Acoustic Instabilities in Aerospace Propulsion

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Lecture - 12
Sound Propagation Through Inhomogeneous Media – 2

Good morning, everybody we are looking at sound propagation through regions of non-uniform temperature and this is quite important for studying some acoustic oscillations in combustion chambers and engines and so on. Because there is always heat and addition to the gas in terms of combustion and then the hot gasses will lose heat because of heat transfer to the walls or cooling and therefore it is very lightly, that the temperature distribution in the combustion or engine will be non-uniform. So, also the area need not be constant. So, due to these reasons we are studying some propagation through a medium with non-uniform temperature non-uniform area.

So, the just to summarize what we talked about last class we looked at some scaling based on a high frequency approximation that would mean that the disturbance's are at a much smaller scale compared to the changes in area or the changes in temperature. So, this is traditionally call w k b approximation.

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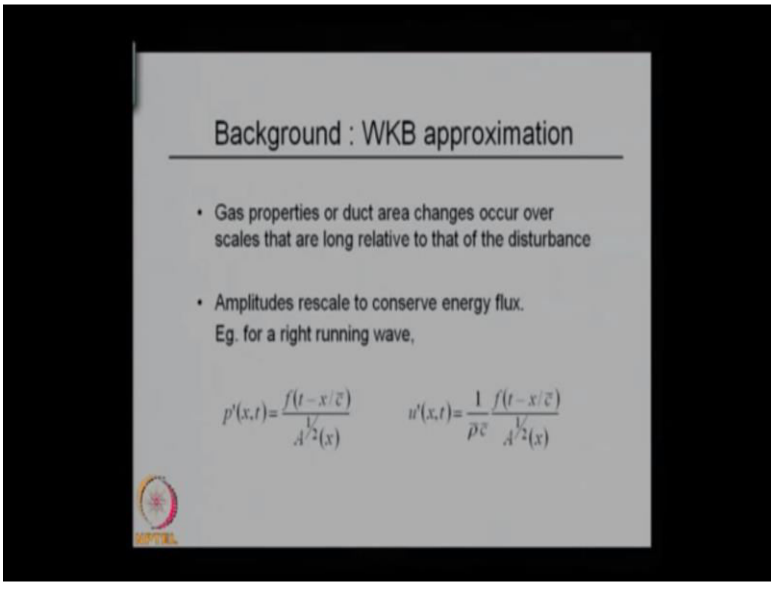

And then, we search that p of x comma t there is a pressure is traditionally it goes like a f of x minus f of t minus x over c, but that gets scale by a factor of A power half or square root of x. Similarly, the velocity gets scale by normally it is one 1 rho c f of t minus x over c but it scales by square root of x and together. If you look at area times pressure times velocity, which is like the power-flow that would be independent of x. So that is the idea.

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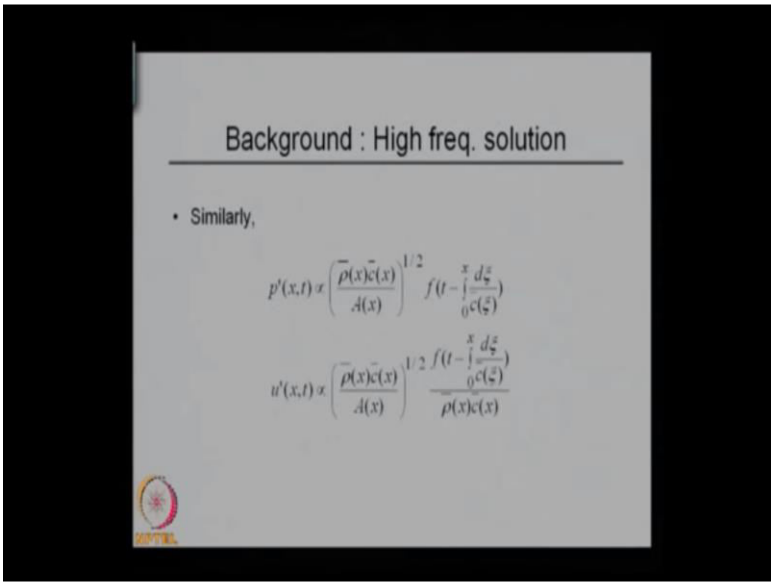

Now, we said that there is similar approach can be taken even when the speed of sound is changing or density is changing and also area is changing. So, then we have to make sure that the product ah p times u times area does not depend on x. So, what you do is to scale with the square root of the area as we saw in the previous case but then you have factor of square root of rho c on both pressure and velocity and of course, when you look at the traditional expression for velocity it goes like p over rho c there is a factor. So, this rho c cancels with this 2 square root of rho c's. So, they cancel with the 1 over rho c and your net power-flow is conserved if density times area is conserved. So, that is the heuristic way we are thinking.

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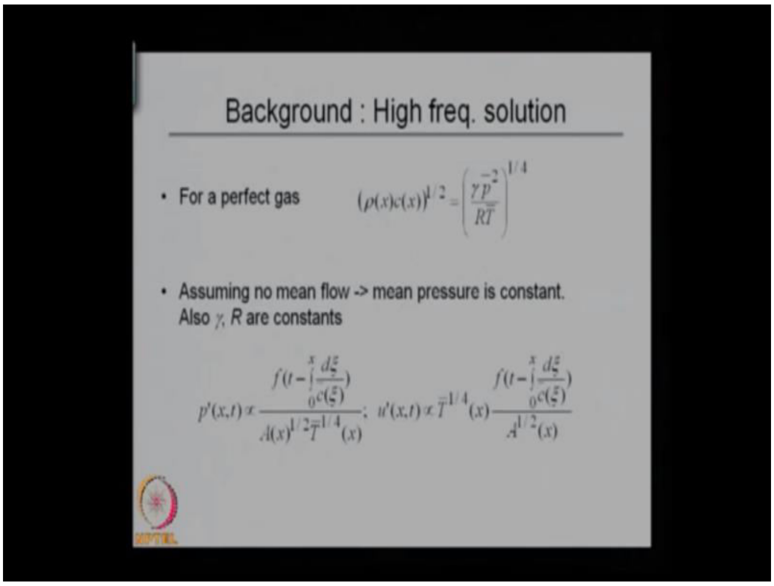

And we then there are perfect gas the relation pressure is proportional to 1 over square root of A and 1 over t power one-fourth and velocity was proportional to t power one-fourth and square root of A 1 over square root of A. So, this is where we stopped

Student: ((Refer Time: 03:05)) a raise to 1 ((Refer Time: 03:10)) and a x raise to 1.

Sorry they are same I should have written this ((Refer Time: 03:15)) sorry about this any other question. So, I have some reference materials on this which you can see.

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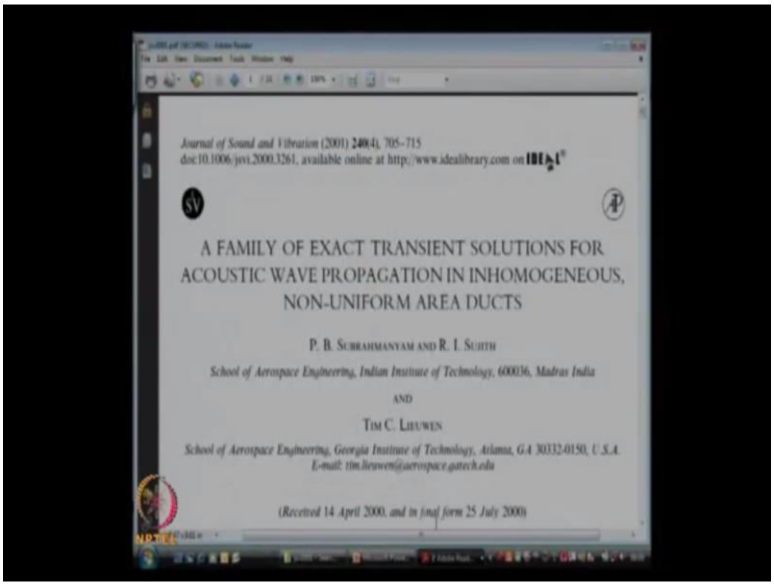

This is a paper written by me in general sound and vibration and Bala Subrahmanyam was my student and we did that, he was in b tech he was in second year undergraduate. The story is he was asked me a question which I did not know the answer to. So, I told him solve a very simple you can work it out yourself and what if he told and he actually worked it out right now. He used to speech the working in c s a r and he is making lot of things very bad guy. He was I think he was the first dual degree guy in the I I T system. So, in this paper if you see. So, we talked about the looking at the sorry.

