Course Name: Combustion of Solid Fuels and Propellants
Professor Name: Dr. Srinibas Karmakar
Department Name: Aerospace Engineering
Institute Name: Indian Institute of technology-Kharagpur
Week: 01
Lecture: 03 Performance Parameters

Welcome back. So, this is lecture number 3. We will continue from where we stopped in the previous lecture. So, if you recall we had talked about the velocity, a flux velocity V_j was dependent on various parameters. So, if you recall the equations we got for V_j was $\sqrt{\frac{2\gamma RuTc}{(\gamma-1)MW}[1-\frac{Pe}{Pc}^{\frac{\gamma-1}{\gamma}}]}$.

Now, if you look at the equation carefully in order to have very high jet velocity or the flux velocity we need to have very high combustion temperature or the high value of T_c is essential.

So, the propellant which can generate very high temperature is going to be beneficial for increase the V_j because V_j has the direct influence on the incremental velocity delta V which we have already seen in the rocket equation. What about the P_c? P_c must be high because you see it is $\frac{Pe}{Pc}^{\frac{\gamma-1}{\gamma}}$.

So, in a sense we can say the in order to get high V_j we need to have high T_c and high P_c or high chamber temperature and high chamber pressure. The molecular weight of the products will have negative influence like sorry the reverse influence like the increase in molecular weight is going to decrease the value of V_j.

So, we want the molecular weight of the products to be in the lower side so that it can give the higher V_j or higher efflux velocity or higher jet velocity. Now, the γ value has some weak influence which one can see through some parametric studies like taking various values of γ keeping other values constant and see how the V_j is going to vary. But eventually if you look at the major influencing parameter here are the pressure and the temperature. So, we can expect that high value of V_j will take place when the combustion of the propellant gives very high value of chamber temperature and at a high pressure. So, the combustion should take place at high pressure and it should generate very high temperature.

Now, I am just referring some of the plots given by Professor Ramamurthy's book which you can get the exact value, but I am just showing only the trends here like if you plot V_j versus chamber temperature which you can try varying from 300 to let us say 4000 or so 4000 Kelvin. So, this is T_c in Kelvin. So, you can refer Professor Ramamurthy's book on rocket propulsion, it is already mentioned in the course document. So, if you look at the velocity of flux or the jet velocity which we can just get it from various ranges starting from let us say 500 meter per second. So, it is in meter per second and it can go as high as like 3000 or so.

So, I just I am just giving you the trend. So, the V_j is going to increase with increase of chamber temperature T_c. Now, if you see the same plot for same effect of V_j with respect to the molecular rate, we may see that the increase in molecular rate is going to actually decrease the value of V_j and that is evident from the equation as well. If you look at the equation it is like 1 by square of 1 by molecular rate. So, the V_j is going to decrease if we have the molecular rate increase in the products.

So, in a sense if we have like a complete combustion of propellants which will give us the higher value of molecular weight that is not conducive in order to generate high value of V_j. So, in many case many occasions what is done slightly fuel rich mixture is used in order to use these advantages of lower molecular of the products. You can take a quick example like if you burn let us say some hydrocarbon Cx Hy + O2 and you are forming like if it is a complete combustion then it is going to give you give you CO2 + H2O. I am not balancing the equation just to give you a glimpse of it, but if it is not a complete if it is not a complete combustion then it can have like let us say this is yCO2, this is y_1CO2 sorry y_1O2 this is going to give CO + CO2 + H2O. Now, if you consider the average molecular weight of these combustion products you may see slightly incomplete combustion is going to generate some CO because CO has a lower molecular weight compared to CO2.

$$C_x H_y + O_2 \rightarrow CO_2 + H_2O$$

$$yCO_2 + y_1O_2 \rightarrow CO + CO_2 + H_2O$$

So, on an average this is going to give us slightly lower molecular weight which will give us the advantage in terms of increasing the velocity V_J. So, in many occasions the mixture ratio is made slightly fuel rich in order to have the V_J. We have just talked about in terms of V_J, but later on we will see that the specific impulse which is one of the important performance parameter is actually very much related to V_J which is actually in a similar unit is actually equal to V_J. So, in a sense we can say the performance of the rocket will very much depend on these things. If we can increase the combustion chamber temperature T_c very high, if we lower the molecular weight that will be conducive to generate high jet velocity or in a sense to increase the rocket performance.

