipe to accelerate drying
- Commercial firm - Pulse Combustion Systems LLC
- www.pulsedry.com



So, the problem in real combustors for example, if you are working on a solid rocket motor or something like that, is that you have particular damping temperature getting mean flow everything together. See each thing will have a different kind of effect on it. We saw what kind of effect the attenuation did to the standing wave you can know what kind of effect admittance condition dusters and we are now saw what kind of effect the temperature variation is to establishing a wave but, each of the solutions when I am studying I am keeping everything else constant.

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For example, when I showed this solution with the when I showed this solution I have kept closed quarter wave to open boundary. So that is no energy coming in going out there is damping when I talked about damping. I kept again a closed end that is what. So, Rajesh was asking whether a equal to b amplitude on the right or left running wave are equal at the reflection at the termination. and I dint have flow or a temperature gradient now in reality there will be all this things mixed together. So, you will have to track carefully on this things.

So, we will as a last part look at the effect of a particulate damping on the Acoustic wave which is kind of a form attentive wave but, we look at it in the presence of temperature gradient. So, the motivation is as I mentioned in rockets you have Alumina in the aluminum in the propellant which burn in alumina form, which is used to damp the combustion instability and the lot of other applications also for example, if you are drying slurries in pulse combustor then the slurry will damp the acoustic wave.

Or if you are using a combustor to set up last even you are they did acoustic drying for increasing the heat and mass transfer process for example, if you are drying milk powder you could you sound to increase the productivity because you can of course, evaporate milk faster by increasing temperature but, only thing is the what to get out maybe spoiled. So, when in food industry cannot increase a temperature beyond some levels the tight control. So, then human beings are greedy we want more and more we have a fancy name call productivity for that. So, then they try to disturb the flow to get productivity with acoustic oscillations or turbulences.

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Particulate damping: Equations

- Momentum:
$$\rho \frac{\partial u'}{\partial t} + \frac{\partial p'}{\partial x} + Ku' = 0$$
$$U(\omega\rho + K) + \frac{dp'}{dx} = 0$$
- Energy:
$$\frac{\partial p'}{\partial t} + \gamma\rho \frac{\partial u'}{\partial x} = 0$$
$$\omega\rho + \gamma\rho \frac{dp'}{dx} = 0$$

Acknowledgement: Karthik Balachandran, Ramakrishnan

So, these equations were derived by Rajesh who is sitting here. So, if you have any question you can ask him. So, here the momentum equations get modified by a little term here and do not get nervous if you cannot derive the solutions. I will not ask them in the exam this particular thing. So, you have a k 'th times u prime I think intuitively you can feel that, if you have a droplet what are drag on the droplet? you must have studied this in school.

Student: Stoke

Stokes yeah how does it go?

Student: πr .

Six π .

R is $R \propto v$ right. So, it is proportional to v basically, and the Reynolds number dependence is c_d equal to $\frac{24}{Re}$ or $\frac{24}{r \cdot v}$ or $\frac{24}{r \cdot \omega}$. So, if the so this sphere as a drag which is proportional to velocity and that is going to take out some momentum. So that is what this term is here right the $K U$ prime and K can be expressed in terms of number of particles and this drag of a sphere.

And of course, if it is a very messy thing with lots of droplets all interacting them it may be harder to actually give the value of K directly from first principles but, does not

matter you can converted into harmonic domain and this fashion, energy equation is staying and modified here, it would get modified if there is evaporation or something in fact I think the question papers which I sent you on those one question about damping by evaporating droplets.

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Wave eqn. with particulate damping

$$\frac{\partial^2 U}{\partial x^2} + \left[\frac{\omega^2}{\gamma R T} - \frac{k \omega}{\gamma \rho} \right] U = 0$$

$$T = \alpha e^{-\beta x}$$

- Exponential profile


$$T^2 \frac{d^2 U}{dT^2} + T \frac{dU}{dT} + \left[\frac{\omega^2}{\gamma R \beta^2 T} - \frac{k \omega}{\gamma \beta^2} \right] U = 0$$

So, you can actually get solutions of this form when there is particular I mean the way the question comes to this form and you have to specific temperature profiles to get a solution. So, for exponential temperature profile get something of this form. And now you can see that the wave number is now a complex stamp, and that is not very unfamiliar because when we saw the attenuation problem we saw that the wave number is getting complex there right. So, do not bother writing down I have mailed you this slides and you can actually get a solution again terms of Bessel functions only this is this nu is.