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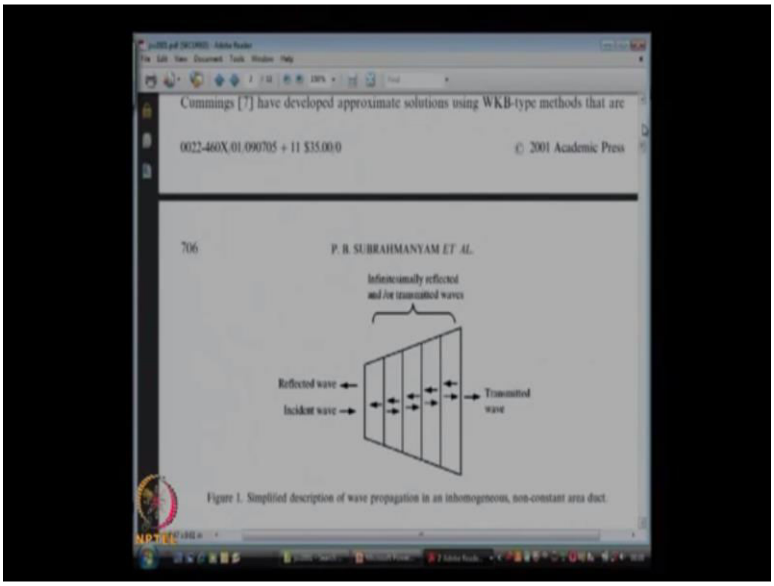

We are talking about looking at the glass. I mean sorry the gas medium as slabs of gas which each of the slab constant property of the but the property varies and how it gets reflected those idea are explained here I can give this paper after this class and then we speak about this w k b approximation and. So, on that is here.

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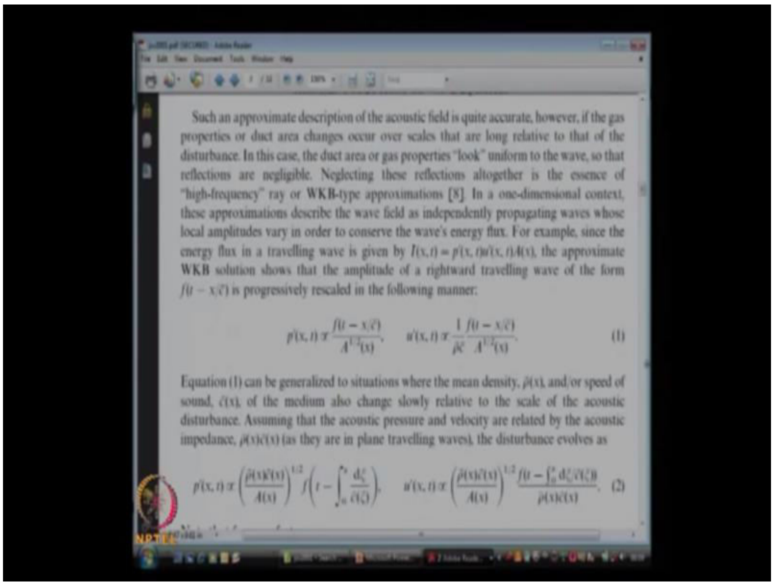
Such an approximate description of the acoustic field is quite accurate, however, if the gas properties or duct area changes occur over scales that are long relative to that of the disturbance. In this case, the duct area or gas properties "look" uniform to the wave, so that reflections are negligible. Neglecting these reflections altogether is the essence of "high-frequency" ray or WKB-type approximations [8]. In a one-dimensional context, these approximations describe the wave field as independently propagating waves whose local amplitudes vary in order to conserve the wave's energy flux. For example, since the energy flux in a travelling wave is given by $I(x,t) = p'(x,t)u'(x,t)A(x)$, the approximate WKB solution shows that the amplitude of a rightward travelling wave of the form $f(t - x/\bar{c})$ is progressively rescaled in the following manner:

$$p'(x,t) \propto \frac{f(t - x/\bar{c})}{A^{1/2}(x)}, \qquad u'(x,t) \propto \frac{1}{\bar{\rho}\bar{c}}\frac{f(t - x/\bar{c})}{A^{1/2}(x)}. \qquad (1)$$

Equation (1) can be generalized to situations where the mean density, $\bar{\rho}(x)$, and/or speed of sound, $\bar{c}(x)$, of the medium also change slowly relative to the scale of the acoustic disturbance. Assuming that the acoustic pressure and velocity are related by the acoustic impedance, $\bar{\rho}(x)\bar{c}(x)$ (as they are in plane travelling waves), the disturbance evolves as

$$p'(x,t) \propto \left(\frac{\bar{\rho}(x)\bar{c}(x)}{A(x)}\right)^{1/2} f\left(t - \int_0^x \frac{d\xi}{\bar{c}(\xi)}\right), \qquad u'(x,t) \propto \left(\frac{\bar{\rho}(x)\bar{c}(x)}{A(x)}\right)^{1/2} \frac{f(t - \int_0^x d\xi/\bar{c}(\xi))}{\bar{\rho}(x)\bar{c}(x)}. \qquad (2)$$

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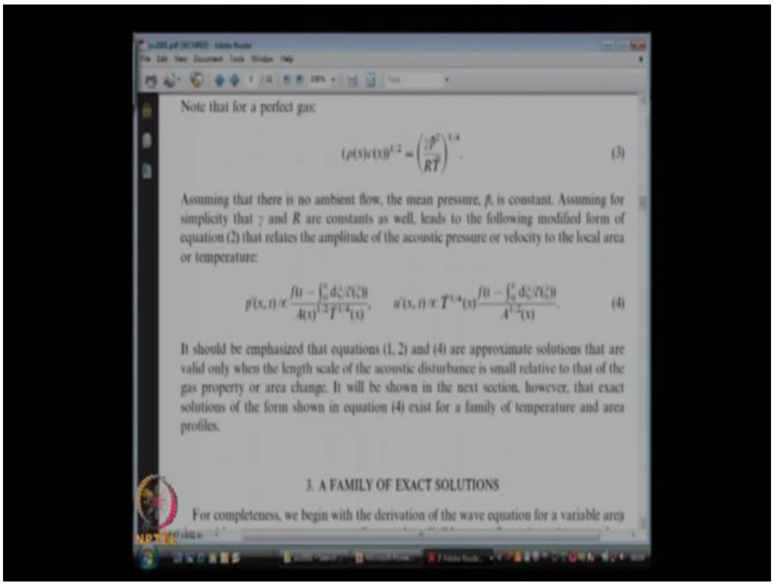
Note that for a perfect gas:

$$(\bar{\rho}(x)\bar{c}(x))^{1/2} = \left(\frac{\gamma\bar{p}^2}{R\bar{T}}\right)^{1/4}. \qquad (3)$$

Assuming that there is no ambient flow, the mean pressure, $\bar{p}$, is constant. Assuming for simplicity that γ and R are constants as well, leads to the following modified form of equation (2) that relates the amplitude of the acoustic pressure or velocity to the local area or temperature:

$$p'(x,t) \propto \frac{f(t - \int_0^x d\xi/\bar{c}(\xi))}{A(x)^{1/2}\bar{T}^{1/4}(x)}, \qquad u'(x,t) \propto \bar{T}^{1/4}(x)\frac{f(t - \int_0^x d\xi/\bar{c}(\xi))}{A^{1/2}(x)}. \qquad (4)$$

It should be emphasized that equations (1, 2) and (4) are approximate solutions that are valid only when the length scale of the acoustic disturbance is small relative to that of the gas property or area change. It will be shown in the next section, however, that exact solutions of the form shown in equation (4) exist for a family of temperature and area profiles.

3. A FAMILY OF EXACT SOLUTIONS

For completeness, we begin with the derivation of the wave equation for a variable area

And the derivation in terms of temperature and area they are here now then we proceed to derive exact solutions for this system without waving hands but actually derived it there is another paper I have it in my paper only.

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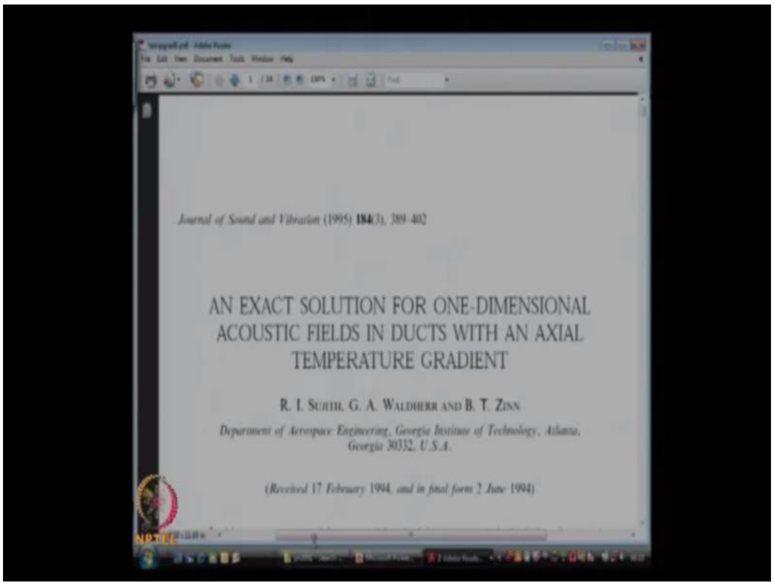

So, one preparation ducks with mean temperature gradient but here I have worked on this problem in the harmonic domain that is we use ((Refer time: 05:07)) with in terms of complex amplitude and this was something ((Refer time: 05:10)) when I was student and this also explains the physics behind and all that detail.

So, I can happily give the p d f files after this and the I have worked out all solutions and am always of the opinion that simple solutions very helpful in understanding rather than very completed numerical stuff.

