Course Name: Combustion of Solid Fuels and Propellants
Professor Name: Dr. Srinibas Karmakar
Department Name: Aerospace Engineering
Institute Name: Indian Institute of technology-Kharagpur
Week: 08
Lecture 40 : Erosive Burning Theories

Hello everyone. So we were continuing our discussion on erosive burning in solid propellant rockets. In the previous lectures we have discussed the basic you know mechanisms behind the erosive burning that once the there is a high velocity cross flow over the propellant surface it cause erosion to the propellant surface and that cause causes erosion erosive burning. We have discussed about various methods for determining the erosive function which is denoted as epsilon which is given as epsilon equal to Re by R0 and we have talked about the various you know laboratory methods and the engine measurement methods. We have shown through pictures we have shown how the each methods they are you know conducting the experiments in order to have the estimate of the erosive function. We have also talked about the disadvantage in laboratory based method compared to the engine measurement methods.

Now I will just give you a quick summary of the you know efficacies of various methods used in you know determining the erosive function. For example, like we have talked about the laboratory based method like x-ray based, photogravy based, probe based then pressure pick up based method. Most of them are like the operating conditions of these methods if you if I just note down I can just tell you before we moving to the theories I will just quickly show you some of the important features of these methods before we move on. So, you know for laboratory based method we have like we have seen x-ray we have to remember this operating conditions it is not the realistic operating conditions.

So, we have created or not we have rather we said that the researchers who had proposed for example, like the Marklund and Lake or Madhav or other researchers or Jucrose method let us say what they have done is they have created some kind of a you know semi realistic approach. So, it is like a not the exactly the it is a semi realistic operating condition. X-ray if you look at the photographic method they are also same thing it is semi realistic operating condition. Of course, in case of x-ray based method the accuracy was not that good accuracy was still poor for photographic technique we could expect like higher accuracy. Strip based method again it is also like operating condition is semi realistic, but again accuracy was still poor.

Pressure pickup pressure pickup method which was used you know tablet based or strip based method still the operating condition was semi realistic and the accuracy was actually better. So, for photographic and pressure pickup cases the accuracy was better than the x-ray or probe based method. So, these are like laboratory based method and then there were like direct you know motor firing methods which is engine measurement methods we said direct motor firing methods. Here we can say that you know condition operating condition was realistic. If you look at the operating condition we can say operating condition was realistic even for let us say interrupted burning.

This was realistic operating condition. The radiographic technique this was also you know realistic because it is done using operating using the same engine operating condition, but again the accuracy of this methods we cannot say very good kind of you know it gives fair accuracy. There are you know probe based method also available there probe based method. There also it is done under realistic operating condition, but it is very difficult to say about the accuracy for few cases like if it is ionization type of probe then the accuracy would be better compared to like thermocouple or conductive conductivity based probe, but more or less you know this probe based method with ionization based probe probably it can gives better burning accuracy results accurate results. But however, we need to remember that they are going to give the average burning rates based on like number of firings.

So, many times firing has to be done and then average burning rate will be calculated from the number of runs. Same thing we can say about this you know pressure pickup probe based method they are also going to give the average burning rate. The photographic based method is going to give us instantaneous burning as we can see it is directly you know filming the burning event. So, we can expect that this is going to give the instantaneous burning rates. Whereas you know x-ray is kind of average or instantaneous depending on like how we are doing it.

So, this is also kind of give us either instantaneous or average burning rates if it is combined with the x-ray I mean cine radiographic technique combined with like x-ray and photography then probably it can give us the instantaneous burning rate otherwise it can give us the average burn rate. So, these are the typical you know features for the laboratory based method and direct motor firing based method. So, I think we have discussed this thing in almost in detail in the previous lectures. One thing we also need to remember that the typical cross section area to the nozzle throat area for like most of the rocket cases port area to the cross sectional area of the throat is in the range of like 2 to you know 2 to 5 values are in the in this range. So, if we have a smaller cross section of the port.

$$\frac{Ap}{At} \approx 2\ - 5$$

So, we can expect that the cross flow velocity is going to be higher and that will cause the erosive burning. So, this number I think we should actually follow from the literature that typically this fall in the category of like 2 to 5. Having said that now let us coming to the erosive burning theories. Now, without consideration given to the type of propellants whether it is a double base propellant or heterogeneous propellant means like homogeneous propellant or heterogeneous propellant the existing theories on erosive burnings are generally categorized into 4, 4 major categories. They are divided into 4 major categories or rather we said like how the you know modeling is done and based on that those modeling these major theories are you know given.

