Course Name: Combustion of Solid Fuels and Propellants

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Lecture: 04 Performance Parameters (continued)

Welcome back. So, we will continue with lecture 4 for the course Combustion of Solid Fuels and Propellants. So, we will resume from where we left off in the previous lecture. So, we are discussing the various performance parameters of the rocket. So, if you recall, we had derived one equation for characteristic velocity or C star and we denoted the characteristic velocity as C star and we got the equation of C star something like,

$$C^* = \frac{\sqrt{R\,Tc}}{\Gamma}$$

where R is the gas constant for that particular gas which is going to be,

$$R = \frac{Ru}{Mw}$$

capital gamma we got as a function of small gamma or the specific heat ratio,

$$\Gamma = \sqrt{\gamma}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

So, this was the equation we already got, and if we recall we can relate the mass flow rate, chamber pressure, and throat area by the relation C star,

$$C^* = \frac{Pc\,At}{\dot{m}}$$

where Pc is the chamber pressure, At is the throat area, and m dot is the mass flow rate.

Now, let us try to understand what exactly the C star is telling us or what is the indication given by the C star. C star is a function of the propellant characteristics and the combustion chamber design. It is not dependent on the nozzle characteristics, that we can say. In other words, we can say that the C star is an index of the performance of the propellant and the combustion chamber design. So, it can tell us how a particular propellant can generate high-temperature gas and raise the chamber pressure.

So, that is, somehow, we can say it is an index of performance. So, overall, we can say that the C star or the characteristic velocity can be considered as a figure of merit or index of performance for comparison of various propellant combinations and combustion chamber design. Now, one can obtain the ideal value of C star or C star ideal, which is you can say it is like the maximum value of C star from you know the ideal gas assumptions like ideal gas assumptions we had or we had followed like the perfect gas assumptions. So, from there we need to find out the value of the ideal value of chamber pressure, mass flow rate, and the throat area. From there, we can get the ideal characteristic velocity.

Now, of course, the actual characteristic velocity may not be equal to the C star ideal. So, there is a parameter defined it is called the eta C star or C star efficiency which is defined as:

$$\eta C^* = \frac{C^* actual}{C^*\ ideal}$$

Now, how do you find out the C star actual or the actual characteristic velocity? We can find out the C star actual from the experimental data like if we get the Pc value or the chamber pressure or the m dot and At from experiment, we can fit this value in the C star equation. So, now, our C star actual will be equal to,

$$C^* actual = \left(\frac{Pc\ At}{\dot{m}}\right) actual$$

Of course, for actual cases means we need to find out the values from the experiment. In many case occasions you know chamber pressure is derived from the sorry chamber pressure is obtained by obtained from the pressure transition data we install during the test.

So, one can see that there may be some pressure transducer install which can tell us the chamber pressure or the value of chamber pressure, the actual chamber pressure and we can fit this data m dot can also be you know, calculated from various measuring technique like if we know the regression rate of the solid propellant from there we can estimate the m dot value and At we know which is the throat area. So, for actual cases, if we know the values of Pc, At, and m dot we can find out the values of C star actual, and from the ideal gas assumptions or perfect gas law from there we can find out the C star ideal by considering the ideal value of Pc, m dot, and At. So, of course, our objective is to make this C star actual as close to the C star ideal. So, in the typical range, if you say a typical range of C star is around, you know, point 0.92 to 0.995 means 92 percent to 99.5 percent. So, it is quite high value the efficiency C star efficiency is quite high value. So, we aim to make it as close to the ideal case. So, that is the value of C star efficiency.