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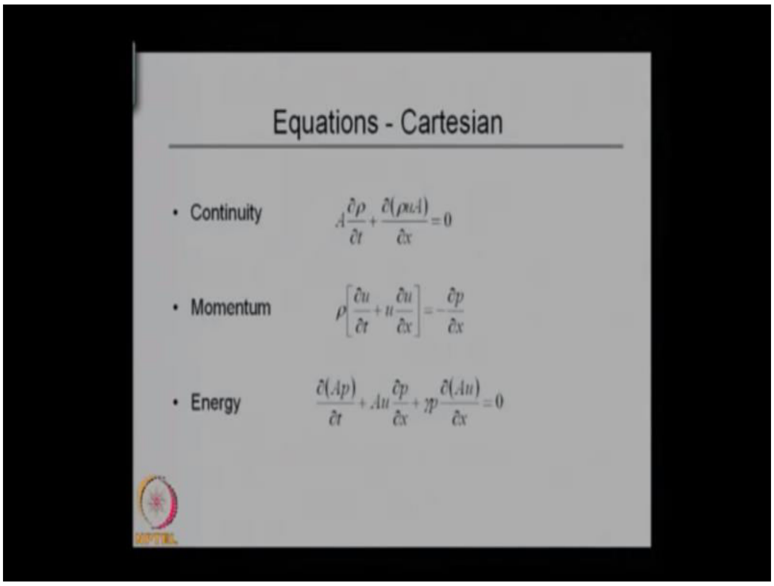

Start with the governing equations. So, we write the Continuity Momentum and Energy equations as is described here and this equation. We have done this as a assignment and if you have any difficulty in this feel free to contact me.

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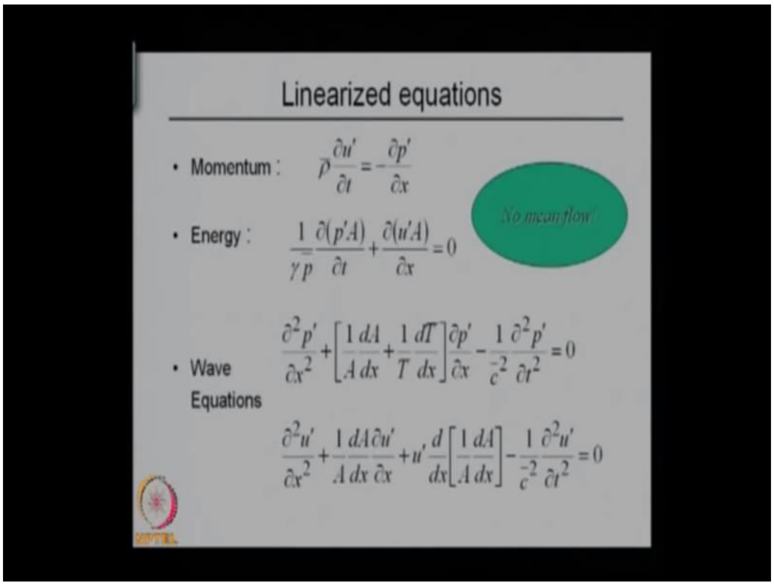

So, we linearize the equations that is the write p equal to p bar plus p prime u equal to u bar plus u prime but then we say u bar is 0 density is o bar plus o prime and. So, forth and then we work out the equations, we differentiate the equations, such that the cross derivative can be eliminated and then we can get wave equation and here are written wave equation.

For pressure and velocity and as you can see this wave equations have they look different as suppose to if the medium is homogeneous is of temperature is constant. It is a same operator operating on pressure and velocity of potential and temperature but here you would have the operator look differently.

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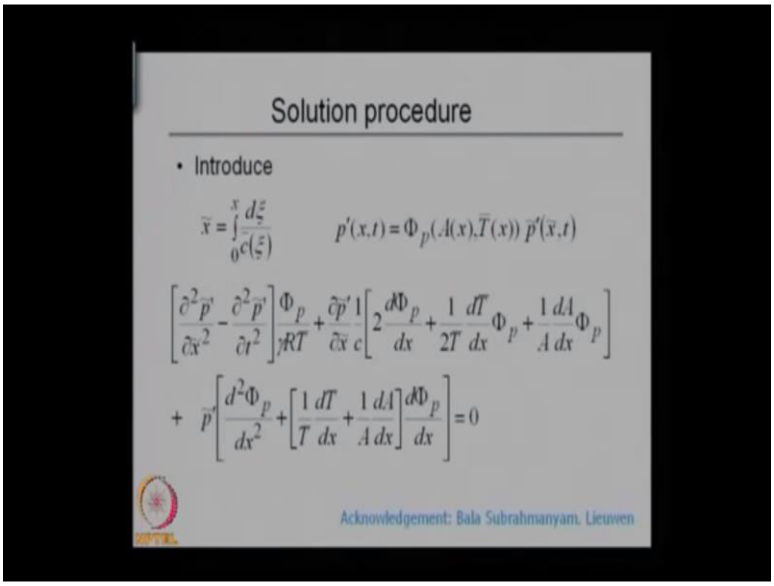

So, at this point we need to think of getting solutions the recurs way to the solution is there something called lead group theory where there is a machinery to actually reduce partial differential equation to the ordinary differential equations but I will not go by the I will go by some intuitive ah understanding. I just want to again tell us some story, generally solutions are just written down by paper just like that and then of course, you can show with lot of theory or intuitive things or it is explanations why these solutions came about.

And so on and these solutions I the first solution I wrote down myself without any reason and then Bala had a lot of elaborate theories on why these solutions and so on. And if you actually read the book there is a very interesting book in the library called the structure of scientific revolutions but this is not a revolution or something solved in the equations but generally he has written that whenever a new thing was there and people knew that it was there and then I mean they just knew. So, in fact I actually wrote this solution.

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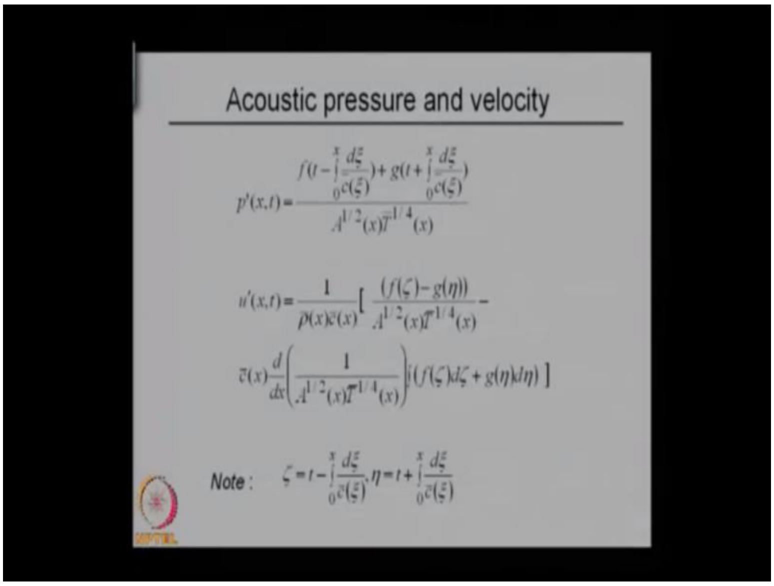

This me I just wrote the solution for this temperature profile just out of the globe and then of course, we had to find explanation and so on. So that is the way it is some and if you work on something long enough your sub conscious crosses things and then suddenly you wake up in the morning with the idea about very middle of road and suddenly solution strikes this important thing is to write down what this solution because while I was doing this.

Actually, I was running in the stadium and I actually fall the solution in front of me and I also knew the temperature of everything. So, I stopped everything and run home by the time I when not that but I only remembered that it temperature had a power of fourth that is all I could remember. So, wrote this down and then I had no time to while contact and I fell sick and small after that for some reason. So, this Bala was around third semester boy. So, he has obvious troubling me thing. So, I told him that four-third should why and he anything students ask just, say that I learnt that it is very trivial if you say that and find out the answer and tell you if you do not know the answer.

So, this is what I told him and then some days later he came and told me this elaborate theory and so on. Has to and cross the solution and this four-third had to be just one of them and so on. So, the way it is written on the paper. Or the way I teach is not necessarily the way you just do things and then something period and this is not that is the way. It is actually when you do original things text books are written after the everything is done after the action is over you write the text books of the way things are given in the text books is not the way things are done. So, I think it is a good idea to read the books revolution by Thomas School. It was really a fascinating book he had, He talks about various discoveries of oxygen or the mechanics, several things and he explains how these things were done and so on. And how people actually did the things you are ((Refer Time: 10:01)) but now he is historian of science. So that much was the out of the camera. So, will think about the solutions we ah said that we have characteristics d x over d t equal to c.

