Acoustic Instabilities in Aerospace Propulsion

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Lecture - 11
Attenuation: Continued

Good mooring everybody, so we are looking at what happens to previous when the attenuated. So, we looked at modern problem where we have some kind of attenuating medium and we are look trying to express, how this standing wave look like. So, we are having a real omega and we are assuming a complex frequent at complex wave number k and then we are trying to express the left running wave and right running wave as in term of this omega and k and so on. And we wrote expression for left running wave and right running wave last class and some student will be that they could not get the final answer, so I will continue with it.

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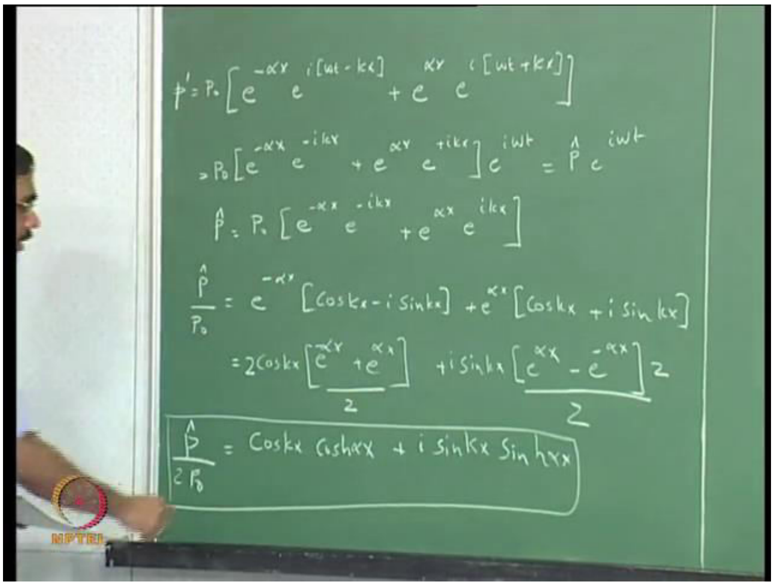

So, putting together we will get p prime equal to p naught times e power minus alpha x e power i times omega t minus k x this was a right running wave plus e power alpha x e power i times omega t plus k x this can be we will expand it for convenient. So, p naught times e power minus alpha x e power minus i k x plus e power alpha x e power plus i k x times e power i omega t, which we can say is p hat e power i omega t where p hat is indeed the complex amplitude.

So, we need p hat is a is a complex number, so it has both amplitude and phase and we will try get expression for these things. So, p hat itself is p naught times e power minus alpha x e power minus i k x plus e power alpha x e power i k x and so we can try to re constitute a more in a convenient way. So, p hat over p naught equal to e power minus alpha x into cos k x minus i sin k x plus e power alpha x into cos k x plus i sin k x this is the fundamental thing and complex numbers.

So, this we can recast as if you take the cos k x out then you will get e power alpha minus alpha x plus p power alpha x plus i times sin k x into e power alpha x minus e power minus alpha x and if you multiply and divide by 2 you would get this. So, we can write p hat over 2 p naught right equal to cos k x cos hyperbolic alpha x plus i sin k x sin hyperbolic alpha x, so this is a neat result. So, from this it is quite trivial to get the actual amplitude of this complex number and the face I will just write the amplitude.

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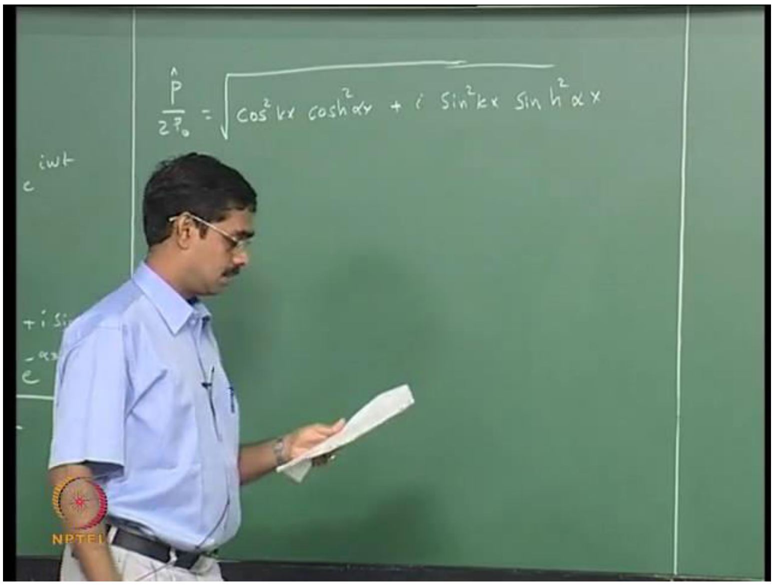

P hat over 2 p naught into square root of we square this square this add them and take this square root that is the same as p hat times p hat on the edge.

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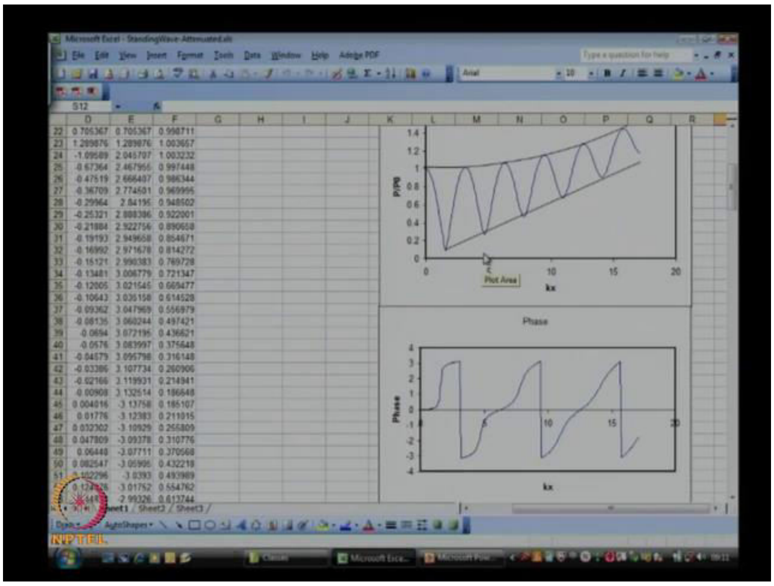

And will be show your plot of this and I mentioned to you can draw this plots with anything you can write a computer program and plot it or you can write a mat lab program or you can do it in mathematical I just have deliberately chosen excel which is really simple sheet for calculating budget and, so on, but we can do it even there. So, this is the variation of pressure I have written p over p naught actually it is p over 2 p naught I have drawn with reference to normalize with reference to with a value at the end.

And you can see clear pattern that is first pattern you notices this minimum value they almost grow up like a straight line right, you can see where the mouse is the minimas are actually growing up like a straight line and the maximas are actually also growing up, but kind of like a curve like a parabola. So, in fact, if you did not have damping what will happen all these minimas will line at a same place. If you had just a standing wave with some admittance and depending on the admittance all minimas will go up or come down together, but if you have damping one thing you would see is the minimas will change.

That is because, at you are having some kind of cancelation at the minima, but the in the case where there is no damping left running wave and right running wave have same strength or if you have a perfect termination and if it is not a perfect termination there are different strengths, but the strengths will be same throughout, wherever it is going here the strength of the left running wave and right running wave are different in different parts of the bulk. So, therefore, the cancelation is also different which is why you get this kind of a shape.

So, on where to check if it is a attenuated wave, so that two waves you can have losses one is this going out through a impendence kind of boundary condition that can have loss all we can have loss like a volumetric damping. So, if it is a volumetric damping on signature to look for is the how the minimum values change and as I told physical relation is because, the standing wave has a different values amplitudes at different location each of the left running wave and right running wave because, the see that these are attenuated in different ways one is attenuated as move to the right one is attenuated move to the left.

The second thing I want to denote is the phase, so if you are starting from the loud speaker and you go towards the hard end as I moving the attenuate at the wave is attenuated. So, amplitudes keeps coming down and then you get reflected and then. So, the incoming wave was a left running wave and outgoing wave was a right running wave, and right running wave gets continue to get even more damp.

So, near the source if you dominatingly going like a travelling wave which is y you see this kind of a variation it is almost straight, but then you see this curves and so on afterwards. And the energy is going from right to left which is y you have a domination of the left running wave here, in this particular system and for the sake of completeness, we will derive the expression for velocity amplitude also any questions yes.

Student: ((Refer Time: 08:21))

You just a face of this expression. So, that will be tan inverse I have to stand this side tan inverse of sin k x sin hyperbolic alpha x divided by cos k x cos hyperbolic alpha x. So, it will be tan inverse tan k x tan hyperbolic alpha x. So, you have analytical formula.

Student: ((Refer Time: 08:45))

Alpha is the complex wave number.