Now, if you look at the influence of the γ because this is another influencing factor γ has a weak influence on the flux velocity or the jet velocity. So, higher the value of γ can have slightly lower value of V_J, but overall it has negligible effect you can say. So, the major effect is playing played by the chamber temperature and the molecular weight you can say. So, I think with that we can understand the influencing parameters which are important for

V_J. Now, we will proceed further and we will try to understand various performance parameter of rocket which we already learned probably in the previous course, but as I said earlier that the first module of this course will just give you some glimpse of what you have already learned from the basic course of rocket propulsion or from the aerospace propulsion and we will try to just revise some of the steps just to set the floor.

So, once we talk about the performance parameters we will first talk about the very important parameter which is thrust. Now, if you recall the thrust equation for turbojet engine or the air breathing engine. We had written the thrust equation as

$$F=\dot{m}_e u_e-\dot{m}_a u_a+(P_e-P_a)A_e$$

This was your equation in case of turbojet engine or air breathing engines because we have some inlet flow stream which has the velocity Ua and we had some inlet mass stream $\dot{m}$, but for a rocket since it is carrying its own oxidizer there is no inlet mass going into the rocket or neither there is any inlet flow stream going into the rocket. So, we do not have this terminology in the thrust equation of the rocket.

So, if you look at the thrust equation of a rocket we can simply write

$F=\dot{m}V_j+(P_e-P_a)A_e$

So, how we are deriving this equation? I will just give you a glimpse of it that if you take this a simple schematic of a rocket. We have the exit area A_e the ambient pressure is working all around the rocket. So, this is the ambient pressure.

So, this is P_a. So, here it is also P_a ambient pressure the exit pressure is P_e throat area may be A_t chamber pressure can be P_c chamber temperature can be T_c the chamber pressure again internally it is acting on inside the rocket everywhere of course, your pressure will be changed because here the throat is there. So, it is. So, this part is throat this is the combustion chamber and this is the exit of the nozzle. So, from here we can actually try to figure out that how we can get this equation that since there is no flow going into the rocket. So, this part is missing if you look at this equation number 1 and equation number 2.

So, this is from your turbojet or in general the air breathing engine. So, this is the air breathing engine. So, this part is missing because we do not have any external the mass stream going into the rocket neither the velocity. So, we have the straight way this equation. Now, what we can say about these two terminology the first one and the second one the first one is the momentum thrust and the second one is the pressure thrust.

I guess this is known to all of us. So, the first one is the moment momentum thrust and the second one is the pressure thrust. Now, if we say that my vehicle is going above the sea level and is going up in the altitude. So, your P_a is changing P_a is the ambient condition because P_a we do not have any control. So, as we go up and up your P_a is reducing.

So, there may be some situation towards the lower altitude that your P_a sorry if the P_e is less than P_a. So, it may be the this term may be negative. So, it has the negative effect. So, if the P_a is greater than P_e it will be the negative effect. So, that means, sometime not desirable also it has some influencing factor if your flow is not going to expand optimally.

So, in a design condition we always say that P_e is equal to P_a. I think you must have already done this thing that for the optimum expansion we say that and that is more desirable anything out of it will be like undesirable. So, that performance will be low, but there are conditions where absolute vaccum conditions where you can say the ambient pressure is 0. So, your thrust equation become $\dot{m}V_j + P_aA_e$. So, that is for case of vaccum for optimum expansion we can simply say $P_e = P_a$.

So, our thrust equation will become $\dot{m}V_j$. In many occasions we are going to follow this optimum expansion case where P_e is exactly equal to P_a. So, there will be no momentum thrust present we can simply write F equal to $\dot{m}V_j$. Now, let us say other performance parameters. So, thrust we begin with the thrust now we can say the total impulse.

Total impulse is denoted as I_t which is $\int_0^t F dt$ for a constant thrust we can simply write the I_t=Ft. Now, one of the important parameter is called specific impulse which is denoted as Isp in some book it is referring as only Is. So, we are referring it as Isp this is the total impulse per unit weight of the propellant. So, we can just simply write this as

total impulse=$\frac{\int_0^t F dt}{g0 \int_0^t \dot{m} dt}$

So, if there is any transient effects like stop and starting there is a change in the flow rate we can actually get the total mass of the or total weight of the propellant in by integrating this and the total impulse we can get it from there if there is a variation of thrust.

