Acoustic Instabilities in Aerospace Propulsion
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Lecture - 10
Admittance, Stability and Attenuation

So, what we did last class was to take a look at the impedance tube technique and we have the solution and we constructed the (( )) expression for the amplitude and phase for travelling ways. And then we took a look at how the solutions, solution here is the amplitude and the phase distributions. How the solutions were for various values of admittance, and we looked at how things were change for real part of admittance, and we also looked at that how things would change for the imaginary part of admittance.

Now, the next thing to I mean, if I were a mechanical engineer. I will be very satisfied with this, because I know how much power is absorbed or how much power is given out, and I would live it here, but working in some more constructs, we have to worry about stability. So, that is the big issue here.

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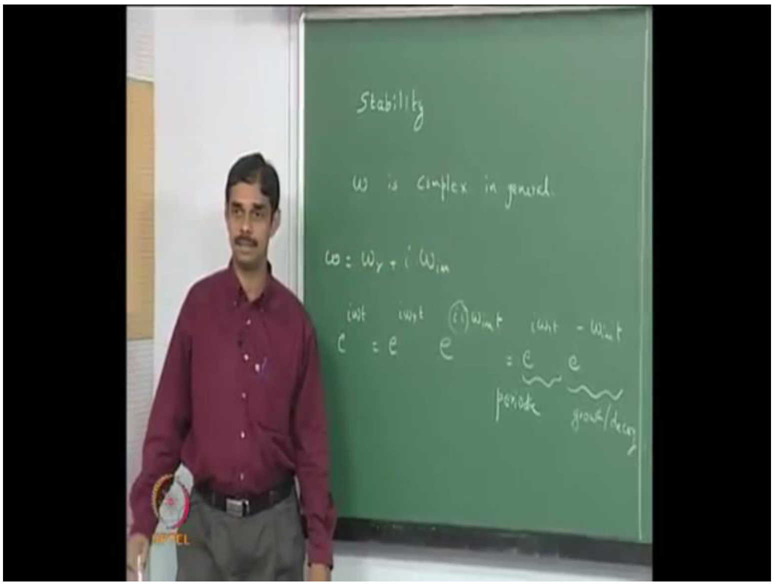

So, our job is to predict stability from this information we obtain. So I think what I will do is to give a simple example where we can try to connect the links between the admittance and stability. I will just give a small example, but it is not like this representative example or something. But you can try to see what happens and from there we can move into more complicated problems in the examination or something. So, just to emphasize we said that omega natural I mean the frequency or we call it also Eigen value. Eigen value means the unique values for which there are solutions. So, this is complex in general.

So, let me ask somebody. Deepak. You are deepak right?

So, what does the real part of omega mean? What does the imaginary part of omega mean or should we use both or should we use only the real part?

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For what?

Student: ((Refer Time.02:25))

And what does the imaginary mean? Manoj what? ((Refer Time: 02.32))

The omega imaginary indicates the growth or decay of the oscillation and omega real indicates the periodic path. And I have spoken about it already. But, let me just emphasize it again because this is perhaps the most important thing in this class. So, omega equal to omega real plus i times omega imaginary. So, e times i omega t equal to e times i omega real t and e times i times i times omega imaginary t, i times i is minus 1. So, we can rewrite this as e power i omega real t times e power minus omega imaginary t. So, this is the periodic path. This is the growth or decay.

And we saw from acoustic analogy that, if energy is coming in boundaries. Acoustic oscillations, the net energy of the system acoustic energy whatever we call the acoustic energy can grow or it goes out of the system, it can decay. Now, we need look at how you can work this out in other and waving sense can we get an expression for this growth rate or decay. So, that is the next thing that we are going to look at.

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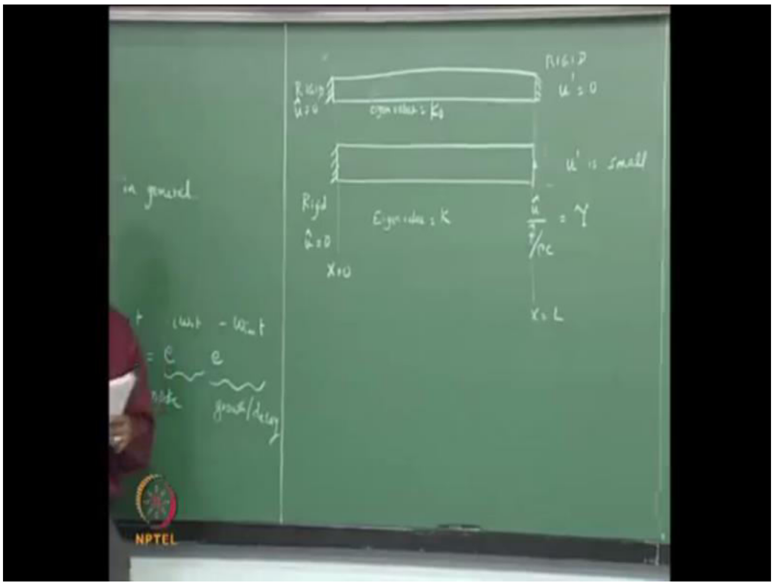