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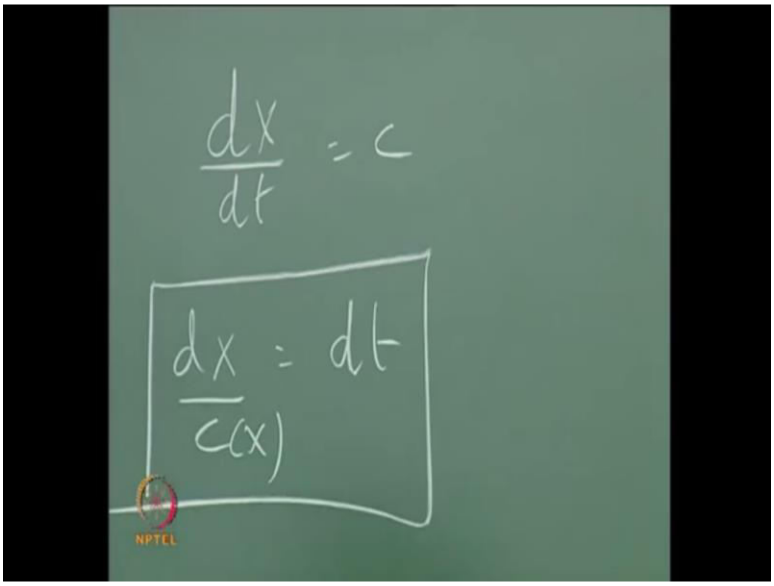

So, we can say that d x equal to d t and our c is not a constant but it is c of x if you integrate this we would get this solution that is here and we kind of say that pressure. Let say we attempt to see if you can write pressure as function of any enterpriser time some other function which do not have this. So, if you can neatly separate this out this saw and then if you substitute the this into the wave into the wave equation at this mess is what that resulted, I mean it is a I mean this I agree that first part is very beautiful. And I pleased to visit but I am left with lot of other stuff, and we have to get rid of it, or we have to learn how to solve the solution and which I have no close.

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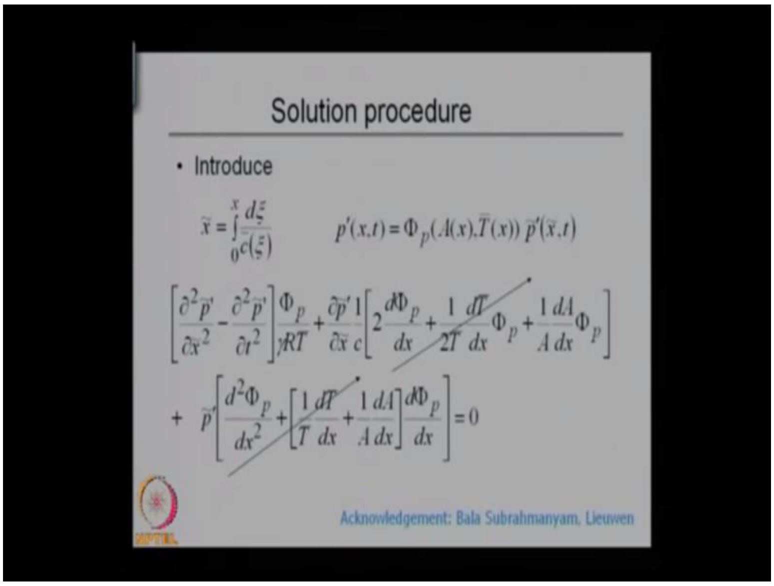

So I thought that I would through this out I cannot deal with it through this out but then you may complaint that. He can you throughout undesirable things we can throughout anything only thing is you can deal with the consequences. That is the thing in life also if something is bothering, you can just get rid of it actually some friend is annoying you and you are now a days you go to find some where you get that how to deal with a annoying friend and the books on it. How to deal the annoying friend I mean now a days we would have just got the different and the there must things were simple but. Then if you get rid over the guy the consequences will be may do some nasty things to you and you must be robust to stand up the or he may not help you which you should be able to but now he. So, in our times we got rid of problems are solved problems. Now you guys manage that for all problems m b a s and all that we want live with this mess and deal with the mess but I am in the old school. So, the mess I am got rid of it and then I face the using as to what is the consequence. So, first the good part.

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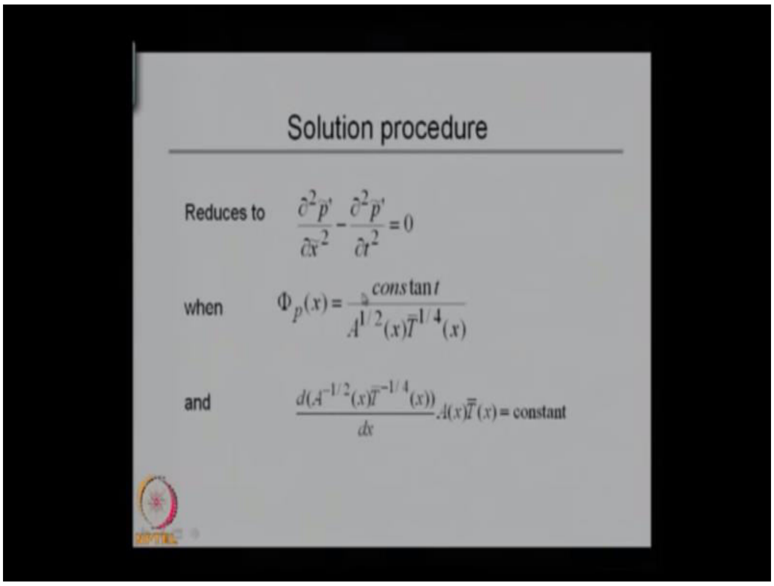

So I will get this slit lovely beautiful most beautiful equation ever seen right here. Which I know the solution like f of x minus c T of g of x plus c T and. So, on here given the then, I have this.

I think this c also gone because it's because the transmutation here it just got up solved into the mean x became non dimensional that is all. So, this is a beautiful solution but then this solution holds good only if you through this things. So, that is what the second statement and third statement says. So, you have this very nice solution but what is only if these two things are 0. So, that is more like a condition which has to be satisfied to be able to get the nice and peaceful solution like your friend is creating hell in your life you want peace. So, you threw him out.

And then you have peace. So, you be able to him or her whatever. So, this first thing is what you want or it is not there and what you want if I knew how to solve the big thing I would be with her everything. I knew how to deal with the messy guy and I m to deal with the full thing but I do not at the moment stupid. So, but I know how to solve this because I learnt it from book. So, whatever I know I will deal with it is like you the same that if you find a hammer you will finds nails everywhere if you have a hammer you will find nails everywhere something like that. So, I know how to solve this equation. So, this is what I will see everywhere in my life of course, the two to that I have this penalty that this condition should be true; the first one is peaceful that says that the p will be of the form square root of a T power one-fourth and that seems to be reasonably because we saw some from somewhat physical arguments that p should be of this form.

So, I am throwing it up and down here. We got p of the form of root A n T power one-fourth, because from our physical insight we said that earlier in the earlier slide that go back to it.

So, we had this factor over here. So, it is quite alright I mean not just alright. I am really pleased that I got this factor out. Now the second one I am not. So, pleased it says that this equation is the is the equation which the area and temperature profile should satisfy. So, that I can get an analytical solution if only my f of x and T of x are in a such a manner as to this. This equation satisfy then I have a solution it is like for example, you said that I can solve this equation if area is linear or it is temperature is linear it is may though it. So, that means there many things possible but one possibility we have a solution right. So, it is something like it is looks more mess is on that but this is we have to have this relation to be able to solve this in close form. So, in the beginning I have already annoyed because I thought this was a restricted but later I got doubt on this it was. So, restrictive.

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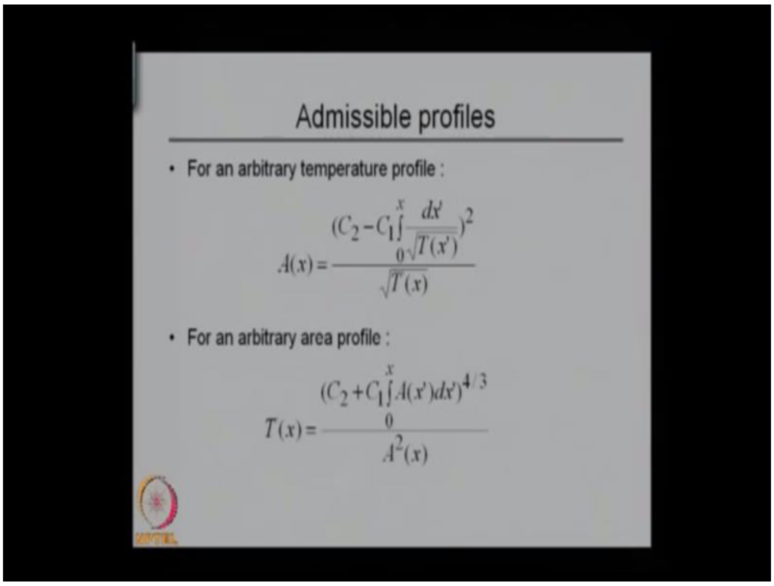

So, for an arbitrary temperature profile. So, let us say I pick the temperature profile any temperature profile then the A of x has to be of this form and then I felt very much of peace because you can treat c 1 and c 2. So, if you have a given area profile then we can actually treat the c 1 and c 2. Such that you can fit the curve into this sort of profile. So, you have a given temperature which is whatever. It is for any given temperature you can actually find a area profile which will give you which will admit solutions but you can't generally say with any general thing. Because we have we actually have a that kind we have to fix the values to profile that we have, but here we have two constants to which it can be fitted. So, it is reasonable and similarly, for an arbitrary area profile you will have a temperature distribution of this form. It is a form it is a like when you say something is linear you still have two constants to play with a x and b.