But for constant thrust and constant mass flow rate of propellant we can simply write

Isp=$\frac{It}{mpG0}$

So, we can say propellant mass here. Now, if you see here that in case of like a higher altitude or outside the gravitational field. So, what is the meaning of this g0? Here we are considering the g0 as the gravitational constant, but acceleration due to gravity. So, what about when there is a outside of the gravitational field? So, in SI system we can say the Isp is expressed just to make this Isp in second if you look at the unit here it will be just in second.

In order to do that g0 is considered to be an arbitrary constant with a value of of course, 9.81 meter per second square. This is equally acceptable for outside the earth gravitational field just to make sure that Isp unit in second. But there are other way of defining it also like we can write this Isp as total impulse by the propellant mass instead of the weight that will give us it will become

$$Isp=\frac{It}{mp}=\frac{Ft}{mp}=V_j$$

If you recall we had the optimum expansion case where in ideal situation we always consider this part that p is going to be equal to p_a.

So, we have the thrust equation as f equal to $\dot{m}V_j$. So, we are considering that and we write down the equation for Isp in terms of total impulse by m_p which is becoming like F by $\dot{m}$ which is eventually become Vj. And once we have Vj this has the unit of velocity which is meter per second. So, this is another way of expressing the specific impulse. Now, what about the other parameters we can keep deriving? One such important parameter is the it is called the characteristics velocity.

So, remember Isp is kind of an important parameter as we consider the the like the down transporation like car or the automobile. So, they have how much you know mileage like how much kilometer is giving per liter of petrol or per liter of diesel. So, it is similar to you know in a rocket literature it is a similar concept that Isp is an important parameter. Another important parameter is called characteristic velocity which is denoted by these symbol C^*. So, now let us try to you know derive the equations for C*. in terms of other parameters.

So, we will begin with the derivation of the C*. by considering the mass flow rate through the throat of the nozzle. As we have already derived the equation for Vj we can we can use the similar manner we can just simply consider the mass flow rate through the throat of the nozzle. So, if you just consider that we can simply write that if the throat area of a nozzle is At the density of the gas is ρt the velocity of the gas at throat is Vt we can simply write the mass flow rate through the nozzle throat is going to be equal to ρt into Vt into At where ρt is the density of the gas at the throat Vt is the velocity at the velocity of the gas at the throat At is the throat area. Now, what we can write about the density we can express density in terms of pressure and temperature. So, we can write $\frac{Pt}{RTt}$ and you may recall that from our ideal rocket assumption we can use the perfect gas law and we can use various equations we used for isentropic flow through nozzle.

So, we can actually apply some of the equations. So, if we apply the perfect we can assume perfect gas and steady one dimensional adiabatic flow in the convergent portion of the nozzle we can simply write the I am just writing from the previous equation I mean we already did it for the derivation of the Vj. So, I am just writing the same thing for considering the throat. So, what I am writing here if you just recall we had something like this. So, if this is your converging part so it is pc, Tc, Vc here we have At we have Vt we have Tt.

So, this is the throat ok. So, we are writing this part. So, if we just write the steady one dimensional adiabatic considering one dimensional adiabatic flow we can simply write

$hc + \frac{Vc^2}{2} = ht + \frac{Vt^2}{2}$

we can take out this part because this is almost negligible. So, we eventually get

$hc = ht + \frac{Vt^2}{2}$

$CpTc = CpTt + \frac{Vt^2}{2}$

This we can write in terms of you know other parameter like for example, like we can use the Mach number here, but let us first simplify little bit.

So, we can just or we can just write $Tc = Tt + \frac{Vt^2}{2Cp} = Tt(1 + \frac{\gamma-1}{2\gamma RTt}Vt^2)$. Now, we know that the sonic velocity is $At = \sqrt{\gamma RTt}$. So, sonic velocity at the throat will become square root of γ R T t which you can see it is here.

