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Lecture 31 : Metal Particle Combustion Regimes

Hello everyone. So, let us continue our discussion on metal particle combustion. In the previous lectures I think we have spent enough time in understanding the various modes of you know metal particle combustion. For example, like either homogeneous combustion or heterogeneous combustion, we have also tried to discuss about the various discriminator based on like the available energy based on like volatility of metal and their oxides. And also the third discriminator which is the inter solubility between the metal and their oxides. We have brought about an important consideration for synthesis of metals or materials by using combustion based method.

We have talked about that cell propagating high temperature synthesis which is kind of important in relevant to like combustion of metal. Although we have mostly you know considered our discussion with respect to like the metal as fuel for ingredients in composite propellants. However, if we think about the combustion of metals as a means for producing materials, then of course, the combustion of metals or mode of combustion of metals is also important for SHS type of method. Then I think we had started our discussion on the metal particle combustion regimes.

There we have talked about that how the metals burn heterogeneously homogeneously earlier and now we want to look at how the time scale of the reactions and the time scale of the mass and energy transports are important. In certain case when the reaction time scale is much larger compared to the transport time scale like mass diffusion or energy transport. In that case we can consider that spatial non uniformity does not exist and reaction is comparatively slower than the species transport or energy transport. In that case we can say that the combustion is kinetically controlled. So, I think we have already said this part that if the reaction time scale is much larger compared to the transports time scale, we can say it is called the kinetic control combustion regime.

So, here we said that the reaction time scale is compared to be higher than the chemical time scale and of course, that will ultimately control the macroscopic features such as the burning rates and ignition delays. Now, in case of the diffusion controlled combustion, the reaction is kind of faster. So, the reaction time scale is much smaller than the chemical time scale where the spatial non uniformity cannot be ignored. What we have done in the previous case, the spatial non uniformity cannot be ignored and the temperature and composition will act is going to be very specially. In that case the transport process is going to control the combustion and that type of combustion is called like diffusion controlled combustion regime.

So, we can say that these are the main two regimes of metal particle combustion. One is the kinetic controlled combustion regime, other one is the diffusion controlled combustion regime. Now, for diffusion controlled combustion regime, we can actually consider as if like similar to like a burning of droplet, what we have we can see in daily basis like various engines, where the droplet of fuel is burning. So, we can actually try to understand that analogy through that you know burning of droplet in a quiescent medium. So, before actually considering the combustion regimes of metal particles in detail, we must first try to understand the mass consumption rate of a metal particles per unit surface area in a quiescent environment.

So, we just want to see how a metal particles. So, if this is a metal particle, how these metal particles burning in a in a quiescent you know oxidizing environment, let us say the oxygen present outside the surrounding, how these metal particles are how a single metal particle is burning. Now, if we compare this with a single fuel droplet in a quiescent environment. So, we are talking about in a quiescent environment here, we can take this analogy by considering a small fuel droplet, which is also burning in a quiescent environment in oxidizing atmosphere. Now, in case of burning single droplet in an oxidizing atmosphere, fuel is evaporated from the liquid surface.

So, we can expect the fuel is going to evaporated from the liquid surface and the fuel vapour diffuses to the flame front. So, if you just consider the flame here, envelope flame here, it will look like something like this. One can actually do this experiment very easily by holding the one small liquid droplet. For example, like if you take a drop of diesel or petrol on a needle just try to do that and just light up with a matchstick you can see nice you know envelope flame what we have just shown here. So, this study was conducted by one of my previous student for you know different fuel, pure JT1, pure butyl butyrate, pure limonene just to give you a glimpse of you know the flame photograph of the single droplet.

You can see he actually ignited using some nichrome wire. So, nichrome was just was put beneath the droplet. In this case droplet was placed on a quartz fibre because quartz fibre is thermally you know minimally conductive. So, the heat transfer from the quartz is considered to be minimum. So, the droplet was ignited by this nichrome wire and it was taken away.

