Course Name: Combustion of Solid Fuels and Propellants
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Lecture 32 : Metal Particle Combustion Regimes (contd..)

Hello everyone, we are continuing our discussion on combustion of metals. And in the previous lecture I think we had started discussing about the metal combustion regimes. Now as we told ourselves that before considering the various you know metal combustion metal particle combustion regimes particularly like diffusion control or kinetic control, we had told ourselves that we want to take the analogy of droplet burning features of normal liquid fuel and we just compare the similarity of the droplet evaporation or burning to the metal particle combustion. In a sense that we compare the burning of single metal particle with that of the droplet of sorry the droplet combustion of single liquid droplet.

Now we had talked about the surface regression of the evaporating droplet and there we have talked about the mass flask with respect to the mass regression rate and we wrote some equation there. So, let me just repeat some of the things there and we want to use the basic equations derived from the droplet combustion features and we want to use the similar you know equations for the burning of single metal particle. So, let us go ahead with what we have done in the previous lecture. I will just repeat some of the things and we just move ahead to learn what are the required for the metal particle combustion. So, as I said we take the similarity to evaporation and combustion of single droplet. So, in order to do that we already consider a single droplet in a quiescent environment as we said. So, the outside is you know oxidizing media is there which is like oxidizer. This is fuel droplet and the droplet is producing fuel vapour. There will be like flame front which is going to provide the heat to the droplet for evaporation of the droplet.

So, droplet will keep on regressing. So, the number I mean the fuel vapour leaves the surface of the droplet. We said that the rate at which the fuel vapour leaves the surface is m dot f and if we want to write that in terms of mass flux at the surface. So, this was we said is the rate at which fuel vapour leaves the surface and corresponding mass flux was denoted as m dot f double prime. So, this was denoted as this was defined as the mass flux at the surface.

So, we can relate this rate at which fuel vapour leaves the surface. So, mass regression rate at the surface we can relate that to mass flux by the equation m dot f equal to 4 pi r square into m dot f double prime where r is the radius of the droplet

$$4\pi r^\wedge 2m^\wedge .f"\ .$$

Now, what it is happening here that the diffusion of the fuel vapour is actually taking place in the medium of fuel vapour and oxidizer vapour. So, there will be the oxidizer vapour here and the fuel vapour is also here. So, there is a media of mixture of fuel vapour and oxidizing vapour and we consider the you know the density comprising the fuel vapour and oxidizing vapour as the rho.

$$Yf\ = \frac{mf}{mf+mo}$$

There is a concentration you know the diffusion will take place due to the bulk flow and the concentration gradient of the fuel vapour. Now, we had denoted the mass fraction of fuel vapour of fuel vapour by Y f and we define Y f as the mass of fuel vapour divided by the mass of fuel

vapour plus mass of oxidizer vapour. This media is consisting of both the fuel vapour and the oxidizing vapour, oxidizer vapour. So, we are considering the mass fraction as m dot f by m dot f plus m dot o sorry m f divided by m f plus m o. Now, if we now write the mass flux equation of the fuel vapour what you can write? We can write that this is going to be equal to Y f into m dot f double prime plus m dot o double prime minus rho D D Y f d r.

$$m''f\backslash\ =\backslash\ Yf\backslash left(m\hat{}.''f+m\hat{}.'o\dashv)-\rho D\{dYf\}\{dr\}$$

So, D Y f d r is basically going to give us the concentration gradient. So, D Y f d r is happening due to the concentration I mean this is the concentration gradient. So, we must need to consider the mass flux due to the fuel vapour due to the concentration gradient we need to incorporate in this equation. So, the term this one is gives the mass flux of fuel vapour due to the concentration gradient D Y f d r. So, we are considering the concentration gradient in the radial direction because this is happening in the radial direction.

So, this is like radial direction. Now, we have considered D is as the diffusion coefficient or the mass diffusivity we have already defined that. So, if we just look at carefully the only the fuel vapour is actually moving out into the ambience. If you look at carefully the fuel vapour is actually going out in the ambience of oxygen. So, we can consider that the m dot o double prime supposed to be like 0.