So, it is nothing, but we can say it is At^2. So, this will become $T_t(1 + \frac{\gamma-1}{2At^2}Vt^2)$. So, what about this V_t^2 by a_t^2? It is nothing, but the Mach number will be M_t^2. So, we can write this as $T_t(1 + \frac{\gamma-1}{2}Mt^2)$. Now, if you recall we assume the value of Mach number at the throat is 1.

So, eventually at throat we have considered the Mach number equal to 1. So, this equation will become$Tc = Tt(1 + \frac{\gamma-1}{2}) = Tt(\frac{\gamma+1}{2})$. So, this is one important equation. So, we can just give this name as equation number 1 here. So, after deriving this now if you considering the isentropic expansion process in the nozzle what you can write? We can write rather we can just go back to this equation instead of giving this equation number 1 we can simply change this and we can say this ratio Tc / T t = (γ + 1) / 2 and that we can say equation number 1.

Now, let us go to the next equation which we can write the isentropic expansion process in the nozzle and we say $\frac{Pc}{Pt} = (\frac{Tc}{Tt})^{\frac{\gamma}{\gamma-1}}$. And, we can now take this equation 2. Now, let us go back and look at this mass flow rate equation. So, let us consider this equation as A. So, we just put this put the value of this Tc / Tt the density equation let us say this is B and we have 1 and 2.

So, we can consider all and we can put in the equation of mass flow rate and we can write down the mass flow rate as we put everything together it will become $\dot{m} =$

$\frac{\sqrt{\gamma}PcAt}{\sqrt{RTc}}(\frac{Tt}{Tc})^{\frac{\gamma+1}{2(\gamma-1)}}$. . So, we can express this in terms of you know we take this γ value other sides and we can write this as what you can write here? We can write here is that$\frac{\sqrt{\gamma}Pc}{\sqrt{RTc}}(\frac{2}{\gamma+1})^{\frac{\gamma+1}{2(\gamma-1)}}$. So, this part is denoted as the Γ and we can write this.

So, this part is denoted as the Γ and you can write this as γ PcAt / RTc. Now, if you look at carefully this RTc / Γ this if you look at carefully the RTc /Γ what about the unit of this one? One can actually look at the unit of this one and we can see the R is the universal gas the gas constant for the particular gas. So, this will be like joule per kg kelvin, T is the temperature, joule can be divided into I mean joule can be further expanded into Newton meter per kg Newton meter then per kg will be there Kelvin will cancel out then finally, this will have a unit of meter per second. One can try doing it and easily find out that this will become a unit of velocity. Eventually this part is actually coming as a you know some kind of a transfer function we can say which is relating the chamber pressure and the throat area with the mass flow rate.

So, this term is called the C*. or the characteristic velocity. So, our m dot is becoming related to the chamber pressure and the throat area with the transfer function C*. or the characteristic velocity. So, this is another important equations we use time to time in case of rocket calculations various calculations using rocket performance. So, C*. will become 1/ Γ we can write R as universal gas constant Ru Tc / molecular weight .

And of course, this is a velocity. So, the unit is meter per second. So, one can actually see how the temperature is going to influence the C*. how the C*. is getting you know affected by the chamber temperature and one can easily see the high temperature means C*. is value is going to be very high and of course, the C*. is going to indicate the potential of the propellant to generate hot gas. So, of course, the various propellants will have a different capacity or the potential to generate hot gas and accordingly the C*. value will be different. So, we can say this is kind of a index of performance it consider as an index of performance ok. So, this is another important parameter we should consider while considering the performance parameters for rocket .

And one can of course, try looking at various chamber pressure keeping the chamber pressure and a t constant then we can look at the C*. how it is influencing one can get the value of various C*. value for a constant γ with a variation of chamber temperature ok. With of course, I mean the specific heat ratio γ will have some weak influence on the C star, but the T c has a major influence and of course, the molecular weight. So, we can say the C*. is also having the similar type of effect of the T c and M w as we said as we have seen already for the case of Vj or the jet velocity. So, basically the molecular weight lower the molecular will give the higher C*. velocity as we have seen for the case of Vj the higher the temperature chamber temperature will have will give the effect of higher C*. velocity.

So, increase in T c decrease in molecular weight will has an positive influence or is going to increase the C*.

So, I think with that we stop here today we will continue with this next topic of this part of the module in the another in another lecture all right. Thank you.