So, we let the droplet to burn and you can see the nice envelope flame is there. So, of course, this is not this is kind of a non spherical flame front, you can non spherical flame you can say it is not a spherical flame, but if the if you make the droplet very very small the surrounding gases can entrain it easily and the relative velocity between the droplet and the nearby gases becomes comparatively small and we can expect the diffusion flames surrounding the droplet is going to be close to like spherical. So, what I am saying is if you burn a larger droplet you can expect that the entrainment will take place and the flame envelope will something like this. So, you know fuel vapour will come out like this. This is the oxidizer vapour, this is the fuel droplet, this is fuel vapour, this is the flame front.

So, idea of bringing these pictures just to you know take an analogy what we are writing here you can see directly from the picture also. You see this is the flame front if you look very closely you can actually see the droplet there. It is like a you know kind of a transparent liquid you can see inside this flame. So, this one is kind of a jetty one is kind of having a lot of suits that is why you can see this yellow hue surrounding the droplet the flame is kind of covered with a yellow colour whereas, in case of butyl butyrate it is kind of like a kind of clean burning and you can see the greener flame near to the bottom of the droplet and you can see this envelope flame is actually covering the droplet. So, and the flame is continuously supplying heat for further evaporation of the droplet and it is providing the fuel vapour and you know heat is transferred from the flame.

So, the fuel vapour is continuously forming it is you know mixing with the oxidizer vapour and keep on burning. So, as time passes you can see the droplet is going to regress and finally, there will be no flame exist. So, this is the way we can analyze the combustion of single droplet. Now, if the droplet diameter is very small we can expect that the surrounding gas can penetrate surrounding gas can entrain very easily and the relative velocity between the droplet and the nearby gases becomes very small and we can expect that the flame can be nearly spherical. So, this is like a non-spherical case and this is a spherical case.

Of course, this is the flame front. So, in this is now this is the fuel droplet and fuel vapour will come out oxidizer of course, surrounding is oxygen. So, this is the oxidizer vapour which will go and react with the fuel vapour. So, this is the difference between the non spherical case and spherical case. Now, while understanding the single metal particle combustion we will try to understand the burning of the single hydrocarbon fuel droplet which is almost similar to like the metal particle combustion.

The rate at which the droplet evaporates and burns is generally considered to be determined by the rate of heat transfer from the flame front to the fuel surface. Now, how the heat transfer is going to take place that will very much determine the rate at which the droplet is going to evaporate and burn. Now, keeping this analogy in mind we will try to you know understand that how a single droplet is evaporating and burning, we will take this similarity and some of the equations we will apply those to you know metal particle combustion. So, what we are going to do is we will first try to see the evaporation of a single fuel droplet. So, if you consider a spherical droplet with a diameter D, let us say the droplet diameter is D diameter D the density of the droplet let us say liquid droplet is the density.

What about the mass of the droplet? We can simply write mass of the fuel droplet is rho L into pi D cube by 6. Now, the rate of vapour generating due to evaporation of the droplet is the rate at which the diameter is changing with time. So, if you just want to find out the you know rate of vapour generation. So, m dot is going to be we can simply just find out the rate at which the diameter is changing with time. So, we can actually do the derivative here we can get rho L pi D square by 2 d dt of dt.

$$m = \frac{\rho l \pi d^3}{6}$$

$$m = \frac{\rho l \pi d^2}{2} \frac{dd}{dt}$$

Now, if you just try to find out the change of the square of the diameter with time we can write d d square dt is going to be equal to we can just get the d d square from this equation and we can get 4 m dot rho L pi D. Now, this is sometime termed as the evaporation constant or evaporation coefficient or evaporation coefficient. Now, this evaporation coefficient is defined by the d square law by the d square law you may be aware of the d square law which is denoted as the equation is denoted as d square equal to d 0 square minus dv t. Now, this expression holds true for you know

$$\frac{dd^2}{dt} = \frac{4m^{\cdot}}{\rho l \pi d} = \beta v = \ evaporation\, const$$

$$d^2 = do^2 - \beta v t$$

burning droplet with mass transfer and heat transfer processes involving chemical reactions. So, we can actually trying to see how the droplet diameter is regression over time and that way we can actually try to understand how the you know burning of the droplet is taking place.

