Course Name: Combustion of Solid Fuels and Propellants
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Lecture: 05 Performance Parameters (Example problems)

Hello everyone welcome back. So, I will continue with this lecture like we want to resume it from where we left with. So, in this lecture we will try to solve some example problems just to understand what we have learnt the performance parameters, how to relate with these some of the numbers and then we will talk about the description of solid propellant rocket, what are the different components and some basics of solid rocket. So, let us try to solve one example problem. Some values are given for this numerical example problem. So, this example problem is related to our performance parameters.

So, this is one rocket in which the value of thrust is given as 6800 kilo Newton. Probably this example has been taken from professor Ramamurthy's book, you may refer to it later on. The chamber pressure is given as 6.65 mega Pascal.

$$F = 6800\,KN\,,\ Pc = 6.65\,Mpa$$

The mass flow rate of the propellant is given as 2600 kg per second. The throat area is given as 0.65 meter square of course, 0.65 meter square. Molecular weight of the product is given as 22 gram per mole or kg per kilo mole.

$$\dot{m}p = 2600\,\frac{kg}{s},\ At = 0.65\,m^2\,,\ MW = 22\,\frac{g}{mole}$$

The chamber temperature is given as 3300 Kelvin. So, you can understand the chamber temperature for rocket is significantly higher compared to the gas turbine engine because gas turbine engine temperature chamber temperature can be limited because of the turbine blades. Because of the turbine blade materials, the chamber temperature the turbine inlet temperature is limited because the turbine blades cannot withstand very high temperature. Whereas, in rocket there are no rotating devices. So, the chamber temperature can be significantly higher.

$$Tc = 3300K\ ,\gamma = 1.22$$

So, you can see it is like 3300 Kelvin. The value of gamma is given as 1.22. Now, it is asking us to find out the value like determine eta C star and I sp. So, we just learnt today that what is eta C star? Eta C star is nothing, but the C star efficiency which relates the actual performance and the ideal performance or as per the definition we know that eta C star is the it relating the ideal C star velocity by the actual C star velocity.

Determine $\eta c *,\ Isp$ Now, our job is to find out the ideal C star from the you know using the ideal parameters like the value of gamma, some of the temperature data, the r value if we relate those probably we can find out the value of C star and the C star ideal and the C star actual can be calculated from the actual value of you know mass flow rate, the throat area, the chamber pressure. So, let us try to do that C star ideal. We write down the equation first which was like R T c by capital gamma. If we recall we had the equation for C star capital gamma. If you recall we had the equation like relating to small gamma which was like small gamma into 2 by gamma plus 2 by small gamma plus 1 to the power gamma plus 1 by 2 into gamma minus 1 this was the equation for capital gamma.

$$\eta c *= \frac{c^{*}ideal}{c^{*}actual}$$

$$c^{*}ideal = \frac{\sqrt{R}Tc}{\Gamma}$$

So, if you place the value of 1.22 for small gamma you can find out the value of capital gamma which is around 0.652. Now, R I think you can find out from the universal gas constant. So, if we do that we can find out the value of R by considering R u by molecular weight.

$$\Gamma = \sqrt{\gamma}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$= 0.652$$

$$R= Ru/MW$$

So, one can find out this put the value of that T c should be 3300 Kelvin. So, if you put the value here let us put the R u by molecular weight here. So, it is like 8314 by 22 then temperature is 3300 Kelvin, gamma we have just calculated capital gamma 0.652. So, one can calculate the actual C star which is going to be 1713 meter per second oh sorry this is not actual I am sorry this is C star ideal because we are using the ideal values here.

$$c^{*}ideal = \frac{\sqrt{\frac{8314}{22}}\times 3300}{0.652}$$

$$= 1713 m/s$$

So, it is ideal C star which is 1713 meter per second and what about our C star actual C star actual can be found out from the actual you know pressure data and mass flow rate and throat area here it is given already. So, we can get this C star actual is going to be equal to something is wrong here. So, I have to put this value back again probably it has been. So, 8314 by 22 into 3300 divided by 0.652.