We looked at the problem of a simple pipe which is closed at both ends. And did you get any growth or decay for this system, we did not. So, we got real Eigen values. What does Eigen values this system n lambda by 2. So, you had to fit multiples of lambda by 2 in the pipes. So, the frequencies were going like n c over 2 l, which is a pure linear number. So, that means if you could manage to setup oscillations in a pipe, somehow mysteriously we could do it by connecting loudspeaker, keep it on and then at some point you turn the loudspeaker off, even though the oscillations will continue to stay there forever. In reality, it does not exist. I mean you can try to do those experiments there will always be losses.

So, let us think of a system where there are small losses and small losses help me use do the algebra easily, in reality the losses can be big as well. So, this is a problem and I have a companion problem which I have perfectly closed here. Somehow magically I have a perfectly rigid huge block being symmetric here or something and I have a pipe of same length. But, let us say I put a hole or something. So, it is almost closed but, there is some small admittance. So, here we have velocity exactly 0.

Here I will have u prime is small, some amount of sound is quote and quote leaked out or something. So, we will not worry about what happens when the sound is leaked out to the outside. So, we will just restrict to what is inside in terms of the admittance and then we will try to link that with the growth rate. That means the problem is, in this first problem when I have perfectly closed conditions, if I have some oscillation there. Let us say if I have oscillations with amplitude 10 pascals or something and if I do not turn on the speaker, I just leave the oscillation 10 pascal to continue to stay 10 pascal forever. But, if some amount is leaking will the 10 pascals come down? It would die down slowly.

What is the rate at which it dies down? So, from this framework it seems like we can fit a exponential decay to it and then how would you relate that to whatever is leaking out? On the contrary, if you had a flame, let us say we have a small flame and the flame amplifies the incoming disturbance. Then you can have a growth. And let us say the flame is a small disturbance amplifying it little bit, then can you find a expression for the growth rate in this case in terms of the admittance of the flame so that is the question. I hope the question is clear to everyone.

So, we need a solution and we have a solution for pressure, which is some a e per i K x plus b per minus i K x and we know by now, how to get the expression for acoustic velocity which is or a particle velocity which is by using the linearized momentum equation or momentum equation for small ((Refer time: 07.40)). So, you get the velocity and then apply the appropriate boundary conditions. Yes. ((Refer time: 07.45))

That is if you are talking about a small particle and if you are being able to add energy as in by heat energy or something to the particle. But, we are not speaking about adding energies there. We are speaking about leaking in acoustic wave. So, in terms of a intensity flux coming in at the boundary. So, that is the way we are thinking about adding energy. We are not really speaking about adding energy in the microscopic sense which yes for example, if you are adding heat you would have you would change this adiabatic condition and then by definition the heat release rate or the heat addition rate of the particle has to appear in the picture.

In fact the amount of acoustic energy added would be the co-relation between pressure and heat released we will show later. But, at the moment so that is like a volumetric dry way, so that is heat released taking place in a volume and how much is the acoustic energy created in this volume wherever the heat released happens. Flame may be thin but, whatever in that flame zone. But, here we are speaking about some wave being pushed in from infiltrated in. I think that is some word we had in newspaper.

So, from outside the sound is created. So, we are not worried about creating sound itself. Somehow there is something here and you send in here could be sound from outside, or it could be loudspeaker coming in or the other problem. The way we look at it is we setup a standing wave sound already exists we are looking at how it leaks out so this question we are side stepping for the moment. But, that is a central question to the course. So, when we finish all these things we will come back to how the heat added to a particle changes the acoustic energy and everything else. Any other question?

So, the boundary condition here would be u hat over t hat over o c equal to y and put the script y, which would mean this is non dimensional admittance and it is very convenient work in terms of non dimensional admittance as you would see that is the research we do. And here it is perfectly rigid that means u hat equal to 0 and in this companion problem again rigid and rigid. So, here also u hat is equal to 0 and my co-ordinate system is such that this equal to x equal to 0 and I would call this x equal to L.