So, what you can get from this equation if we just write if you rewrite this one in terms of like the m dot f double prime which is m dot f by 4 pi r square. If we just simplify this and write this in terms of m dot f we can get minus 4 pi r square rho D by 1 minus Y f into D Y f d r

$$m\cdot f\ =\ \frac{-4\pi r^2\rho D}{1-Yf}\frac{dYf}{dr}.$$

I think we have done this one in a previous class and then we have said we have also simplified it further in terms of like D Y f d r sorry not D Y f d r we had said that we want to do the we want to take this D Y f in one side. So, I think we had written minus D Y f by 1 minus Y by f that was equal to m dot f by 4 pi rho D into d r by r square. So, we just you know modify these equations

$$\frac{-dYf}{1-Yf} = \frac{m{\cdot}f}{4\pi\rho D}\frac{dr}{r^2}$$

Now we also said that if the concentration of fuel vapour at the droplet surface. So, which is like at r equal to r s which is the droplet surface we can denote the concentration of fuel by Y f s and at r equal to infinity means far field. We can denote the concentration of fuel as f infinity. Now if you integrate this equation what we can write we can write this is going to be integrate from Y f s to Y f infinity minus D Y f by 1 minus Y f and this is going to be like surface of the droplet to the infinity m dot f by 4 pi rho D d r by r square. Let us say this is equation b.

$$\int_{Yfs}^{Yf\infty} \frac{-dYf}{1-Yf} = \int_{rs}^{\infty} \frac{m{\cdot}f}{4\pi\rho D}\frac{dr}{r^2} - (B)$$

$$\ln \quad \frac{1-Yf\infty}{1-Yfs} = \frac{m{\cdot}f}{4\pi\rho Drs}$$

If you do this integration using the boundary condition what you can get? You can get ln 1 minus Y f infinity by 1 minus Y f s is going to be equal to m dot f by 4 pi rho D by r s, r s is the radius of the droplet. Now from here we can write the mass flow rate of the fuel vapour from the droplet surface we can write m dot f is going to be equal to 4 pi into rho into D into r s into ln 1 minus Y f infinity by 1 minus Y f s. Now if we look at carefully this term 1 minus Y f s by 1 minus Y f s sorry 1 minus Y f infinity divided by 1 minus Y f s is kind of a gives us an indication that this is kind of a transport of fuel vapour transport of fuel vapour from the surface of the droplet. So this is you know we can simplify by denoting one transport number, number b and we define it as Y f s minus Y f infinity by 1 minus Y f s and if we incorporate this transport number into this equation of the rate of generation of fuel vapour. If we write that equation by incorporating this transport numbr we can write m dot f is going to be equal to 4 pi rho D r s ln 1 plus b.

$$m{\cdot}f = \ 4\pi\rho Drs\ln(1+B)$$

This is one of the important equations which we can use later on for you know understanding the evaporation of the droplet. We can try to find out the evaporation rate constant and how we can do that we will just show in a minute. So if you recall what we already defined that evaporation parameter or evaporation constant you know under steady state vaporizing conditions for a particular you know in a particular ambient condition for which if we are given the rho D in fact the density of liquid transport number if they are specified we can say the value of this beta V which was defined as the evaporation parameter is kind of coming out to be the constant and that is why this is also known as evaporation constant. Of course the value of beta V is going to increase as the temperature of the droplet increases we can also say that the beta V is going to be related with the you know that D square law we can get it from D D0 square by Dt is going to be equal to beta V. So if we consider this equation we can actually try to find out the time taken time taken for the evaporation by you know denoting the time let us say by TV.

$$\beta v = \frac{ddo^2}{dt}$$

So what you can say the time taken for evaporation TV it basically is going to be like when all the droplet you know all fuel is evaporated. So basically d0 will is going to be none. So if you integrate this from D0 to nil or none we can actually get the time taken for the evaporation. So simply we can get TV is going to be equal to D0 square by beta V. And what about our beta V? We had already looked at in the previous equations we already defined the beta V there where we had incorporated the beta V in terms of the 4 into m dot f divided by rho L.