The other parameter another important concern is the thrust coefficient is Cf denoted as Cf. To understand the thrust coefficient we have to recall the thrust equation we had in our previous discussions like thrust equation was,

$$F = \dot{m}V_J + (Pe - Pa)Ae$$

where we already said that V_J is the jet velocity or the efflux velocity, Pe is the pressure at the exit of the nozzle, Pa is the ambient pressure, Ae is the exit area, and m dot is the mass flow rate. Now, if you look at this equation the principle of thrust generation is the principle by which the efflux of propellant is taking place the efflux of propellant gas is taking place or the momentum going out causes the reaction force, which is the thrust. Now, since the flow is supersonic the exit pressure at the nozzle does not necessary to be always equal to the ambient pressure. So, Pe doesn't need to always be equal to Pa as you all already learned from our previous courses like high-speed aerodynamics or gas dynamics course where we have seen that Pe may not be equal to Pa in a case where Pe is less than Pa we consider this case as over-expanded nozzle.

So, of course, this will have some you know issues with flow separation and since it is not the optimum expansion your performance will decrease, it will decrease the performance. In an ideal case what you want we wish that Pe is equal to Pa which is the optimum expansion and in all our ideal rocket assumptions we said that the gases exiting the nozzle are optimally expanded which means, our exit pressure is equal to the ambient pressure. So, the influence of the you know flow separations decrease in performance will be even considerably low or can be neglected. So, that is the ideal cases in most of the you know following discussions we generally consider the optimum expansion for the simplicity of the analysis, but we should actually know that what are the in real case when the Pe is not equal to Pa. So, in case of Pe less than Pa which is like you know near to sea level cases where your exit pressure is less than equal to the ambient pressure.

So, that case is called the over-expanded nozzle there will be an issue with the flow separation. So, that will decrease the performance in case of an under-expanded nozzle. So, your rocket is flying higher and higher. So, at a higher altitude, the ambient pressure decreases. So, in that case, your Pe will become greater than Pa.

So, in that case, it will not expand properly. So, that case is called an under-expanded nozzle. So, of course, the expansion of gas is incomplete and of course, the performance will decrease in this case as well. Anyway, so, these two scenarios I think you are well aware of these because in the previous course either in your elective course or in the basic courses, if you have taken the compressible flow, if you have taken the gas dynamics course or high-speed dynamics course you must have you know encountered this phenomenon like over-expanded nozzle, under expanded nozzle and the optimum expansion. So, the discussions underlying theories I think since those were already discussed in those courses.

So, the participant may revise these things from their, you know, notebooks or the reference textbook. So, we are keeping it aside from the particular course. Now, let us try to see what we can get from this thrust equation, how much simplification we can do and we try to understand how the thrust coefficient is important in terms of the thrust data. So, if you recall, we had the equation of the V_J, we had derived in one of the lectures which we already got the equation of V_J as

$$V_J = \sqrt{\frac{2\,\gamma\, Ru\, Tc}{(\gamma-1)Mw}\left[1-\left(\frac{Pe}{Pc}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

So, that was the equation for V_J or the efflux velocity, we can write this Ru by Mw equal to simply R, because you know that will save some space in writing the equation.

$$V_J = \sqrt{\frac{2\,\gamma\, R\, Tc}{(\gamma-1)}\left[1-\left(\frac{Pe}{Pc}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

Now, let us recall the equation for m dot, we had the relation of m dot with respect to the chamber pressure throat area and the characteristic velocity by this equation like

$$\dot{m} = \frac{Pc\ At}{C^*}$$

Now, if you put the value of C star here, we can write Pc At divided by the equation for C star.

$$\dot{m} = \frac{Pc\ At}{\frac{\sqrt{R\ Tc}}{\Gamma}}$$

$$\dot{m} = \frac{Pc\ At}{\frac{\sqrt{R\ Tc}}{\sqrt{\gamma}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}}}$$

I think we should remember these equations all the time because in many equations we have to recall the equation for C star and the V_J. So, it is very these are very important equations.