So, we are solving for the Eigen value and let us say Eigen value of this problem is K and Eigen value here is K naught. And you can already see the conspiracy, we will try to express K in terms of K naught, then some simplification will happen. So, we say that K is K naught plus K prime which is why I spoke, I starred with a rigid plate and then said this is small departure from that and then the algebra becomes simple. So if you have a expression it is easy to explain this and then you can move to solving a more complex problem numerically and so on. So, I hope the problem is setup clearly. It is a well defined problem.

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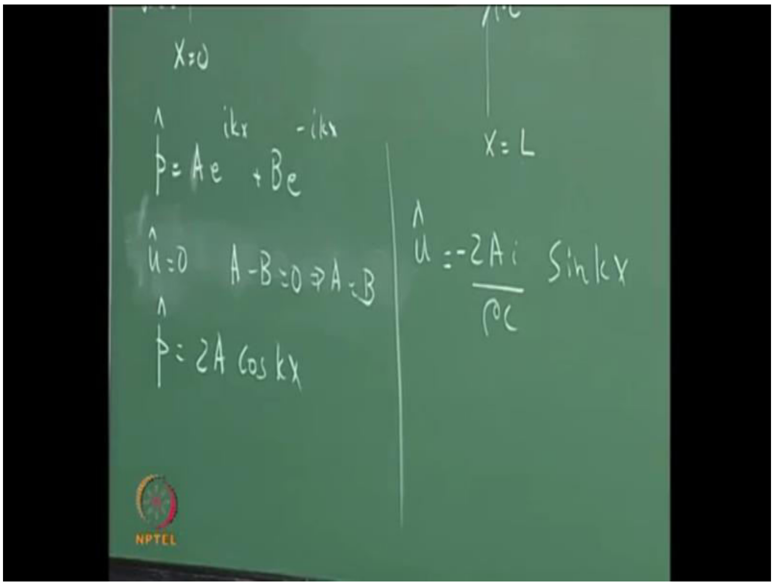

So, the pressure p hat equal to it is a complex amplitude A power i K x plus B power minus i K x. And if you have a rigid end at x equal to 0 what happens, A will be equal to B. So, u hat is 0 and u hat would be like A minus B. So, you should get p hat equal to 2 A cos K x. You can write this as e power i K x. Take the A out so e power i K x plus e power minus i K x multiply and divide it by 2. And then you call the exponential term as cos K x and there is a 2 outside.

And u hat, can you derive an expression to this and just copy and I will write the answer, just see if you get it. So, if you take d p by d x i equal to i omega u hat and from there we can get this. Can you just check if the any sign mistakes or something? This should be a minus. Are you getting the same answer?

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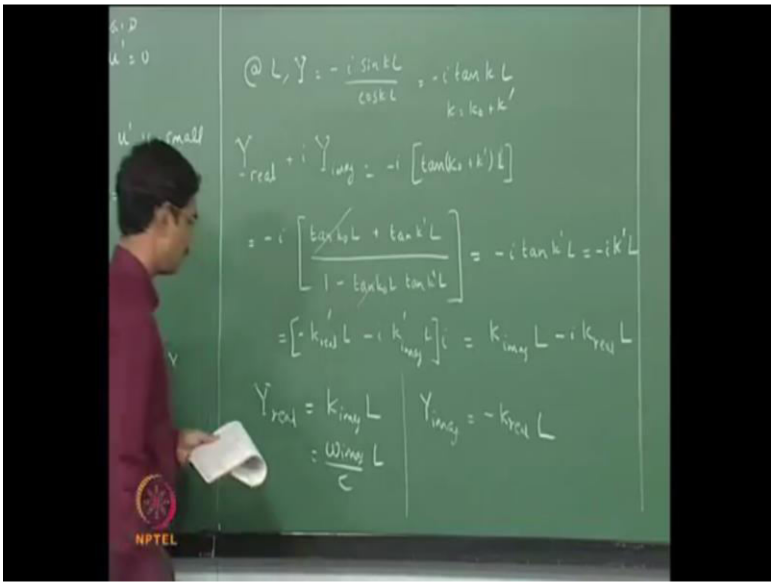

At L Y, that is admittance equal to minus i sin K L divided by cos K L equal to minus i tan K L. So, why the non dimensional admittance consists of the real and imaginary path? And we will try to separate out the real and imaginary path. And I will say this K equal to K naught plus K prime, where K naught is the Eigen value of p corresponding closed duct K naught is the Eigen value of this closed duct. So, this will be tan K naught plus K prime and L.

This could be expanded out as minus i times tan K naught L plus tan K prime L divided by. And at this point I have not made any assumptions about what sort of quantity is K prime? So, in general it is a complex quantity and only assumption I am making is that K prime is very small. But, magnitude of K prime is very small compared to K naught so that, I can do the tail expansion kind of thing.