You have y equal to a x plus b you can play with a and b and fit any two points in strait line right. So, it is a same thing. So, e and if you can't actually then you can actually fit it into certain segments and within each segment you can fit this curve with. It is look reasonable but you can fit this curve if it looks reasonable and if it does not fit it you can make the segments smaller and try to fit. So, that is the idea. So, it is not very bad.

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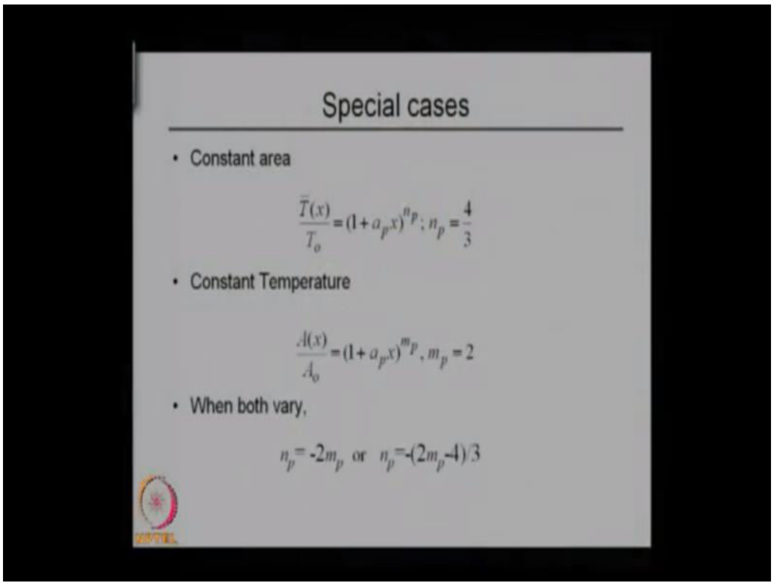

And what is interesting is if you have a constant area you get this T over T naught is 1 plus X or four-third and as I told in my story this is what I originally started with and it came after the blow well. It was given and a constant temperature if we have a temperature constant, it will give you a area 1 plus a x square. It is like a quadratic distribution for which solution exist now. This is actually standard solution actually I terms of for vibration of rods we have the same equation because when then longitudinal vibration have lost same compression and gradient fraction propagating axially.

So, if you end of the same kind of difference equation and the solution was not there it was not something new that we discovered but this four-third is something new that we discovered and then if both are varying then one polynomial profiles you just have to have some kind of relation between this n and m. So, that you can get they have to be put this way but you can still treat the constant. So, this is what we get and ((Refer Time: 18:32)) in it. So, while we do all this wonder that can't you just solve the equations. We have ordinary differential equations cant we solve them with some numerical technique Runge Kutta yes that is yes but each time you solve you get a different solution each solution is different but if you have a even for a very special case if your analytical solution you can certainly see some general of pattern.

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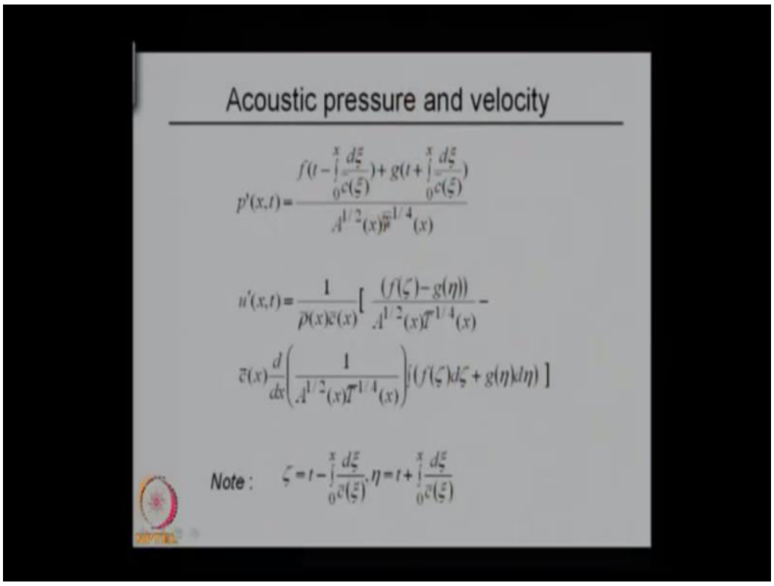

So, the solution here is pressure is a for half time. If for one-fourth in the denominator then it can be dependents on f of this is right running view and this is a left running view instead of f of t minus x over c we have t minus integral d psi over c from 0 to x and then understandable.

Because we have this characteristic equation and if c is not constant it cannot be integrated directly you have to do a kind of integral. So, the good thing is although we did what might appear to be right on the world algebra or something. I mean earlier I was hand waving but this factor came out of the globe. So, this kind of dependents I mean now we know it is there sure only thing is the velocity dint go as per my hand waving. The first term was the only term that was there but there was a this another term which is also present. So, this term second term would not be present had been had w k b up. So, if you actually ensure that disturbances are of very small length scale compared to the length scale, in which the area that changes or temperature changes this term would not have significant contribution but otherwise. So, the key factor I am trying to tell is that if you have a converging area the amplitude will go up if you have a temperature distribution which is going up pressure will come down but the velocity will go up.

So, that is the cuts in the matter and this like say if you have formula it seems to see this if you have numerical thing you have to plotten curves and look at them and I see this pattern. So, it must be what what is happening. So, that is a different approach and I personally think analytical things are eloign, but it is hard to get . So, I hope you got the correct of the matter and I will give you this slides and that original article which has much more detail in whatever I am saying. So, in this excise I did not manage to get a general solution I only managed to get solutions what this is called class of solution. A class of temperature profile some class of area profile for which analytical solution I get this but this does not mean that there is no general solution could be there and if one of you can solve it. You can immediately publish it and that becomes the sheet of the art. So, in the assignment one of the assignment will actually solve this equation numerically.

So, where modern world you need numerical techniques and its very handy to waved no other. So, we can take this equations solve them and plot the temperature distribution and so on. but at the same time I mean if you use your mind and can do some analysis I think you get much better insight and. So, I am for using both analytical techniques plus numerical techniques together I think you will have much better understanding.

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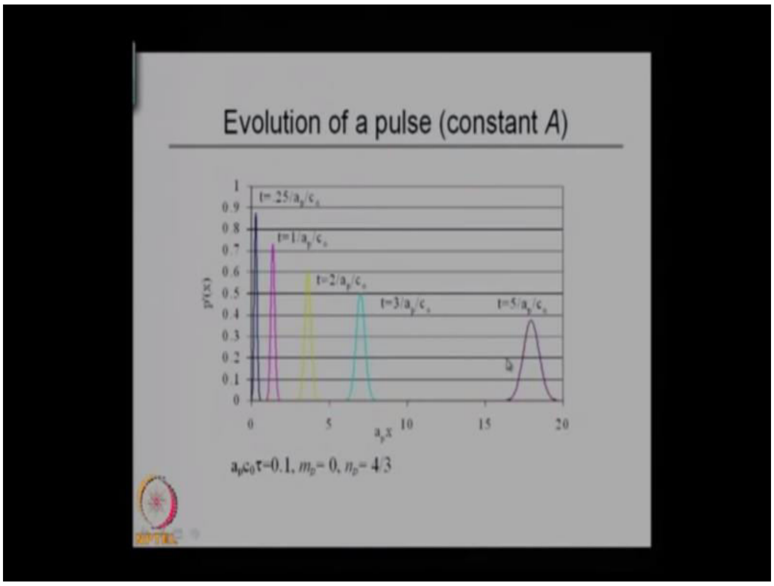

So, will look at what happens to the evolution of a pulse. Let's first look at constant area case and temperature is varying. So, here temperature is varying as per this profile. So, the temperature is increasing and the pressure amplitude is dropping I think that is what we saw from our formula then temperature increases pressure will drop and there is another thing earlier in the.

I think those guys they are not here today they always object to my term classical wave equation. I do not know what is the other term for it regular wave equation we go to Mac Donald's or all this sued shops, they will say regular, extra large, medium, small something. So, I still want to call as classical. So, if you guys agree we will meet the those two guys. So, what not sure happen continue to refer right us classical waves. Anyway in the classical wave equations what happens is we get a solution p prime is f of x minus c t plus g of x plus c t. Now hidden in this thing is that, the shape of the wave does not change, but here actually the shape of the wave is changing, it is slimming down that is a rescaling, but scaling bulk here. You see it is not just a reduction in the height or rescaling in the amplitude, but actually this thing is getting bulkier can we see physically feel why it is happening lets hidden in this thing actually.

Student: ((Refer Time: 24:00)). So, because of that ((Refer Time: 24:05))

Don't know I think we are thinking too fancy what happens when something is hot? what happens to wavelength? For example, Student: ((Refer Time: 24:19))

Wave length increases, ((Refer Time: 24:22))

So, that what it is when you go when the wave goes to a hotter part it is wavelength is just try to stretch when it comes to a colder part if you are propagating from right to left it will shrink let's clear. But, now mean most of you guys are from here. So, if a wave by itself without any change in medium or something temperature is on.