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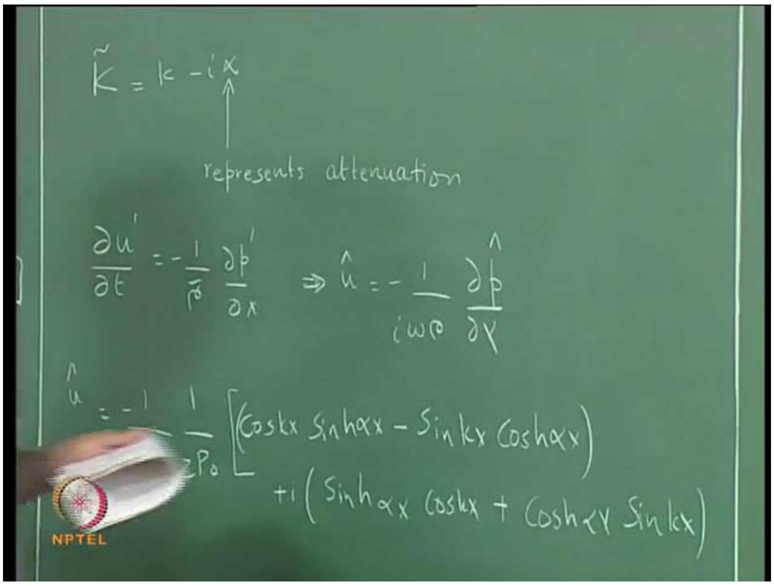

So, k equal to, so alpha is the complex part of the wave number that is what represents this attenuation. So, I will replace what he said alpha is not cos attune attenuation alpha represents attenuation. Something physical cause of attenuation, we are trying to represent that now, what exactly happens it can be as there is no pointed out whatever happened and the system which takes away the energy or you can have some kind of props take the energy or you can have some kind of chemical engineering system like a pad belt or observing porous medium like a stack in thermo acoustic engine.

So, the indudal physics of the attenuation can be different, in different circumstances, but alpha we are representing that within the linear frame work you are seeing this alpha. Student: That is clear. You have a I remember you wrote a paper on this damping all time years back. So, if anybody has any question you can ask Rajesh, Rajesh as published something on this more complex problem it is any other questions.

So, velocity is likewise quite simple to get, again we go back to the linearised momentum equation. So, we can from this you will get you have equal to minus 1 over i omega rho p hat bar x. So, if you just need to differentiate this expression. So, you will get terms like cos k x sin hyperbolic alpha x and cos hyperbolic alpha x times minus sin k x and so on. So, each of this terms when you differentiate becomes two terms.

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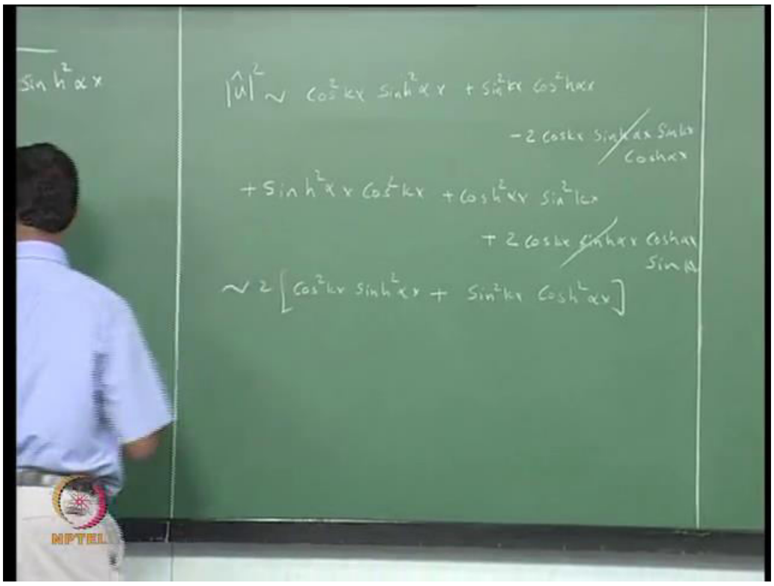

So, this is what you would get so, it goes like if you square this we square this term and square this term I will not bother to write the constants. So, you will get cos squared k x sin hyperbolic squared alpha x plus sin squared k x cos squared hyperbolic alpha x minus I will write this here 2 cos k x sin h alpha x sin k x cos h alpha x plus we squared these terms. So, you will get, so this term will cancel with this and these are, these are the same terms actually. So, this would be equal to 2 right cos squared k x plus sin squared k x. So, it is a proportional because, I have this minus 1 over i omega rho 2 p naught. So, if you take the modulus of this you will get omega square rho squared 4 p naught also there.

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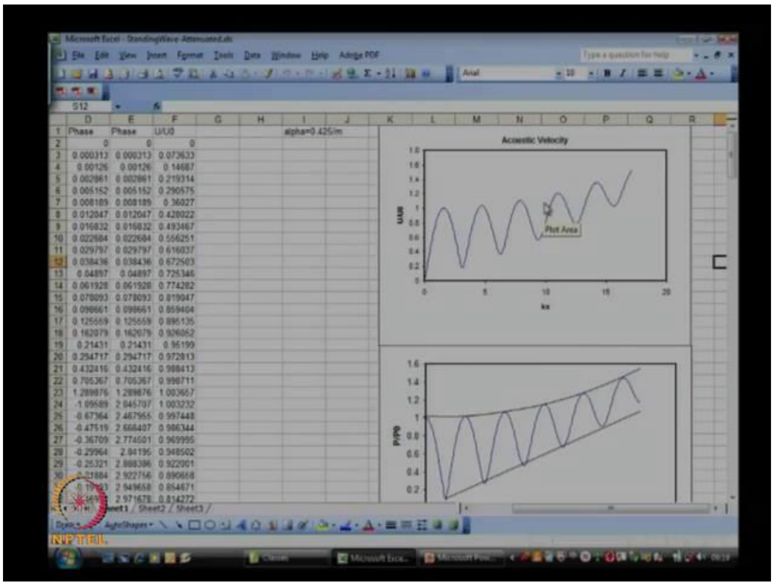

So, this expression is what I have plotted in here this excel graph as u over u naught versus k x it also follows similar pattern remaining must keep going up and they maxima is also going up, but the minimas go up kind of linearly this 1 goes more like a parabola. And even those things you can show apart with approximate ((Refer Time: 14:27)), but I will stop here. So, in summary we have taken a look at how where standing wave found by 2 damp rate can be represented.

As suppose, earlier case where we studied we did not have the various being damped we the waves while reflecting loss some amplitude that is all. In the in the earlier case we studied when we are dealing with the impedance tube. So, you can actually in principle do impedance tube technique with damping also. So, now, I assume that the one end is perfectly rigid.

So, I said reflection happens perfectly, but you can also relax that and apply appropriate admittance boundary condition there and work out this problem and it should not be too difficult I will let you do that. So, work out this problem for a general admittance and having this alpha as attenuation that is a homework.

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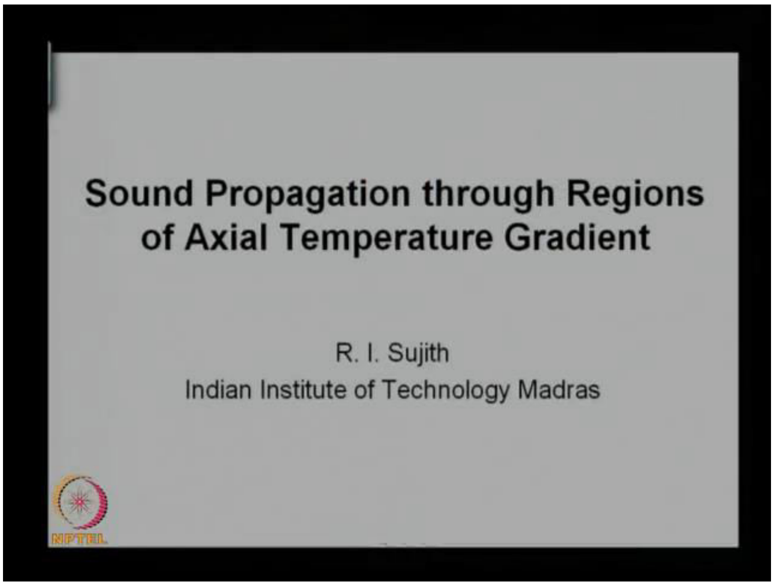

So, I will pause for a minute if you have any questions I can answer that and then will move to next topic. I will write it down.