So, that gives us the value of 1713 meter per second this is my ideal C star if you do the actual C star actual C star will be like we can put the value of PC 80 by m dot PC was given already which was 6.65 into 10 to the power 6 that much Pascal 80 I think it was given 80 was given as 0.65

meter square divided by m dot we have to be consistent with the unit. So, accordingly we can write the values here if you calculate this will come as 1663 meter per second. So, now we can calculate the eta C star as per the definition C star ideal by C star actual.

$$c^{*actual} = \frac{PcAt}{m^{\cdot}} = \frac{6.65\times10^{6}\times0.65}{2600} = 1663\ \frac{m}{s}$$

So, this will become like 1663 by 1713 and we can multiply with 100 to make it percentage. So, you will get 97 percent. So, remember we already told our self that the C star efficiency values generally varies from 92 to even as high as 99.5 percent. So, you can see here also it is coming very close to the high values like 97 percent.

$$\eta c^{*} = \frac{c^{*}ideal}{c^{*}actual} = \frac{1663}{1713}\times 100\% = 97\%$$

So, it is quite high. So, the aim of the designer is to make it in such a way that the actual performance should be closer to the ideal performance which was calculated from the you know considering the ideal rocket assumptions. So, here it is following that it is quite high value 97 percent. Now the second question was asked is the ISP. So, what about the ISP? ISP as per the definition we know ISP is the specific impulse which is the like total impulse by the propellant mass or the you know we can write this as thrust by mass flow rate.

So, we can put the value of thrust here 6800 kilo Newton. So, we can make it Newton and then we can put the mass flow rate which is 2600. So, that will give us the ISP value in terms of you know 2710 Newton second per kg. If you figure out the unit here it will become the unit of meter per second. So, your ISP is eventually is going to be equal to your m dot v j.

$$Isp = \frac{It}{mp} = \frac{F}{m^{\cdot}} = \frac{6800\times10^{3}}{2600} = 2710\ \ \frac{N.s}{kg}$$

$$Isp = m^{\cdot}\,Vj$$

So, that was the first example problem. Let us take another example problem very simple one we will try to do this. In a given problem following data are given. So, let us take another one that C star was given as 1220 meter per second m dot was given as 73 kg per second C f now we are introducing C f here.

$$c* = 1220\ \frac{m}{s},\ m^{\cdot} = 73\ \frac{kg}{s}\ ,\ Cf = 1.50$$

So, C f is given as 1.50, A t is given as 0.0248 meter square. So, it is asking us to find out the value of you know chamber pressure, jet velocity and the specific impulse. So, it is asking us that what is the value of chamber pressure, what is the value of thrust, what is the value of jet velocity, what is the value of specific impulse. So, these are the questions we have to answer.

$$Pc\ =?,\ F =\ ?\ ,\ Vj =?,\ Isp =?$$

If you recall that in our performance parameters discussions we have said that we have to remember the you know combine form of the equations how we are relating the one performance parameter with another then it will be very very easier for us. Because we know the thrust equation was related to mass flow rate into the jet velocity for a optimum expansion case we can simply write f equal to m dot into v j where p equal to p a. The chamber pressure is related to our C star as C star equal to p c A t by m dot. So, there we are relating the chamber pressure the throat area and the mass flow rate. The I s p I s p is nothing, but this is going to be equal to directly v j if we want to write in terms of the unit second for I s p then we can simply divide that by g.

So, I will just show you how to do this. So, this is like a using the formula putting the values you will get the answer. So, if you just simply write the equation for C star you will get p c A t by m dot. So, what is asked it is asked that find out the value of p c. So, you can directly get the value of p c from this equation.

So, it will become C star into m dot divided by A t. So, what are the values given C star is given as 1220 into m dot divided by A t. You know I am doing this example problem in class just to make sure that later on once the assignments are given it will be very very easier for you to understand what to do and how to do. So, that is why I am just doing one one two example problems just to be in the same side what we have already discussed theoretically. So, if you do a calculation you can find out the value of p c as 3.591into 10 to the power 6 Pascal. So, which is like 3.591 mega Pascal. So, that is our first answer which was asked p c what was the value of p c.

$$c^* = \frac{PcAt}{m^\cdot} \rightrightarrows Pc = \frac{c^* m^\cdot}{At} = \frac{1220 \times 73}{0.0248} = 3.591 \times 10^6 Pa = 3.591 Mpa \qquad \text{....Ans(1)}$$

So, we did it. So, this is answer 1. Next question is what is the value of thrust? So, we know the thrust equation already we need to know the thrust equation as C f into p c into A t. Now, we have already found out the value of p c here. So, we can simply put the values here C f is already given you see 1.5 into p c we just got 3.591 into 10 to the power 6 A t is given as 0.0248. So, it is actually a small rocket. So, 0248 that much of Newton. Make sure you use the consistent unit then you will get the correct results.

$$F = CfPcAt = 1.5 \times 3.591 \times 10^6 \times 0.0248\, N$$

In case if you do any mistakes in the calculations you should question yourself for example, like the V j value if you get like couple of meter per second then of course, you know that you are doing something wrong. The jet velocity cannot be like 20 meter per second or 30 meter per second since it is a rocket. So, we are expecting the jet velocity order of some 1000 like 1400, 1300, 1600 meter per second at least. For a larger rocket you may expect the jet velocity can be as high as like 2000, 3000 meter per second. So, we should always you know check the numbers what we get out of the calculation.