So, we should remember these. Now, if you come back to the thrust equation, we can replace the V_J and C star by the previous relation we got for V_J we got this one and for m dot, we got this one. So, we can replace m dot and V_J in our thrust equation. So, let us call this equation as 1, this one we can call it as 2, we can call it 3. So, let us use the relation of V_J and m dot which is like 2 and 3 into equation 1 and we can write F equal to,

$$F = \left\{\frac{\sqrt{\gamma}Pc\ At}{\sqrt{R\ Tc}}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}\sqrt{\frac{2\gamma R\ Tc}{\gamma-1}\left[1-\left(\frac{Pe}{Pc}\right)^{\frac{\gamma-1}{\gamma}}\right]}\right\} + (Pe - Pa)Ae$$

Here (Pe-Pa)Ae is pressure thrust. So, we can simplify it further, we can write this in the form like this F equal to let us take out the Pc and At outside.

$$F = Pc\ At\ \sqrt{\frac{2\,\gamma^2}{\gamma - 1}\left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{\gamma-1}}\left[1 - \left(\frac{Pe}{Pc}\right)^{\frac{\gamma-1}{\gamma}}\right]} + (Pe - Pa)Ae$$

So, Pc and At it is just like a you know form of a force because you see Pc is the pressure At is the area. So, it is a multiplication off chamber pressure and throat area. So, eventual it is becoming a force.

So, let us put other thing inside or maybe we can do it little later first let us try to simplify this one because there are so many you know gamma R inside the square root here and here also. Let us do some simplification and then we will do the next thing. So, we will write in this form

$$F = Pc\ At\ \left\{\sqrt{\frac{2\,\gamma^2}{\gamma - 1}\left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{\gamma-1}}\left[1 - \left(\frac{Pe}{Pc}\right)^{\frac{\gamma-1}{\gamma}}\right]} + \left(\frac{Pe}{Pc} - \frac{Pa}{Pc}\right)\frac{Ae}{At}\right\}$$

So, Ae by At is nothing, but the area ratio sometime it is denoted as epsilon if you recall which is the ratio of exit area by the throat area. So, this terminology which is inside the second bracket or the curly bracket is known as the thrust coefficient it is called thrust coefficient or C_F denoted as C_F.

So, eventually this thrust equation will become

$$F = Pc\ At\ C_F$$

So, this is another important equation in terms of rocket performance which we can use from time to time while calculating the thrust. So, typically the values of C_F is varies from around.

So, C_F ranges from 0.8 to 1.9. Now, if we see the how to determine the value of C_F we can write C_F in terms of thrust as

$$C_F = \frac{F}{Pc\ At}$$

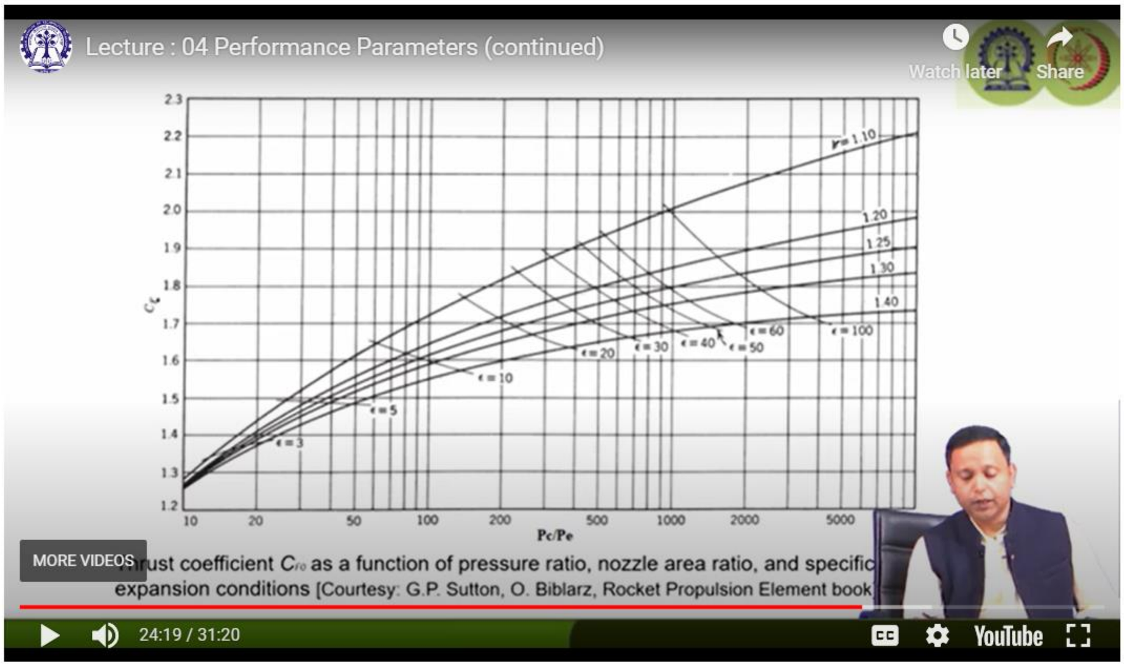
What we can see here is that experimentally if we are able to measure the thrust, the chamber pressure, and throat area we should be able to determine the value of C_F. Now, if you look at carefully about this equation what exactly it is doing the thrust coefficient is as if like a doing some amplification to this force part because Pc At is the force Pc into At pressure into area which is kind of a force part. So, the thrust coefficient can be considered as the amplification of thrust due to the gas expanding in the supersonic nozzle as compared to the thrust generated by the chamber pressure if it is acting over just on the throat.