If you recall what we had earlier this was given as 4 into m dot f divided by 4 pi sorry 4 mb by rho pi D this was like liquid. So if you put this in terms of this equation you will finally will get if you replace the values we can get instead of radius if you write in terms of diameter we can get the value of sorry the time taken for evaporation will come something like an equation d0 square into rho L by 8 rho D and then 1 plus b. So basically we are replacing m dot f from this equation and we are writing r as in diameter terms which is D0 square which is D and if you just simplify the

$$tv = \frac{do^2}{\beta v} = \frac{do^2 \rho}{8 \rho D \ln(1 + B)}$$

equation you will get this type of relationship. So which tells us that the evaporation time taken for the sorry this is we can there is a mistake in the equation sorry I guess we can check this one there is a mistake from this one you simplify this by replacing m dot f in this equation and you will end up getting this rho D. Yeah you are right I think it was correct it was denoting the D0 square by this thing.

So it should be like D0 square into rho L by 8 times rho into the diffusivity into ln 1 plus b yeah this was this was correct. So what you can say that the time taken for different droplets is going to vary if the as the droplet diameter decreases the time taken is going to reduce. Now if we consider the droplet evaporation at a high pressure it may take like longer period time in many occasions if we consider the droplet evaporation for typical you know droplets in a spray which is of the order of like 100 micron or so the droplet evaporation time can be the tens of milliseconds. Now we can proceed further by taking this analogy that in case of droplet combustion of course there is a you know flame exist there and flame is going to provide the heat to the evaporation of the droplet and generation of fuel vapour. So if you just look at it further for the you know combustion of a droplet what can we say because ultimately the idea behind doing this exercise is to try to understand and use the similar equation for a single particle.

So what we are trying to do here is we try to understand what is going to happen for single metal particle. So we are trying to use this equation for droplet combustion in the context of single metal particle combustion. So let us look at if we just consider this combustion of a droplet. For combustion of a droplet what you can say the droplet is there, there will be oxidizer. So this is our fuel, there will be oxidizer and of course the flame zone will be there somewhere over here.

This is let us say flame zone. So flame is going to transfer heat to the droplet for further evaporation. We can say that this is the radius of the droplet let us say Rs one can actually understand the radius of the flame also because we are considering the spherical droplet and from the flame we can say the products are forming. So the mixing of fuel vapour and oxidizer vapour is taking place. So in the similar way we can extend this our understanding and we can modify the transport number what we have just done for the evaporation case.

We can simply do the transport number for the combustion of a fuel droplet as well. Now what we can see here once the combustion is taking place we can considering that the spherical symmetry is maintained. So the flame is kind of spherical around the droplet. From here also we can say the fuel vapour is vaporizing away from the droplet. What we can consider here is that it is maintaining a stoichiometric mixture of fuel vapour and an oxidizer vapour.

So let us say we consider the fuel by oxidizer vapour stoichiometric is maintained there. So if we consider that we can modify the transport number and we can write in terms of including this fuel by air fuel by oxidizer ratio which is like the mixture ratio stoichiometric. So the oxidizer vapour can stretches you know to infinity where we can say the temperature is T infinity the surface temperature we can say Ts we can consider the surface temperature Ts the temperature at the infinity or the far field is the T infinity. Now the product of combustion are going to come out from the flame and they can mixed in this zone. The heat which is required to vaporize the fuel droplet is actually coming from the enthalpy of the ambient and the heat release from the stoichiometric flame zone.

So basically the heat is coming via you know T infinity minus Ts plus some you know some chemical energy generated due to the combustion. So if we say that heat release taking place due to the combustion and heat transfer is taking place to the fuel surface for further vaporization. So that way we can actually modify the transport number and we can say that for this it is going to be a coupling function between the it is going to be coupling function based on the coupling function between the energy and the oxidizer species. So we are writing based on coupling function between the energy and oxidizer species and we can modify this as fuel by oxidizer ratios stoichiometric into oxidizer mass fraction into there is one you know heating value or calorific value or heat of reaction of the fuel. So let us say we denote that heat of reaction of the fuel plus the enthalpy Cp into T infinity into Ts this divided by the latent heat of vaporization.