So, both K real and K imaginary would be small compared to the value of K naught. So, that is the only assumption that I am making. But, otherwise it is in general complex. So, now you can see why I have made this assumption, that all the separation that K is K naught plus K prime. So, you can tan K naught L will be 0, because velocity is 0 at this end and therefore, you can write this as minus i tan K prime L which I can further simplify which if i expand out would be minus K real prime times L minus i K prime imaginary times L times i. So, this if I multiply, it would be K imaginary L. This would be minus i times K real times L. We will pause for a moment to let you verify if this is indeed correct because otherwise the answer would be wrong.

Correct. Fantastic!

So, this is y real plus y imaginary. So, we can actually write separate formulas for y real and y imaginary. So, we can write this as 2 real equations. This complex equation y real in terms of K imaginary and y imaginary in terms of K real, so, we wanted to first take a look at the growth or attenuation. So, for that we need to look at the K imaginary. So, K imaginary is nothing but, omega imaginary over C times L.

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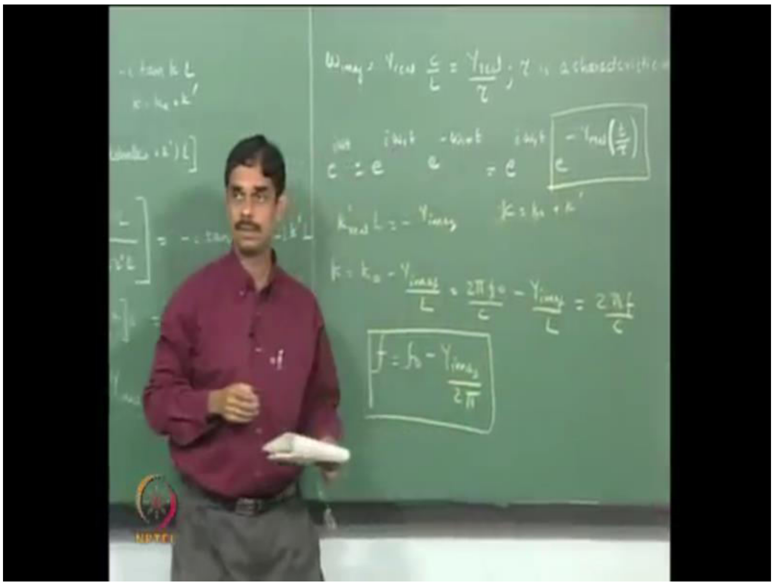

So, you can get omega imaginary would be equal to Y real multiplied by C over L which we can think of as L over C is like a characteristic time. What is L over C? L over C is a time required for sound wave to traverse through the length of the pipe. So, that is the characteristic time tau equal to tau, it is a natural time scale in our problem because we have length L and the sound wave is going. So, L over C is a time taken for a sound wave to traverse.

So, if you look at our term e power i omega t. We have we have written here that e power i omega t is equal to e power i omega real t times e power minus omega imaginary t. So, we are exponential term growth or decay comes straight out in terms of y real. So, if y real is a positive quantity that would mean that we remember from last class that yes what does it means, energy wave for left to right. So, energy is going from here out. So, you should have a decay and that is what you are getting and if y real is negative energy is coming in this direction and therefore, you will have it should be something like a flame which traverse or a loudspeaker whatever so, you should be having growth if you have the opposite sign. So, is this part cleared this point.

So, the other question that is still to be done which is what does y imaginary mean? That is a question I think you guys asked earlier. So, you can see the answer by writing it out. So, this equation K prime real times L equal to minus Y imaginary. So, K equal to K naught minus, this K prime is K minus K naught. So, I can bring the K naught to the other side. K prime equal to K equal to K naught plus K prime plus what I did.

So, if you know K prime. You can bring K naught to the other side, except that we are only dealing with the real part here. This would be equal to 2 pi f naught over C minus Y imaginary over L this and K would be 2 pi f over C. So, if you multiply out we will get f equal to f naught minus Y imaginary over 2 pi ((Refer Time: 22.53)) tan theta is small, goes like theta. So, I am assuming that K prime L is small quantity ((Refer Time: 23:06)) No, you can have a expansion of sin theta. What is the expansion of theta minus theta q? Therefore, theta plus that goes for example, complex thing ((Refer Time: 23:30)) the magnitude of theta will be small, that is all there is some convergence criteria.