So, numbers are not just numbers we have to relate with the physical you know implication all right. So, if you calculate this you will get the thrust as 133590 Newton ok. So, 133 kilo Newton 133.6 kilo Newton you can say. So, that is our answer for question number 2.

$$F = 133590\, N$$

$$F \approx 133.6K\,N \quad .... \quad \text{Ans} \quad (2)$$

Then we were asked the value of V j. So, what about the V j? V j we can simply write from the thrust equation other form of thrust equation because we know thrust was f equal to m dot into V j. So, V j is f by m dot you put the value here 133590 by m dot which is 73 you do the calculation you will get meter per second. You see the value of you know V j should be in the range of that high. So, if you get the values in the lower side we should always check that am I doing anything wrong here. So, that one can easily check it from the answer ok.

$$Vj = \frac{133590}{73} = 1830\ \frac{m}{s}$$

Here of course, you should write that we are considering the optimum expansion which is P e equal "$pe = pa$ "to P a we should always write that one as an assumption ok. What about the specific impulse? Specific impulse is simply the you can write directly as V j, but the unit will be in meter per second, but if you wish to write this in terms of in second. So, we can write I s p equal to f by m dot into g 0 you know remember that g 0 is acceleration due to gravity, but once it is outer space out of the gravitational field this is this g 0 is merely an arbitrary constant the value of which is like 9.81 meter per second square.

So, we can use that constant. So, we will put the value of thrust 133590 divided by m dot was given as 73 into 9.81. So, we will get the value of I s p as 186.5 second ok. So, this is our so, this was our answer 3 and now this is the answer 4 ok.

$$Isp = Vj\left(\frac{m}{s}\right) \rightrightarrows Isp = \frac{F}{m\cdot \times go} = \frac{133590}{73\times 9.81} = 186.5 \ldots\ldots Ans\ (4)$$

So, with that I think you can easily you know do some example problems yourself and even the assignments are given to you can calculate the values very easily if you understand the performance parameters correctly and how to relate one parameter with another parameter we can do that we have to remember the optimum expansion if it is not an optimum expansion if the P e value is different than P a then of course, you have to use the value of. So, here I should I should better write C f 0 because this is the case of optimum expansion, but if it is not an optimum expansion then you have to use the formula for C f considering the P e and P a as well ok. You remember we had written the long equation for C f which is the actual equation considering that P e is not equal to P a, but when the P e is equal to P a for the case of optimum expansion we can use the equation C f 0. In this example problem C f 0 value was given, but if it is not given then we have to recall the formula and we have to calculate our self ok and then we have to do it all right. So, I think if you do it correctly if you are consistent with the units then I think there would not be any problem you should be able to do it very easily only thing after doing the calculation you must check yourself are those numbers are physically significant are those numbers are making sense considering the actual performance of the rocket.

So, that way you can actually realize that whether you are doing any wrong in the calculations ok. So, I think with that we close the discussion on the performance parameters. Now, let us move on to another topic which is the major part of our course like remember our course is on combustion of solid fuels and propellants. So, we are eventually moving towards the discussion on the solid propellant rocket in this course we will not talk about any of the liquid propellants or any of the any parts of the liquid propellant here just liquid propellant rocket here just because our discussions is restricted to the combustion of solid fuels and propellants. But in order to understand the combustion process we also need to know the part of the I mean the basics of solid propellant rocket how what is the solid propellant rocket is all about and then what are the various propellants used in solid rocket then only we can really understand the combustion process.

So, just to set the you know background we are doing all these things to be in the same side that we actually know the basics of rocket propulsion. Now, we will move towards the basic discussion on the solid rocket and then we will talk about the various things related to solid propellant rocket. So, if you look at this figure it has been taken from again Sutton's book which is another reference book we are using here. So, the major you know components of the solid propellant rocket or the rocket motor another name is like rocket motor. So, this is the predominantly used for you know launching satellite vehicle satellite launch vehicle the booster stage which is predominantly the solid rocket because it can generate very high thrust it may be sometime augmented with the cluster of rocket remember we talked about the staging of rocket and clustering of rocket.

So, staging of rocket is done by like serially you know firing one after another in order to get the high incremental velocity instead of at once we get the incremental velocity step by step like every step every stage is going to give some incremental velocity and combinedly it is going to give higher incremental velocity. But in summary if you say that the booster stage or the first stage is predominantly the solid rocket it has the high density propellant compact placed in the smaller volume because the high density propellant can be stored in a smaller volume, but it is a huge amount of propellant you can say because the take off thrust requirement is very very high. One can actually easily see some numbers from the different satellite launch vehicles data set and they can find out these requirement of the take off thrust is significantly higher. So, if you look at the solid rocket it is containing a solid block of I mean propellant which is called like propellant grain. So, if you look at this picture there is a figure in grey colour and it is termed as the propellant grain you know this is a rocket terminology.