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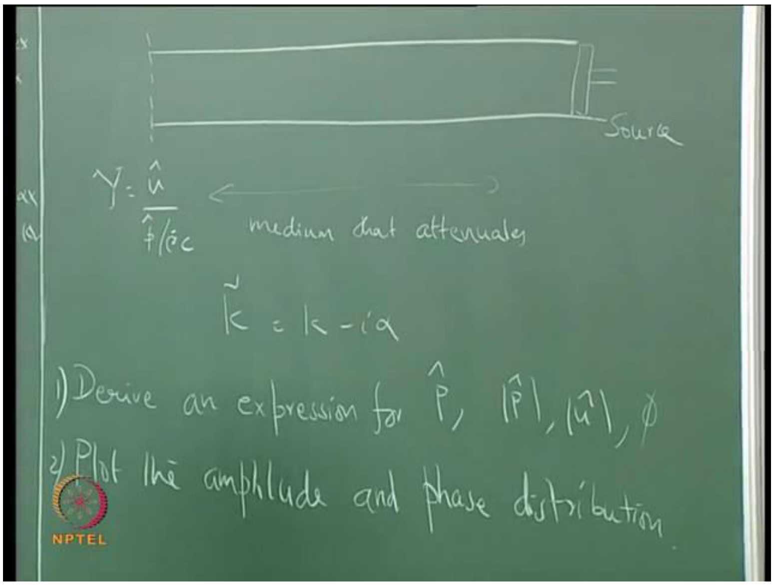

We had some kind of source and we have a rigid plate and we have a medium that attenuates. So, we had k delta as the wave number which is complex which k minus I alpha. So, we keep all that, but we. So, we do the same problem, but we remove this rigid plate and we say here I have of admittance boundary condition equal to and then work out how the way would look like. So, once you have p hat amplitude you can also get the amplitudes and the complex amplitude you can get the amplitude and the phase. So, this is 1.

So, like I said we now, or know by looking at something what is the boundary condition and, so, on. The way to know is to here the sound or measure the sound in the standing wave you get the acoustic feel and from that we can reduce what kind of loss it is. So, you could have a loss purely in the form of admittance loss or gain also or you can. So, that is like a surface if you remember, we had this p prime, b prime over integrated on surface.

So, you can have that kind of term or you can have like a loss in the volume a loss or you can also have a gain like Vishnu was saying if you had heat addition, then you would have a gain. So, you can have a volumetric losses or gains as well as surface losses or gains both are possible. So, the way to get the feel as the engineer is to have a good fell of the forward problem.

So, given a perfect condition how does the wave look like, given a imperfect and admittance condition how does the wave look like, given a perfect end condition, but there is damping how does the wave look like, given now the look at the next step you given a imperfect and or a admittance condition and at damping how does the wave look like. And then you step up given a temperature again how does the wave look like that is what we are going to look next given temperature gradient and damping how does the wave look like given temperature gradient damping and losses at the boundary how does the wave look like.

So, you have to build this feel by playing with the equations and then you also look at the actual experiment and then you see how things are you can never see a end and tells right straight away, eventually when you have got experience you can look at the system and the just say what causing I just tell you a story this may to be editor out of this video thing, but we were trying to make a thermocrosic engine and c s i r and the idea was to actually collect solar lights set up a temperature gradient and drive this engine.

And whatever, and this was big director general of c s i r was coming to see this. So, that and it has to worked well. So, that you get funny and this thing was not working. So, the my former student bala is now, scientist since here. So, he came here and took some of our students are there and the right away said that a he has to reseal the whole thing the seal is not perfect and he said does I mean nothing is going out it is perfect, but then they said insisted that the seal is not very, very good and I have losses.

Yes, actually this is true when you reseal the whole thing will thing make sound. So, that was this p prime, b prime, time average taking out all the energies are put and that it was not able to grow. So, you this there was some kind of acoustic energy being added by the interaction between heat addition and the sound feel, that is like a driving temp, but equally important is the losses the losses were taking out the whatever was put in.

So, you never had oscillations and how did this guy my student tell that I was there eventually they feel, but initially you play with all this simple expressions and also with the play with the wave field at the tube that is you put microphone and keep microphones measures them and how the wave looks like and, so on and then you develop a feel and then you have like a you can tell just by looking that this is what happened I think there was a book I read some time back blink or sometime by ((Refer Time: 21:05)) markem clock well.

The same guy wrote the book out layers, you said paper you have good judgment they can just tell answer in a jiffy. So, but you have to develop it does not get I am a star and I will judge everything in millisecond yes they expects judge in milliseconds, but they have worked years on this to develop this, but you cant you have to go through this process this is like tie a shoe lace or learning how to control a football.

And it will be like a hockey ball or playing ball in a middle come over and night and putting night over and all that what you have to deal with these thing you have to be with the thing and eventually like the martial art people say the weapon is not something beyond you, but it is like part of a body. So, something other. So, that kind of feel comes by writing the expression see how you can simplify like, see whether you can actually get a linear whether the minimas are going linear liable actually the answer is, but I am not going to show you, but I hope on personal that is to it.

And how the maxima will go up and play with the numbers and say. So, that is quite critical to be able to get a feel for waves it is this exercise is very useful because, it just not for sound in a duct or anyways I mean it can be a vibration of string or some other lam waves, so on or electromagnetic wave is this is fundamental phenomena that we are looking at and the fundamental mechanism which will build down each other and cancel on each other. So, I really hope that you will play with this.

In the next portion is the very first they have a round wherever I asked a question that I was trying to derive a wave equation assuming constant temperature and I think those guys over there, made some mastic commend saying that it combers how can temperature will be constant absolutely right. And that is one I told my boring story about the professor and the ring and, so on. So, whatever I can do I will do. I have to tell you one more story I that lot of stories I will tell and the, but I must acknowledge some of my collaborates from home I learned a lot I will show this stuff.

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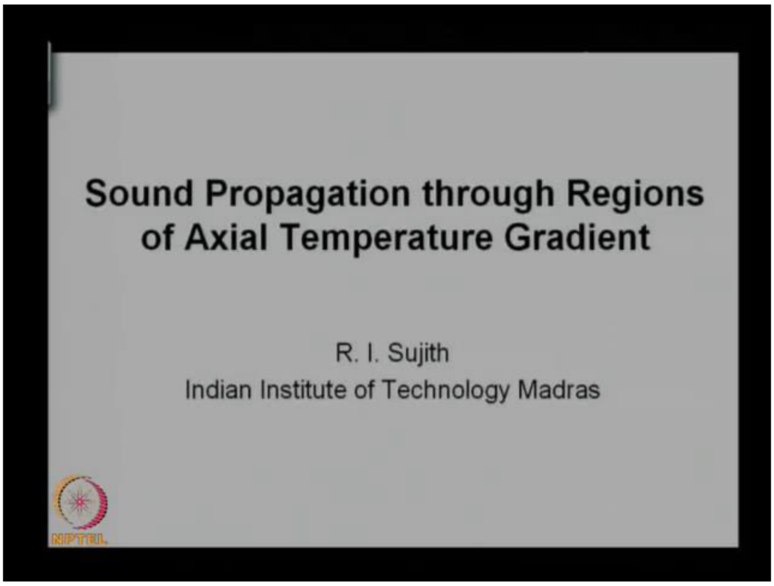

So, we are going to speak about sound propagation through gradient or axial temperature gradient. And this is quite important for some acoustic because, some acoustic deal with temperature changes heal, lease and, so on.

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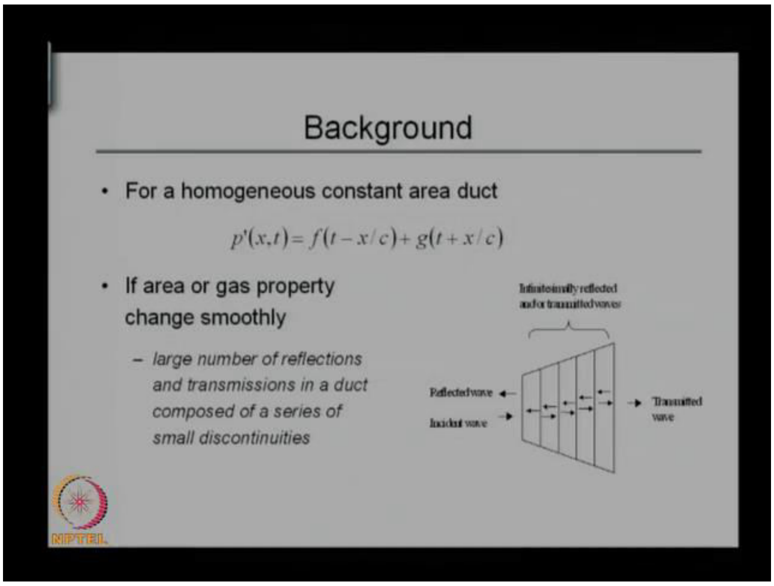

And you can never have combusted and heat added without change of temperature I mean it does not exist whatever be the assumption you make it just does not hold water. So, will revise or go over some of the things that we did earlier. So, for a homogenous constant area duct we had the solutions p prime is f of t minus x over c plus g times g of t plus x over c.

So, if you have a duct and you do not change gas properties or characteristic impedance is rho c which means that a wave will keep on going it does not reflects unless it sees some change in the impendence, which is what we call as reflection and transmission and, so on. Otherwise if it just a medium of properties rho c you would have the wave continue to go, a left running wave continue to go at the left and right running wave continue to go at the right as long there is tube, a tube ends will starts getting reflected back.

Alternately if we have media property changes you can have reflection. So, if you look at this, I have to use the mouse because, that is they are recording directly from this. So, I will not point there. So, please pay attention of most. So, we can think of gas property I mean gas property really changing smoothly, but we can think of it has a large number of reflection and transmission in duct composed of a series of small discontinuities.