I think it should converge, it will converge unconditionally. It actually works for a small and there is no restrained on real or complex. The question here is that, whether this expansion for tan theta in terms of theta would work for complex numbers? And the answer is: It will, now back to this. So, if we said that equal to f naught minus Y imaginary over 2 pi. So, Y imaginary is positive, that means that should come down. That means you can actually pack more waves into it. And if Y is negative, actually increases that means, fit less number of waves or something like that. I mean it is not like many more waves but, it stretches that way. So, I hope this answers the question and it stretches this way or that way depending on where exactly the reflection happens.

So, this is a concern in terms of what is the actual frequency change but, more importantly if you think about the growth rate. I think this term, it tells the whole story. So, we can relate the impedance or the admittance of the boundary to what is the standing wave, the growing in time or decaying in time. So, this is very straight forward. Any questions, I will write a general result.

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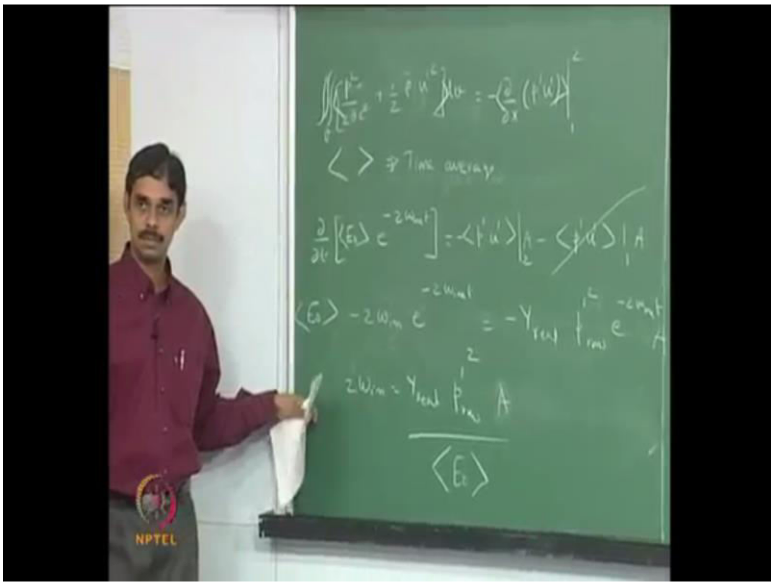

So, we apply it to this particular problem. Let us call this as end 1 and this is end 2. So, this is the acoustic energy corollary and we will time average this so that, the triangle bracket would mean equals time average. So, if we do this, let us we can write it off the form e power minus 2. This 0 because this at 1 it is a perfectly rigid condition. So, if you differentiate this you will get times e naught. So, we can write this as 2 omega imaginary equal to Y real times P prime r omega square. I forgot this area. So, this times A divided by E naught. So, we may find this kind of expressions in some books or papers.

So, with this exercise I try to tell you the reason, why we are interested in measuring admittance of flame and so on. Because one thing it will, the direction would be determined by the sign of the real part of admittance and also it translates to growth rate or decay rate. Any questions? If there are no questions, I wish to move to a next topic which is acoustic attenuation.

We wrote our solution in terms of f and g and we said acoustic pressure is f of x plus g t and acoustic velocity is f of x minus c t. There is right running wave and left running wave g of f x plus g t. And similarly, we wrote expressions for velocity as 1 over rho c times f of x minus c t minus g of x plus c t and by definition this wave does not change shape and this is not able to take care of any attenuation of the wave and so on. But, in reality attenuation does happen and sometimes it deliberately happens and one so, in combustion stability class, let us speak about, how, what would be the practical means to have attenuation in a combustor?

So, let us think about most of you are aerospace students. So, we have a solid docked motor and it does have sort of motors are very prone to instability and we have to have some end of counter measure or some kind of safety against instabilities occurring. And I guess some of you who did propulsion class would know, what does the easiest thing to do to get rid of instability, ((Refer Time: 30:12))

Add aluminum: Aluminum is added anyway to sort propellant, to increase the performance because metals give high specific impulse and so, aluminum also burns and forms alumina, which is initially molten and as goes through the nozzle it may start solidifying. Those alumina particles have a certain size and they have a certain drag because the particles when they are in a sound field the particle will try to move along with the sound field. Sound field would drag the particle and in the process the sound will lose some energy, because when it is trying to drag the particle that means it is giving some of it is energy to the particle or the droplet and it is giving energy.

So, it is actually losing and the particle is gaining energy and if there are lots of particles. If it is like one particle in the sound wave, it probably does not matter. You can think of it as one by coupling. But, if there are very large numbers of particles, the sound will start the sound wave or acoustic field wave will start losing some amount of energy so, it is like now a two way transfer. So, this would live to acoustic attenuation and that is what the solid propellant motor designers try to make use of are some kind of insurance or protection against combustion and stability and you must have seen attenuation in some form or the other.