So, they are divided into 4 major groups. So, model based on you know the phenomenal phenomenological based method like phenomenological heat transfer based method which will actually give more importance to this one. So, this is phenomenological heat transfer theories. This is one category, second category is integral boundary layer theory, then modification of combustion mechanism or you know there is another method used by the researcher is chemically reacting boundary layer analysis. Although these 4 major groups are given in literature we I mean you know because of the simplicity of the analysis we will just give importance to the discussion because this phenomenological heat transfer based theories were well popularized or widely used and this method was given by the first developed by the 2 scientists called Linier and Robillard and popularly it is known as the LR method or rather I would say LR theory.

So, we will restrict our discussion only to this you know LR theory because this has been widely used in rocket performance calculation. Of course, if you look at the literatures there may be some you know criticisms to this LR theory and also there are some literatures available which can support these theories. Nevertheless we will give importance to our discussions on this theory because this is you know considered with the simplicity in the modeling part. So, this theory is based on like the phenomenological heat transfer analysis. So, we will look at this theory and we will just restrict our discussion only to this theory part.

Now the participants are requested to look at other theories to you know enhance their knowledge but just to you know focus our discussion and within the scope of the course we will just discuss only this heat transfer based theory. So, let us look at how these theories are you know major features of this theory. So, mainly 2 mechanisms were proposed here like from the primary burning zone which is kind of you know independent of the gas velocity and that is only function of pressure. So, basically this LR theory these were mainly 2 you know mechanism of heat transfer sorry let us write here 2 mechanism of solid gas to solid heat transfer were proposed in this theory. So, what are those? One is like you know from the primary combustion zone.

So, this part is actually independent of the gas velocity, independent of the core gas velocity and it is basically is a function of pressure, function of only pressure, only pressure plus it is a function of only pressure. So, this is the from the primary combustion zone and the second one is from the core hot combustion gases. Of course, that depends on the gas velocity. So, following this approach the total burning rate the total burning rate is given as if we just express this in terms of let us say total is R which is given as R 0 plus R e where you know this is the pressure dependent normal burning component, normal burning component which is basically you know we already know that that R 0 equal to A p to the power n and this second one. So, we are writing actually R 0 here.

$$ro = ap^n$$

$$r = ro + re$$

So, R 0 is this pressure dependent normal burning component whereas, R e is the erosive burning component. So, R e is erosive burning component. Now in order to analyze this further you know we have to consider the heat flux from the gas flow to the wall surface of a flow channel. We can have like the circular cross section or the depending on the port geometry we can have like different type of cross sections, but in general if we say the heat transfer in order to understand this theory I think we need to consider the heat transfer analysis. So, we have to see that how the heat transfer is actually through a boundary layer is taking place.

So, we can actually see that if we consider this heat transfer through a boundary layer the burning rate of a propellant which was just represented by only R equal to A p to the power n is no longer valid because since there is a cross flow is present there will be some component due to the cross flow which is happening in terms of like erosive burning. So, the heat flux transfer from the cross flow to the burning surface of a propellant will increase the burn rate and that is reflected through this erosive component or erosive burning component. So, we cannot directly use that R equal to A p to the power n when there is a situations high gas velocity or high cross velocity. Now in general as I said the heat flux from the gas flow to the wall surface if we know that if the heat transfer is Q we can relate that with the equations what we know that Q equal to you know H g into t g minus t s. Now, if we know that heat transfer coefficient along the flow channel if we want to find out we can simply refer to the literature that the heat transfer coefficient along the flow channel in a circular port most of us we use this cylindrical port where the internal port is actually in circular cross section.

$$q = hg(Tg - Ts)$$

So, we can use this semi empirical correlation. So, basically we use this semi empirical equation where it gives H 0 equal to 0.0288 sorry c g into rho g c g into rho g sorry mu g to the power 0.2 Prandtl number to the power minus 0.667 K into g to the power 0.

$$ho = 0\frac{.0288cg\mu g^{0.2}\ \mathrm{Pr}^{-0.667}\quad k\,G^{0.8}}{L^{0.2}}$$

8 divided by L to the power 0.8. So, here g is the mass flux in the circular port which is mostly given by the m dot divided by port area. K is experimentally determined constant and L is the distance from the leading edge to the fluid flow, leading edge of the fluid flow. Now, if you wish to find out more details about this correlation one can refer to the paper by Rashdan and Kuo this was exactly related to erosive burning of solid propellant solid propellants in AIA book series one can find this through the literature it is available sorry I think it is 1984 volume 90 chapter 10 page number they can refer to is like 515-598 so, some important information they can find out from here.