So, you can think of each you can discreetise the duct in to small elements, but we can also have variable area duct that is also a possibility. And you have a different sections having different properties and different areas, but we can think of in each section we have a constant area, where the incident wave coming and each of these interfaces you can think of it has a reflection although in reality it is a continues process, but I mentioned this just to bringing the idea that we can I guess imagine that some times ((Refer Time: 25:53)) difficulty in believing that why will a gas reflect you are moving.

So, we can imagine that it was a discontinuity in the medium you can have reflections, right I think that is imaginable. So, if you have a series of discontinuities and each of the discontinuity would reflect. So, you can approximate a continues variation to a series of discontinuities, no matter how approximate it is. So, it is in principle we have reflection throughout the duct right is that it.

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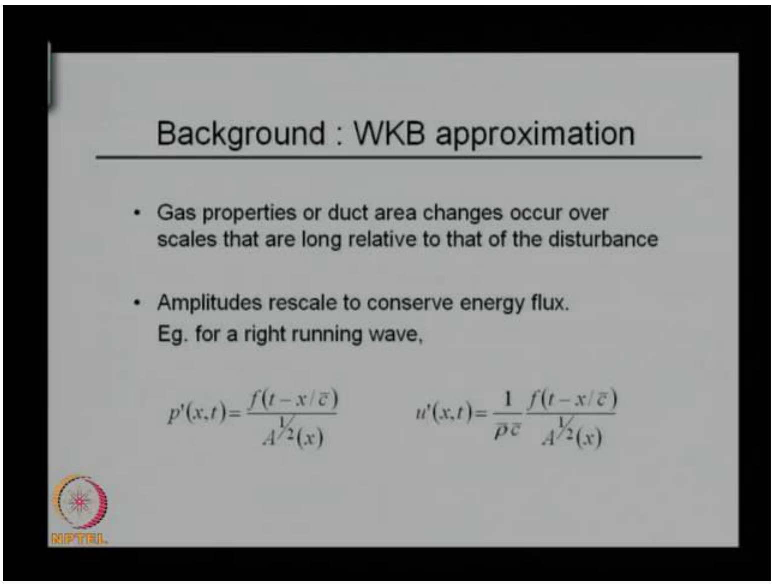

So, we can think of what is called the W K B approximation, it actually comes from quantum mechanics. I forgot the expansion of W K B, but it is also called W K B j approximation. So, we say that the gas properties are duct area changes occur over scales that are long relative to that of disturbance. So, let us that; that means, relative disturbance things are changing very, very slowly the scales are very long. So, then what happens is the amplitude would rescale to conserve energy flux, that is let us imagine we are having a right running wave which is f of t minus x over c. Now, if you think that the area is changing for example.

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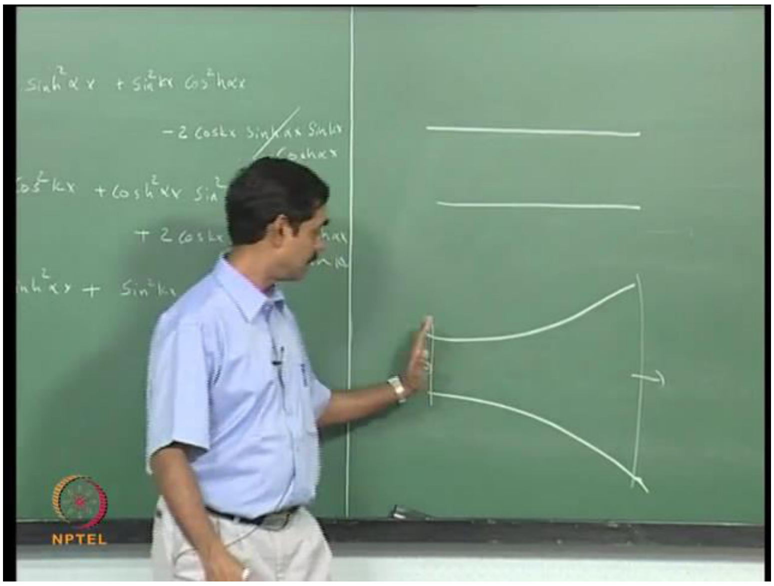

So, you can have one case where the duct is like this another case where you have like a halve or something the area is changing. So, we know that if something is coming in the same thing should go out. So, if energy is conserved and you do not produce any driving here whatever power that comes in what is power, power has a precise definition, acoustic power has a precise definition. You can look at the notes and see what it is p prime, u prime times area absolutely. So, that should be conserve right.

So, we have let us say p prime, u prime and here you have p prime, u prime, but the area is different. So, both p prime, and u prime has to change such that p prime, u prime a should be constant. I mean things would be differentiae some power was, energy was generated in to that or taken out or something, but that not happening whatever power is flowing here, should flow over through here.

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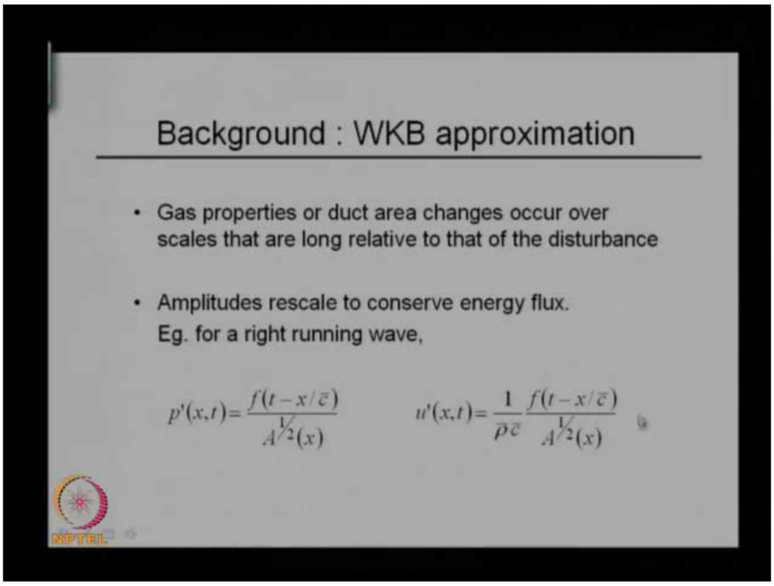

So, this would mean that you will have same kind of f of t minus x over c, but let us say we have a area term square root of A and here you have u prime of x is going normally, it goes like one over rho c times f, but let us put A power half. So, if you multiply this out you have area times this term, times this term. So, let me rewrite.

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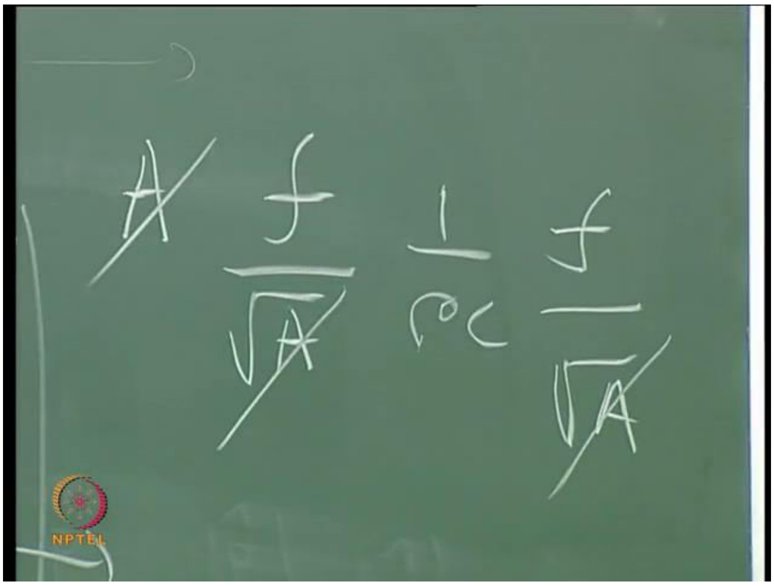

A times f over square root of A times 1 over rho c f over square root of A. So, this square root square root and a can cancel. So, you will get f square over rho c and you will get the power to be conserved.

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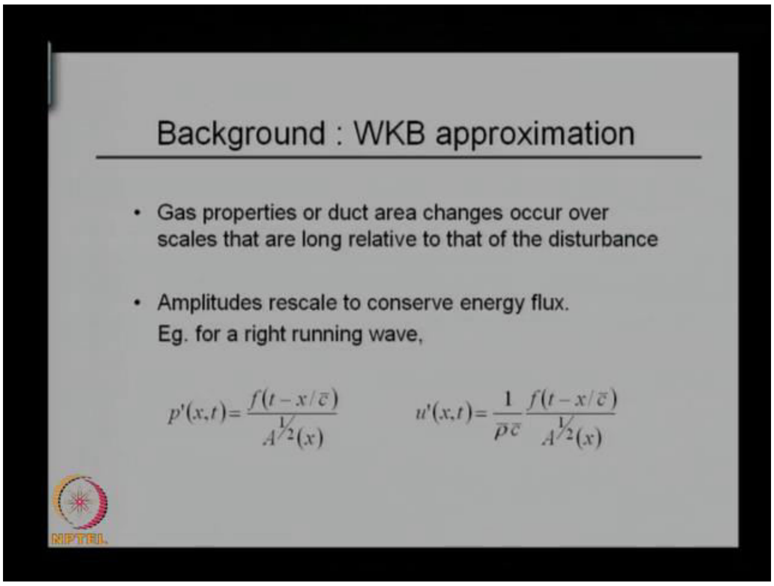

So, this is a hand varying explanation, but you can actually do it mathematically this W K B approximation and get this formula that is possible I am not attempting to do that, but this idea is to get you the physical principle. So, is this part clear, if this is not clear there is no point in going forward everybody is writing. So, is there everything was clear and they are summarizing or nothing is understood. So, whatever I said has to be written down. So, that you can figure out later. Understood. So, I will pass over 30 seconds for you to finish your summary.