So, this is not only true for evaporation of the single droplet, but also is going to hold true for burning of droplet with mass and heat transfer processes involving chemical reaction. Now, what is the analogy you are going to do here we will try to see the evaporation and burning of single liquid droplet and we will compare that same thing we will just try to use those equation for a single metal particles because metal particles can be compared with the you know evaporation and combustion of single droplet. So, excuse me if you look at this picture again here we are actually looking at the same thing we are actually trying to see how the droplet is regressing over time.

If you see at a different time instance the droplet flame is presented here, but if you want to see the how the droplet diameter is regressing over time we can actually conduct this experiment by illuminating with a high intensity LED light so that it can nallify the flame you can it can only show the droplet. So, in that case if we just conduct the experiment by you know if your droplet is here your droplet flame is something like this if you just use some high intensity light backlight and take with some you know high speed camera you may see that it will nullify the flame light it will just only show the droplet as the some black dot and over time how it is regresses that will be depicted for different time instances t 1 t 2 slowly it is going to regress over time.

So, one can actually get the time varying droplet diameter. So, eventually one can actually try to plot the droplet diameter regression over time. So, the discoil law what I have just shown here we can have the initial droplet diameter we can get any instantaneous droplet diameter by comparing the you know time varying droplet diameter from this you know images and this is certainly done by many researchers if you look at any droplet combustion papers you can actually find this one how it is done. So, this is just to bring you know some ideas referring to the combustion of single droplet. Anyways having said that our target is to understand the combustion of single particle.

So, we will try to figure out how we can use the analogy of burning a single droplet and try to figure out some of the equations. So, let us say the fuel droplet is surrounded by the oxidizer vapor and let us say we have the droplet is there which is surrounding surrounded by the you know oxidizer vapor. Now, as the fuel droplets vaporize it gets surrounded by its own vapor you know it is going to surrounded by its own vapor also because it is vaporizing. So, let us say that is we say it is fuel vapor this is oxidizer vapor at any instant if you say the droplet radius let us say we just say the droplet radius at any instant let us say r. Now, if we consider the spherical symmetry the rate at which the fuel vapor you know leaving the droplet surface how the fuel vapor is forming rate at which is leaving the droplet surface let us say we denote that by m dot f.

So, if you write in terms of you know mass flux at the surface if you say the mass flux at the surface is like m dot double prime f we can write this that m dot f is going to be equal to 4 pi r square into that mass flux which is m dot double prime f. Now, diffusion of fuel vapor is going to take place in the medium of fuel vapor and the oxidizing vapor oxidizer vapor. So, we said that this is the oxidizer vapor. So, the diffusion of fuel vapor and oxidizer vapor is going to take place. Now, the diffusion will take place by the bulk flow of the concentration and the due to the bulk flow and the concentration gradient of the fuel vapor.

$$m\hat{}\{\hat{}.\}F \;=\; 4\pi r\hat{}2m\hat{}\{\hat{}\{."\}\}F$$

$$m\hat{}.F = 4\pi r\hat{}2m\hat{}\{."\}F$$

So, here the fuel vapor is more here is the less. So, it will just keep on you know diffusing through the oxidizing media. Now, if you just get the mass fraction of fuel vapor and you denote that fuel vapor. So, we are actually denoting the mass fraction of the fuel vapor in the mixture of the fuel vapor and the oxidizing vapor. So, basically the mass fraction of the fuel vapor can be denoted as the mass of fuel vapor divided by mass of fuel vapor plus mass of oxidizing vapor or oxidizer vapor.