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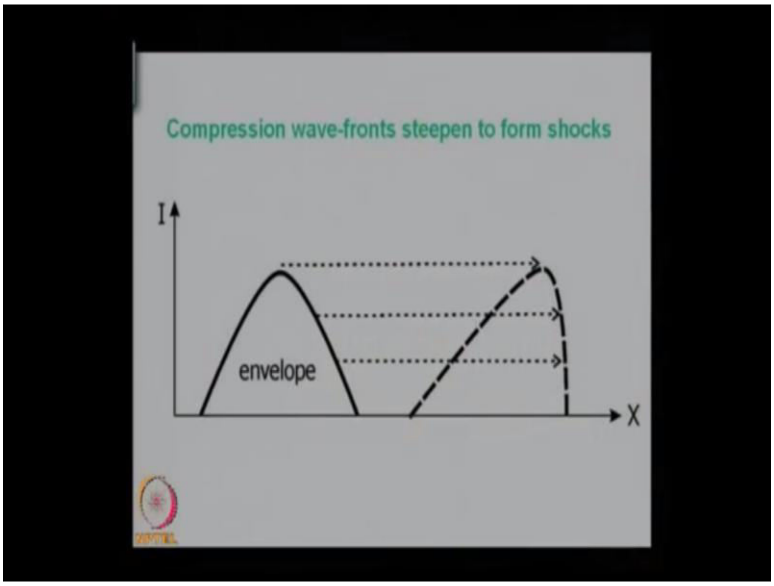

So, what happens let's say this is a wave and this progresses now this if you have a compression wave at the front, let's say moving to the right what have what that the compression wave do to the gas properties? Increase temperature, increase pressure and the temperature increases the part of the wave that comes behind. See it is a higher temperature when temperature is increasing what happens to the propagation speed increases. So, what is coming from behind catches up and then it will tend to form a shock. So, a normal disturbance a regular disturbance will become shock anyway well. Now the anyway has to be qualified what is the qualification here? If it becomes a rock it will become a shock before the length scale in which we discuss axe after. So that is the thing oh if you think about if you are in a frame work of oiler frame work we cannot really deal with this equation. So, I will in the moment where frame work oiler. So, we just have to make this condition. But, if we have to actually find out whether, the shock is formed or not.

You have to account for the viscous losses and heat conduction losses and then see at the wave forms by then but I mean that many practical examples like the crackles what you get here the shock wave. And I spoke a word shock wave leathotropes you where send a ((Refer Time: 26:10)) tip and form a shock, and if you see musical instruments like the brass instruments such as trombone. And trumpet at the exit and they are very bright instruments found as very bright and I do not know this bright what makes sense to you it sounds very clear right. They are they compare to let us say flute or a violin. So, that is because of the instruments are long relatively compared to other instruments and this or compared to regular string and instruments like guitar and all or veena. So, this waves propagates for a distance and it actually steep and form and people have done ((Refer Time: 26:58)) graph looked at this thing shock wave and sound now what happens so.

So, this but this is not linear acoustics this theory does not deal with it. So, our theory says that the wave can steep and but it’s in a linear frame work you reverse the track and it will go by the other way.

Let us for a moment exposed think about what happens to a wave moving into a region of increasing temperature the wavelength increases to the wave actually axes it is press out right. Now if you are coming other direction if you are coming towards the cold direction the wavelength actually the wave actually tends to steepend this opposite of axe.

So, if you are moving into a cold region what happens? The front part of the wave starts slowing down because it sees the cold one, the back part is still coming fats. So, you are steepening will be favor and if you are moving into a hot region in the wave is moving into a gas which is hot, it will tend to elong at. So, it will attempt to relax. So, then there is a balance there is a natural ((Refer Time: 28:10)) tendency versus the effort to relax because its wavelength.

So, I mean you can still form frog but there is a balance between these two things if you are pass for a minute I hope you understood, there is a very interesting thing topic and this things can be competitively calculated at least for one dimensional ((Refer Time: 28:34)) very interested you can see may privately. And you can give you some material. So, once again gas would tend to relax if you are going to the wave will tend to relax. If you are going to you know high temperature and if you tend to steep and if you are going to region of low temperature. Now if you are going to a region of decreasing area then the shock formation will be accelerated because the the wave is sending to steepen and if you are going into a diverging area then tendency to relax. So, again if you are talking about non-linear acoustics then it is just natural tendency to form a shock versus the tendency to relax or steepen. So, there is balance.

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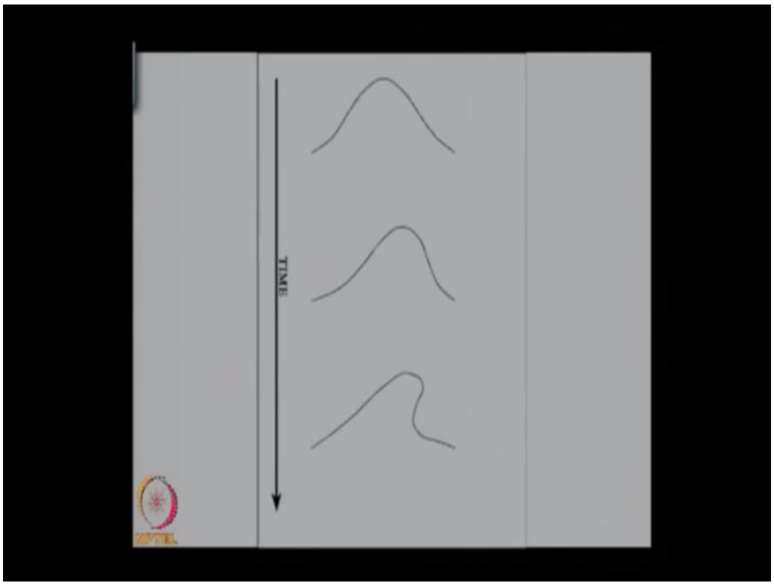

Now, of course, on if you look at sea waves I am not expert on this. So, I cannot speak much on it.

But, you can there also you can see this kind of formation that the wave is steepening but then it over turns the wave is breaks, but in gas atomics we cannot have over turning in fluid mechanics, because at every point. You have one property that is the way you fluid mechanics structure that every point have one property where as this wave at every point you can have two or three different heights. I mean the wave is over turning but we do not have that possible in gas. So, when wave steepens it becomes short and then we have to use the shock conditions rugen hur jump conditions and the continue beyond that if you want to calculate.

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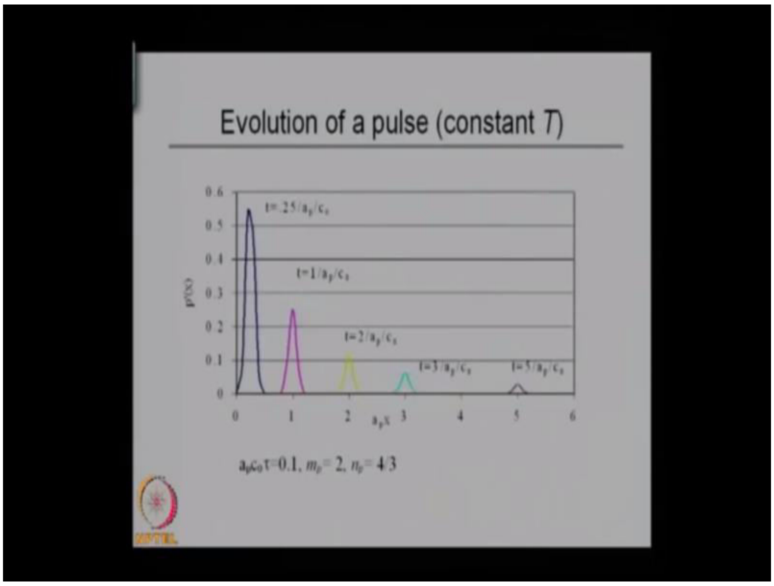

Now, we look at the evaluation of the pulse at constant temperature of course, is there any question some non-linear I'm very happy to answer your quests with the pulse way the constant temperature you can see there is only amplitude base scaling. So, this is a case where the area's increasing. So, the amplitude comes down because we saw it is like one over square root of a n the area is decreasing the amplitude will go up.

Because you are steepening and the same turn will happened for pressure and velocity where as for the case of non-linear temperature pressure will go one way and velocity will go the other way but here it will be a similar. So, we have to come for some kind of conclusion about what happens to sound wave in the time domain kind of approach will pass for a minute and see if you have any questions.

Student: Sir in out speakers come cones to spread with some. So, in that case if area increases pressure ammeter decreases. So, what is happening?

What does the interest of a person using out speakers? Is this interest gives the sound out or to keep the sound in the duct? When you are talking about this horn type speakers.

So, if your interest is in keeping getting the sound out or keeping it inside a duct.

Student: Sir, getting it out.