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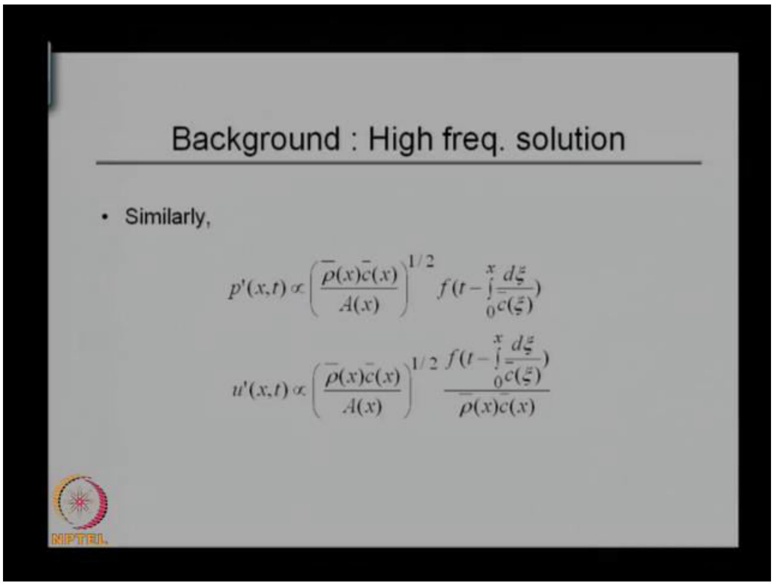

So, let us now, step up the complexity let us say we have I mean, I started with area change because, that this is it very trivial situation to imagine that we can write by hand and the answer.

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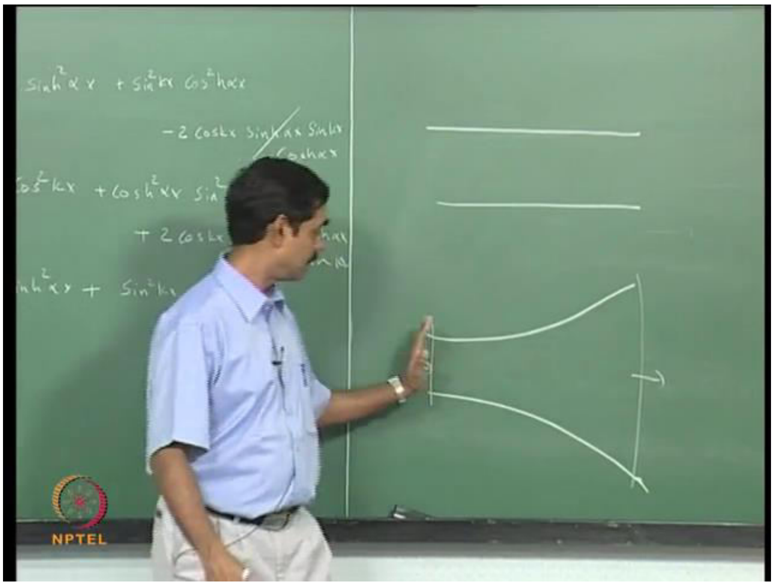

So, let us say where area is changing and density is changing and periods are to changing, density and periods change will, change with temperature. Now, usually when the mean mutt number is more, we must have studied from gas in the mixed at p plus gamma m square is a constant. So, as long as m is quite small, like I do not know what this small 0.45 over 0.1 or something, pressure would not change much. And again how small is small we will not get in to that, but let us for the time being assuming that pressure is constant, in the limit there is no flow pressure will surely be a constant right.

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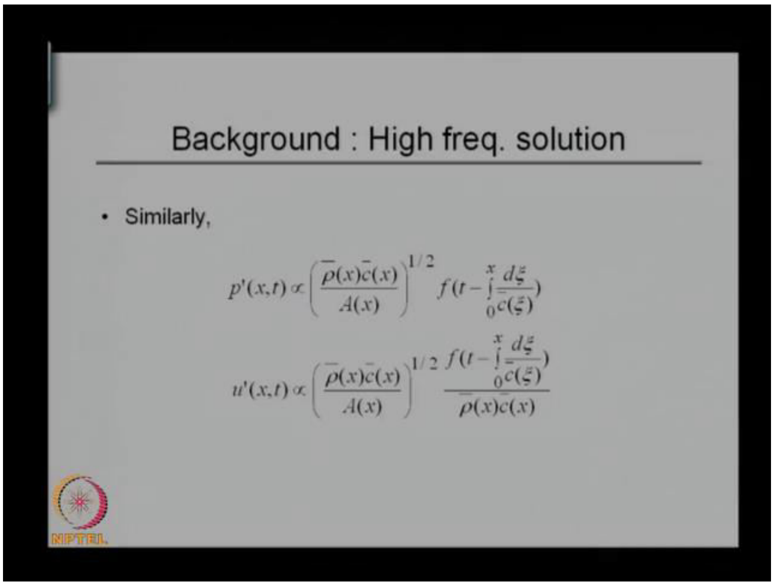

So, here in or your rho is changing c is changing. So, we have to cancel those also out. So, that your final intensity is not a function of x at all. So, if you for example, take p goes like make a conjecture, and like I said you can derive some of these vigor sly, but I would not do that right at the moment. So, let us p is rho bar x c power half and there is edge are here and u also has a rho c here power half over A.

Now, when you take p prime u prime times A, now the A will cancel with this one over root A and one over root A and a one over rho c will cancel with there is square root of rho c this is square root of rho c. So, what is remaining is these things of course, with the units being appropriately putting. So, you take out the dependence on x or temperature is that clear. So, this is the at the moment I conducted, but like I said high frequency assumption you can derive this with that assumption.

There is something is babu. Now, we will see how this depends on temperature in a moment, but I will just I think it is good to write this things rather than just see it as some slide show and all that because, once you write you get a feel of this things. Now, one more thing I wish to point out that normally in the previous slide we wrote t minus x over c. Now I am writing t minus some phi over c integrals from 0 to x. Now, why is that.

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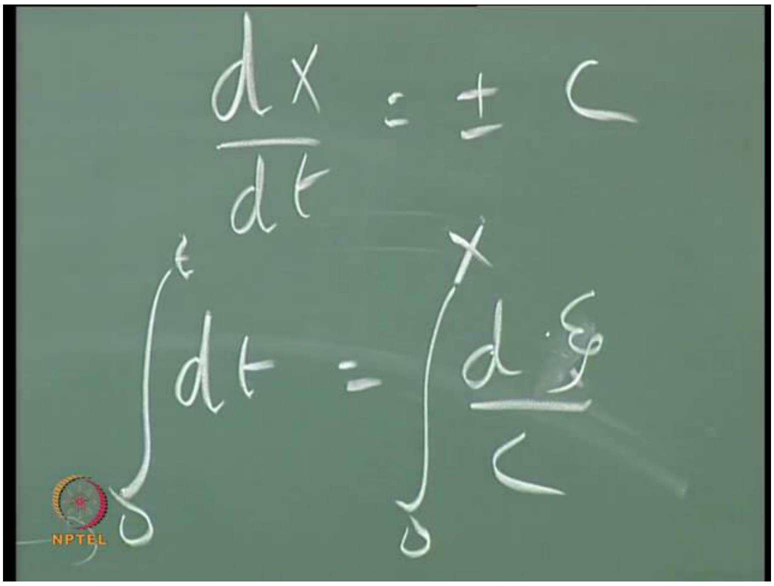

Because, all if you see my characteristic is d x over d t equal to plus or minus c. Now, c is a constant then x is equal to c t plus constant, but if c is not a constant then I have to my equation becomes d t equal to d x over c. So, I integrate this from let us say 0 to t and 0 to x and use this phi as a dummy variable and I note explicitly that speed of sound is varying with distance.