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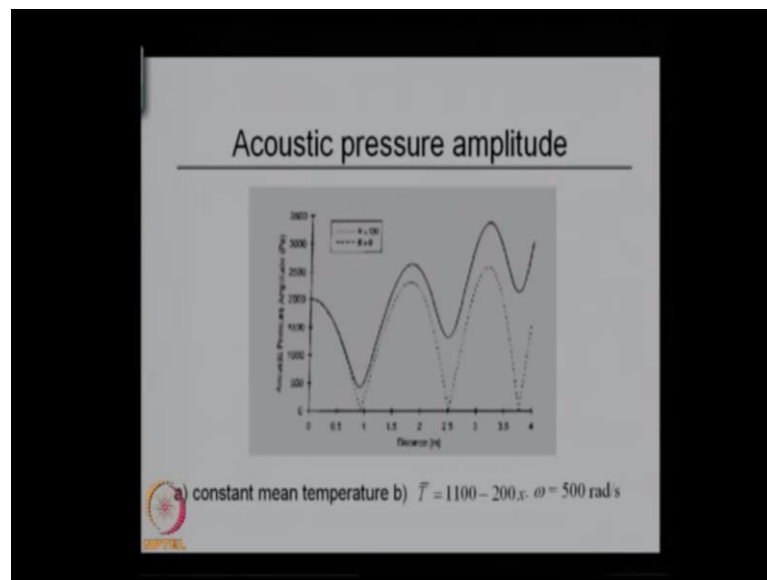
Particulate damping : Solution

Substitute $s = \frac{1 - 4\omega^2}{\sqrt{T} \gamma R \beta^2}$

$$\frac{d^2U}{ds^2} + \frac{1}{s} \frac{dU}{ds} + \left[1 - \frac{v^2}{s^2} \right] U = 0$$
$$U(s) = c_1 J_\nu(s) + c_2 J_{-\nu}(s)$$


So, if you make this transformation and we can get in terms of Bessel function general Bessel function. Only thing in this nu is actually complex and. So, this is in terms of complex Bessel function.

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Now, look at the pressure distribution. So, I still have a closed end here on the left side I'm still holding on to some simple things but, the dotted line is when there is no damping but, only temperature gradient. So, you see that goes up. This is same solution

we saw earlier but, when there is damping the minima's will start going up the maxima's will also change as we saw in the earlier derivation.

So, this combination will come now. So, the minima's are going up the maxima's are going up in a different way and this is they are there, and to actually do the inverse problem if you have to determine the admittance and damping cohesion by given the solution if you have to do you have to actually measure the standing wave up to at least two minima's because the pattern for this is the minima's will continue to go up. if you are having energy flow from one side and no attenuation in the medium then the minima will go up evenly all minima's will go up by the same level. But, if you have attenuation in the medium each minima will go up differently.

So, you have to measure multiple wavelengths to this is the velocity amplitude. So, you see that minima's are continue to go up the maxima's are staying constant, this is not that there is anything which is maxima stay constant because the temperature gradient try to bring it down but, the damping makes goes the other way. So, between those two it kind of roughly canceled in this particular example that is all. So, you have to be careful when you are really worry about this way or that way. So, one should be quite careful about when when the combination effects are there I think this is where I will stop with this ah effect of non-uniform media now what I also want to do I have stop for a moment ask questions if there any questions there any question?

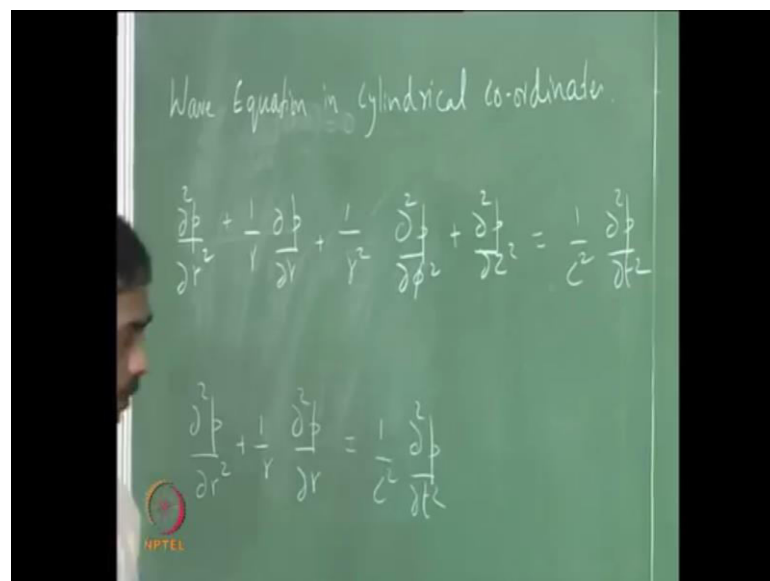
So, we will have a assignment will have two assignments. First one is take these two solutions take these two equations and solve them numeric way with fourth order Runge Kutta and you can then compared with the solutions that argument by the closed form solutions. So that is assignment one is that clear and you have to write a computer program after you understand what is Runge Kutta technique and you can program in any language, you like you can do it with mat lab or Fortran c is a simple thing. So, does not matter where you do whichever you like, you can also do it mathematical any anywhere you can and do it numerically with Runge Kutta and of course, in mat lab and mathematic you do not need to write the Runge Kutta routine it will be there I think it will be nice side if you write Runge Kutta routine.