Let me say it again. So, it is just like an amplification to the force generated by the chamber pressure over the throat only. If you look at only this part it is like as if your chamber pressure is acting over the throat that is the generating force which is Pc At. So, if that is the thrust which was generated by the chamber pressure if acting over the throat which is Pc At that is amplifying, but by this parameter called C_F. So, that is why the thrust is amplified by this parameter. So, this parameter that is why it is called kind of a amplification factor.

Now, to understand the effect of chamber pressure the altitude variation in a nozzle configuration or to you know correct the sea level results for flight altitude condition it is very important that we consider this C_F it is convenient to use this thrust coefficient parameter. Now, of course, the maximum value of thrust will be obtained for the optimum expansion as we already told earlier which is going to be like Pe equal to Pa. In that case, the optimum thrust coefficient can be defined as we can write the optimum thrust coefficient denoted as C_{F0} which is for optimum expansion that means, our Pa equal to Pe equal to P an optimum expansion case. So, we can modify the equation and we can get C_{F0} as,

$$C_{F0} = \sqrt{2}\,\gamma \frac{2}{\gamma+1}^{\frac{\gamma+1}{2(\gamma-1)}} \frac{1}{\sqrt{\gamma-1}} \sqrt{1-\left(\frac{Pe}{Pc}\right)^{\frac{\gamma-1}{\gamma}}}$$

So, this is the modified equation for C_F which is C_{F0} or the optimum thrust coefficient. So, we can easily write down this equation from the previous one and we can take out this term because Pe equal to Pa. So, this will be neglected. So, we will simply have this one. So, from there we got the thrust coefficient C_{F0}. Now, if you look at this plot this is a plot showing the variation of C_{F0} or the optimum thrust coefficient with respect to the ratio of chamber pressure by exit pressure Pc by Pe.



It has been taken from G P Sutton and Biblarz's book on Rocket Propulsion Element. So, this is one of our textbooks used in this course. One can easily see it in the book as well. So, it is showing the optimum thrust coefficient as a function of pressure ratio, nozzle area ratio, specific heat ratio for optimum expansion conditions. So, you can see that epsilon are nothing, but the area ratio nozzle area ratio Ae by At.

So, for different nozzle area ratio it has been given for different pressure ratio Pc by Pe. So, as you can see the once the pressure ratio is increasing the thrust coefficient is actually increasing and for various value of gamma, it has been given also. For as the for low value of gamma it is going to be the higher value of C_{F0} whereas, as the gamma value increases it is actually dropping down. So, this is another important consideration. One can actually do this calculation by taking out the effect of gamma separately, effect of Pc by Pe separately just to understand how they are influencing on the value of C_{F0}.