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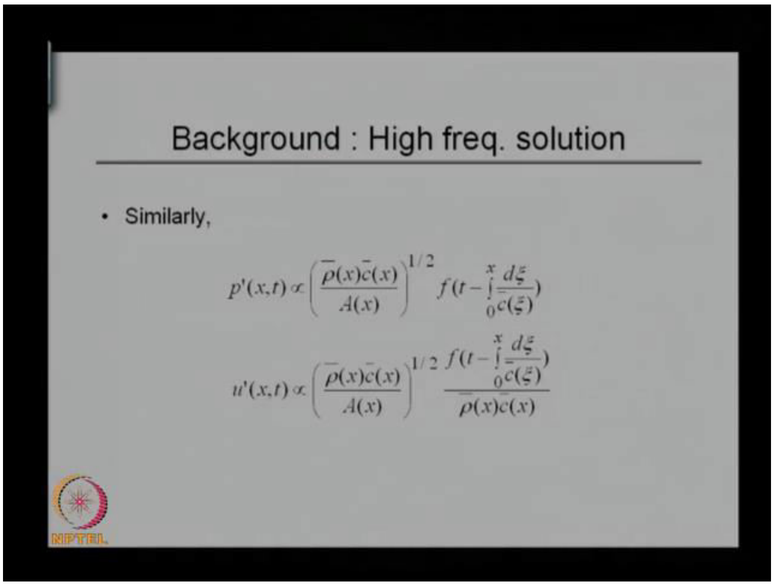

So, this explain this fancy looking argument in side, but there is nothing fancy about it is just that speed of sound is changing and, so x minus c t has to be appropriately modified to do that is this.

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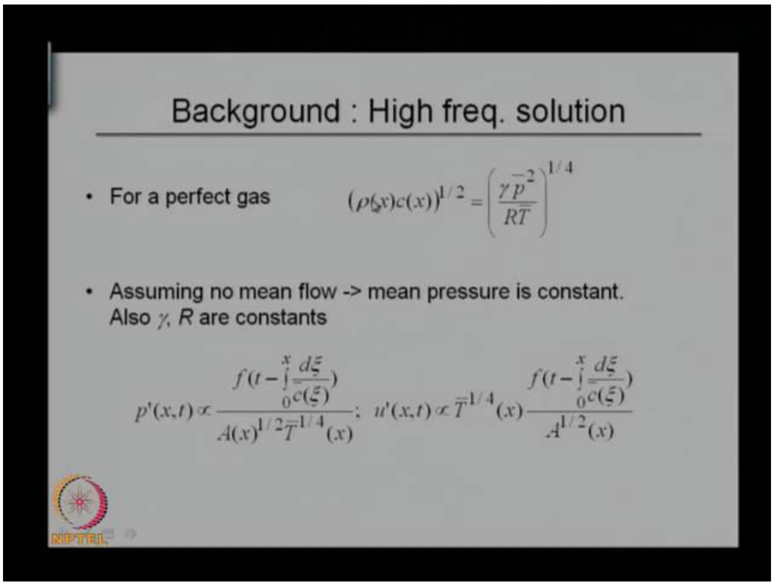

So, now we can say that rho c rho will be p over R T you can try to sorry you can try to write this one rho will be p over R T and c will be square root of gamma R T. So, if you multiply this out you would get this gamma p bar square power 1 4'th times R T for a perfect gas. So, let us say we assume that the there is no main flow r very low elastic mean flow because, you cant sustain a temperature getting really without having mean flow I mean for a long time.

So, we, but we say at mean flow can be neglected and therefore, I mean our objective is to save that the mean pressure is constant. And also gamma and are constant, now this is a questionable assumption for somebody like sorrow Groton all that you it is of proportion last semester where we hard quite a bit on how gamma can change and how r can change how the molecular way can change and sound, but in a gas rewind type of combustor where you bond quite lean as I think this is reasonable assumption because, if you are not having that much significant change in gamma or c p and c v and, so on but, in a rockets this is a bad assumption.

So, if you make these assumptions you can say p is f of t minus d x over c integral times where the square root of n and t power 1 over 4'th because, this rho c a term becomes this. And you goes like t power 1 over 4 on the numerator that is because, we have 1 over rho c which you plug by can in terms of temperature and you would get a t power 1 4'th here times this f of t minus integral d is i over c 0 to x times square root of A. Now, I will just pass a few moments. So, that you can whenever you see equation it is important to stop at the equation smile at the equation and then the equation will also smile by at to you. So, if you are not smile I think time to smile.

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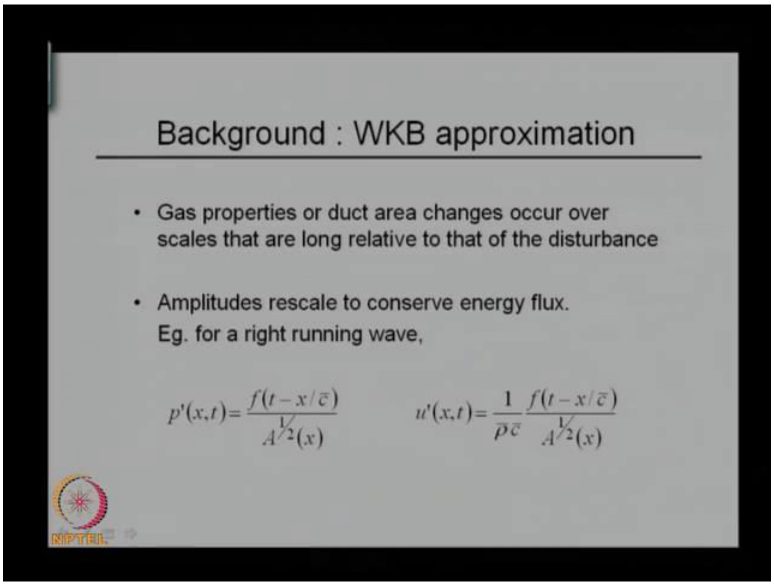

This screen is a indigent screen just things that I want to change with it change within and, so on will annoyed this. So, you can see that a earlier we had a root A here.

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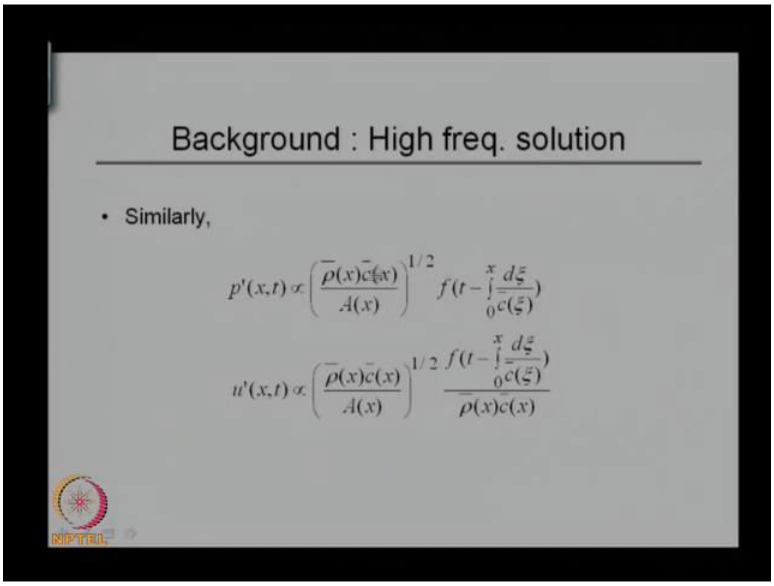

And then we said that we have to rescale with rho c over A term.

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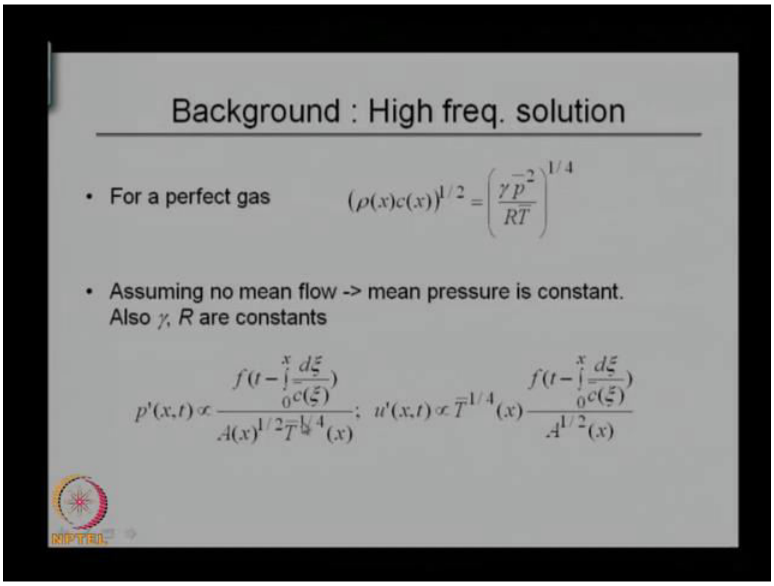

And, so it in the end we want to talk in terms of temperature because, we are burning something we are getting a temperature then you may have heat transfer after that. So, the for example, if you have a combusting non end there will be rapid temperature raise, but then a as the heat transfer happens the temperature will fall down also. Now, so you will have the amplitudes of rescaling such that there is a 1 over square root of A dependence at the denominator and there is a very interestingly there is a T power 1 4’th here and this T power 1 4’th a numerator here.

So, you would see that pressure amplitudes are coming down if you increase temperature, but actually velocity will go up if you increase temperature. And if you decrease temperature pressure will go up, but velocity will come down. So, pressure amplitude if you look at the standing wave and you see the maximas will go up and all that, do not conclude immediately that there is driving or if maximum or falling down conclude that or there is a attenuated wave and, so on.