Now, the heat transfer coefficient is also related with the physical properties and the flow properties of the fluid. So, we can bring this Stanton number here because Stanton number you know correlate with the heat transfer to the flow to the you know thermal capacity of the fluid. So, the Stanton number comes into the picture that actually relates the basically Stanton number is given by the you know ratio of heat transferred heat transferred to the fluid by the you know heat

$$St = \frac{heat\ transfered\ to\ the\ fluid}{thermal\ capacity\ of\ the\ fluid}$$

capacity or rather we can say thermal capacity of the fluid of the fluid. And another correlation comes once we relate with the physical property and flow property of the fluid. So, we have this Stanton number in terms of this is again a correlation 0.0288 Reynolds number to the power minus 0.2 Prandtl number to the power 0.667. So, you can see this actually coming from here only. So, this is actually coming directly from here.

$$St = 0.288 Re^{-0.2}\, Pr^{0.667}$$

So, basically as a Prandtl number I think we all know this is relating to like momentum diffusivity by thermal diffusivity so, it is written as like nu by alpha denoted as this. So, it is basically relating the flow parameters along with the you know properties of the fluid and it is combining in this empirical equations. And if you just want to express this numbers then we have to involved by the you know physical properties of the fluids in the boundary layer. So, in many occasions this Stanton number will come as this H0 by rho g U g into C g as I said that it is heat transfer to the fluid. So, the heat transfer coefficient divided by rho g U g into C g.

$$Pr = \frac{momentum\ diffustivity}{thermal\ diffustivity}$$

$$Pr = \frac{\upsilon}{\alpha}$$

$$St = \frac{ho}{\rho g U g C g}$$

So, it is relating the you know heat transfer to the fluid to the thermal capacity of the fluid which is denoted as the C g part here you can get it here. So, in a number of you know experimental studies it has been shown that the heat transfer coefficient with cross flow it is written as it is expressed as H0 e to the power minus beta rho p r by g. And this is given you know beta is given as the blow parameter. And it is generally determined from the experiment ok. So, what we can see that of course, the Prandtl number is a function of the physical properties of the you know cross flow I mean the gas passing through the passing on the surface.

$$h = ho\,.\ \exp^{\left(-\frac{\beta \rho g r}{G}\right)}$$

And Reynolds number is a function of you know kinetic viscosity of the gas and the flow velocity. So, and the heat flux is also given as a function of you know the wall temperature, the gas temperature, the gas velocity and the physical property of the product gases or the combustion gases. So, that way one can assume that the burn rate which is increased due to the cross flow of the gas is going to be given as the total burning rate will be the function of the erosive component of the burning and the non-erosive component or the pressure dependent normal burning component. So, R0 is the burning rate without cross flow whereas, Re is the erosive component of burning. Now, this equation was further given in terms of by incorporating these parameters and this equation was further sorry.

$$r = r0 + re$$

$$= ap\^{}n + kho \exp \^{}\{ (-\{\beta \rho r\}\{G\})\}$$

So, we can just write it here. So, this equation was further elaborated and it was given as R equal to as we said already that R equal to R0 plus Re. So, R0 was simply Ap to the power n and this one was you know somehow modified and it was given as relating these component like K into H0 e to the power minus beta rho p into R by g as you have seen here it is propellant density rho p is the propellant density. So, once you are writing this we can incorporate all these equations what we had here these empirical correlation are added there and the final form is generally written in terms of like Ap to the power n plus alpha g to the power 0.8 divided by L to the power 0.2 into e to the power minus beta rho p R by g where this alpha is given with a correlation

$$= ap^n + \alpha \left(\frac{G^{0.8}}{L^{0.2}} \right)$$

0.0288 C g into U g into Prandtl number to the power minus 0.667 into that is K that K we have said already that experimentally determined function. So, K is a constant which is dependent on the interaction of the flow between the parallel to the surface and the blow off gas from the burning surface. So, what I am seeing is the blow off means you should remember I am talking about this one this is the blow off. So, the gas is coming out from the propellant.

$$\alpha = 0.288 cgUg \times \mathrm{Pr}^{-o.667} \quad k$$

So, this is the burning propellant this is the propellant surface and the cross flow is coming from here. So, here the cross flow which is U g and this is the blow off. So, blow off parameter is going to give us from the experiment that how much you know blow off is happening from the burning surface. So, alpha is actually relating this one sorry beta is the blow off parameter is relating this one and alpha is a combining with the liquid you know physical property the sorry fluids physical property and the flow properties. So, this is the combined form of the equation.