So, when your const your duct the radiation efficiency is your poor. So, you are actually making in this form. So, that actually your nature from plane wave to kind of radiating waves and then the radiation efficiency of these kind of geometries much more than that of what is being I mean what is being happening from a plane wave now of course, if you have a kind of expanding thing. For example, if you expanding like a cone your pressure you can show mathematical at the pressure and velocity goes over like one over r which is like one over square root area it does happen but even if let us consider a case where there is no duct and you are speaking and sound is propagating. So, one over r decay because is a same power nothing is created or put in from anywhere. So, the same thing if more people has to take it has to be.

I mean if have 100 rupee and give it to one person he gets 10 rupee but if I am giving it to ten people sorry if I have hundred rupee give it to one person he gets 100 rupee but if I give it to ten people each will get ten, if I give to hundred people each will get only one. So, as the area increases the person and velocity has to come down but it is a question of conservation of power. So, I ask you said the pressure and velocity will decrease lets go to the area but there the objective is not to worry about it but to make sure that you read it out. So, your constant area duct everything will almost everything was stay inside or if you are converging that even more serve will stay inside. Any other questions? Yes, Manoj.

Student: Sir, when the area is changing the velocity will change so.

Absolutely.

Student: Then the steepening will batten off.

Student: It is not as in ((Refer Time: 33:40))

Here the area is increasing. So, it will rescale down the amplitude will come down. So, amplitude is coming down

Student: The velocity of the velocity of the constant series

Velocity of propagation I think we have to.

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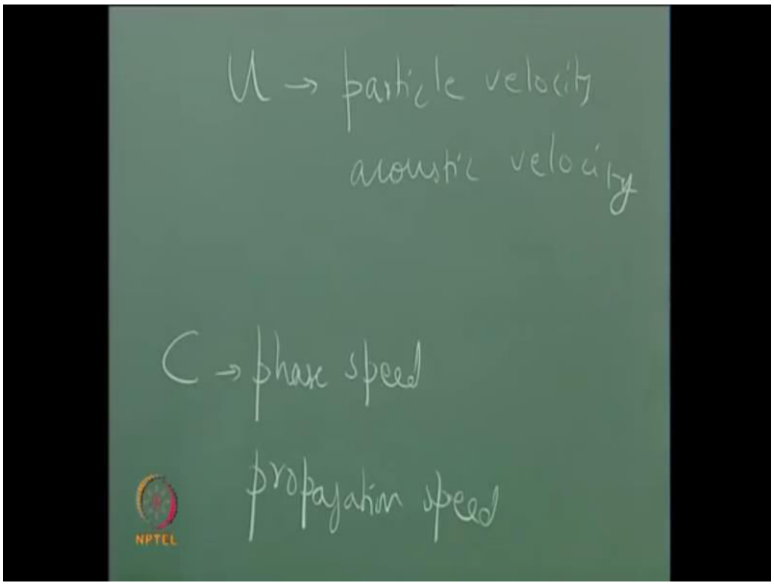
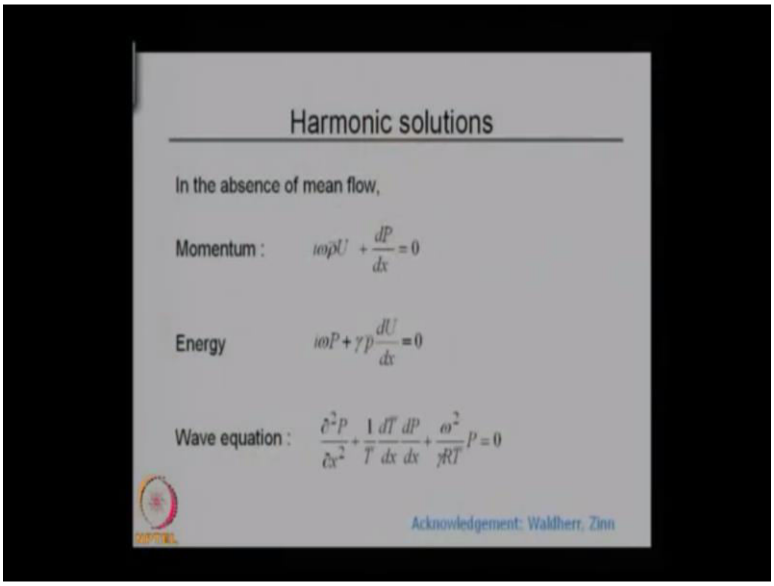


So, there are two things where as u with what we call particle velocity or acoustic velocity c, which is what you call phase speed or what I mean the propagation speed. I think this question up before you started attending it. So, the u actually means how much the particle is moving and that is not the speed at which the wave will go. So, just to give a example, if you consider a line for taking a ticket in the movie theatre or something and their push the person front and he pushes the person front.

And he pushes the person front and. So, forth eventually they will push the other end of the line and I am not basically moving from here to the end of the line and pushing the person front the movement of each of the person's. That is analogous to the particle velocity but the push itself is travelling at the speed probably much faster and that is the phase speed or the speed of the particle is moving. So, here as temperature is constant C which is the speed of the wave is constant therefore, the wavelength will not increase of decrease unless the temperature changes. Now if you have non-linear acoustics naturally we comparison way will heat up the gas. So, the back part the wave will see a higher temperature and if you reduce fraction it will cool down the gas and the back part of the wave will see a cool a gas or and it will go slowly but in linear acoustic this effects are not there and solution of linear acoustics. So, basically speed of sound is constant but if speed of sound varying.

Then, you will have which the provision to vary, that is non-linear acoustics then yes steepening can happen even in a constant area even in a variable area duct in fact if you decrease the area the steepening tendency will increase. If you increase the area steepening tendency will be coming down but then it is a interplay between the inherent tendency of wave to steepening versus relaxing tendency of increasing area for example, in a trumpet. So, what the exit is diverging but still actually a short forms there. So, diverging is to for radiation efficiency for the sound to come out but still I can actually form a shock very nice question any other question.



So, the next issue is to get harmonic solutions and harmonic solutions are very convenient easy to deal with and the other thing is many situations we put a loudspeaker on at a constant frequency and experiment and then you have own frequency.

And then might us will write the harmonic domain solution if the convenient. So, in the absence of mean flow we had written the equations in the time domain. Now you substitute pressure will equal to some complex amplitude times e power I m p negative velocity of some complex amplitude time e power I m p negative and so forth. So, your momentum equations reduces to I omega rho U plus d P by d x equal to 0 and the energy equation reduces to I omega P plus p bar d U over d x is 0. One change from the earlier notation we had used P amp for complex amplitude here is capital P for complex amplitude and then you can get the wave equation of this form and I wish to point out two differences between the traditional wave equation or Helmholtz wave equation this equation. If you look at the first half it is identical d square P by d x square and in fact I think this is not needed you could have just written d square P over d x square there is a middle term here did not exist the case of the classical Helmholtz equation because there was no d T over d x, d T over d x is 0. So, this term will vanish.

So, it is consistent with what we learnt earlier plus omega square gamma R T bar earlier gamma R T bar I mean gamma R T bar c square. So, omega squared over c square will be k square and our solution was d square P, d x square plus k square P equal to 0. Let me just write that down we have.

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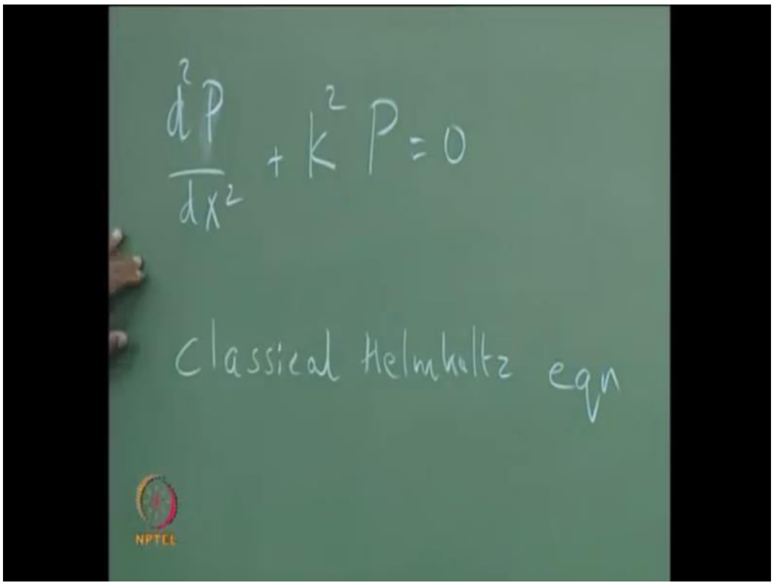

So, I use classical in the sense that a there is no temperature gradient and so on. So, actually we can recover this equation from this equation that I have written here by putting T equal to constant. The first term will drop out and this omega squared over gamma R T squared it is squared. Only thing is in the classical sense k squared is a constant and your solution was k sine, k x plus because k x. Now can you guess what is the solution would this be given that we saw? That is I think Rajesh you cannot answer you know the answer given that the temperature is changing and we saw that the wave goes up the amplitude goes up.

And we also saw that the temperature increases, wavelength increases, temperature decreases, wavelength decreases. What kind of solution would be percent because I just give one more hint if you look at what does the thing hockey in cricket when they when you hit a boundary it shows how it bounces.