So, in that case we can write the mass flux of the fuel vapor is going to be equal to m dot double prime f is the mass fraction times the this plus oxygen minus rho d dy f dr, where this is the you know diffusion coefficient d is the or sometime it said the mass diffusivity or the gas mass diffusivity excuse me. What is essentially giving the if you look at this term this is like a rho the rho is now the density of the medium this is like the density of the medium which consisting of consisting of fuel and oxidizer vapor. So, if you look at this term this term is basically giving that mass flux of the fuel due to the concentration gradient dy f dr. So, this is the concentration gradient radial radial concentration gradient dy f dr of the fuel. So, this is essentially giving the mass flux of the fuel vapor due to the concentration gradient.

$$\dot{m}''_F = Y_F\left(\dot{m}''_F + \dot{m}''_o\right) - \rho D\frac{dY_F}{dr}$$

Now, what about the what about the m dot o since only the fuel vapor is moving out in the ambience oxygen we can expect that this is considered to be 0 because only the fuel vapor is moving out in the ambient oxygen in the ambient oxygen in the surrounding oxygen surrounding oxidizer. So, we can modify the equation we can rewrite the equation of the mass flux in terms of mass flux at any radius r is going to be equal to we said already the mass flux was m dot f equal to m dot f by 4 pi r square. This is important because we want to know that how the mass flux how the mass is regression of the fuel droplet is taking place. So, we can actually get the m dot f equal to 4 pi r square m dot this I think we have already seen this in the previous slide. We have already written somewhere here did we yeah here it is written already m dot equal to 4 pi r square m dot double prime f m dot f double prime.

$$\dot{m}''_F = \frac{\dot{m}_F}{4\pi r^2}$$

So, from there it is being written again. So, if we just look at that equation what you have just said that we can put m dot o double prime as 0 we can modify it we just want to write this equation in terms of m dot f only m dot m dot f double prime only. If you write from the equation a what you can write since m dot o double prime is going to be 0 we can simply write m dot f it will come this side. So, 1 minus m dot f 1 minus y f will come in the denominator. So, we can simply get the equation for m dot f double prime.

$$4\pi r^2\left(\frac{-\rho D\frac{dY_F}{dr}}{1-Y_F}\right) \rightrightarrows \frac{-dY_F}{1-Y_F} = \frac{\dot{m}_F}{4\pi\rho D}\frac{dr}{r^2}$$

So, if we simply write this let us say we want to just write this in terms of m dot f double prime what we can get we can get this will come this side. So, minus rho d dy f dr by 1 minus y f is not it we can write this. Now, what it is telling us here what is m dot f. So, we can just simply put the value of m dot f double prime here. So, it is like 4 pi r square m dot f double prime was minus rho d dy f dr minus rho d dy f dr divided by 1 minus the mass fraction of fuel.

So, this was the equation. Now, what can we do here we can simply integrate the we can take out this one we can write dy f dr by 1 minus y f or let us do it other way we can just simplify this in terms of minus dy f by 1 minus y f is going to be equal to we can simply get m dot f by 4 pi rho d dr by r square dr by r square we can write this. Now, if the concentration of the fuel vapor if we know the concentration of the fuel vapor at fuel vapor concentration we can say that at r equal to at the surface let us say r equal to r s we can have the fuel vapor as y f s and far from the droplet the concentration of fuel vapor we can denote as y f infinity. We can actually now you know try to see that how the mass regression or the mass flow rate of fuel vapor is going to happen from the droplet surface we can get the equation if we integrate it from surface to the far field or to the far from the surrounding. So, I think if we just apply this boundary condition we can actually try to integrate this by knowing these conditions if you try to integrate I think we can get the mass flow rate of fuel vapor in terms of the concentration gradients of fuel the mass diffusivity the vapor density and of course, the radius of the droplet. So, I think we will try to do this integration in the next class and we will try to understand further on this part and from there we can take the corresponding equation and we try to understand the metal particle combustion with respect to single droplet combustion and we take those equations and we move out I mean move further in order to understand the various combustion regimes.

Mainly we are we are considering the diffusion control regime here that is why you are taking the analogy of you know liquid fuel droplet hydrocarbon liquid fuel droplet all right. So, till then you can revise this stuff I think we can discuss this in the following lecture ok. Thank you.