It could be that there is a temperature gradient. And, so you have to look at both temperature and sorry you have look at both the acoustic pressure and acoustic velocity only then we can conclude what is happening. So, we should not jump in a haste about this. Now, this variation of how the propagation is being influenced by temperature, as lot of implication not just in term of acoustic and lot of other subjects.

For example, this ultra sound by imaging for people who study breast cancer or trying to image a unbound babies there you send sounds and we look at how the sound is being scattered or reflected by of course, it is a 3 D problem more complicated, how the impendence changes in our body or affecting that. And if you have like cancers tissue it will show up in a different way if you have a assistant over your something which is due to certain illness.

Then you would actually see it because, those things have a different impendence characteristic impendence compared to the medium around it. So, any time some new growth happens inside the body it would reflect the sound waves that are set in. So, that is the reason, why we are able to image you also use this for imaging the what is the people do study the things hidden cities and all that not all this ecologist. So, they try to find out. So, if that is a city under that.

So, what they will do is a away they will do a small explosion and they send the waves and try to image the waves back and look if some Mohan jadav or Harappa or something like this hidden under Neath and then they can try to image that with this imaging technology. So, you have lot more applications and there are even more applications while I mention, you must have heard about shock wave lithotripsy. So, the waves are affected by change in impendence also the area they there it should a cylindrical medium.

So, cylindrically you can converge as you impart the waves. So, the if you send an acoustic waves as they converse to the kidney stone you, actually the wave Stephen just like you have this square root of A. So, if A is coming. So, it is not delay variable as a duct, but its like a 2 pi r kind of conversion. So, all is coming down. So, you will have a square root of r kind of term there. So, your amplitude will go up and as amplitudes go up the wave will then go in to non-linear edge and make a shock and the shock will destroy the shock, will have high temperature and then it can break up the kidney stone.

So, this is really not a very specific thermo acoustic subject to anything although I am that is what I have studied. So, I will teach all those things, if you were blue glasses were not it appeared blue. So, if you appear if you should had thermo acoustic glasses everything in the world appears as a thermo acoustics, but I had friends. So, did this study of shock waves and I had a friend who use to study how to do this shock wave lithotripsy on very fat people.

So, because, they cannot fit in the machine and then how to do that and then, so on it. Then you will have to put them in a water tub which will have water will have same kind of impendence as our human body and, so on. So, I have some idea about these things I also have a doctor friend used to scan tigers. So, tigers also have the same problem as human being. So, she was specialized in doing out of some scanner tiger, but tiger has to be made unconscious first.

So, I have learnt quite about this things from non aerospace kind of people. So, what I am teaching is in general must more general than a what is just for the combusted and, so on. So, now we go from the equation. So, far I did hand waving. So, now, we are going to go with the equations. So, I think you had a homework to derive these equations I trust everybody has derived I see some naughty smiles which means many of you have not derived or that coming in the exams do not care.

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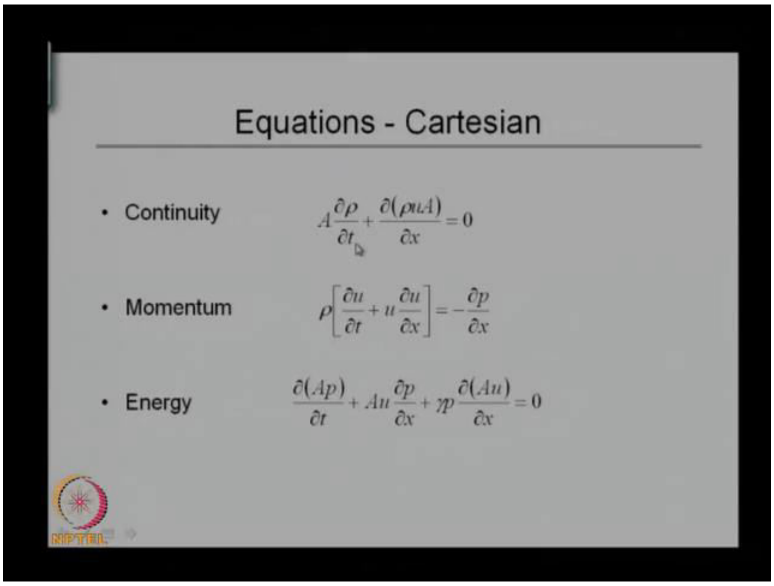

So, at least you know first 1 is continuity you heard this, what is continuity equation what is continuity equation can you give the equation manoj what is continuity equation just close your eyes and say, what is it should I jump out of the window or go out and jump out mani what is continuity equation density time, area time.

Student: velocity. It is a constant.

Yes anyone coming to jump out with me. So, rho A v if it is constant you will not hear me speak. So, the question was what is conservative mass and the answer was rho v was a constant and I am trying to propose that if rho A v if god made rho A v constant you would not hear me speak anything. What do you say why, ganesh what do you think I am right or wrong big smile means you are you can be either with mani or with me or what you know with both of as she can be right or she can be wrong, what is continuity equation you yourself. So, what your name.

Student: Kadiravan.

Kadiravan what is continuity equation is rho A v constant is it continuity equation is it momentum equation or is that what is continuity equation. So, you see the some other term in front. So, we cannot through this away this now, here is it mention that this term is I mean if you are having a compressive medium and if you through with this term and then this really no sound. Similarly, if you ask how to write momentum equation again people would probably through away the first time, but without that there is no there is no sound I mean.

So, for sound is unsteady phenomena and we have to keep track of unsteady terms. So, all that god said this when he created the world he said if mass is coming, in and mass is going out whatever is coming in minus whatever is going out equal to whatever is remaining there, he did not say nothing should remain there did god say that in any religion did not he said if something is coming in and something is going out balance will stay inside that is all he said.

So, why is a rho A v constant. In fact, I asked you mani the same question sometimes mani will give same answer. So, this is what is called conditioning because, this elephants they are chained when they are small babies and when the elephant becomes big they can peacefully break the chain, but it thinks or it cannot because, it is always having the chain it has to have the chain. So, rho A v has been always constant because, we probably do not know how to deal with the unsteady times, so therefore, we keep it.

So, I think it is very important I think I do not know you have to probably print the slide and keep it in your room or something with dou rho by dou rho t written in red letter and with some yellow paint I round it to something and same as that to get un do the conditioning I am serious. So, we have these equations. So, I hope we will because, the if we by instinct come out b y rho A v is constant then there is nothing to do actually after that. Because, the sound is I mean the classical wave equation 1 over c square dou square p by dou t squared equal to dou squared p by dou x square if there is no dou squared p by dou d squared then there is no, there is nothing left actually.

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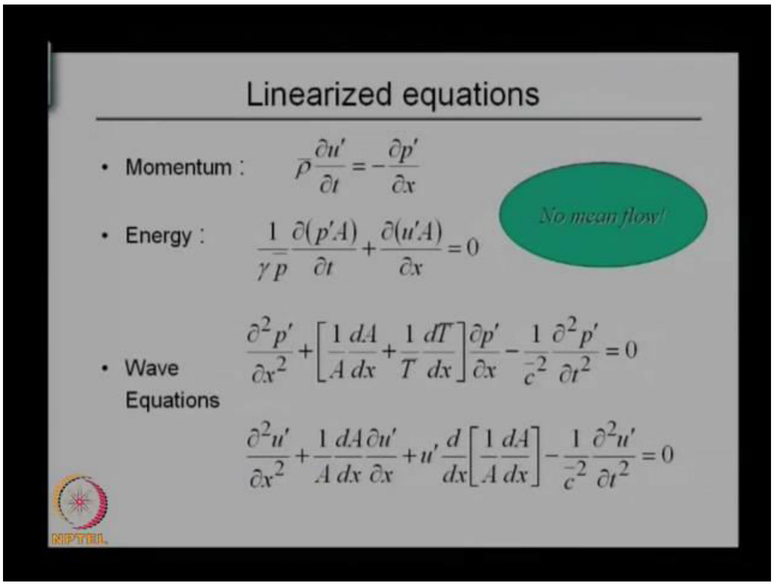

So, we then have to line arise the equations, which is what we studied we in the second class or third class. So, if you line arise the equation and for a change I am working with momentum and energy equation and I explain that for the classical case it does not matter because, continuity equation would boiled down to same as a energy equation, but here it wont look exactly alike, but you can chose to work with which ever one you want I choose to work with energy because, it is convenient as you would see.

So, the first one is the linerised momentum equation the second one is energy equation. So, if you differentiate, if you do the same algebra that we did. So, if you eliminate the term d square dou over dou t squared sorry d square d dou over dou x dou t from these two terms you will get a wave equation in terms of pressure. And if you eliminate pressure.

So, it is you have this if you differentiate it such that you have a term d square p over d x d t and here also you differentiate this with x. So, that you have term d squared p or d squared d t if you eliminate this term you would get a wave equation in terms of velocity. So, this is the wave equation for a in homogenous media I mean as in homogenous as in temperature is changing or density is changing and, so on. Now, and I have we have used p bar is constant.