For example, if it is raining and you shout, I mean the rain drops will attenuate the sounds and you will feel that you really have to shout loud or if you are attending a open air concert and the rain drops are coming, you would suddenly feel a reduction in the volume. Of course, the two effects you get simultaneously, only the sound does not end itself and the rain drops take away, it will dam the oscillations.

The other example of damping I can think of is, if any of you are musicians here and when the music band practices, when they setup the instruments and in the concert hall and they will set the amplitudes at or the amplifier at certain level and everything would be quite loud. But, then they wait for the. So, they will setup everything, tune the instruments and make sure everything is ok and then when people come in and then, they start playing and the sound level would not be enough at all. It is almost like the volume went away somewhere.

So, the people and their clothes are damping. So, they take away the energy. So, actually I have to jack up the amplifier once the people come in. So, there are a lot of places where you see damping. You see in concert halls, the special panels like we have here in this studio. You this panels they are meant to damp because this holes they actually absorb the sound and take away some energy from the incident sound field so, there are a lot of damping.

So, we will not look at a general case but, we will look at attenuation happening to one dimensional wave standing waves and we will see how this can be studied? And a simple way to think of it is, if you have a tube filled with pebbles or something or packed bed, some of you with chemical engineering background would know, what is a packed bed? A packed bed is nothing but, a column in which lot of packing's.

Packing's are like material, like pebbles or something but, they have the good packing's of high fraction that means they have lot of free space and they are made to improve the contact between let us say some two fluids or something like that. So, let us say you have you can think of a tube filled with foam or packing something like that and that would be a good example for a only attenuated travelling wave or you can other example is to think of a droplet laid and gas inside a tube.

Attenuation will depend on the medium and its properties and sometimes the flow that is setup and even if there is nothing inside, there will be attenuation because the molecules have vibration and relaxation and so on and they take away the energy. So, for example, I mean if you have a simple tube even in that the air itself will absorb and in fact you would notice that on humid days there will be more attenuation compared to days which are dry. So, we will not speak or go in detail what causes attenuation but, we will describe the attenuated wave. I think, I have restricted my objective just to describe the attenuated wave and describe how the amplitude variation is? How the phase variation is?

But, then the attenuation itself is depended on the physical phenomena. For example, if it is a droplet layden gas, like in the case solid docked motor, then it is related to the droplet's properties, the size of the droplets. For example, certainly affects the drag, and the frequency will come into play because the time scales involved, they are now strong hold number, reinhold's number involved. Those numbers will come into play. In fact when you use or when you try to design a solid dotted motor, ideally we would like to adjust the aluminum size, such that the alumina droplet size would be the correct size needed to damp the oscillations. For every frequency you would need a certain particular droplet size distribution to damp the oscillations.

So, ideally a good designer, I do not know whether they work with the equation or they go by field but, they will actually treat that this is one parameter which is available for leaking and that is why whatever works for a rocket of a certain size may not work for rocket of other. For example, if you have a rocket of certain size, let us say 10 meter and everything works fine and the same rocket with same propellant and same diameter it just extend into double. And then suddenly it may develop, there have been cases where suddenly gives instability and that is because the droplet size was just fine to kill the develop frequency. Now the frequency has come to be a lower value and now this aluminum size that you have which results in certain alumina size, droplet size that is not tuned to attenuate frequency in this big motor or vice versa.

So, yes this is a important thing and I will not attempt to model that aspect but, I will talk about a description of the standing wave in presence of alumina or in the presence of particles or drag. So, if this is the attenuation, how will the wave look like? What will be its structure? Now, doing that a connecting that attenuation to your specific system depends on the specific systems properties, I hope it is clear. I gave a very long answer for a short question. But, let us say a tricky point we can discuss in private. I have worked out some such problems in the past, from a research.

Now, I forgot to mention another case where attenuation is very important is when you are talking about propagation and narrow tubes or narrow ducts. For example, in a term acoustic engines, the term acoustic engine is device where you use temperature gradient across a thing called stack which is a sheet of plates and use that to dry sound the engine. So, in that the stack has smaller plates which are narrowly spaced so, there attenuation is quite important. So, in general when you have narrow ducts attenuation gets very important. So, we will study this in idealized geometry.