Because this is an kind of an amplification factor in the thrust equation. Now, after getting all these relations we can try to combine some of these equations together in order to have some convenience in calculations. You know we can just look at what we have done here and in the previous lectures we can combining the performance parameters. See what we understood in the discussions or even in previous courses that the thrust develop by a given propellant would you know depend on how much high temperature gas is generated in the combustion chamber and thereby you know increasing the pressure. And how low molecular weight of the product gas is being formed because if you recall the equation of V_J we have said that it is going to be proportional to 1 by square root of molecular weight of the gas whereas, it is proportional to the square root of chamber temperature.

So, basically the if we look at the propellant side if the propellant has the capacity to produce high-temperature gas and thereby you know producing you know generating the or sorry pressurizing the combustion chamber this will give the high performance. From the molecular mass side or the molecular weight side we are expecting that if the molecular weight of the of the product gases are in the lower side that will actually enhance the performance because V_J is proportional to 1 by square root of molecular weight. And also there are other things in terms of nozzle performance we had said the capacity to expand the gases in the nozzle if it matches with the exit pressure if it matches the exit pressure is matches with the ambient pressure we call it as an optimum expansion and that gives us a better performance in terms of thrust. So, combining all the performance parameters we have learnt so far if we just start writing it what you can write we can write C star equal to,

$$C^* = \frac{Pc\ At}{\dot{m}}$$

that we have already just seen in earlier class and today also we wrote. We had written f equal to, the amplification factor Cf into Pc into At.

$$F = C_F\ Pc\ At$$

We had already said the specific impulse Isp is going to be equal to,

$$Isp = \frac{F}{\dot{m}}$$

So, combining this we can write that Isp is going to be equal to,

$$Isp = C^*\ C_F$$

So, what we can say is that this specific impulse is kind of a composite index which is you know relating the capacity of the propellant and the capacity of the nozzle or the I would say the expansion capability of the nozzle because C star is very much dependent on the propellant combinations or type of propellants which has the capability to produce high-temperature gas. So, that is why the Isp which is a multiplication of C star into Cf is a composite index of performance which you know relating the capacity of the propellant to generate high-pressure gas and it also relates the expansion of the gas of the gases I would say gases because since it is a combustion products it is a mixture of gases. So, the expansion of gases in the nozzle that is why it is kind of a composite index of performance.

And of course, the Isp will vary with Pc and Pe. One can actually generate some you know performance parameter plot by keeping some of the parameters constant and just varying the others and generate the how the Isp is actually varying with the Pc by Pe by considering the Isp equation and keeping some of the parameters constant one can generate some parametric study sorry the parametric plots. Now, looking at further at Pa equal to 0 which is like a vacuum condition we can actually write the thrust equation as,

$$F = \dot{m}\, V_J + Pe\ Ae$$

So, from there you can write the equation for Isp vacuum is going to be equal to,

$$Isp\ vacuum = V_J + \frac{Pe\ Ae}{\dot{m}} = V_J + \frac{Pe\ Ae\ C^*}{At\ Pc}$$

So, we actually are combining the other parameters into this equation. So, of course, you can understand that Isp vacuum is higher than the Isp sea level because at Isp sea level or the where Pe equal to was Pa.

So, which has like Isp sea level we said it was equal to like V_J where we had Pe equal to Pa, but in case of vacuum, it is Pe is going to be equal to 0. So, we can have the ISP vacuum as this equation. So, some of these relations if we combining them together it will be easier for us to understand and also if we do some numerical problems this will be very easier to recall some of these equations. So, next what I am going to do is I will try to bring some example problems and just to show how we can use the relationship to understand the performance of a rocket. So, I will just solve one two example problems in the next class alright. Thank you.