And you can actually, write d t over d x in terms of d rho over d x also some people write in that form, but the first thing to notice is this was given as a homework I think sometime back right this equation is different from this equation. The equation for pressure appears different from the equation from velocity. So, whereas, earlier in the classical case we had a operator dou squared p all in write that down.

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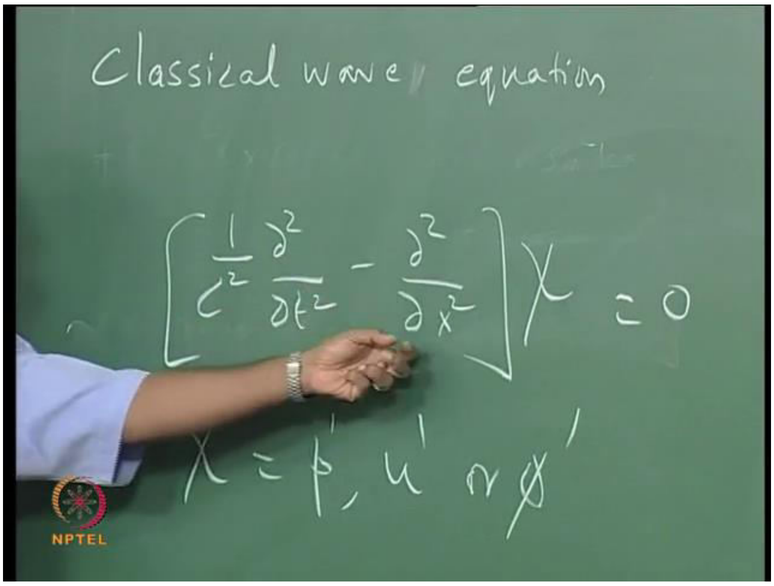

So, you would have a operator that is a wave operator 1 over c square dou square by dou t squared minus dou squared by dou x squared times some variable kie is 0 and kie can be unsteady pressure I mean acoustic pressure acoustic velocity or potential.

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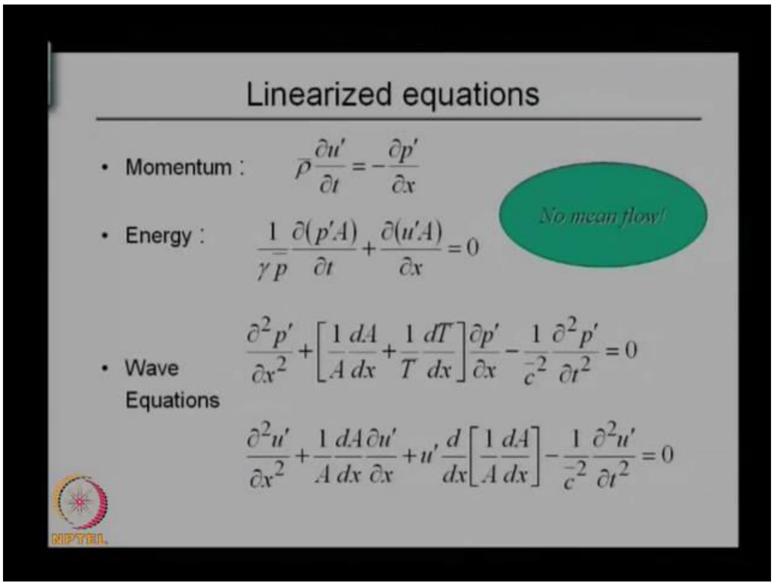

But, now you see that you cannot use the same operator and which I have here, and you see it on the velocity because, they are different is a clear. I think it is a important point you should make a note this. Of course I just remember that I assume no mean flow and which lead to pressure constant and that is without it the equation gets much more mastum.

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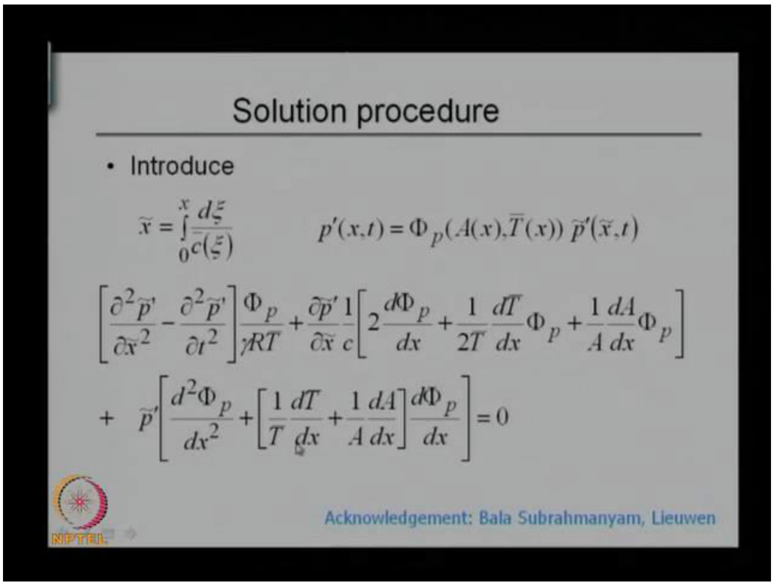

Now, we have to have a solution procedure. So, what are this is like a introduce procedure. So, we say we know that d x over d t equal to the characteristic.

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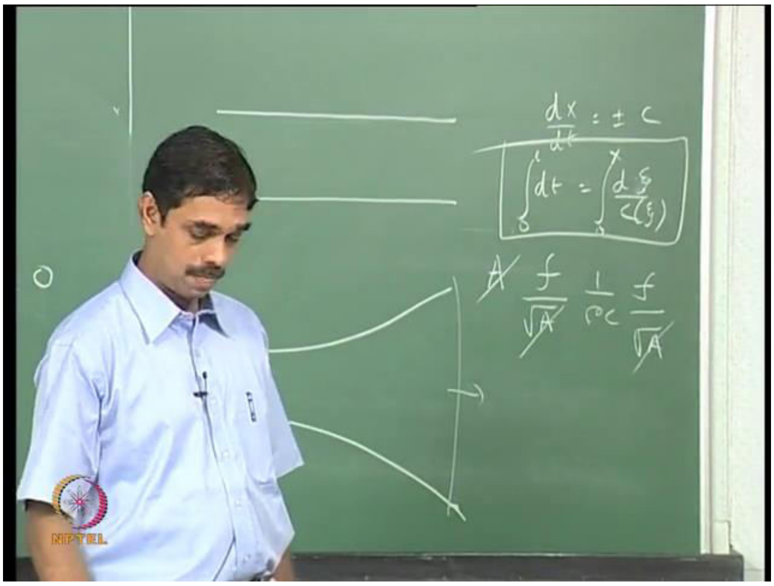

So, we can have this transformation and lets say.

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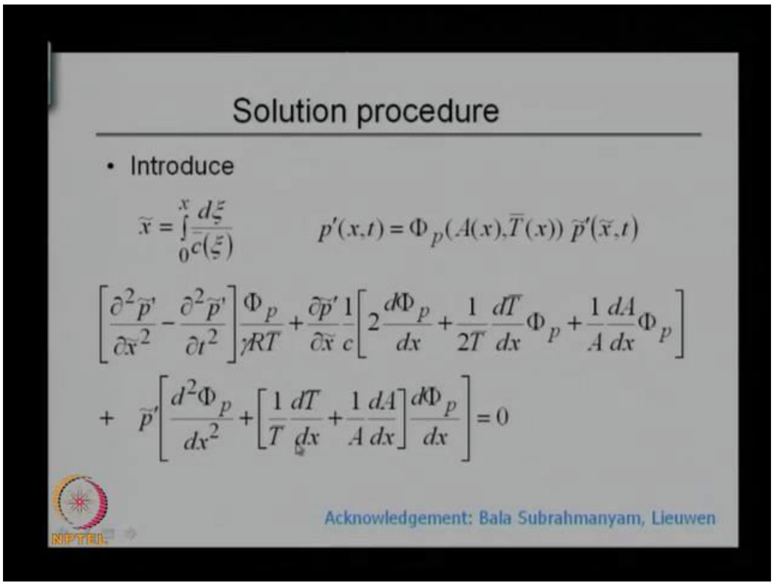

We just hope that pressure is some kind of function which is depend on area and temperature times some pressure which does not depend on area and temperature, some kind of scaled pressure. So, we are hoping for a rescaling to occur, so that we get a simpler equation. So, then if you substitute theses two in to the earlier equations, the wave equation what you get is this term d square this is the classical wave operator acting on this rescale pressure time something plus there is another term here which is operating at d p prime over d x plus there is another term over here which is operating on p itself.

And this looks. So, complicated that there is I mean I do not see any possibilities of solving it. So, how to solve it we will do in the next class. So, what we did is to derive a wave equation for media which has non uniform temperature and both the pressure and velocity. Now, then we are attempting to solve for pressure. So, we are hoping that we can rescale the pressure such that we can get the dependence on are and temperature out and then we can perhaps solve it in terms of the classical wave equation or some such thing. And, but it is not coming out, so easily, but is there a way to solve this that is what we will examine let us stop here.

Thank you.