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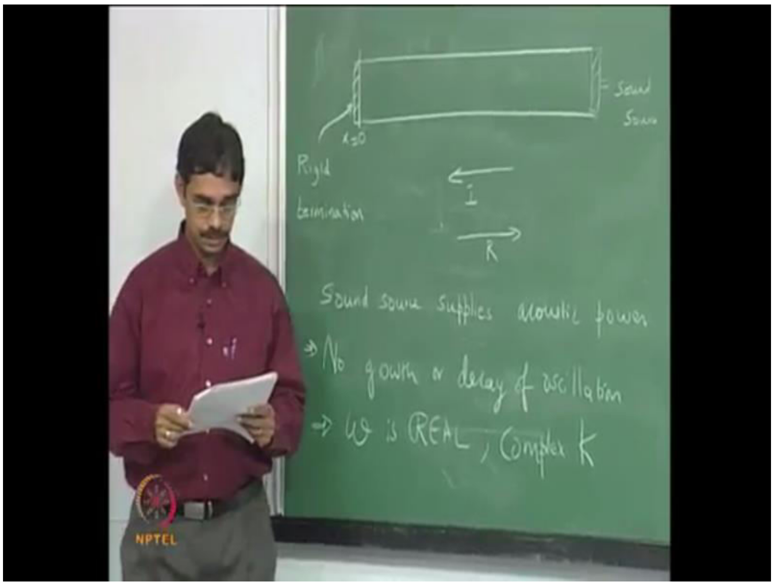

And the good description of this is given in the textbook by Kinslaven Fray. We have I, R. I is the incident wave and R is the reflected wave. And let us say we have some kind of sound source here. It can be a piston moving back and forth or a loudspeaker something like that. So, let us for the present assume that, we have perfect termination or a perfectly rigid termination that means the amplitude incident wave is equal to amplitude of the reflected wave.

In the earlier problem, we saw a minus b equal to 0, a equal to b that kind of situation. So, this sound source supplies the acoustic power which will be consumed in the acoustic appropriate. So, we will look at such a case. So, we will not look at a complex omega, we will setup a real omega that means there is a sound source and we have crossed the transience and we have reached in some kind of steady standing wave, stationary standing wave. There will be no growth or decay of oscillation that means omega can be chosen to be real. But, we look at the wave number which can be complex that means the wave in the standing wave, you will not have same maxims for all the maxims that can be growing or decaying and that will count in terms of wave number.

Let me write these things now. But, we will have a complex K, we will have a complex wave number. So, we will write this complex number as something real plus something imaginary.

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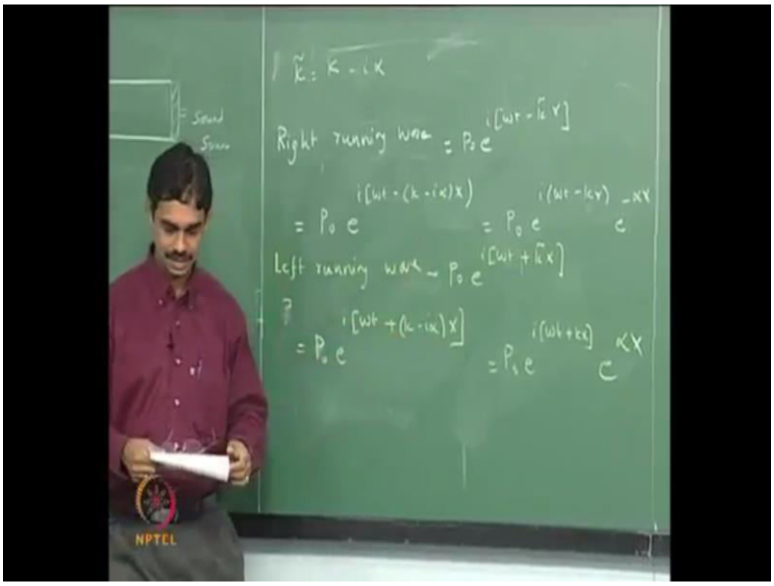

So, let us call the wave number is K till tau, which is K minus i alpha and it will be clear why it is convenient to choose such a form. Now we will think of how to write the expression for the standing wave. So, we will write it first for right running wave and then for left running wave. So, that must be of the form P naught e power i times omega t minus K theta x.

Now, we substitute this expansion for K theta, this would be P naught e power i times omega t minus, this would be P naught e power times minus terms minus this, plus i times i is minus 1. So, this would be e power minus alpha x. So, this is a right running wave. So, when a wave is running to the right, it should decay to the right. Left side should have more amplitude then right side and that is so, we will assume alpha as positive and then this description looks good for a right running wave.

Now, we will have to look at a left running wave in the same manner. This is another color. So, it will be, it should go like P naught e power i times omega t plus K theta x here. So, this would be p naught e power i times omega t plus K minus i alpha times x. So, this would be P naught e power i times omega t plus K x times e power i times i is minus 1 e power alpha x. So, a left running wave is what is described here. So, it will appear as if it is growing but, if you think about a wave from right to left it is actually decaying. So, this decays to the left, this decays to the right. So, it is a little tricky. So, the total pressure can be written as sum of these 2 things.