$$B = \frac{\left(\frac{F}{O}\right) st\, Yo\infty\, \Delta hrf + cp(T\infty - Ts)}{Lvap}$$

So let us define this parameter what we just said where this is B is the mass transfer number. We can define the newer parameter what we just said fuel by oxidizer is the mixture ratio at the stoichiometric condition or stoichiometric mixture ratio. T infinity is the gas temperature far from the surface far from the surface Cp specific heat. This delta H rf is the heat of reaction of the fuel. Of course this is given on the basis of per unit mass Ts is the surface temperature.

What else? This L vaporization is the latent heat of vaporization. So if we just want to use the same thing same equation for a particle with heterogeneous surface reaction for example, like boron particles what we say that there will be like heterogeneous surface reaction. For example, like carbon particles is going to have like heterogeneous surface reaction. So what we can say there that due to the heterogeneous surface reactions we can modify this equation a bit and also we can say that there is no volatility of the fuel at the surface. So yfs is going to be equal to so this is basically based on the oxidizer and energy.

If we say that because we have considered this based on coupling function between the energy and oxidizer. So if we want to use this for the same thing for the fuel based on the coupling function between the fuel and oxidizer species equation then we can modify it slightly. So this is remember this is based on the coupling function between the energy and the oxidizer species. If you do the same thing for a particle with heterogeneous surface reaction. Let us go to the next page and we can write there.

So if we do the same thing for a particle with heterogeneous surface reaction. We can define B based on coupling between fuel oxidizer species equation. Then what we can define b o f which we did earlier for oxidizer and energy q denotes somewhat like the indication of energy. So it is energy and oxidizer species. Here we are defining based on the fuel and oxidizer species and we can define this as B o f fuel by oxidizer stoichiometry into y o infinity plus y fs divided by 1 minus y fs.

$$Bof = \frac{\left(\frac{F}{O}\right) st\, Yo\infty + Yfs}{1 - Yfs}$$

This was similar to what we did in the earlier case for evaporation of single droplet. It is almost similar to that we have defined this, but there is a case when there is no if there is no volatility. What does that mean that we can say the mass fraction of the fuel at the surface is going to be equal to 0. So in that case we can take out this is 0 this is 0. So eventually the b o f is going to be equal to simply the mixture ratio stoichiometry into the mass fraction of oxidizer at the far field or far from this particle surface.

$$Bof = \left(\frac{F}{O}\right) st\, Yo\infty$$

And accordingly one can actually get the consumption rate per unit surface area one can actually get the consumption rate per unit surface area and we can get like m dot by 4 pi r p square. So here I am writing r p, r p is the particle diameter in terms of you know rho into d by r p ln 1 plus 1 plus

b. So 1 plus b means now we are saying in terms of oxidizer and fuel species. So it is becoming 1 plus b means this one. So it is f by o stoichiometry into y 0 y o infinity.

$$\frac{m^{\cdot}}{4\pi rp^2} = \frac{\rho D}{rp}\left[1 + \left(\frac{F}{O}\right) st\,.Yo\infty\right]$$

So this is you know mass consumption rate per unit surface area that we can write it for the particle. Now having this background our next task is to find out the combustion time scale combustion times for the diffusional control combustion regime and we have to find out the time combustion time for the kinetic control combustion regimes. Because if you recall we had started our discussion for metal particle combustion regimes for these two cases one is the diffusional control another is the kinetic control. So in our next module I think we can begin this discussion with this you know time for the diffusional control combustion and kinetic control combustion. And then we will try to understand how the combustion regimes is going to be different for different particles.

And of course our basic understanding on the classification of metal combustion like the heterogeneous combustion and homogeneous combustion or the gas phase combustion versus the heterogeneous combustion like we have taken the example of aluminum and boron. So we will try to understand the burning mechanism of single boron particles and the burning mechanism of aluminum single aluminum particles. Because they are the representative of heterogeneous and homogeneous combustion mode. So I think we can start that discussion in the next module.

Till then you please revise this stuff ok. Thank you.