So, R equal to Ap to the power n this and it is written in this form. If you refer to the literature you will get the similar expressions given by that and this was typically the LR theory proposed by this two gentlemen Leonor and Robillard and that was the theory widely used for you know understanding the rocket performance calculations. So, we will actually consider this theory for this purpose, but as I said the other theories we mentioned can be referred to it, but however, for the simplicity of the theories we will just follow this LR theory only for our discussion. Now, one can actually argue with that that the ok. So, let us first conclude this part that the burning equation with cross flow is derived based on the experimental data and the equations given here is known as the LR equation or Leonor Robillard equation.

So, we will just follow this one for calculating the burn rate when there is a cross flow situation. Now, one more comments or situations may arise in our mind that is there any situation when there will be a negative burning you know can it be situations where there will be sorry negative erosion like what we just said that erosion is always positive that can there be some composition of propellant combination and at a particular pressure chamber pressure scenario when there is a you know negative erosion is going to take place or rather the burning rate starts to you know decrease it is going to reveal that there are some situations for a propellant combinations AP and polyurethane. It has been shown experimentally that when AP ammonium per plate with polyurethane type of binder has been you know burned under a cross flow velocity at about you know pressure about 3.3 MPa. It has been observed that the burning rate is decreased with increasing cross flow velocity and it reaches to the burn rate reaches to minimum at a velocity of about 370 meter per second.

So, it is about around it is in the literature is shows that about 23 percent decrease in burn rate is observed and you know it is actually not observed for any type of you know plate to burning for double base propellant or APHTB based propellant. So, this typical you know APPU based propellant it has been observed with a cross flow situations that at a velocity 370 meter per second burn rate is actually decreased, but after that velocity once the cross once the cross flow velocity is increased then the burn rate again start increasing. Now, what has been I mean what has been analyzed there that the burning surface during the I mean the close observation of the burning surface it was revealed that the burning surface was partly covered with the molten layer from the binder. So, whatever the polyurethane binder was there part of the polyurethane binder was covered on the propellant surface which actually prevents the decomposition of ammonium particles further and that actually leads to you know reduce in the heat flask coming from the gas phase to the condensed phase and due to which the burning rate was decreased. But once the cross flow was increased further the molten layer which was covered the propellant surface was actually removed by the high shear exerted by the flow and that actually removes the that molten layer of the binder and again the decomposition of AP starts doing I mean start increasing and then further the burning rate is going to increase.

So, this is just to you know give you an example that there are some situations where negative erosion can take place what we have just told so far that only you know erosive burning is going to give add to or give increase to a burning rate that is not true for certain case of combination of you know oxidizer and binder at and certain scenario at a particular pressure and with the cross flow we can actually see that burning rate is going to decrease for a certain cross flow velocity. But again once we increase it further it can actually increase and the possible reason for that one is that that binder was actually covering or the molten binder was covering some of the ammonium per plate crystals which was actually not decomposing further and due to which the burning rate was decreased. So, we can actually remember this one that this is a typical example of you know negative erosion. I think with that we pretty much covered our discussion on the erosive burning and I would suggest that you please refer to the some of the literature as I mentioned in the lectures I mentioned it there it is already given in some of the you know references also. So, please look at literature to have you know further ideas or information on this part some of the experimental techniques we have mentioned regarding the determination of erosive function one can actually look at the current scenario or what are the different you know current experimental techniques with in improved you know experimental facilities are conducted so that they can get further information on that.

With that I think we can conclude this module. So, we are almost coming to the end of this course on combustion of solid fuels and propellants. Before we sign off I would like to introduce our team without their tireless effort it was not have been possible by me to complete this course. So, I would like to first call our TAs and they will briefly introduce themselves just by saying their name.

So, first I would like to call Mr. Ritesh Dubey. So, let us meet our TA members Ritesh Dubey, Mr. Sogato Mandal, Mr. Sumit Kumar Gupta and then we have our technical personnel without their support it would have been a very very you know difficult job for us. So, they always keep working in the background we do not generally show I mean show them here, but they are you know doing you know tireless job for preparing the you know course materials like after the video lectures, posting the videos once we give them the assignment they are creating the assignment in the proper format, publishing it and so many. So, I would like to introduce our technical personnel

Mr.Devapriya Chakraborty and Mr. Swarab Bhattacharya. With that we come to the end of this course. So, please practice all the assignments revise all the videos lecture materials properly and I wish you all the best for the exams of this course. Thank you.