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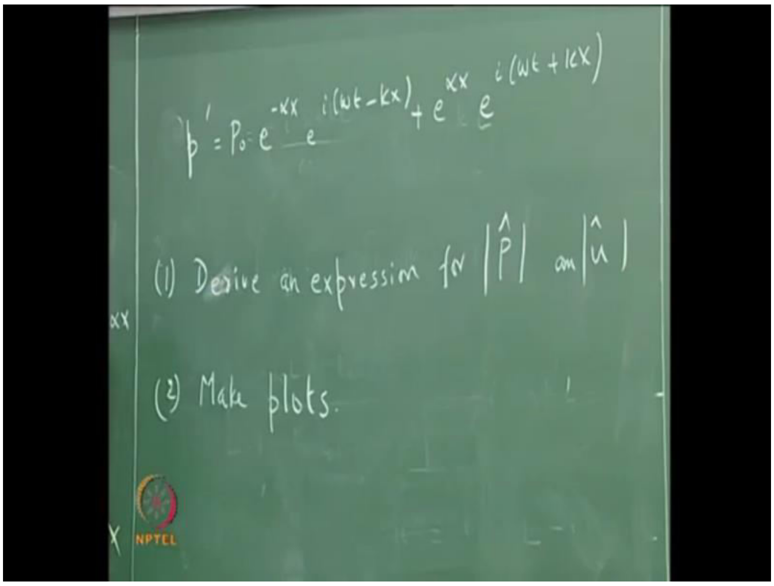

This is the way total pressure. So, I will give you some homework at this point, because I do not think that you will have time to complete it. We can look at it in the next class. So, first thing would be, derive a expression for the pressure amplitude distribution. So, and u hat and second thing would be make plots and I will show you some sample plots now.

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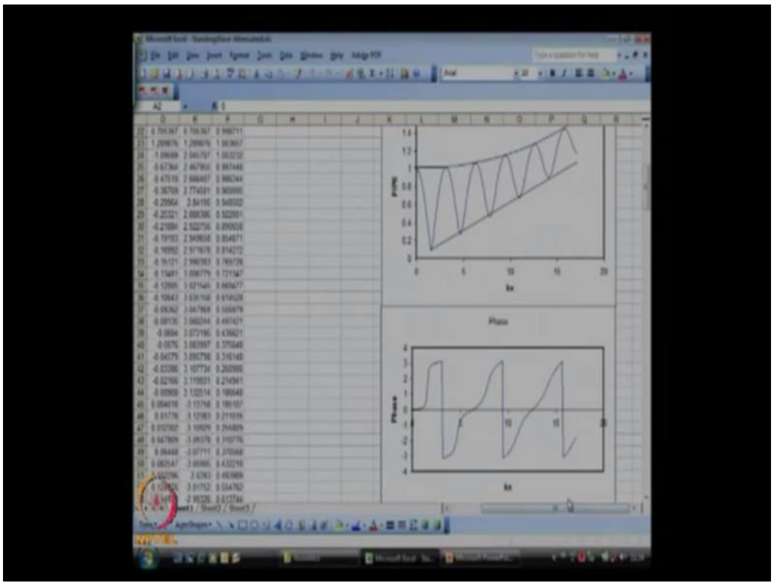

So, over here is a, would be a plot of pressure amplitude versus distance and when you write K x, it is non dimensional because K is wave number and this would be the plot or phase. So, please make or try to calculate this. I have done it using excel which is quite simple but, you can write your program in fortran or C or math lab or anything of that sort and try to plot. And this is how the acoustic velocity would look like and again I have normalized the velocity and pressure with their values, that x equal to 0.

And last thing, try see if this interpretation of phase agrees with what you think because if you have a sound source here and energy is coming to it and left running wave is decaying and so, eventually it will come here and then what is going here will also start decaying as you come here. So, near the sound source here, it will almost be like a travelling wave because you had a wave, a left running wave in here and then it reached here and it lost lot of its amplitude and then it starts coming as a right running wave and by again continuing get attenuated. So, here it will be predominantly a left running wave, if it is damped well and as you come more to it says you have both of them coming and then the phase should be such that we have looked at the phase. So, the particles here should vibrate first before the particles here moving because the energy flow is in this direction.

So, we had some intuitive of understanding how the phase relates to the direction of intensity flow and see if this is consistent with that and please do this at home then we can discuss this topic tomorrow.