So, the propellant is consisting of both fuel and the oxidizers and they are in the solid form. So, the name itself suggests that it is a solid propellant rocket. So, you know this is a solid block of material. So, it is similar to what we have already you know drawn earlier. So, this is consisting of some kind of a you know solid block of propellant with some perforation you see it is written here that it is having cylindrical perforation.

So, it is consisting of a cylindrical hole. So, if I just draw a half of it you will see it will something like this it will having some it is going to have some this type of perforation. So, this is typically the inside of it you know this is like it is a cylinder. So, once the burning initiate once you ignite it if you look at there are various parts mentioned here like the solid propellant grain has to be contained or has to be placed in the motor casing. So, if you see the outer part is the motor case. So, here it is written here written here it is a motor case body.

So, that is containing the solid propellant it will have you know some forward skirt it will have some up skirt it will have some insulation because it is subjected to very high temperature and of course, very high pressure as well. So, the material should withstand that you know high temperature high pressure it needs to have some kind of a liner just to handle the very high temperature. So, insulating material is required the nozzle is actually connected to the end part of the you know rocket. So, you can see this is the nozzle here is the nozzle throat the nozzle exit is also given here. So, once the combustion takes place, but of course, the combustion has to be initiated by the igniter.

So, the igniter will play an important role by igniting the initial layer of the propellant. So, if you look at the propellant surface the if I draw the 2D view of it let us say this is the rocket. So, if you look at the 2D view of it. So, igniter is going to give us give us some kind of a plume you know.

So, this is let us say igniter. So, igniter is going to create some kind of a hot plume to the initial surface of the propellant. So, this is our propellant grain if you look at this is our propellant grain. So, the igniter is giving the initial heat required to initiate the ignition process. So, very first layer of the propellant will be heated up by this igniter. So, igniter is nothing, but this is another you know small squib of propellant.

If it is a small rocket then this igniter small igniter propellant can be sufficient to generate enough heat to you know ignite the propellant surface, but if it is a huge rocket. So, the small amount of igniter material may not be sufficient in order to initiate the ignition initiate and sustain the ignition. So, there a small rocket itself is used as the igniter for the big rocket. So, there are two type of igniters are generally used one is called pyrotechnic igniter, one is pyrotechnic igniter another is called pyrogen igniter. In case of pyrotechnic igniter it is a simple like some squib of igniter material which is you know ignited by initially you know igniter has to be ignited also.

So, it is squib of small amount of propellant again it is a igniter propellant we can say it is a different type of propellants of course, it may carry the similar type of oxidizer and fuel with some readily ignitable materials some metal powders like boron, potassium nitrate are used as a igniter propellant. So, that will be initiated by some you know you have to supply some voltage that will initiate the some small ignition coil may be there just like it you know you can say like nichrome wire which is going to heat up which will ignite the ignite ignition material that will create some kind of a plume which will initiate the heating of the propellant surface and that will you know decompose that will pyrolyze and the flame is going to form. But for a bigger rocket this plume may not be sufficient to initiate the ignition. So, there a small rocket itself will be used as the igniter. So, if it is a huge rocket a small rocket itself can be used to ignite the propellant you know this plume should come from a small rocket.

I will show you better picture of this one. So, this is like the pyrogen igniter this is the pyrotechnic igniter. So, this is the igniter you can see igniter is an important part there. So, once the ignition initiated the gas will start forming. So, the initial combustion will start forming, but that will not be sufficient to you know hold the combustion because it needs to increase the pressure also. So, the pressure will start you know building up slowly it is going to fill the you know poured volume the chamber will fill with the product gases the pressure will increase and then it will start flowing through the you know expanding through the nozzle.

So, once the combustion is sustained the flame will be established and the product gas will keep on going outside of the nozzle by expanding it. So, this is the major you know process of operation of the solid propellant rocket the various parts are involved there are many things involved there. So, what we will try to do here is we will try to give some brief description of these solid propellant rocket in terms of its functionality in terms of some of the you know internal ballistics of the solid propellant rocket. Just to be you know in the same platform that we have some basic knowledge of solid propellant rocket in order to understand the combustion processes later on. And of course, we must need to learn the various type of propellant otherwise you know whatever the combustion process we learn for what type of propellant.

So, we need to definitely need to know the different types of propellants and there are different ingredients involved in those particular propellants. So, I think we will spend some time on this one regarding the description part in the following lecture. So, till then take care. Thank you.