Do it but, if you can use in-built thing that is also fine as long as you learn how to use it. So, this is assignment. So, once you do this you should forget some resource compare it

with theoretical solution and write a nice report and give it to me and I think maybe in three weeks time it can be done I will I do not have a calendar here. So, next day I will tell the process did and once you write once you finish the solution if you write a report and explain what you have done? Why you are doing? What you doing and results and so on. And if I guess everybody knows some kind of programming. So, you should be able to do it in anything you have access to I mean you know some programming language Fortran c, c plus plus, mat lab, mathematic anything is you use it to if you do not if.

If you do not know mat lab maybe it is a good idea to it mat lab then you can learn met lab or if you mat lab try to do it mathematic way you can learn it because very simple assignment. So, you can use the chance to learn something else anything else. So, the next topic is this multi-dimensional situations where you have a in cylindrical tube, or we have square box and. So, on I will work out some cases and I will start but, in the exams only till here but, I would in ask you to solve a different equation with tricks and so on.

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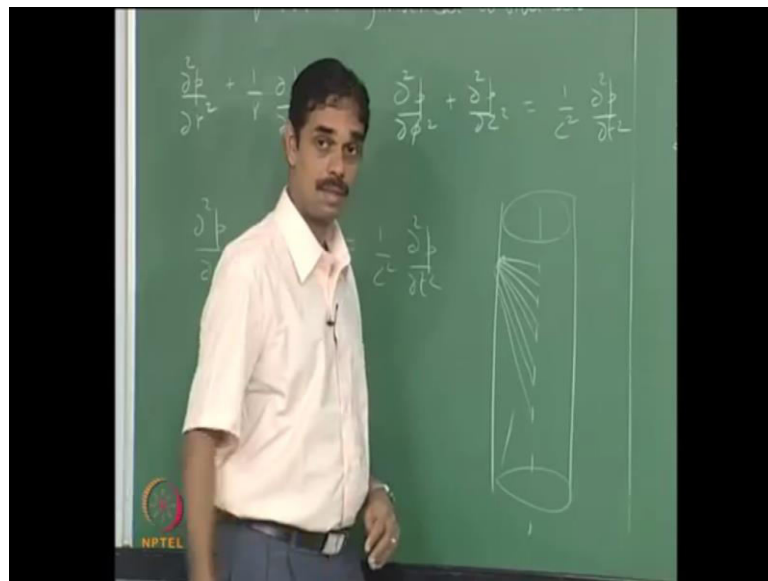
So, let us ah now look at wave equation in cylindrical co-ordinates, Let us see next topic. So, this is the wave equation in cylindrical co-ordinates, I will not derive it you can either try to expand del square. And then try to understand solution or derive it from start either way anyway you like and derive and if I just want to look at the radial variations. So, I will remove, I will say the problem is cylindrically symmetric. So, I will remove

the dependence on peak I will also say I do not want to study the dependence on the $e z$. So, will just look at the r pi. So, you have a cylindrical pipe and you're having solutions. So, can you write a solution f of r minus $c T$ or f of r plus $c T$ scale b y some r dependence like can you get something of the form if you have a spherical wave equation you know the solution its f of r minus $c t$ over r or g of r plus $c t$ over r can you get some such thing here.

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I would guess that you should have a solution of the form f of r minus $c t$ or r plus $c t$ over square root of r or some other function of r but, it turns out that nobody was ever able to derive the solution. So, it does not worth that way and then there is a reason given here. So, two d is nothing but, like three d in finitely wrong right. So, if you think about a cylinder region in finitely wrong and you think about a lines source.

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So, if you are looking at somewhere here and you are looking at sound coming from here. So, you can get sound coming here and of course, it is spreading out as you r is increasing but, sound from everywhere also keep on coming and it actually will keep on coming forever.

So, this is what precludes us from deriving a solution of the form f of r plus or minus $c t$ divided by root r . So, there is of course, if you do it good ca chance for at least becoming

famous I do not know about no press will you will be it is something which has eluded paper, and I think there is good reason why it is eluded but, that does not mean you cannot solve it. So, what does a next best thing you can do. So, we cannot get a solution and time domain in cylindrical co-ordinates you can get time and domain there is scaling is one over r for pressure and velocity ha a in spherical yeah but, the exam this has homework problem I mean problems of that in cylindrical it is not possible to get and.

And. So, what can you do we have to do in harmonic domain that is no other way it its always elegant to have f of x plus c t x g of x minus exact that kind of solution you cannot get it you cannot get it. So, then we write it in ah you have to do separation of able you try to do separation of variables that is also need not work. So, p equal to p of r times t of t and substitute and you can get solutions and for this. So, this is what we can try to do it at home and come back and tell me what is the kind of solution you get tomorrow.