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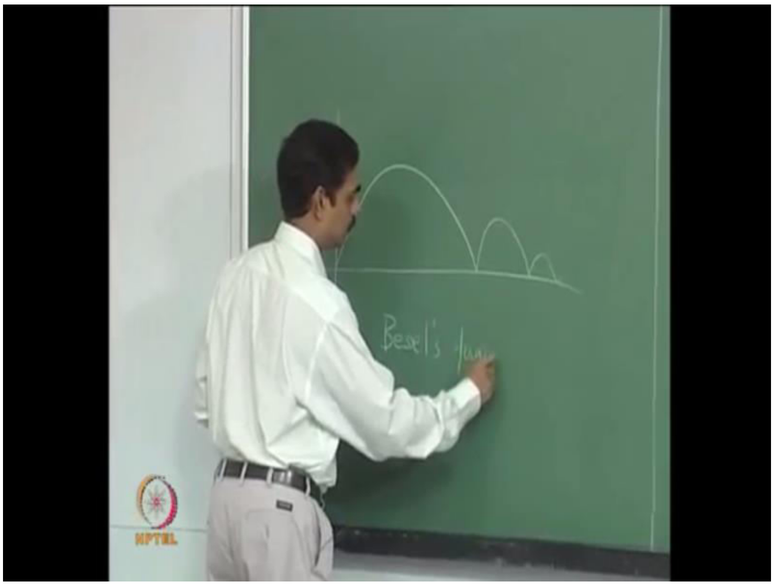

So, if I hit a boundary if I go by what the problems in j e physics this is the way the balls goes but as if anyone of played cricket would know that the ball actually goes this way right. So, what kind of functions? I mean definitely this is in school physics reality is kind of like this right. So, you have this amplitudes scale as well as the distance between the minima’s also.

Student: Bessel

Absolutely right. Perfect. So, and generally Bessel Functions are found in cases where you have cylindrical.

Or spherical geometries in cylindrical geometry you must have studied conduction or something like that in cylindrical geometries your radius is having a one over r kind of one over r square one over r it is a cylindrical or spherical. So, then you have Bessel. Here temperature is giving some such sense. So, that is why it is coming but still not obvious from this. So, what to do that is the question.

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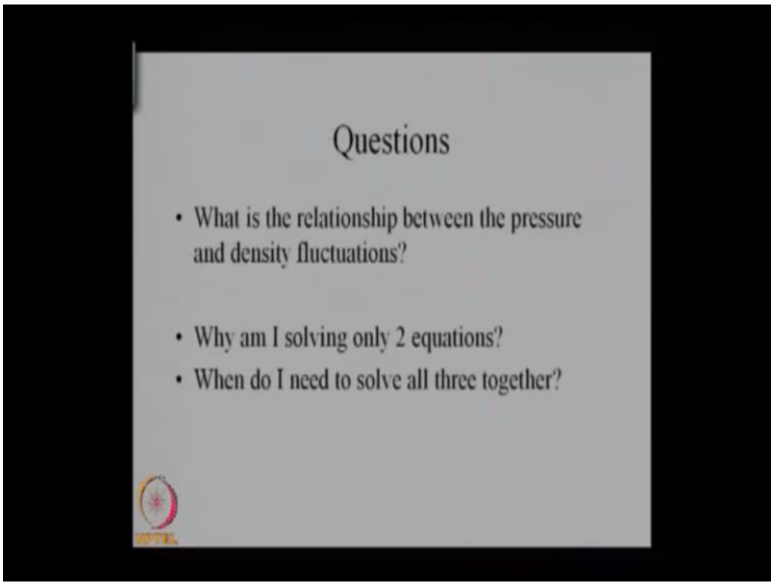

Before this I want to ask you I want to take a time out commercial break and ask you some questions. What is the relationship between pressure and density fluctuations? Just to this is quiz I do not have any chocolates to give you if you answer but I'll bring some next class, shubrish right. What does a relation between pressure p prime and rho prime.

Student: (( )).

P square p prime you are going to.

C squared.

So, we will take a poll here is this right or is this wrong now you are having a it is not a constant temperature here you are having varying temperature right is this hold that is a question. So, if you have any question it will. So, if I had a question I would not think I know the equation I will jump and answer. So, can we take a minute you can solve it in a minute.

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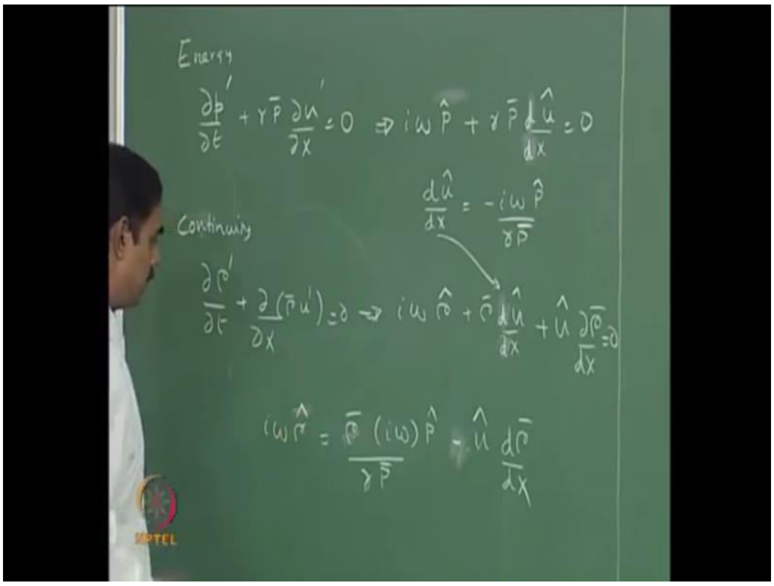

Now we are having capital P but let us still put P here when you write on the board this is what equation is this. Energy equation for fluctuation and let us write continuity, what is it? So, I should say d when I go this sorry the now if you rewrite this, I can say d u bar over d x equal to minus I omega P hat over gamma P bar substitute that here. So, minus will go.

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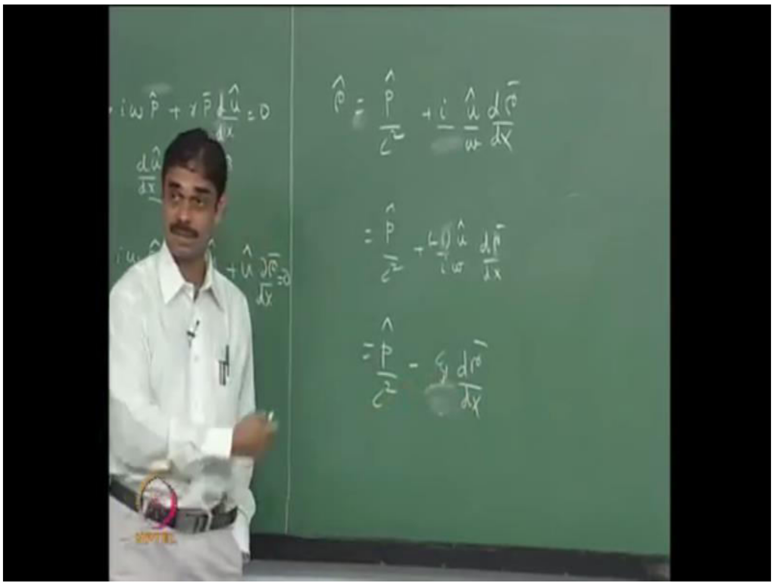

So, now if I simplify this I will get rho hat equal to I can remove I omega. So, just tell me if there is a mistake this gamma P bar over rho bar equal to c squared. So, minus 1 is

I squared. So, I squared divided by i will be i plus i over omega. Did you is this correct? Just check. So, I do not have this P over c squared as rho hat is not P hat over c squared but there is extra term. Now what causes this physically? Vishnu, what is the meaning this term?

Student: (( )) yes

So, u is correlated to density. Let us say and then what happens if you are moving into region of higher density then you can actually have a change in density because of this isentropic comparison or whatever plus there is extra change simply but transport of quantity there is a transport of I mean, gradient tendency let us say.

So, I mean sorry fluctuation can transport that property and therefore, a particle which is coming a from a region higher density to a linear of lower density will have some effects if it is coming from a region from lower to a higher it is also a another effect in fact isn't it. So, this is the transport happening because a fluctuations are moving things from a with higher properties to lower properties. So, that also will contribute. If you write it in terms of displacement what happens maybe you can see it better displacement is let us see right. So, if you say e power i m p t, i omega psi hat equal to u right. So, u over i omega psi hat is that right? So, I will write I times. So, if I multiply top and bottom both by I will get minus 1 here by I omega u hat p over d x equal to P hat over c squared minus this problem makes sense.

So, I think again we should not blindly believe that P prime over c squared is rho prime when the temperature is mean temperature is constant. You could have something of this form I have two more questions, why m I solving is our momentum of or in the beginning. We were in the first class base does with momentum equation and continuity equation. Here I actually use the momentum and energy you can use momentum and continuity also. Why do I have only two equations? And why am I solving all 3 together? If you know the answer often you can let me know otherwise you can think over it and come back next class. So, stop it for today.